

# Analysis of jet engine modulation effect with extended Hilbert-Huang transform

J.H. Park, H. Lim and N.H. Myung

The jet engine modulation (JEM) effect is widely used for radar target identification. The Hilbert-Huang transform (HHT), a high-resolution time-frequency technique, is extended in order to make an effective JEM analysis. The simulation and measurement results show that the extended HHT not only extracts jet engine features but also definitely exhibits sinusoidal waveforms of rotation-induced micro-Doppler signatures. It also has better applicability for signatures corrupted by noise than other conventional time-frequency techniques.

**Introduction:** The jet engine modulation (JEM) effect, a micro-Doppler phenomenon generated by a rotating engine compressor, plays an important role in radar target identification [1]. To effectively interpret JEM, its application to time-frequency analysis via the short-time Fourier transform (STFT) was presented in a recent study [2]. However, time-frequency techniques using window function integration, such as the STFT, generally have resolution limitations and vulnerability to noise. Although accurate micro-Doppler analysis should be based on high-resolution time-frequency techniques, there has been a lack of work carried out on JEM analysis using such methods.

In this Letter, the Hilbert-Huang transform (HHT) [3], a time-frequency technique with high resolution, is employed and its extended form is proposed for effective JEM analysis. For mathematically simple demonstration, the analysis mainly focuses on the first chopping harmonic in which the substantial JEM energy resides [4].

**JEM signature model and extended HHT:** JEM undergoes a large amount of scattering from rotating blades in different rotor stages. These electromagnetic interactions make an exact mathematical description of JEM unfeasible. For this reason, a JEM signature is expressed by resorting to the JEM parametric model developed by Bell and Grubbs [1]. The JEM signature with radar frequency  $f_0$  consists of the amplitude modulation (AM) part and the phase modulation (PM) part as follows:

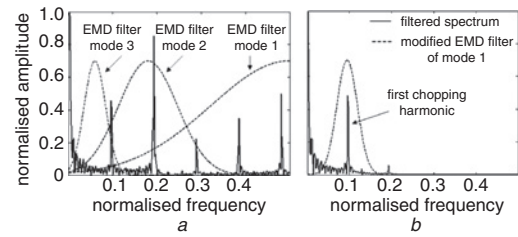
$$s_{JEM}(t) = \left\{ \sum_{n=-\infty}^{\infty} a_n e^{j2\pi n f_r t} \right\} e^{j \sum_{n=-\infty}^{\infty} \beta_n \sin(2\pi f_r n t)} e^{j2\pi f_0 t} \quad (1)$$

where  $n$  is the JEM harmonic index,  $a_n$  and  $\beta_n$  are the Fourier series coefficients associated with the AM and PM, respectively, and  $f_r$  is the engine rotation frequency. From (1), the JEM spectrum is composed of a number of harmonics with the fundamental frequency  $f_r$ . According to the JEM theory, the JEM spectrum has relatively dominant amplitude at chopping harmonics whose fundamental frequency is  $Nf_r$ , where  $N$  is the number of blades in the first rotor stage [4]. The instantaneous frequency corresponding to the first chopping harmonic ( $N$ th JEM harmonic) is derived from the PM part such that

$$f_i(t) = \Delta f_N \cos(2\pi f_r N t) \quad (2)$$

where  $\Delta f_N$  is the phase modulation index associated with the JEM.

To facilitate efficient analysis, the JEM analysis performed in this study chiefly concentrated on the first chopping harmonic. The HHT is a method for individual analysis of this component. It decomposes a signal into several intrinsic mode functions (IMFs) via the empirical mode decomposition (EMD). The Hilbert spectrum, a joint time-frequency plot, is finally obtained using the IMFs of the JEM mode [3]. However, since the EMD functions as a filter bank where a filter of mode  $k + 1$  roughly occupies a half-band of the filter of mode  $k$  ( $k > 1$ ) [5], the EMD filter 3 corresponding to the first chopping harmonic may fail to capture the JEM spectral component, as shown in Fig. 1a. This can result in distortion of the time-frequency localisation in the instantaneous frequency. To enhance the original HHT, we developed the extended HHT by inserting a pre-processing lowpass filter (LPF) to complement the EMD. We selected a filter cutoff frequency and an attenuation level to make the filtered spectrum cover an effective range of the first chopping harmonic. As shown in Fig. 1b, the LPF is expected to help the EMD filter have a valid range and explicitly assign the first chopping harmonic to the EMD filter of mode 1 since it equivalently operates as a bandpass filter to the filtered spectrum.



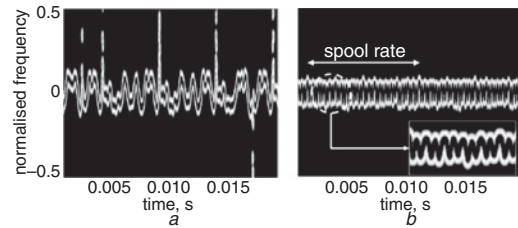
**Fig. 1** Conceptual EMD operation as filter bank

a EMD operation to original JEM spectrum  
b Modified EMD operation to filtered spectrum

**Analysis with simulation:** The JEM signature from a realistic jet engine CAD model was examined. Information on the model is summarised in Table 1. The simulation [2] was based on the shooting and bouncing ray (SBR) algorithm with ray density  $\lambda/10$ , where  $\lambda$  is the wavelength of the radar signature. The pulse repetition frequency (PRF) and  $f_r$  are given as 80 kHz and 100 Hz, respectively. To give an insight into the advantage of the extended HHT, Hilbert spectrums in Figs. 2a and b depict the instantaneous frequency of the first chopping harmonic using the original HHT and the extended HHT, respectively.

**Table 1:** Information on simulated jet engine model

Original model	1st rotor stage		2nd rotor stage		3rd rotor stage	
	$N$	$L$	$N$	$L$	$N$	$L$
F 136	40	12.7 $\lambda$	88	10.5 $\lambda$	96	10.5 $\lambda$



**Fig. 2** Hilbert spectrums of JEM signature from jet engine model

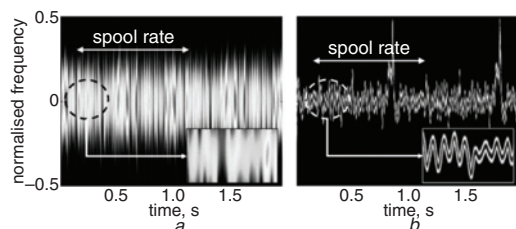
a From original HHT  
b From extended HHT (expanded between 0.001 and 0.004 s)

As shown in Fig. 2a, the result from the original HHT shows an inaccurate time-dependent characteristic of JEM since the EMD filter fails to cover the proper range of the first chopping harmonic. However, the extended HHT in Fig. 2b exhibits a clear sinusoidal modulation as expressed in (2) and thus it effectively supplements the original HHT.

In the overall view of Fig. 2b, a partial group of the instantaneous frequency repeats every spool rate indicating one revolution period. This is due to the fact that the spool rate gives the fundamental frequency to the JEM spectrum. The expanded view definitely reveals the chopping rate that denotes the period when a blade moves to its adjacent position. This result confirms that the extended HHT can provide information on the number of blades in contrast to other resolution-limited techniques. The estimated number of blades gave quite a good match with the real blade count. Note that, in the JEM phenomenon, it is commonly difficult to observe the JEM by stages beyond the first rotor stage.

**Analysis with measurement:** The extended HHT was applied to the measured JEM signature to prove its applicability. The experimental jet engine model was fabricated with three rotor stages and 42 blades were equipped in the first rotor stage. With a PRF of 1.8 kHz, the JEM signature was measured from the model rotating at a rate of 1 Hz. A detailed description is also available in [2].

In general, the JEM signature is severely deteriorated by background noise when measured in real environments. To validate the superiority of the extended HHT, the STFT was used as a representative for window integration-based techniques and was compared in terms of accuracy and noise adaptivity. The resulting STFT spectrogram and Hilbert spectrum are expanded in Figs. 3a and b.



**Fig. 3** Analysis results of measured JEM signature

*a* Spectrogram (expanded between 0.3 and 0.5 s)

*b* Hilbert spectrum (expanded between 0.3 and 0.5 s)

The spectrogram exhibits such a complicated result that extracting JEM parameters is very difficult, while the Hilbert spectrum remains a sinusoidal waveform. Although it is slightly deviated by residual noise, the extended HHT turns out to be more powerful than the STFT in the sense that it has improvement in both accuracy and robustness to noise. In addition, it retains computational efficiency since the EMD operates on the JEM spectrum preprocessed by the LPF.

**Conclusion:** This Letter has presented an effective analysis of JEM using an extended HHT in the time-frequency domain. The extended HHT has high-resolution characteristics and can extract the JEM parameters from a sinusoidally-varying instantaneous frequency. It also remains valid when applied to the measured JEM signatures. The proposed method is expected to be complementary to other existing JEM analysis methods and significantly helpful in radar target identification.

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J.H. Park and N.H. Myung (*Department of Electronic Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-Gu, Daejeon 305-701, Republic of Korea*)

E-mail: dydynoel@kaist.ac.kr

H. Lim (*Agency for Defense Development, POB 305-5 Yuseong Post Office, Yuseong-Gu, Daejeon, Republic of Korea*)

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