# **Predicting Popularity of YouTube Video**

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#### Abstract

In this work we address the problem of ranking two objects given their features. The problem is applied to a context of guessing the more popular one out of two given YouTube videos. We formulate this problem into two different approaches, namely, binary classification and linear regression. Experiments are conducted in our data set crawled from YouTube for one month. Results show that both approaches yield a slightly above 50% accuracy. We conclude that bag-of-words features with linear classifiers are not efficient in determining videos' popularity.

### 1 Introduction

YouTube is one of the most popular video-sharing online platform in the Internet. It attracts billion of unique user visit monthly and has diverse topics in their video content. YouTube users can earn money from the number of views of their uploaded videos. Hence, understanding the secrets of making a popular YouTube video is essential for people who want to make benefits from the site. There are many factors to determine popularity of a video, but in general we can pin down to have the following two: having good content and good marketing plan. One of the techniques to attract more viewerships is to make the video's "visual appearance" look appealing via catchy keywords, informative cover picture, etc.

The ranking problem (aka. Learning to rank<sup>1</sup>) is a typical supervised learning problem of predicting the rank of a set of items regarding to a set of criteria. It has a numerous applications in a broad domains such as web search, multimedia retrieval, recommender systems, etc. Results from such problems can bring benefits to Internet users, such as saving their time by introducing the most relevant products/articles/web pages to their interests. In YouTube context, ranking problems can be raised to recommend most relevant videos to a given video. Another application is to predict the ranking of videos regarding to their popularity.

**Problem statement.** Given a set of videos, each is associated with a set of bag-of-word and numeric features extracted from their metadata. A pair of videos is said to have an order on their popularity by comparing their number of views. Video with more viewership is considered to be more popular than the other. Given two videos with their features, the video ranking problem is to construct a model to accurately predict which one has more views.

<sup>1</sup>http://en.wikipedia.org/wiki/Learning\_to\_rank

# 2 Ranking by Classification

Our goal is to construct a prediction rule f that can rank the videos with respect to their popularity using their meta-data as feature inputs. Since there are only two outcomes in ranking two videos, we choose to model the output of f as a binary class label. Hence, we can formulate the ranking problem as a binary classification problem as in Section 2.1. This approach poses two problems. First, it does not take into account the magnitude in the ranking, meaning there is no difference between pairs of videos which are significantly different in their viewerships and pairs which are just slightly different. We address this problem by borrowing ideas of re-weighted training set from the Boosting technique in Section 2.2. Second, video viewership also depends on the video's "age", i.e. number of days passed since it was uploaded to YouTube to the day it was crawled. It is not comparable to rank popularity of a video uploaded years ago with recent ones. Therefore, we bin videos according to their "age", and learn a distinct classifier f for each group. We also construct an ensemble of these time-specific classifiers in order to enhance the overall predictive performance. All are described in Section 2.3.

### 2.1 Problem Formulation

We can reformulate the ranking prediction problem between two videos as a binary classification problem. To be specific, let  $X_i \in \mathbb{R}^D$  and  $X_j \in \mathbb{R}^D$  be feature vectors of two videos i and j correspondingly. Each video pair (i,j) is associated with a binary label  $Y_{ij}$  defined as follows

$$Y_{ij} = \begin{cases} 1, & \text{if $\#\_$of\_views\_i$} \geq \#\_$of\_views\_j\\ 0, & \text{otherwise} \end{cases}$$
 (1)

We can form a representative vector of the pair (i, j) as follows

$$\mathcal{X}_{ij} = k(X_i, X_j), \tag{2}$$

where  $k:(\mathcal{R}^D,\mathcal{R}^D)\to\mathcal{R}^{D'}$  is a feature transformation function. There are several options for k.

- Difference between two feature vectors:  $\mathcal{X}_{ij} = X_i X_j$
- ullet Concatenation of two feature vectors:  $\mathcal{X}_{ij} = [X_i, X_j]$  (Matlab notation)
- Kernel functions, e.g.  $\mathcal{X}_{ij} = ||X_i X_j||^2$

At the moment, we cannot find any theories/signals to indicate which form of k is the most appropriate. For the scope of the project, we choose to represent  $X_{ij}$  as difference between  $X_i$  and  $X_j$ , meaning D = D'. The kernelized version is left in future work.

The function form of classifier  $P(Y_{ij}|\mathcal{X}_{ij},\mathbf{w})$  as follows

$$P(Y_{ij} = 1 | \mathcal{X}_{ij}, \mathbf{w}) = \frac{1}{1 + \exp(w_0 + \sum_d w_d \mathcal{X}_{ij}^d)} = \frac{1}{1 + \exp(\mathbf{w}^T \mathcal{X}_{ij})}$$
(3)

The model parameters  $\mathbf{w} \in \mathbb{R}^D$  can be learned using MAP.

$$\hat{\mathbf{w}}_{MAP} = \arg\max_{\mathbf{w}} \prod_{(i,j)} P(Y_{ij}|\mathcal{X}_{ij}, \mathbf{w}) P(\mathbf{w}) \qquad (P(\mathbf{w}) \sim \mathcal{N}(0, \tau^2 I))$$

$$= \arg\max_{\mathbf{w}} \prod_{\{(i,j)|Y_{ij}=1\}} P(Y_{ij}|\mathcal{X}_{ij}, \mathbf{w}) P(\mathbf{w}) \qquad (Y_{ij} + Y_{ji} = 1, \mathcal{X}_{ij} = -\mathcal{X}_{ji})$$

$$= \arg\max_{\mathbf{w}} \sum_{\{(i,j)|Y_{ij}=1\}} ln P(Y_{ij}|\mathcal{X}_{ij}, \mathbf{w}) + ln P(\mathbf{w})$$

$$= \arg\max_{\mathbf{w}} \sum_{\{(i,j)|Y_{ij}=1\}} ((1 - Y_{ij}) \mathbf{w}^T \mathcal{X}_{ij} - ln(1 + \exp(\mathbf{w}^T \mathcal{X}_{ij}))) - \lambda_w ||\mathbf{w}||_2^2 = l(\mathbf{w})$$

We can optimize Equation 4 (a concave function) using Gradient Ascent algorithm with the following update rule

$$w_d^{t+1} \leftarrow w_d^t + \eta \frac{\partial l(\mathbf{w})}{\partial w_d} = w_d^t + \eta \left( \sum_{\{(i,j)|Y_{ij}=1\}} \mathcal{X}_{ij}^d ((1 - Y_{ij}) - \frac{\exp(\mathbf{w}^T \mathcal{X}_{ij})}{1 + \exp(\mathbf{w}^T \mathcal{X}_{ij})}) - \lambda_w w_d \right)$$
(5)

Since the size of training set is less than the number of features, we opt to using Stochastic Gradient Ascent algorithm (described in Algorithm 1) with L2-regularization.

$$w_d^{t+1} = w_d^t + \eta(\mathcal{X}_{ij}^d((1 - Y_{ij}) - \frac{\exp(\mathbf{w}^T \mathcal{X}_{ij})}{1 + \exp(\mathbf{w}^T \mathcal{X}_{ij})}) - \lambda_w w_d)$$
(6)

### Algorithm 1: Stochastic Gradient Ascent Algorithm

```
Initialize {\bf w} as zero-valued vector. Initialize epoch=1. Initialize \lambda=0.01. Initialize \eta=1. while epoch < maxIter\ {\bf do}

for \forall i,j\ {\bf do}

Update {\bf w} using Equation 7 epoch++;

Return {\bf w}.
```

# 2.2 Extension 1: Re-weighting the features

By definition in Equation 1,  $Y_{ij}$  can only capture which video has more viewerships, but not how much their viewerships differ. Hence, the classifier f cannot weight correctly important features in determining video's popularity. Assume that we have three YouTube videos  $v_1$ ,  $v_2$  and  $v_3$ , each attracted 1000, 10 and 1 views after one day since they were uploaded to the web. Also assume that we only use bag-of-word features and there are five words in our dictionary  $\{w_1, w_2, w_3, w_4, w_5\}$ . The following table contains all necessary information for this example.

Video	Title	Number of views	Bag of word features
$v_1$	$\{w_1, w_2, w_3\}$	1000	$\{1, 1, 1, 0, 0\}$
$v_2$	$\{w_2, w_3, w_4\}$	10	$\{0, 1, 1, 1, 0\}$
$v_3$	$\{w_3, w_4, w_5\}$	1	$\{0,0,1,1,1\}$

As the above scheme in Section 2.1, we can represent the pair  $(v_1, v_2)$  and  $(v_2, v_3)$  with feature vectors  $\mathcal{X}_{12} = \{1, 0, 0, -1, 0\}$  and  $\mathcal{X}_{23} = \{0, 1, 0, 0, -1\}$ . When training a classifier f on these two vectors with both  $Y_{12}$  and  $Y_{23}$  equal to 1, we can see no differences between the weights (i.e. model parameters) on feature  $w_1$  and  $w_2$ ,  $w_4$  and  $w_5$ . Hence, f cannot correctly rank a pair of videos with titles of  $\{w_1, w_4\}$  and  $\{w_2, w_5\}$  correspondingly.

A general idea to solve this problem is to incorporate the magnitude in difference between two videos' number of views into f. Basically, we want to have more weights on features that are frequently attached with popular videos. In this work we propose two ad-hoc solutions: 1) Augmenting the representative feature vectors  $\mathcal{X}_{ij}$  and 2) Re-weighting the gradient.

# 2.2.1 Augmenting the representative feature vectors

In this approach, we scale the representative feature vectors  $\mathcal{X}_{ij}$  for pairs of videos (i,j) in the training set, and train a classifier f on these augmented features. The scaling factor is determined based on the ratio of numbers of views of the corresponding video pair i and j. To avoid the overflow problem caused by high variance in number of views (2 billions versus 10), we transform the number of views into log space before computing the ratio. Let consider the pair  $\mathcal{X}_{12}$  in the above example. The scaling factor is computed as follow:  $\alpha = \frac{\log 1000}{\log 10} = 3$ . Hence the augmented representative features  $\mathcal{X}_{12} = \{3, 0, 0, -3, 0\}$ .

There are other various options to compute the scaling factor  $\alpha$  such as using a different log base, or normalise all the view counts, etc. However, all these approaches are akin to the proposed one, and hence do not have a theoretical guarantee on the overall performance. We demonstrate one way of using ratio of log base 10 in this work.

### 2.2.2 Re-weighting the gradient

Another way to tackle the above problem is to re-weighting the gradient. Similarly as the first approach, we also compute a scaling factor  $\alpha$  from the two videos' number of views. The only difference is instead of augmenting the representative feature vectors  $\mathcal{X}_{ij}$ , we re-weight the gradient in Equation 6 as follows

$$w_d^{t+1} = w_d^t + \eta \alpha (\mathcal{X}_{ij}^d ((1 - Y_{ij}) - \frac{\exp(\mathbf{w}^T \mathcal{X}_{ij})}{1 + \exp(\mathbf{w}^T \mathcal{X}_{ij})}) - \lambda_w w_d)$$

$$(7)$$

The idea is borrowed from the AdaBoost technique by re-weighting the training data points. However, the difference is that AdaBoost keeps updating weights for every iteration, meanwhile we fix the weight. Our goal is to stress the importance of pairs having high variance in their viewerships by forcing the corresponding parameters to update more gradients of these pairs.

### 2.3 Extension 2: Ensemble of time-specific classifiers

Number of views of a video also depends on time passed since it was first uploaded to YouTube. It is more comparable to measure the popularity between videos sharing the same passed days. Such videos are clustered into a bin and a distinct classifier f is trained on them. For the above example, we have 3 different bins,  $B_{1000} = \{v_1\}$ ,  $B_{10} = \{v_2\}$  and  $B_1 = \{v_3\}$ , with corresponding classifiers  $f_{1000}$ ,  $f_{10}$  and  $f_1$ . Given a new video pair  $v_4$  and  $v_5$ , we first find the bin that both videos belong to according to their age, and use the corresponding classifier to compare their popularity.

This approach has a drawback which makes the learning model suffers from data sparseness. Since our data is a one-month sample from YouTube repository, it is possible to have keywords do not exist in the observed video pairs of a bin. To overcome this issue, we suggest to construct an ensemble of all bins' classifier to enhance the predictive performance.

According to [4], ensemble methods are learning algorithms that blend results from different hypotheses to perform some prediction tasks on new data points. There are two main reasons behind these methods. First is statistical. Basically, we often do not have sufficient data to identify the best, but equally accurate, hypotheses. By taking the majority votes or average results of these hypothesis, we can reduce the risk of choosing the wrong hypothesis. Secondly, different hypotheses may have different starting points and explore different local optima. Hence an ensemble of these hypotheses can give a better approximation than any of individual hypothesis. In literature, Bagging and Boosting are common ensemble algorithms.

The above two reasons motive us to construct an ensemble of bin-specific classifiers with selection of the majority weighted voting scheme. First, it is possible to have a keyword w does not exist in an arbitrary bin's training data, but can occur in other bins. Hence we can borrow information from those bins to weight w. Second, to incorporate the fact that the video's viewcount growth rate will be diminishing over time, the weights from bins further away from the selected bin must have smaller impact to its neighbouring bins. We therefore introduce a majority weighted voting scheme as follow. Let  $f_t$  be the classifier trained on data of bin  $B_t$ , containing videos of t-days old. Assume that we have T such bins. For a pair of videos i, j in the bin  $B_t$ 's testing set, we compute the probability of ranking as follows

$$P(Y_{ij}^{t} = 1 | \mathcal{X}_{ij}^{t}, f_{1}, \dots, f_{T}) = \sum_{t'} \frac{1}{1 + |t - t'|} P(Y_{ij}^{t} = 1 | \mathcal{X}_{ij}^{t}, f_{t'})$$

$$= \sum_{t'} \frac{1}{1 + |t - t'|} P(Y_{ij}^{t} = 1 | \mathcal{X}_{ij}^{t}, \mathbf{w}^{t'})$$
(8)

We can compute  $P(Y_{ij}^t = 0 | \mathcal{X}_{ij}^t, f_1, \dots, f_T)$  similarly. The class with higher probability is selected as final prediction.

The two above extensions are workarounds for the Classification approach to incorporate the magnitude in video rankings. Since we rank the videos based on their view counts, it may be more efficient if we directly model the number of views instead of the orders. This can be treated as a simple regression problem, as described in the next section.

# 3 Ranking by Regression

Another approach to the ranking problem is to predict the number of views for each video and then rank the videos based on predicted values, rather than comparing two videos directly. This approach has two advantages over the above approach of classification. First, it can take into account the magnitude of difference between two videos' viewership. Secondly, we can use it to predict a number of views of a video for a specific time in future given their current status. We treat this problem as an ordinary linear regression problem.

### 3.1 Problem Formulation

Our linear function assumes that are labels  $Y_i$  come from our input  $X_i$  plus some noise  $\epsilon$ :

$$Y_u = X_u \beta + \epsilon, \tag{9}$$

where  $Y_i$  is the number of views of video i,  $X_i$  is the associated feature vector. We therefore seek a function of the form

$$f(X) = X\beta \tag{10}$$

and attempt to minimize the mean squared error loss function, giving

$$\hat{\beta} = \underset{\beta}{\arg\min} \, 1/n(A\beta - Y)^T (A\beta - Y) \tag{11}$$

where  $A = [X_1...X_n]^T$  and  $Y = [Y_1...Y_n]^T$ , n is the number of training data points.

In order to solve this problem, we can use either the closed form or Gradient Descent to learn the  $\beta$  parameters. However, since our feature space may be quite large, we opt for the latter. We therefore initialize  $\beta^0$  to 0, and thereafter use the update step

$$\beta^{t+1} = \beta^t - \eta A^T (A\beta^t - Y) \tag{12}$$

Since our context is a bad conditioning problem, similarly to the Classification approach, we opt to Stochastic Gradient Descent with L2-Regularization. The new update function

$$\beta^{t+1} = \beta^t - \eta(x_i(x_i\beta^t - Y_i) + \beta^t) \tag{13}$$

After the learning stage is complete, the predicted ranking can be done as follows

$$\hat{Y}_{uv} = \mathbb{I}(\beta X_u > \beta X_v),\tag{14}$$

where  $\mathbb{I}$  is the indicator function, return 1 if the expression as argument is true, and 0 otherwise.

### 3.2 Comparing Order-of-Magnitude

It is important to decide what we will consider as being "close to correct". We use least-squared regression, where we minimize the total of the squares of the differences between our prediction and the true value. However, we must deal with the gigantic variance in our observed data.

Ideally, we wish to consider orders of magnitude rather than direct counts, and for this we will set our loss function equal to the square of the difference between the log of our prediction and the log of the observed value. The motivation is that we wish to reflect the human intuition that there is more difference between the popularities of two videos with 10 and 1,000 views (respectively) than between two videos with 1,000,000 and 1,001,000 views. This will prevent petty variations among the most popular videos from drowning out the differences in all others.

We therefore deal with the number of views in log scale for the regression (though we also tried running our analysis without using log scale). One anticipated effect of this is that features will be expected to contribute multiplicatively, rather than additively, to the popularity of a video.

# 4 Experimental study

#### 4.1 Dataset

We crawled and extracted our data from YouTube for one month (Oct - Nov, 2015). The crawling strategy is akin to bread-first search. First we initialize the crawler with several random "seed" videos, which mostly are in the *Movie* and *Music* category, and recursively explore all other related videos recommended by YouTube recommendation systems. For each video, we extract all of its metadata such as title, uploader and his number of subscriptions, description, upload date, number of views/likes/dislikes, video length, and a number of other attributes, as well as a list of around 30 videos YouTube recommends as being similar.

### 4.1.1 Data Statistics

Although the crawler was suspended twice due to technical issues and upgrades, we have gathered a sizeable amount of data, with tremendous variation in the observed values:

• Number of videos crawled: 1.432.213

• Number of uploaders: 628,072

• Number of "bag of word" features (extracted from the titles): 2,447,603 entries

• Video length: [1...107, 373] (seconds)

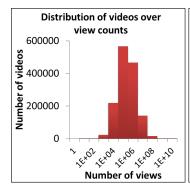
• Range of number of day passed since uploaded: [1...3423] (days)

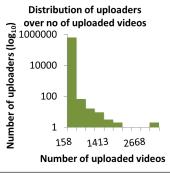
• Range of number of views: [7...2, 104, 518, 656].

• Range of number of likes: [1...8, 639, 650].

• Range of number of dislikes: [1...4, 184, 769].

We plot in Figure 1 different histograms reflecting characteristics of our data. In the red-colorer histogram, we look for the distributions of videos and their view counts. Interestingly, the distribution is in the shape of Gaussians, instead of following the Power Law distribution as frequently observed in social networks. We guess this observation is due to the way we sampled the data, by following the recommended links. It is intuitive since videos with high view counts are usually recommended together. The Gaussian distribution also encourages us to perform some standardisation on the data, such as subtracting the label to the mean of the Gaussian and predict the left amount.





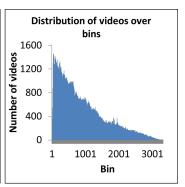


Figure 1: Red-colored histogram: Distribution of videos over view counts. Blue-colored histogram: Distribution of uploaders over number of uploaded videos. Green-colored histogram: Distribution of number of videos in each bin.

As the statistics show, we have a large magnitude in ranges of number of view, likes and dislikes. This motivates us to do some preprocessing on data, such as feature normalization or data standardization, to ensure the numerical stability and good speed of convergence on the learning algorithm for logistic regression. We discuss more on this step in the Section 3.2.

### 4.1.2 Feature extraction

In this section we discuss the feature engineering process in the project. First, we build a dictionary mapping the uploader to the number of videos they have uploaded and the total number of views there videos have. We also take care to prevent "cheating": In order to ensure that our predictor has only such information as would be available before the video's publishing is ever used, we temporarily reduce these number of video-views and the total number of video uploads for the uploader according to the publish date of the video under current consideration.

We considered many features, which ultimately include:

- Features extracted via a bag-of-words model on the title, using TF-IDF.
- The number of videos uploaded by the uploader prior to the current video's upload date.
   Because of our desire for caution against "cheating", we count only those videos that we have crawled.
- The total number of views for the uploader due to videos released prior to the current video's upload date. Again, we count only those videos that we have crawled. This dateconscious counting is particularly important because there are many cases where there is only one video crawled for a given uploader, meaning that this feature would become a nearly perfect predictor.
- The number of subscribers for the uploader. We lack sufficient data to know how subscribers changed over time, so we simply had to keep this constant.
- The runtime of the video, in seconds.
- The age of the video at the time of crawling, in days.
- The number of likes/dislikes. Since these are expected to scale with the number of views, we forbade ourselves from using the number directly, but we did allow certain combinations, such as the log of the like-vs-dislike ratio.
- Various combinations of the above features (for example, the log of some other feature, or the ratio between two features).

# 4.2 Evaluation Metrics

Normally, a rank correlation  $^2$  is a common metric in ranking problems. Since our problem is reformulated as a classification and regression problems, we evaluate our methods using corresponding metrics such as Root Mean Square Error (RMSE) for Regression and  $F_1$ -score and Accuracy for Classification.

#### 4.2.1 Classification

The error matrix (or confusion matrix) is a common way to measure performance of any arbitrary classifiers. Several metrics derived from this matrix are Accuracy, Precision-Recall,  $F_1$ -score or Receiver Operator Characteristics (ROC). In our work we employ two of them: Accuracy and  $F_1$ -score  $^3$ .

The accuracy can be equivalently stated as a 0-1 loss function over total number of pairs in testing set.

$$0/1_{loss} = \sum_{(u,v)_{test}} \frac{1}{|(u,v)_{test}|} \mathbf{1}[\hat{Y}_{uv} - Y_{uv} == 0]$$
(15)

# 4.2.2 Regression

To Joseph: Can you put your least square formula here? The first few paragraphs in section 4.3.2 can be put here. And show the equations as well.

<sup>&</sup>lt;sup>2</sup>http://en.wikipedia.org/wiki/Rank\_correlation

<sup>&</sup>lt;sup>3</sup>http://en.wikipedia.org/wiki/F1\_score

### 4.3 Results

Due to our limitation in computing resources, we can only present our approaches for the first 30 bins (meaning videos with age ranging from 1 day to 30 days). For each bin, we create 5 folds cross-validation with 80% of our data for training, and the other 20% for testing. The average results over 5-folds are reported.

### 4.3.1 Classification Results

The results of our logistic regression comparing paris of videos are

### 4.3.2 Regression Results

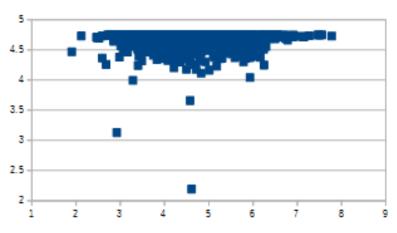
The linear regression can be evaluated in more than one way.

The first way to measure our results is to look at its predictions directly. Our mean squared error indicated that our prediction was was typically 0.93 orders of magnitude away from the true result, but it is hard to say whether we should be impressed.

The second form of evaluation is to compare the results for each pair of videos and then use the **0-1 loss function**, so that results can be compared to the logistic regression results. Not surprisingly, logistic regression performed better (having been trained explicitly for this task). What is surprising, however, is that our accuracy is not significantly greater than 50%, which indicates that our linear regression holds no measurable value for the ranking problem.

A likely cause is that this problem may not be linear in nature – in fact, it would admittedly be surprising if it were. We tested our accuracy on the training data as well as on the testing data, and found that it performed just as poorly, indicating that the weak results on our testing data did not stem from over fitting.

In order to investigate further, we plotted our output against the true number of views (for a random subset of our data):



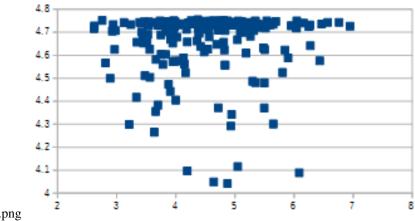
regression training.png

Figure 2: Training data: true value vs prediction (in log-scale).

We observe a strong tendency towards the same output value (roughly 50,000 views), which corresponds to the average number of views – this would be the typical outcome when linear regression is inadequate.

### 5 Related work

Our problem is a special case of the more general **Learning to rank** problem, which is widely used in many applications such as Information Retrieval, Data Mining, etc. [1] gives a short survey of



regression testing.png

Figure 3: Testing data: true value vs prediction (in log-scale).

state-of-the-art approaches to the problem of ranking documents relevant to user queries. Most of them are implemented in Ranklib<sup>4</sup>.

Another form of the problem is ordinal classification (a.k.a. ordinal regression). There are several approaches, mostly adapted from classification techniques, such as Support Vector [2], Gaussian Processes [3], etc. Although two problems are related to predicting the objects' orders, they do have some similarities and differences. First, both of them are considered as supervised learning tasks, meaning from the given features and labels, trying to predict the orderings of testing objects. The difference is in the level of orderings. As [1] discussed, Learning to rank cares more about the accurate ordering of objects, while the latter focuses on the ordered categorization. For example, a list of top-K relevant documents retrieved from a Learning to rank algorithm would be sorted according to the relevance to a given query. Meanwhile, ordinal regression would try to categorize movies into one out of five stars, reflecting how much the movies match with users' preferences. There are no specific rankings of movies receiving same stars.

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<sup>4</sup>http://people.cs.umass.edu/ vdang/ranklib.html