A The Type System of λ_i^+

Definition 7 (Type erasure).

$$
\begin{aligned}
|\mathsf{Nat}| &= \mathsf{Nat} \\
|\top| &= \langle\rangle \\
|A \to B| &= |A| \to |B| \\
|A \,\&\, B| &= |A| \times |B| \\
|\{l : A\}| &= \{l : |A|\}
\end{aligned}
$$

Definition 8 (Meta-functions $[\![\cdot]\!]_\top$ and $[\![\cdot]\!]_\&$).

$$
\begin{aligned}
[\![[]]\!]_\top &= \mathsf{top} \\
[\![\{l\}, \mathcal{L}]\!]_\top &= \{l : [\![\mathcal{L}]\!]_\top\} \circ \mathsf{top}_{\{l\}} \\
[\![A, \mathcal{L}]\!]_\top &= (\mathsf{id} \to [\![\mathcal{L}]\!]_\top) \circ ((\mathsf{top} \to \mathsf{top}) \circ (\mathsf{top}_\to \circ \mathsf{top}))
\end{aligned}
$$

$$
\begin{aligned}
[\![[]]\!]_\& &= \mathsf{id} \\
[\![\{l\}, \mathcal{L}]\!]_\& &= \{l : [\![\mathcal{L}]\!]_\&\} \circ \mathsf{dist}_{\{l\}} \\
[\![A, \mathcal{L}]\!]_\& &= (\mathsf{id} \to [\![\mathcal{L}]\!]_\&) \circ \mathsf{dist}_\to
\end{aligned}
$$

$\boxed{A <: B \rightsquigarrow c}$ *(Declarative subtyping)*

$$\frac{}{A <: A \rightsquigarrow \mathsf{id}}\ \textsc{S-refl} \qquad \frac{A_2 <: A_3 \rightsquigarrow c_1 \qquad A_1 <: A_2 \rightsquigarrow c_2}{A_1 <: A_3 \rightsquigarrow c_1 \circ c_2}\ \textsc{S-trans}$$

$$\frac{}{A <: \top \rightsquigarrow \mathsf{top}}\ \textsc{S-top} \qquad \frac{A <: B \rightsquigarrow c}{\{l : A\} <: \{l : B\} \rightsquigarrow \{l : c\}}\ \textsc{S-rcd}$$

$$\frac{B_1 <: A_1 \rightsquigarrow c_1 \qquad A_2 <: B_2 \rightsquigarrow c_2}{A_1 \to A_2 <: B_1 \to B_2 \rightsquigarrow c_1 \to c_2}\ \textsc{S-arr} \qquad \frac{}{A_1 \,\&\, A_2 <: A_1 \rightsquigarrow \pi_1}\ \textsc{S-andl}$$

$$\frac{}{A_1 \,\&\, A_2 <: A_2 \rightsquigarrow \pi_2}\ \textsc{S-andr} \qquad \frac{A_1 <: A_2 \rightsquigarrow c_1 \qquad A_1 <: A_3 \rightsquigarrow c_2}{A_1 <: A_2 \,\&\, A_3 \rightsquigarrow \langle c_1, c_2\rangle}\ \textsc{S-and}$$

$$\frac{}{(A_1 \to A_2) \,\&\, (A_1 \to A_3) <: A_1 \to A_2 \,\&\, A_3 \rightsquigarrow \mathsf{dist}_\to}\ \textsc{S-distArr}$$

$$\frac{}{\{l : A\} \,\&\, \{l : B\} <: \{l : A \,\&\, B\} \rightsquigarrow \mathsf{dist}_{\{l\}}}\ \textsc{S-distRcd}$$

$$\frac{}{\top <: \top \to \top \rightsquigarrow \mathsf{top}_\to}\ \textsc{S-topArr} \qquad \frac{}{\top <: \{l : \top\} \rightsquigarrow \mathsf{top}_{\{l\}}}\ \textsc{S-topRcd}$$

$\boxed{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}$ *(Inference)*

$$\frac{}{\Gamma \vdash \top \Rightarrow \top \rightsquigarrow \langle\rangle}\ \textsc{T-top} \qquad \frac{}{\Gamma \vdash i \Rightarrow \mathsf{Nat} \rightsquigarrow i}\ \textsc{T-lit}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A \rightsquigarrow x}\ \textsc{T-var} \qquad \frac{\begin{array}{c}\Gamma \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e_1 \\ \Gamma \vdash E_2 \Leftarrow A_1 \rightsquigarrow e_2\end{array}}{\Gamma \vdash E_1\, E_2 \Rightarrow A_2 \rightsquigarrow e_1\, e_2}\ \textsc{T-app}$$

$$\frac{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}{\Gamma \vdash E : A \Rightarrow A \rightsquigarrow e}\ \textsc{T-anno} \qquad \frac{\begin{array}{c}\Gamma \vdash E_1 \Rightarrow A_1 \rightsquigarrow e_1 \\ \Gamma \vdash E_2 \Rightarrow A_2 \rightsquigarrow e_2 \qquad A_1 * A_2\end{array}}{\Gamma \vdash E_1\,,,\,E_2 \Rightarrow A_1 \,\&\, A_2 \rightsquigarrow \langle e_1, e_2\rangle}\ \textsc{T-merge}$$

$$\frac{\Gamma \vdash E \Rightarrow A \rightsquigarrow e}{\Gamma \vdash \{l = E\} \Rightarrow \{l : A\} \rightsquigarrow \{l = e\}}\ \textsc{T-rcd} \qquad \frac{\Gamma \vdash E \Rightarrow \{l : A\} \rightsquigarrow e}{\Gamma \vdash E.l \Rightarrow A \rightsquigarrow e.l}\ \textsc{T-proj}$$

$\boxed{\Gamma \vdash E \Leftarrow A \rightsquigarrow e}$ *(Checking)*

$$\frac{\Gamma, x : A \vdash E \Leftarrow B \rightsquigarrow e}{\Gamma \vdash \lambda x.\, E \Leftarrow A \to B \rightsquigarrow \lambda x.\, e}\ \textsc{T-abs} \qquad \frac{\Gamma \vdash E \Rightarrow B \rightsquigarrow e \qquad B <: A \rightsquigarrow c}{\Gamma \vdash E \Leftarrow A \rightsquigarrow c\, e}\ \textsc{T-sub}$$

$\boxed{A * B}$ *(Disjointness)*

$$\frac{}{\top * A}\ \textsc{D-topL} \qquad \frac{}{A * \top}\ \textsc{D-topR} \qquad \frac{A_2 * B_2}{A_1 \to A_2 * B_1 \to B_2}\ \textsc{D-arr}$$

$$\frac{A_1 * B \qquad A_2 * B}{A_1 \,\&\, A_2 * B}\ \textsc{D-andL} \qquad \frac{A * B_1 \qquad A * B_2}{A * B_1 \,\&\, B_2}\ \textsc{D-andR}$$

$$\frac{A * B}{\{l : A\} * \{l : B\}}\ \textsc{D-rcdEq} \qquad \frac{l_1 \neq l_2}{\{l_1 : A\} * \{l_2 : B\}}\ \textsc{D-rcdNeq}$$

$$\frac{}{\mathsf{Nat} * A_1 \to A_2}\ \textsc{D-axNatArr} \qquad \frac{}{\mathsf{Nat} * \{l : A\}}\ \textsc{D-axNatRcd}$$

$$\frac{}{A_1 \to A_2 * \mathsf{Nat}}\ \textsc{D-axArrNat} \qquad \frac{}{\mathsf{Nat} * \{l : A\}}\ \textsc{D-axNatRcd}$$

$$\frac{}{\{l : A\} * A_1 \to A_2}\ \textsc{D-axRcdArr} \qquad \frac{}{\{l : A\} * \mathsf{Nat}}\ \textsc{D-axRcdNat}$$

$\boxed{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow C}$ *(Context typing I)*

$$\frac{}{[\cdot] : (\Gamma \Rightarrow A) \mapsto (\Gamma \Rightarrow A) \rightsquigarrow [\cdot]}\ \textsc{CTyp-empty1}$$

$$\frac{\begin{array}{c}\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_1 \to A_2) \rightsquigarrow C \\ \Gamma' \vdash E_2 \Leftarrow A_1 \rightsquigarrow e\end{array}}{\mathcal{C}\, E_2 : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow C\, e}\ \textsc{CTyp-appL1}$$

$$\frac{\begin{array}{c}\Gamma' \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e \\ \mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Leftarrow A_1) \rightsquigarrow C\end{array}}{E_1\, \mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow e\, C}\ \textsc{CTyp-appR1}$$

$$\frac{\begin{array}{c}\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_1) \rightsquigarrow C \\ \Gamma' \vdash E_2 \Rightarrow A_2 \rightsquigarrow e \qquad A_1 * A_2\end{array}}{\mathcal{C} \,,, E_2 : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_1 \,\&\, A_2) \rightsquigarrow \langle C, e\rangle}\ \textsc{CTyp-mergeL1}$$

$$\frac{\begin{array}{c}\Gamma' \vdash E_1 \Rightarrow A_1 \rightsquigarrow e \\ \mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow C \\ A_1 * A_2\end{array}}{E_1 \,,, \mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow A_1 \,\&\, A_2) \rightsquigarrow \langle e, C\rangle}\ \textsc{CTyp-mergeR1}$$

$$\frac{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow C}{\{l = \mathcal{C}\} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow \{l = C\}}\ \textsc{CTyp-rcd1}$$

$$\frac{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow C}{\mathcal{C}.l : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow C.l}\ \textsc{CTyp-proj1}$$

$$\frac{\mathcal{C} : (\Gamma \Rightarrow B) \mapsto (\Gamma' \Leftarrow A) \rightsquigarrow C}{\mathcal{C} : A : (\Gamma \Rightarrow B) \mapsto (\Gamma' \Rightarrow A) \rightsquigarrow C}\ \textsc{CTyp-anno1}$$

$\boxed{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Leftarrow B) \rightsquigarrow C}$ *(Context typing II)*

$$\frac{}{[\cdot] : (\Gamma \Leftarrow A) \mapsto (\Gamma \Leftarrow A) \rightsquigarrow [\cdot]}\ \textsc{CTyp-empty2}$$

$$\frac{\begin{array}{c}\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma', x : A_1 \Leftarrow A_2) \rightsquigarrow C \\ x \notin \Gamma'\end{array}}{\lambda x.\, \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Leftarrow A_1 \to A_2) \rightsquigarrow \lambda x.\, C}\ \textsc{CTyp-abs2}$$

$\boxed{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow C}$ *(Context typing III)*

$$\frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1 \to A_2) \rightsquigarrow C \qquad \Gamma' \vdash E_2 \Leftarrow A_1 \rightsquigarrow e}{\mathcal{C}\, E_2 : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow C\, e} \text{ CTYP-APPL2}$$

$$\frac{\Gamma' \vdash E_1 \Rightarrow A_1 \to A_2 \rightsquigarrow e \qquad \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Leftarrow A_1) \rightsquigarrow C}{E_1\, \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow e\, C} \text{ CTYP-APPR2}$$

$$\frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1) \rightsquigarrow C \qquad \Gamma' \vdash E_2 \Rightarrow A_2 \rightsquigarrow e \qquad A_1 * A_2}{\mathcal{C} \,,,\, E_2 : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1 \,\&\, A_2) \rightsquigarrow \langle C, e \rangle} \text{ CTYP-MERGEL2}$$

$$\frac{\Gamma' \vdash E_1 \Rightarrow A_1 \rightsquigarrow e \qquad \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_2) \rightsquigarrow C \qquad A_1 * A_2}{E_1 \,,,\, \mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow A_1 \,\&\, A_2) \rightsquigarrow \langle e, C \rangle} \text{ CTYP-MERGER2}$$

$$\frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow C}{\{l = \mathcal{C}\} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow \{l = C\}} \text{ CTYP-RCD2}$$

$$\frac{\mathcal{C} : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow \{l : B\}) \rightsquigarrow C}{\mathcal{C}.l : (\Gamma \Leftarrow A) \mapsto (\Gamma' \Rightarrow B) \rightsquigarrow C.l} \text{ CTYP-PROJ2}$$

$$\frac{\mathcal{C} : (\Gamma \Leftarrow B) \mapsto (\Gamma' \Leftarrow A) \rightsquigarrow C}{\mathcal{C} : A : (\Gamma \Leftarrow B) \mapsto (\Gamma' \Rightarrow A) \rightsquigarrow C} \text{ CTYP-ANNO2}$$

$\boxed{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Leftarrow B) \rightsquigarrow C}$ *(Context typing IV)*

$$\frac{\mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma', x : A_1 \Leftarrow A_2) \rightsquigarrow C \qquad x \notin \Gamma'}{\lambda x.\, \mathcal{C} : (\Gamma \Rightarrow A) \mapsto (\Gamma' \Leftarrow A_1 \to A_2) \rightsquigarrow \lambda x.\, C} \text{ CTYP-ABS1}$$

$\boxed{\mathcal{L} \vdash A \prec: B \rightsquigarrow c}$ *(Algorithmic subtyping)*

$$\frac{\mathcal{L} \vdash A \prec: B_1 \rightsquigarrow c_1 \qquad \mathcal{L} \vdash A \prec: B_2 \rightsquigarrow c_2}{\mathcal{L} \vdash A \prec: B_1 \,\&\, B_2 \rightsquigarrow [\![\mathcal{L}]\!]_{\&} \circ \langle c_1, c_2 \rangle} \text{ A-AND}$$

$$\frac{\mathcal{L}, B_1 \vdash A \prec: B_2 \rightsquigarrow c}{\mathcal{L} \vdash A \prec: B_1 \to B_2 \rightsquigarrow c} \text{ A-ARR} \qquad \frac{\mathcal{L}, \{l\} \vdash A \prec: B \rightsquigarrow c}{\mathcal{L} \vdash A \prec: \{l : B\} \rightsquigarrow c} \text{ A-RCD}$$

$$\frac{}{\mathcal{L} \vdash A \prec: \top \rightsquigarrow [\![\mathcal{L}]\!]_\top \circ \mathsf{top}} \text{ A-TOP}$$

$$\frac{[] \vdash A \prec: A_1 \rightsquigarrow c_1 \qquad \mathcal{L} \vdash A_2 \prec: \mathsf{Nat} \rightsquigarrow c_2}{A, \mathcal{L} \vdash A_1 \to A_2 \prec: \mathsf{Nat} \rightsquigarrow c_1 \to c_2} \text{ A-ARRNAT}$$

$$\frac{\mathcal{L} \vdash A \prec: \mathsf{Nat} \rightsquigarrow c}{\{l\}, \mathcal{L} \vdash \{l : A\} \prec: \mathsf{Nat} \rightsquigarrow \{l : c\}} \text{ A-RCDNAT} \qquad \frac{}{[] \vdash \mathsf{Nat} \prec: \mathsf{Nat} \rightsquigarrow \mathsf{id}} \text{ A-NAT}$$

$$\frac{\mathcal{L} \vdash A_1 \prec: \mathsf{Nat} \rightsquigarrow c}{\mathcal{L} \vdash A_1 \,\&\, A_2 \prec: \mathsf{Nat} \rightsquigarrow c \circ \pi_1} \text{ A-ANDN1}$$

$$\frac{\mathcal{L} \vdash A_2 \prec: \mathsf{Nat} \rightsquigarrow c}{\mathcal{L} \vdash A_1 \,\&\, A_2 \prec: \mathsf{Nat} \rightsquigarrow c \circ \pi_2} \text{ A-ANDN2}$$

B The Type System of λ_c

$\boxed{\Delta \vdash e : \tau}$ *(Target typing)*

$$\frac{}{\Delta \vdash \langle\rangle : \langle\rangle} \text{ TYP-UNIT} \qquad \frac{}{\Delta \vdash i : \mathsf{Nat}} \text{ TYP-LIT} \qquad \frac{x : \tau \in \Delta}{\Delta \vdash x : \tau} \text{ TYP-VAR}$$

$$\frac{\Delta, x : \tau_1 \vdash e : \tau_2}{\Delta \vdash \lambda x.\, e : \tau_1 \to \tau_2} \text{ TYP-ABS} \qquad \frac{\Delta \vdash e_1 : \tau_1 \to \tau_2 \qquad \Delta \vdash e_2 : \tau_1}{\Delta \vdash e_1\, e_2 : \tau_2} \text{ TYP-APP}$$

$$\frac{\Delta \vdash e_1 : \tau_1 \qquad \Delta \vdash e_2 : \tau_2}{\Delta \vdash \langle e_1, e_2 \rangle : \tau_1 \times \tau_2} \text{ TYP-PAIR} \qquad \frac{\Delta \vdash e : \tau \qquad c \vdash \tau \rhd \tau'}{\Delta \vdash c\, e : \tau'} \text{ TYP-CAPP}$$

$$\frac{\Delta \vdash e : \tau}{\Delta \vdash \{l = e\} : \{l : \tau\}} \text{ TYP-RCD} \qquad \frac{\Delta \vdash e : \{l : \tau\}}{\Delta \vdash e.l : \tau} \text{ TYP-PROJ}$$

$\boxed{c \vdash \tau_1 \rhd \tau_2}$ *(Coercion typing)*

$$\frac{}{\mathsf{id} \vdash \tau \rhd \tau} \text{ COTYP-REFL} \qquad \frac{c_1 \vdash \tau_2 \rhd \tau_3 \qquad c_2 \vdash \tau_1 \rhd \tau_2}{c_1 \circ c_2 \vdash \tau_1 \rhd \tau_3} \text{ COTYP-TRANS}$$

$$\frac{}{\mathsf{top} \vdash \tau \rhd \langle\rangle} \text{ COTYP-TOP} \qquad \frac{}{\mathsf{top}_\to \vdash \langle\rangle \rhd \langle\rangle \to \langle\rangle} \text{ COTYP-TOPARR}$$

$$\frac{}{\mathsf{top}_{\{l\}} \vdash \langle\rangle \rhd \{l : \langle\rangle\}} \text{ COTYP-TOPRCD}$$

$$\frac{c_1 \vdash \tau_1' \rhd \tau_1 \qquad c_2 \vdash \tau_2 \rhd \tau_2'}{c_1 \to c_2 \vdash \tau_1 \to \tau_2 \rhd \tau_1' \to \tau_2'} \text{ COTYP-ARR}$$

$$\frac{c_1 \vdash \tau_1 \rhd \tau_2 \qquad c_2 \vdash \tau_1 \rhd \tau_3}{\langle c_1, c_2 \rangle \vdash \tau_1 \rhd \tau_2 \times \tau_3}\ \textsc{cotyp-pair} \qquad \frac{}{\pi_1 \vdash \tau_1 \times \tau_2 \rhd \tau_1}\ \textsc{cotyp-projl}$$

$$\frac{}{\pi_2 \vdash \tau_1 \times \tau_2 \rhd \tau_2}\ \textsc{cotyp-projr} \qquad \frac{c \vdash \tau_1 \rhd \tau_2}{\{l : c\} \vdash \{l : \tau_1\} \rhd \{l : \tau_2\}}\ \textsc{cotyp-rcd}$$

$$\frac{}{\mathsf{dist}_{\{l\}} \vdash \{l : \tau_1\} \times \{l : \tau_2\} \rhd \{l : \tau_1 \times \tau_2\}}\ \textsc{cotyp-distRcd}$$

$$\frac{}{\mathsf{dist}_{\rightarrow} \vdash (\tau_1 \rightarrow \tau_2) \times (\tau_1 \rightarrow \tau_3) \rhd \tau_1 \rightarrow \tau_2 \times \tau_3}\ \textsc{cotyp-distArr}$$

$\boxed{e \longrightarrow e'}$ *(Small-step reduction)*

$$\frac{}{\mathsf{id}\, v \longrightarrow v}\ \textsc{step-id} \qquad \frac{}{(c_1 \circ c_2)\, v \longrightarrow c_1\, (c_2\, v)}\ \textsc{step-trans}$$

$$\frac{}{\mathsf{top}\, v \longrightarrow \langle\rangle}\ \textsc{step-top} \qquad \frac{}{(\mathsf{top}_{\rightarrow}\, \langle\rangle)\, \langle\rangle \longrightarrow \langle\rangle}\ \textsc{step-topArr}$$

$$\frac{}{\mathsf{top}_{\{l\}}\, \langle\rangle \longrightarrow \{l = \langle\rangle\}}\ \textsc{step-topRcd}$$

$$\frac{}{((c_1 \rightarrow c_2)\, v_1)\, v_2 \longrightarrow c_2\, (v_1\, (c_1\, v_2))}\ \textsc{step-arr}$$

$$\frac{}{\langle c_1, c_2 \rangle\, v \longrightarrow \langle c_1\, v, c_2\, v \rangle}\ \textsc{step-pair}$$

$$\frac{}{(\mathsf{dist}_{\rightarrow}\, \langle v_1, v_2 \rangle)\, v_3 \longrightarrow \langle v_1\, v_3, v_2\, v_3 \rangle}\ \textsc{step-distArr}$$

$$\frac{}{\mathsf{dist}_{\{l\}}\, \langle \{l = v_1\}, \{l = v_2\} \rangle \longrightarrow \{l = \langle v_1, v_2 \rangle\}}\ \textsc{step-distRcd}$$

$$\frac{}{\pi_1\, \langle v_1, v_2 \rangle \longrightarrow v_1}\ \textsc{step-projl} \qquad \frac{}{\pi_2\, \langle v_1, v_2 \rangle \longrightarrow v_2}\ \textsc{step-projr}$$

$$\frac{}{\{l : c\}\{l = v\} \longrightarrow \{l = c\, v\}}\ \textsc{step-crcd} \qquad \frac{}{(\lambda x.\, e)\, v \longrightarrow e[x \mapsto v]}\ \textsc{step-beta}$$

$$\frac{}{\{l = v\}.l \longrightarrow v}\ \textsc{step-projRcd} \qquad \frac{e_1 \longrightarrow e_1'}{e_1\, e_2 \longrightarrow e_1'\, e_2}\ \textsc{step-app1}$$

$$\frac{e_2 \longrightarrow e_2'}{v_1\, e_2 \longrightarrow v_1\, e_2'}\ \textsc{step-app2} \qquad \frac{e_1 \longrightarrow e_1'}{\langle e_1, e_2\rangle \longrightarrow \langle e_1', e_2\rangle}\ \textsc{step-pair1}$$

$$\frac{e_2 \longrightarrow e_2'}{\langle v_1, e_2\rangle \longrightarrow \langle v_1, e_2'\rangle}\ \textsc{step-pair2} \qquad \frac{e \longrightarrow e'}{c\, e \longrightarrow c\, e'}\ \textsc{step-capp}$$

$$\frac{e \longrightarrow e'}{\{l = e\} \longrightarrow \{l = e'\}}\ \textsc{step-rcd1} \qquad \frac{e \longrightarrow e'}{e.l \longrightarrow e'.l}\ \textsc{step-rcd2}$$