Identification of Volterra model parameters in wireless systems

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***Abstract*— This paper reports the identification of nonlinear models for wireless communications systems. The procedure relies on a novel complex-valued Volterra series (CVS) representation to provide a sparse representation based on statistical hypothesis testing and the Bayesian information criterion (BIC). The approach has been experimentally evaluated with the front-end of a communications transmitter taking into account the possible nonlinear impairments associated with the I/Q modulator. The proposed identification method showed an excellent performance in terms of model complexity reduction and dynamic range.**

***Index Terms*— Behavioral model, Volterra series, model identification, power amplifiers, wireless systems.**

I. Introduction

One important application of Volterra series to wireless communication systems is the design of linearizers based on digital predistorters (DPD) evidenced by the great interest of the current research in the joint predistortion of the transmitter I/Q modulator and power amplifier (PA). A good DPD design depends on the availability of an adequate behavioral model and the conventional baseband Volterra models, such as the full Volterra (FV), the memory polynomial (MP), or the generalized MP (GMP), are specific for PAs, lacking accuracy to represent more general nonlinear systems with complex-valued input signals. To overcome this limitation, several proposals that include a second Volterra series memory polynomial whose input signal is conjugated have been advanced [1]–[3]. However, there is no guarantee that these techniques are able to represent a general complex-valued system, as it is the case of the system composed by the I/Q modulator and the power amplifier connected in cascade.

In this communication, a general procedure for model identification in the context of a Volterra series representation is presented. It is based on the complex-valued Volterra (CVS) model, which has been demonstrated mathematically in [4] and implemented in a proposal of DPD design in [5]. The formal deduction is based on Wirtinger calculus, which allows to consider the complex-valued system dependent on the input $x(k)$ and its complex-conjugate $x^*(k)$, and to operate these variables as real-valued. Then, it can be viewed as a two input system and analysed using a double Volterra series approach. The final result is the general CVS representation, which provides a complete set of regressors to describe any nonlinear block of the communications field and also Volterra systems in other disciplines. The rise of the number of coefficients in this complete set makes necessary a suitable procedure to identify the model parameters in an efficient and robust manner, as in [6]–[7], where a technique for model-reduction using the sparse structure of Volterra kernels was introduced. With respect to other reduction techniques, this procedure directly apply the pruning to the regressors matrix, and the extra processing required to calculate the matrix transformation is avoided.

Here we present a novel approach to the identification of behavioral model parameters, and results are illustrated with the CVS model to successfully model a communications transmitter and its performance is verified in terms of order-reduction, precision and robustness. We also demonstrate the different features of the FV and CVS models. Finally, we confirm the robustness of the proposed identification approach by verifying that the set of parameters identified at a given power level are applicable to estimate the system output with an outstanding accuracy over a wide dynamic range.

II. Proposed Identification Procedure in Volterra Models

In the context of Volterra series representation, a general system with complex-valued input is described by the CVS model [4]. In particular, the the input-output relationship for the complex envelopes in a wireless communications system, $x(k)$ and $y(k)$, can be expressed as

$$y(k) = h_{0,0} + \sum_{n=1}^{\infty} \left\{ \sum_{\mathbf{q}_n=\mathbf{0}}^{\mathbf{Q}_{n,0}} h_{n,0}(\mathbf{q}_n) \prod_{r=1}^{n} x(k-q_r) + \right.$$
$$+ \sum_{\mathbf{q}_n=\mathbf{0}}^{\mathbf{Q}_{0,n}} h_{0,n}(\mathbf{q}_n) \prod_{s=1}^{n} x^*(k-q_s) +$$
$$+ \sum_{m=1}^{n-1} \sum_{\mathbf{q}_n=\mathbf{0}}^{\mathbf{Q}_{n-m,m}} \sum_{\mathbf{p}_m=\mathbf{0}}^{\mathbf{P}_{n-m,m}} h_{n-m,m}(\mathbf{q}_{n-m}, \mathbf{p}_m) \times$$
$$\left. \times \prod_{r=1}^{n-m} x(k-q_r) \prod_{s=1}^{m} x^*(k-p_s) \right\}, \quad (1)$$

where $h_{n-m,m}[\mathbf{q}_{n-m}, \mathbf{p}_m]$ is a bivariate Volterra kernel of order n, $\mathbf{q}_{n-m} = [q_{1-m,m}, q_{2-m,m}, \cdots, , q_{n-m,m}]^T$ for $m = 0, \cdots, n$ is a vector of delays of the nth-order term, and $\mathbf{Q}_{n-m,m} = [Q_{n-m,m}, Q_{n-m,m}, \cdots, Q_{n-m,m}]^T$ is the vector of maximum delays. The vectors $\mathbf{p}_m$ and $\mathbf{P}_{n-m,m}$ are defined in a similar way. The product of the input signal samples is denoted as $\prod_{r=1}^{n} x[k-q_r] = x[k-q_1]x[k-q_2]\cdots x[k-q_n]$ and the same notation is used for the product of the image samples $x^*(k)$.

The CVS structure is composed of two standard Volterra models dependent on $x(k)$ and $x^*(k)$, respectively, and also includes a third model with cross products of $x(k-q_r)$ by $x^*(k-p_s)$, not all present in [1]–[3]. Observe that it contains even-order terms not present in the FV model. The general character or the CVS model provides a complete set of regressors with a high number of parameters, although it also means an associated suitability for a pruning procedure to search the most significant contributions.

Using matrix notation, the relationship (1) can be expressed as

$$\mathbf{y} = \mathbf{X} \cdot \mathbf{h} + \mathbf{e}, \tag{2}$$

where $\mathbf{y}$ is a column vector with the samples of the output, $\mathbf{X}$ is a measurement matrix composed of the model regressors, $\mathbf{h}$ is a vector that arranges the normalized Volterra coefficients, and $\mathbf{e}$ is a Gaussian noise process. The number of parameters of a direct least-squares (LS) solution can be relatively high and proposals to reduce the model complexity have been published based on compressed sensing methods [7]–[8]. The procedure [7], based on the Orthogonal Matching Pursuit (OMP) algorithm and the Bayesian information criterion, denoted here as BIC1, will be used as a reference.

The alternative procedure developed in this communication is basically a detection problem in presence of Gaussian noise where the coefficients are obtained by the LS algorithm based on statistical hypotheses testing. Focusing on the ith element of the estimated coefficients vector $\hat{\mathbf{h}}$, the signal will be divided into N segments, each one yielding an independent measurement of this particular coefficient. We can define the vector $\tilde{\mathbf{h}}_i$ constructed with the N different realizations of this random variable to decide whether or not the ith coefficient has to be incorporated as an active parameter of the model. We use the Neyman-Pearson (NP) approach to make the decision, based on two hypotheses [9]: the measurement is produced by noise,

$$\mathcal{H}_0 \text{ if } \tilde{\mathbf{h}}_i = \mathbf{w}_i,$$

or the measurement is produced by the presence of a model coefficient plus noise

$$\mathcal{H}_1 \text{ if } \tilde{\mathbf{h}}_i = \mathbf{h}_i + \mathbf{w}_i.$$

Assuming that $\mathbf{w}_i$ is a zero-mean complex-valued white Gaussian noise (CWGN) with variance σ^2, the probability density function under $\mathcal{H}_1$ is

$$p(\tilde{\mathbf{h}}_i; \mathcal{H}_1) = \frac{1}{\pi^N \sigma^{2N}} e^{-\frac{1}{\sigma^2}(\tilde{\mathbf{h}}_i - \mathbf{h}_i)^H (\tilde{\mathbf{h}}_i - \mathbf{h}_i)} \tag{3}$$

where H represents the Hermitian transpose operation. A similar expression holds for the probability density function under $\mathcal{H}_0$, $p(\tilde{\mathbf{h}}_i; \mathcal{H}_0)$. For a given probability of measurement produced by noise, the NP theorem states that the probability of true detection is maximized if the detector decides the hypothesis $\mathcal{H}_1$ when the likelihood ratio exceeds a given threshold γ, i.e.,

$$L(\tilde{\mathbf{h}}_i) = \frac{p(\tilde{\mathbf{h}}_i; \mathcal{H}_1)}{p(\tilde{\mathbf{h}}_i; \mathcal{H}_0)} > \gamma. \tag{4}$$

Equivalently, we decide $\mathcal{H}_1$ if

$$\mathrm{Re}(\mathbf{h}_i^H \tilde{\mathbf{h}}_i) = \mathrm{Re}\left(\sum_{n=0}^{N-1} h_i^*(n) \tilde{h}_i(n) \right) \geq \gamma' \tag{5}$$

Since any coefficient is an unknown constant $h_i(n) = h_i$, we can estimate it as the average value

$$h_i \approx \bar{\tilde{h}}_i = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{h}_i(n) \tag{6}$$

and operating with (5), we can decide $\mathcal{H}_1$ if

$$\mathrm{Re}\left(\bar{\tilde{h}}_i^* \frac{1}{N} \sum_{n=0}^{N-1} \tilde{h}_i(n) \right) = \mathrm{Re}\left(\bar{\tilde{h}}_i^* \bar{\tilde{h}}_i \right) = |\bar{\tilde{h}}_i|^2 \geq \frac{\gamma'}{N}. \tag{7}$$

The proposed procedure is as follows. Once the set of active coefficients has been estimated by the OMP algorithm for each segment, the mean value $\bar{\tilde{h}}_i$ and the hypothesis test (7) are computed for all coefficients. Starting from a high value, the lower the threshold level, the more coefficients (regressors) will be incorporated to the model and the BIC criterion can be used to decide the optimum number of regressors n_c.

In this paper, we express the BIC rule with its explicit dependence on the normalized mean square error (NMSE). If the NMSE is expressed in dB, the variance is given by

$$\hat{\sigma}_e^2 = \left(\frac{1}{M} \sum_{m=0}^{M-1} |y(m)|^2 \right) \times 10^{\mathrm{NMSE}/10} \tag{8}$$

and substituting in (23) of [7], the BIC rule becomes

$$n_{c0} = \arg\min_{n_c} \left\{ \mathrm{NMSE} + \frac{n_c}{M} 10 \log(2M) \right\}. \tag{9}$$

This novel procedure is referred to as BIC2. Compared to BIC1, BIC2 requires the calculation of the average model and thresholding to get the reduced model, which is not significant in terms of computation time.

III. Experimental Results

The test bench was composed by a commercial I/Q modulator (SMU200A of Rohde & Schwarz) followed by the evaluation board of a PA based on Cree's CGH40010 GaN HEMT, and a vector signal analyzer (PXA-N9030A from Agilent Technologies). To test the procedure, 15-MHz OFDM signals were generated according to the LTE-downlink standard at 3.6 GHz. The maximum average input level was +6 dBm, corresponding to a measured output average level of +19 dBm and a peak power

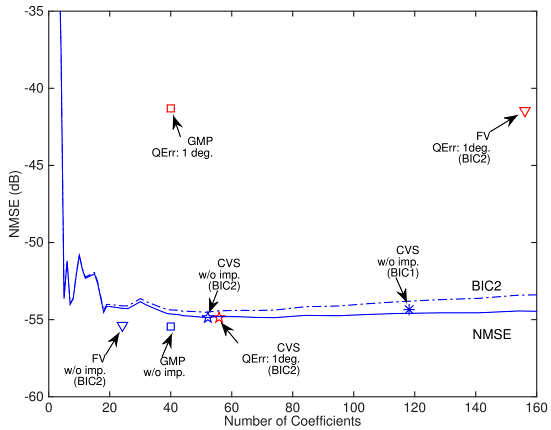

Fig. 1. Performance of the identification procedure for the CVS model using a signal generated by an I/Q modulator without impairments and with a quadrature error of 1 degree.

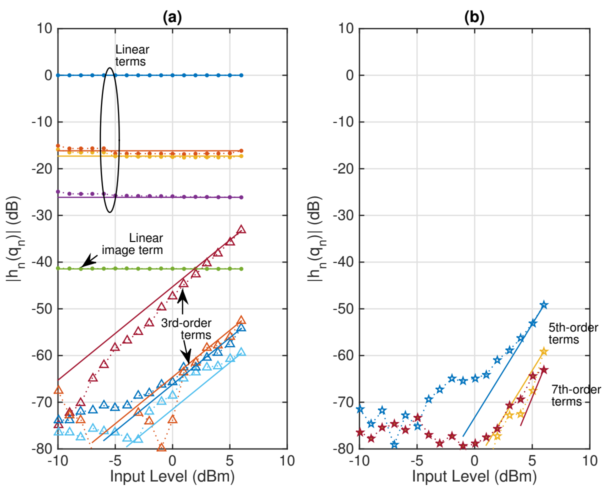

Fig. 2. Dependence on the average input power level of a) the normalized first- and third-order coefficients, and b) the normalized fifth- and seventh-order coefficients for the CVS model (third-order terms: triangles, fifth-order terms: pentagram and seventh-order terms: filled pentagram).

of about +30 dBm. The models were configured with thirteenth order, setting a maximum delay $Q = 3$ for orders 1 to 5 and memoryless (ML) for the higher orders. Initially, the CVS model [5] was pruned through the use of the BIC1 procedure as proposed in [7], showing an NMSE= -54.3 dB with 118 coefficients (see Fig. 1). The procedure was also validated with a second acquired signal, demonstrating the robustness of BIC1.

To evaluate the BIC2 technique, the acquired samples of the output complex envelope were processed on a symbol-by-symbol basis. A maximum number of coefficients $n_{max} = 200$ per symbol were incorporated by

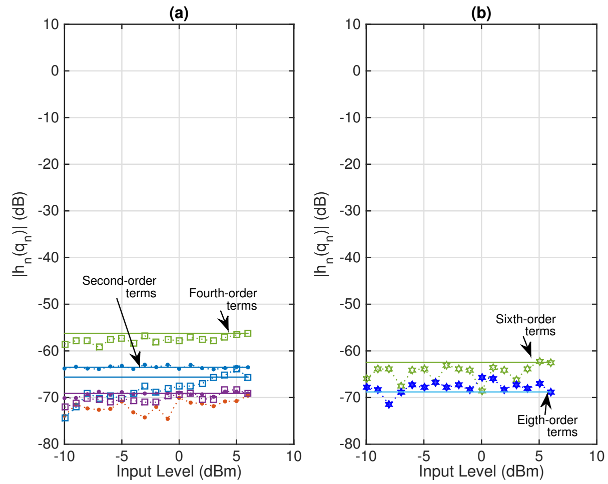

Fig. 3. Dependence on the average input power level of a) the normalized second- and fourth-order coefficients, and b) the normalized sixth- and eigth-order coefficients for the CVS model (second-order terms: dots, fourth-order terms: squares, sixth-order terms: hexagram and eighth-order terms: filled hexagram).

the OMP algorithm, yielding 56 sets of kernel vectors with 200 non-zero elements from a total of 1370. The mean value of these sets was computed and compared to a given threshold level, allowing only the n_c coefficients with magnitude above this threshold to be selected for the model. The threshold was swept from -50 to -85 dB below the coefficient of maximum absolute value (the ML linear coefficient). For each threshold, a distinct number of coefficients were included in the kernel vector allowing computation of the corresponding NMSE and BIC rule. The BIC2 procedure selects the number of coefficients that minimizes (9), yielding 52 coefficients for the CVS model and NMSE= -54.9 dB, thus demonstrating a performance and order-reduction superior to BIC1, as shown in Fig. 1. The same approach was applied to the FV model and for comparison, the results of the standard GMP model are also plotted. Their NMSE values were -55.4 dB with 24 and 40 coefficients, respectively. The foreseen weakness of the GMP and FV models is revealed when the identification procedure is repeated for a quadrature error of 1 degree forced in the I/Q modulator. The GMP model accuracy worsens in about 14.1 dB, as the square marks of the same figure show. In the case of the FV model (triangular marks), not only the accuracy is reduced by 13.9 dB, but also the number of coefficients is increased to 156. The robustness of the CVS model is evident as it maintains an exceptional NMSE=-54.9 dB with 56 coefficients, thanks to its direct dependence on $x^*(k)$. It is important to note that these results have been supported by measurements with different LTE waveforms.

The coefficients of the CVS model were identified for

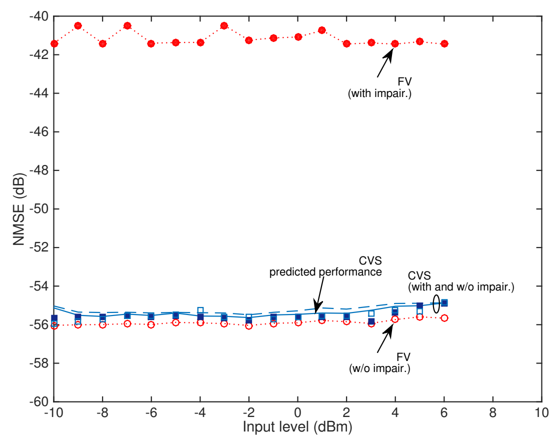

Fig. 4. NMSE of the model predicted with the coefficients extracted at $+6$ dBm (FV model: circles. CVS model: squares. Modulator impairment: 1 degree quadrature error).

the case of 1 degree of quadrature impairment in the modulator and the energy of each regressor, normalized with respect to the linear ML term, is displayed in the graphs of Figs. 2 and 3 as a function of the average input level. Focusing on the CVS odd-order terms, plotted in Fig. 2, the linear coefficients are signal level independent and the slope of the other coefficients increases accordingly to the nonlinear order n, following a $(n-1)$-dB by dB rate, as a result of the normalization. Recall that not all the odd-order coefficients have an equivalence in the FV model. For example, a noticeable linear $x^*(k)$-term emerges in the CVS model, labelled as 'linear image term', caused by the quadrature error. This coefficient is almost imperceptible for the system without impairments. Although not represented to avoid cluttering the figure, other novel higher-order terms are active in the CVS model. More significant are the even-order terms plotted in Fig. 3. This group of non-FV regressors is exclusive of the CVS model and shows no dependence on the input signal level, which is an evidence of its origin being the baseband nonlinearity present in the I/Q modulator and not the bandpass nonlinearity of the PA. At the same time these terms, not included in the conventional PA models, secure the superior performance of the CVS model when imperfections are present in the modulator, and consequently in the transmitter.

The consistency of the proposed procedure was tested by extracting the coefficients in a dynamic range from -10 dBm to $+6$ dBm and calculating the corresponding NMSE. The results in Fig. 4 demonstrate an excellent accuracy in the no-impairment case, with NMSE near -56 dB for the FV model (circles) and the CVS model (squares). On the contrary, when the quadrature error is present, the FV model accuracy worsens to about -41 dB (filled circles) while the CVS model maintains its superior performance (filled squares). Furthermore, the model extracted at $+6$ dBm was reused to extend the PA coefficients to other power levels in a range of 16 dB, which has been represented as straight lines in Figs. 2 and 3. The prediction of the output waveforms exhibited a remarkable precision for the cases with- and without impairments, as the plotted lines in Fig. 4 show.

IV. Conclusion

This paper has reported a reliable identification procedure for behavioral models of nonlinear communication systems based on a complete model (CVS) and a careful processing technique to identify the model parameters in a consistent manner. As it could be expected, the superiority of the CVS model against the reference FV or GMP models is evident as it keeps excellent performance even under the presence of I/Q modulator impairments. Experimental results also confirmed the reliability of the approach, as the reduced set of identified model coefficients provide a very good accuracy in a wide dynamic power range.

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