Computational study becomes more and more sophisticated in various subjects, thanks to faster and cheaper computing facility. On the other hand, the computing resource at one's disposal will be scooped out soon when the level of complexity and accuracy is increased, in this respect, model order reduction(MOR) is indispensable at certain stage. In this course work, we are not aimed at applying MOR to a specific research problem, instead, we would pursue a education-oriented approach, i.e. build toy examples and apply different MOR techniques to see what happens(some contents in this report are refereed to the course slides and book by Prof. Grivet).

In this work, we have considered a heated bar discretized by Galerkin finite element method, in time domain, the discretized system is then integrated with the so called θ scheme, which generalizes the forward/backward Euler and Trapezoidal method. A pair of Dirichlet and Neumann boundary conditions are imposed at the two ends of the heated bar, Fig. 1 has demonstrated the time evolution of the temperature distribution on a bar subjected to a Gaussian heat illumination.

Our domain PDE and the corresponding dicretized algebraic system has the form,

$$
\begin{aligned}
\frac{\partial T(x,t)}{\partial t} &= \frac{\partial^2 T(x,t)}{\partial x^2} + h(x)u(t) \\
E\frac{dz(t)}{dt} &= Az(t) + Bu(t) \qquad (1)
\end{aligned}
$$

Where E is the so-called Mass matrix in FEM language, which is diagonal in finite difference language, A is named stiffness matrix, and B is the load vector, the state vector $z(t)$ can be interpreted as the unknown nodal values of the discretized PDE system.

Now our purpose is to do MOR for this system, but we should first ask ourselves why it is possible to do MOR for this toy example. The reasons come from both analytical and numerical observations, from the analytical view point, this PDE would have quasi-exact solution if no strange

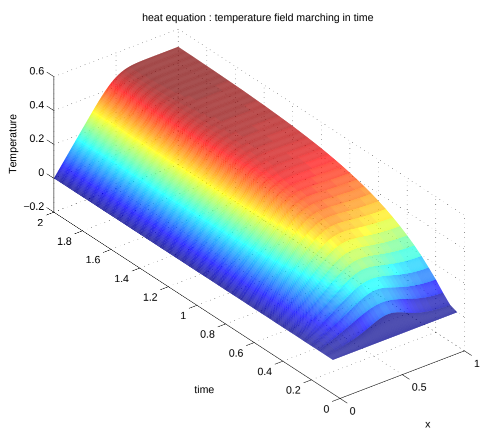

FIG. 1. Time evolution of temperature distribution, Dirichlet B.C. at left, Neumann B.C. at right.

source term is present, i.e. we can solve the system with Fourier based technique, perhaps a few planewaves would suffice, while on the numerical side, once we build up the finite element matrices, we can observe some patterns keep repeating themselves, indicating substantial redundancy in the algebraic system.

In the following, we will realized the MOR using mainly two techniques, namely, model truncation and moment matching based on Krylov subspace projection. Model truncation is indeed intuitive appealing, the basic idea is to decouple the mode through eigenvalue decomposition, and then select relatively important modes for the specific system, finally a reduced analytical model can be derived. We will not present the derivation here but directly go to the result, as shown in Fig. 2 and Fig. 3, with choice of a small q(here $q = 12$) for the slow modes, the time domain results, for both $u(t) = 1, t > 0$ or a transient signal $u = sin(0.01t)$, the consistency with the full system is reasonably good. However, when we compute the frequency response, the reduced model by model truncation fails even if a large q is chosen.

Moment matching based MOR is at a more sophisticated level, this family consists of Pade

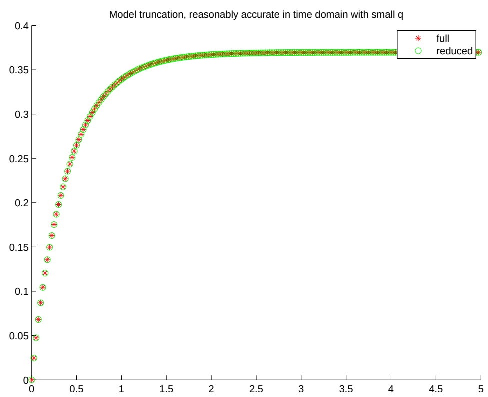

FIG. 2. Output(average temperature) of the system, constant source term($u(t) = 1, t > 0$).

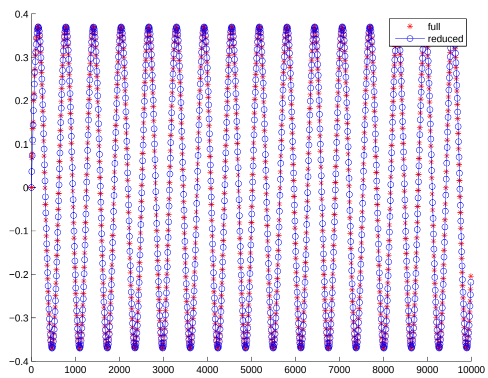

FIG. 3. Output(average temperature) of the system, sinusoidal source term($u = sin(0.01t)$).

approximation via explicit moment matching, complex Frequency Hopping, and reduction by projection. We will demonstrate the application of a implicit Krylov-subspace based moment matching to our system. Starting from original state-space model(Eq. (1)), we build up the "tall and thin" matrix projection through W_q and V_q,

$$\begin{aligned} EV_q\dot{x}_q(t) &= AV_q x_q(t) + Bu(t) \\ W_q^T EV_q\dot{x}_q(t) &= W_q^T AV_q x_q(t) + W_q^T Bu(t) \\ y(t) &= CV_q x_q(t) \end{aligned} \tag{2}$$

Now we have to find the projection matrix, in this work, we choose specifically the Galerkin projection, i.e. $W_q = V_q$ and the reduced system matrices are,

$$E_r = V_q^T EV_q, A_r = V_q^T AV_q, B_r = V_q^T B, C_r = CV_q \tag{3}$$

Instead of expanding at $s = 0$, we can expand at $s = \tilde{s} + s_h$, and reformulate our problem as

$$\begin{aligned} (\tilde{s} + s_h)Ex &= Ax + bu \\ \tilde{s}(A - s_h E)^{-1}Ex &= x + (A - s_h E)^{-1}bu \end{aligned} \tag{4}$$

therefore we consider a prjection matrix whose columns span the corresponding Krylov subspace

$$im(V_q) = span\{(A - s_h E)^{-1}b, ..., ((A - s_h E)^{-1}E)^{q-1} \cdot (A - s_h E)^{-1}b\} \tag{5}$$

In Fig. 4, we plot the frequency response of the reduced system with respect to the full system, we have used two expansion point and $q = 1$ for both real and imaginary parts,

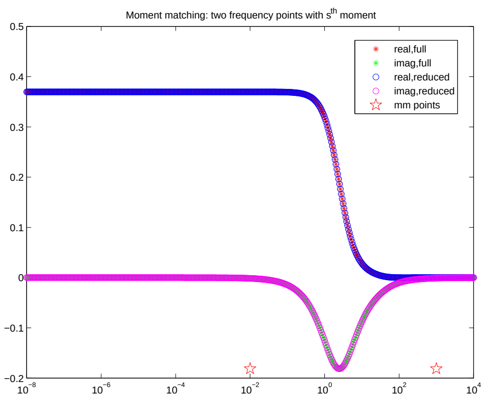

FIG. 4. Frequency response comparison, moment matching based MOR gives very good result.