High Contrast Grating Solver

A User Guide to the Matlab® Program

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A high contrast grating solver (HCG Solver) is developed to solve the reflection and transmission coefficient of a plane wave incident onto a single layer of high contrast grating (HCG). It uses plane waves and waveguide array modes as the eigen modes to expand the electromagnetic field outside and inside the HCG respectively. By matching the boundary conditions at the HCG input plane and output plane, the electromagnetic property of HCG can be solved.

The program is developed by Weijian Yang, and optimized by Vincent Wang and Weijian Yang, in Prof. Connie Chang-Hasnain's Group in University of California, Berkeley.

This program was originally uploaded at https://light.eecs.berkeley.edu/cch/hcgsolver.html in March 2014**.**

Citation

The formulations used in the program are based on the following paper:

C. J. Chang-Hasnain and W. Yang, "High contrast gratings for integrated optoelectronics," *Adv. Opt. Photon.* **4**, 379-440 (2012).

Please reference this program and its authors in any publication for which you used it, by citing the above paper, and the webpage of this program.

Contents

This document provides a tutorial of the Matlab program of the HCG Solver. It first set ups the problem, and defines the parameters. The Matlab functions used in the program are then explained. Finally some examples are given. The users are encouraged to run the example programs to get familiar with the program.

1. Parameter Definitions

Figure 1 shows the schematics of HCG. The HCG is uniformed in the *y* direction, and the period is infinite in the *x* direction. The incident light onto the HCG is in the *xz* plane, and the incident angle is θ. In Fig. 1b, we define three different regions: **region I** as input region, **region II** as HCG region, and **region III** as exit region. The $z = 0$ plane is defined as **input plane**, and the $z = t_g$ plane is defined as the **exit plane**. Table 1 defines the parameters shown in Fig. 1b.

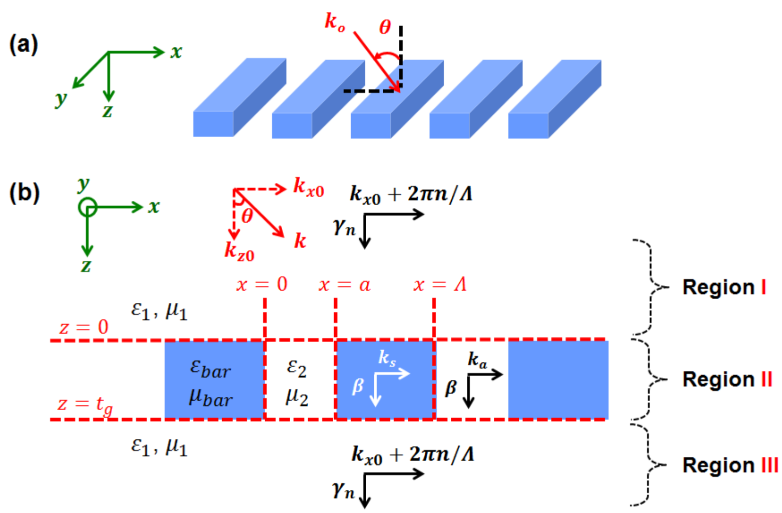

Fig. 1. Schematics of HCG. The various parameters shown in (b) are defined in Table 1.

Table 1. Parameter definitions

Symbol	Definition	Unit	Variable Name in Matlab Code	Note
Λ	HCG period	m	period	User defined
a	HCG air gap (or the low index material gap)	m	a	User defined
s	HCG grating bar width	m	s	User defined
t_g	HCG thickness	m	tg	User defined
η	HCG duty cycle	--	DC	User defined
ε_1	Relative permittivity in region I and III	--	e1	User defined
ε_2	Relative permittivity of the air gap of region II	--	e2	User defined
ε_{bar}	Relative permittivity of the grating bar	--	ebar	User defined
μ_1	Relative permeability in region I and III	--	u1	User defined
μ_2	Relative permeability of the air gap of region II	--	u2	User defined
μ_{bar}	Relative permeability of the grating bar	--	ubar	User defined
n_{bar}	Refractive index of the grating bar	--	nbar	User defined
λ	Incident wavelength in vacuum	m	lambda	User defined
θ	Incident angle	rad	theta	User defined
N	Total order of plane waves used in region I and III: 2*N*+1. *n* is used to label the mode orders: *n*=-*N*, -(*N*-1), …-2, -1, 0, 1, 2…*N*.	--	N	User defined
M	Total order of waveguide array modes used in region II: *M*. *m* is used to label the mode orders: *m*=0, 1, 2…*M*-1.	--	M	User defined
k	Incident wave wave-vector	rad/m		Derived
k_{x0}	Incident wave x-direction wave-vector	rad/m	$\mathrm{k_{x0}}$	Derived
k_{z0}	Incident wave z-direction wave-vector	rad/m		Derived
k_{xn}	Reflection wave or transmission wave x-direction wave-vector for region I and III. "*n*" labels different order, *n*=-*N*, -(*N*-1), …, -2, -1, 0, 1, 2…*N*. $k_{xn} = k_{x0} + 2\pi n/\Lambda$	rad/m	$\mathrm{k_{xn}}$	Derived
γ_n	Reflection wave or transmission wave z-direction wave-vector for region I and III. "*n*" labels different order, *n*=-*N*, -(*N*-1), …, -2, -1, 0, 1, 2…*N*	rad/m	gamma	Derived
β	Propagation constant of the waveguide array mode in region II. "*m*" labels different order, m=0, 1, 2…*M*-1	rad/m	beta	Derived
k_s	x-direction wave vector inside the grating bars of the waveguide array mode in region II. "m" labels different order, m=0, 1, 2…*M*-1	rad/m	$\mathrm{k_s}$	Derived
k_a	x-direction wave vector inside the air gap of the waveguide array mode in region II. "m" labels different order, m=0, 1, 2…*M*-1	rad/m	$\mathrm{k_a}$	Derived
H	H field overlap integral between the plane waves in region I/III and the waveguide array modes in region II. The matrix size is *M* by (2*N*+1)	--	H	Derived
E	E field overlap integral between the plane waves in region I/III and the waveguide array modes in region II. The matrix size is *M* by (2*N*+1)	--	E	Derived
TE/TM	Polarization of the incident light. TE: electrical field polarized in parallel to the grating bar; TM: electrical field polarized perpendicular to the grating bar.	--	isTM	User defined

2. Calculation Procedure and Matlab Sub-Functions

The Matlab programs are broken into several sub-functions. This allows the users to get access to various derived parameters. Table 2 lists these sub-functions.

Table 2. Matlab sub-functions

Function Name	Functionality
solveKaKsBeta_OA.m	Solve β, k_s and k_a.
solveHE_OA.m	Solve γ, H and E.
solveRT_OA.m	Solve the reflection and transmission coefficient
solveHCG_OA.m	Contains "solveKaKsBeta_OA.m" "solveHE_OA.m" "solveRT_OA.m" Directly solve the reflection and transmission coefficient
solveFieldProfile_OA.m	Contains "solveKaKsBeta_OA.m" "solveHE_OA.m" "solveRT_OA.m" Directly solve the reflection and transmission coefficient Solve the E field and H field profile inside and outside HCG

(1) Define the parameters for the HCG, such as grating period, duty cycle, thickness, input light wavelength, incident angle, polarization, etc. These parameters are labeled as "user defined" in Table 1. Define the order using the variable "ord", and N and M are automatically defined as N=ord, M=2*ord+1. In this way, the matrixes involved in the calculation are square, and this makes it easier for the inverse matrix calculation in the program.

(2) Solve the eigen modes in region II. At this step, we treat region II infinitely extended in the z direction, and solve β, k_s and k_a. Use the following function:

```
[ka, ks, beta] = solveKaKsBeta_OA(a, s, lambda, eBar, uBar, e1, u1, e2, u2, theta,
M, isTM)
```

The above function returns k_s, k_a and β. All of them are a vector with M elements, representing M modes.

(3) Solve the eigen modes in region I and III, as well as the field overlap integral at the HCG input plane and exit plane. At this step, we solve γ, H and E. This function also returns β. Use the following function:

```
[H, E, beta, gamma] = solveHE_OA(ka, ks, a, s, period, eBar, uBar, e1, u1, e2, u2,
lambda, theta, N, M, isTM)
```

The above function returns γ, a vector with 2N+1 elements, representing 2N+1 modes. H and E are matrixes with a size of M by (2N+1).

(4) Match the boundary conditions at the HCG input plane and exit plane, and solve the reflection and transmission coefficient. Use the following function:

```
[powRefl, powTrans, fldRefl, fldTrans, superModeSingleTripPhase] =
solveRT_OA(H, E, beta, gamma, tg, lambda, e1, u1, period, theta, N, M, isTM)
```

"powRefl" and "powTrans" are the power reflection and transmission coefficients. "fldRefl" and "fldTrans" are the field reflection and transmission coefficients. They contain all the reflection and transmission orders that are not evanescence wave. All of them are vectors with 2*N*+1 elements, corresponding to the orders of *n*=-*N*, -(*N*-1), …-2, -1, 0, 1, 2… *N*. Thus the middle element is the 0^{th} order reflection and transmission coefficient.

The "superModeSingleTripPhase" describes the single trip phase of the supermode inside the HCG. It is defined as the following inside the function.

```
modePhase=angle(eig(rho*phi))
```

This can be used to plot out the resonance curves on the $t_g - \lambda$ reflection contour plot.

- The above steps are warped into the following function:

```
[fldRefl, fldTrans] = solveHCG_OA(a, s, eBar, uBar, e1, u1, e2, u2, lambda, tg,
theta, ord, isTM)
```

This function directly output "fldRefl" and "fldTrans" -- the field reflection and transmission coefficients. They contain all the reflection and transmission orders that are not evanescence wave. All of them are vectors with 2*N*+1 elements, corresponding to the orders of *n*=-*N*, -(*N*-1), …-2, -1, 0, 1, 2… *N*.

- To solve the E field and H field profile in region I, II and III, use the following functions:

```
[powRefl, powTrans, H_all, E_all, incidentWaveAmp] = solveFieldProfile_OA( s, a,
tg, eBar, uBar, e1, u1, e2, u2, lambda, theta, ord, isTM, x, z )
```

x and *z* is the mesh grids in the *x* and *z* direction. The span of *x* should be one period, and the span of *z* should cover that from region I to region III. "H_all" and "E_all" are H field and E field defined on this grid coordinate. "incidentWaveAmp" is a 1x2 vector, and contains the amplitude of the incident H field and E field. They can be used to normalize "H_all" and "E_all".

"powRefl" and "powTrans" are the power reflection and transmission coefficients. They contain all the reflection and transmission orders that are not evanescence wave. All of them are vectors with $2N+1$ elements, corresponding to the orders of $n=-N$, $-(N-1)$, …-2, -1, 0, 1, 2… N.

3. Example Programs

Based on the Matlab sub-functions listed in Table 2, the users can get access various property of the HCG. The $\omega - k$ diagram of the waveguide array modes, the HCG reflection and transmission coefficients, as well as the E field and H field profile of the HCG are the items in common interest. Table 3 lists several Matlab programs to address these. **They serve as examples, and can be run directly by the users.**

Table 3. Example programs

Program Name	Functionality
OA_HCG_Omega_k_Diagram.m	Plot the $\omega - k$ diagram of the waveguide array modes
OA_HCGReflectionSpectrum.m	Plot the HCG reflection spectrum (0^{th} order)
OA_HCGReflectionContour.m	Plot the HCG reflection contour (0^{th} order) versus HCG thickness and incident wavelength
OA_HCGFieldProfile.m	Plot the E field and H field profile of the HCG

The above sub-functions and programs are developed for any incident angle. For surface normal incidence, another set of sub-functions and programs are developed, which are more efficient in computation. They are with the same name with those in Table 2 and Table 3, but without suffix or prefix "OA", listed in Table 4.

Table 4. Matlab sub-functions and example programs for surface normal incidence

Function Name	Functionality
solveKaKsBeta.m	Solve β, k_s and k_a.
solveHE.m	Solve γ, H and E.
solveRT.m	Solve the reflection and transmission coefficient
solveHCG.m	Contains "solveKaKsBeta.m" "solveHE.m" "solveRT.m" Directly solve the reflection and transmission coefficient
solveFieldProfile.m	Contains "solveKaKsBeta.m" "solveHE.m" "solveRT.m" Directly solve the reflection and transmission coefficient Solve the E field and H field profile inside and outside HCG
Program Name	**Functionality**
HCGReflectionSpectrum.m	Plot the HCG reflection spectrum (0^{th} order)

HCGReflectionContour.m	Plot the HCG reflection contour (0^{th} order) versus HCG thickness and incident wavelength
HCGFieldProfile.m	Plot the E field and H field profile of the HCG

4. Examples

(1) TM Broadband reflector at surface normal incidence.

Run "TM_BroadbandReflector_SurfaceNormal_HCGReflectionSpectrum.m"

The HCG parameters are: $\Lambda = 0.77\ \mu m$, $t_g = 0.455\ \mu m$, $\eta = 0.76$, TM polarized light incident at surface normal incident angle. The HCG is fully suspended in air. The grating bar refractive index is $n_{bar} = 3.48$.

The 0^{th} order power reflectivity and the reflection phase calculated by the program are shown in Fig. 2.

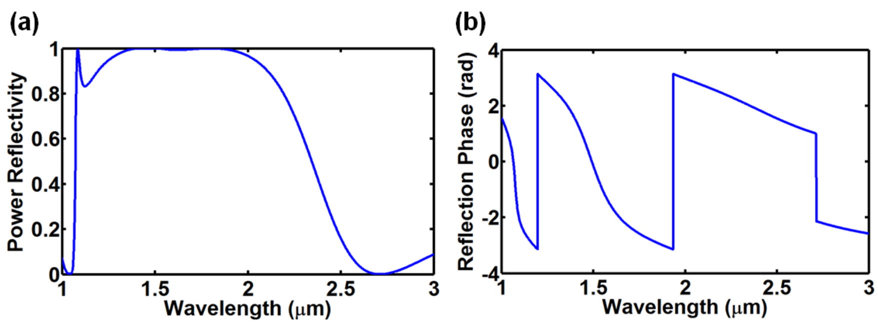

Fig. 2. 0^{th} order power reflectivity and reflection phase of a broadband TM reflector.

(2) TE HCG reflection contour.

Run "TE_ReflectionContour_theta50_OA_HCGReflectionContour.m"

The HCG parameters are: $\eta = 0.6$, TE polarized light with an incident angle of 50^o. The HCG is fully suspended in air. The grating bar refractive index is $n_{bar} = 3.48$.

The 0^{th} order power reflectivity contour plot versus HCG thickness and incident wavelength calculated by the program is shown in Fig. 3.

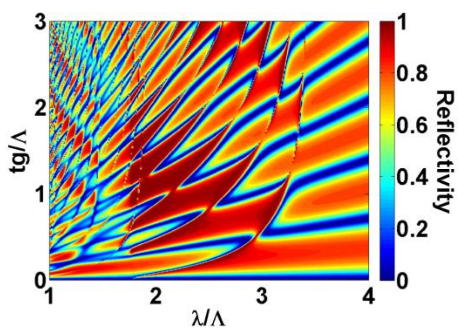

Fig. 3. 0^{th} order power reflectivity contour versus HCG thickness and incident wavelength (normalized by HCG period) of a TE HCG.

(3) TE HCG resonator
Run "TE_Resonance_HCGFieldProfile.m"

The HCG parameters are: $\lambda/\Lambda = 2.3294710618$, $t_g/\Lambda = 0.84180$, $\eta = 0.70$, TE polarized light incident at surface normal incident angle. The HCG is fully suspended in air. The grating bar refractive index is $n_{bar} = 3.48$. Order "Ord" is set to be 8.
The 0^{th} order power reflectivity and E field profile calculated by the program are shown in Fig. 4.

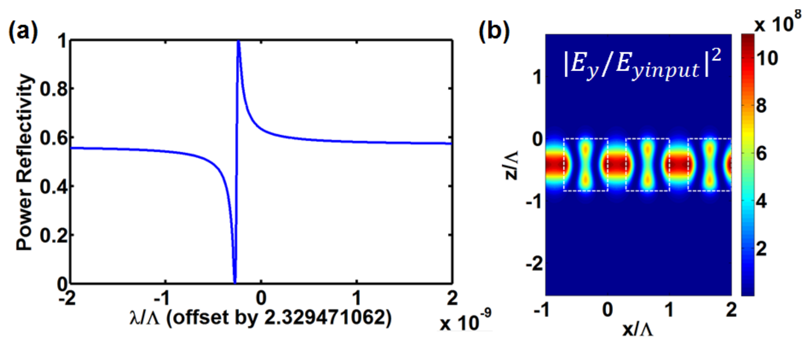

Fig. 4. 0^{th} order power reflectivity spectrum and the E field profile of the HCG at a TE HCG resonance.

(4) $\omega - k$ diagram of the TE waveguide array modes
Run "TE_wkDiagram_theta50_OA_HCG_Omega_k_Diagram.m"
The HCG parameters are: $\Lambda = 1\ \mu m$, $\eta = 0.5$, TE polarized light incident at an incident angle of 50^o. The HCG is fully suspended in air. The grating bar refractive index is $n_{bar} = 3.48$.

The $\omega - k$ diagram of the TE waveguide array modes calculated by the program is shown in Fig. 5.

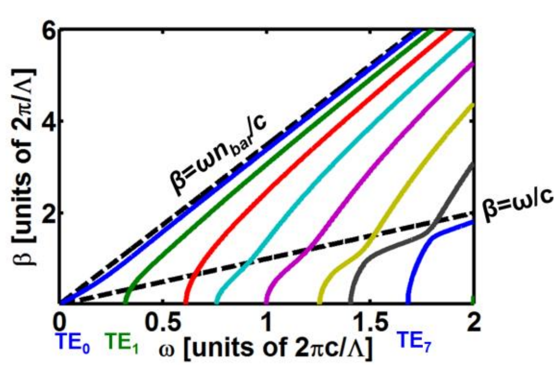

Fig. 5. The $\omega - k$ diagram of the TE waveguide array modes.