List of Formulas

Derivatives

$$\frac{d}{dx}(x^n) = n\,x^{n-1} \qquad \frac{d}{dx}(a^x) = a^x \ln a \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Product and Quotient Rules

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx} \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

l'Hôpital Rule

If $\lim\limits_{x\to c}\frac{f(x)}{g(x)}$ is of indeterminate form, then $\lim\limits_{x\to c}\frac{f(x)}{g(x)} = \lim\limits_{x\to c}\frac{f'(x)}{g'(x)}$

Integrals

$$\int x^m\,dx = \frac{x^{m+1}}{m+1} + c \text{ for } m \neq -1 \qquad \int \tan x\;dx = \ln|\sec x| + c$$

$$\int \frac{1}{x}\,dx = \ln|x| + c \qquad \int \cot x\;dx = \ln|\sin x| + c$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + c \qquad \int \sec x\;dx = \ln|\sec x + \tan x| + c$$

$$\int \sin x\;dx = -\cos x + c \qquad \int \csc x\;dx = -\ln|\csc x + \cot x| + c$$

$$\int \cos x\;dx = \sin x + c \qquad \int \frac{1}{k^2+x^2}\,dx = \frac{1}{k}\tan^{-1}\frac{x}{k} + c$$

$$\int \sec^2 x\;dx = \tan x + c \qquad \int \frac{1}{k^2-x^2}\,dx = \frac{1}{2k}\ln\left|\frac{k+x}{k-x}\right| + c$$

$$\int \csc^2 x\;dx = -\cot x + c \qquad \int \frac{1}{\sqrt{x^2 \pm k^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm k^2}\right| + c$$

$$\int \sec x \tan x\;dx = \sec x + c \qquad \int \frac{1}{\sqrt{k^2-x^2}}\,dx = \sin^{-1}\frac{x}{k} + c$$

$$\int \csc x \cot x\;dx = -\csc x + c \qquad \int u\,dv = uv - \int v\,du$$

Fundamental Theorem of Calculus

If F satisfies $F'(x) = f(x)$ for $a < x < b$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

n-th Root of a Complex Number

$$z_k = r^{1/n} \exp \mathbf{i}\left(\frac{\theta + 2\pi(k-1)}{n}\right) \text{ for } k = 1, \dots, n$$

Geometric and p-series

$\sum r^k$ converges for $|r| < 1$, diverges for $|r| \geq 1$; $\sum \frac{1}{k^p}$ converges for $p > 1$, diverges for $0 < p \leq 1$

Ratio Test

$\sum a_k$ is convergent if $L = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| < 1$, and divergent if $L > 1$ or $L = \infty$

Dot and Cross Products

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \cdot \|\mathbf{v}\| \cdot \cos\theta = u_1v_1 + u_2v_2 + u_3v_3 \qquad \mathbf{u} \times \mathbf{v} = (\|\mathbf{u}\| \cdot \|\mathbf{v}\| \cdot \sin\theta)\,\hat{\mathbf{n}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Distances

$$d_{\text{point-point}} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \qquad d_{\text{point-line}} = \left\|\overrightarrow{\mathrm{PQ}} \times \hat{\mathbf{v}}\right\|$$

$$d_{\text{point-plane}} = \left|\overrightarrow{\mathrm{PQ}} \cdot \hat{\mathbf{n}}\right| = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Discriminant of $f(x, y)$

$$D_f = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

Area and Volume as Double Integrals

Area of $D = \iint\limits_D dA$ and Volume of $S = \iint\limits_D f(x,y)\, dA$ where S is the solid under the surface $z = f(x, y)$

Double Integrals over Simple Polar Regions

$$\iint\limits_D f(x,y)\, dA = \int_{\alpha}^{\beta} \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

Ordinary Differential Equations

$M(x,y)dx + N(x,y)dy = 0$ is exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

The general solution of $y' + P(x)y = Q(x)$ is $y = \frac{1}{\mu(x)}\left[\int \mu(x)\, Q(x)\, dx + C\right]$ where $\mu(x) = e^{\int P(x)\, dx}$

Wronskian of f and g: $W(f,g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix}$

The general solution of $ay'' + by' + cy = 0$ is $\begin{cases} y = C_1e^{r_1x} + C_2e^{r_2x} & \text{if } r_1 \neq r_2 \text{ (real)} \\ y = e^{rx}(C_1 + C_2x) & \text{if } r = r_1 = r_2 \\ y = e^{\alpha x}(C_1 \sin\beta x + C_2\cos\beta x) & \text{if } r_1 = \overline{r_2} = \alpha + \beta\mathbf{i} \end{cases}$

The particular solution of $y'' + P(x)y' + Q(x)y = F(x)$ is $y_p = V_1y_1 + V_2y_2$, where

$$V_1' = -F(x)\,\frac{y_2}{W(y_1, y_2)} \text{ and } V_2' = F(x)\,\frac{y_1}{W(y_1, y_2)}$$