Novel Detection Model for MIMO Radar in Multipath Environment

Junpeng Shi, Guoping Hu, Hao Zhou, and Mingjun Chen

Abstract—**In this paper, a novel detection model for Multiple Input Multiple Output (MIMO) radar based on entropy theory in multipath environment is discussed. The proposed model, which defines the concept of Multipath Distance Difference (MDD) in depth, studies the variance and correlation of multipath scattering coefficient with MDD systematically according to the multipath received signal model. We illustrate that our model can use the multipath echo power to improve the detection performance by introducing the uncertainty analysis into the model. The simulation results demonstrate that the proposed method can achieve performance improvement due to the realistic physical model.**

Index Terms—MIMO radar, Multipath Distance Difference, entropy theory, scattering coefficient

I. INTRODUCTION

MULTI-INPUT multi-output (MIMO) radars, which can capture the spatial diversity of the target's radar cross section (RCS), have potential advantages over the traditional radars [1]-[2]. It has been demonstrated that MIMO radar shares more degrees of freedom for transmission beam forming, improved detection performance and higher sensitivity for moving targets [3]-[5]. Utilizing MIMO radar for targets detection in multipath or non-homogeneous environment has drawn a great deal of attention in recent years [6]-[14]. However, the detection algorithms in multipath environment take no account of the multipath parameter or simplify it as chi-square distribution, resulting from the bad reliability [12]-[14]. As a result, to improve the multipath detection performance, it is important to analyze the multipath received signal model in the three dimensional space. In this scenario, multipath effect will affect the phase shift of the received signals tremendously [15]-[17], which have a close relationship with the location of the target and antennas.

In this work, a novel multipath received signal model for MIMO radar is proposed to fully consider multipath effect on the scattering coefficient and correlation coefficient. To the authors' best knowledge, the model is analyzed for the first time [18]-[21]. Furthermore, the distribution diversity of multipath received signals is conducted based on the entropy theory, which is demonstrated to be effective in the uncertain case [22]-[24]. To achieve this work, we build a statistical MIMO radar model in space objectively. The rest of the paper is organized as follows.

In Section II, the multipath received signal and multipath correlation coefficient are built in multipath environment. In Section III, we develop a target detection model based on entropy theory, and the new algorithm can make the distribution more effective. Numerical simulations are presented in Section IV. Finally, the Section V concludes the whole paper.

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II. MIMO RADAR MULTIPATH RECEIVED SIGNAL MODEL

We assume that a far field and low-angle target is located at some point (x_0, y_0, h_0) in space, and the target has a rectangular shape with a dimension $\Delta x \cdot \Delta y$. M and N elements are arranged with same spacing between adjacent antennas for transmitting and receiving arrays, respectively, which is located vertically. Referring to Fig.1, we denote by (tx, ty, h_k), k=1,…,M the location of transmitters and by (rx, ry, h_l) , l= 1,…,N the location of receivers. In order to describe the multipath effect, we define Multipath Distance Difference (MDD) as the difference value between direct path and reflected path from antennas to target.

A. Multipath Distance Difference (MDD)

Denote by

$$d(x, y, x', y') = \sqrt{(x-x')^2 + (y-y')^2}$$

the distance between (x, y) and (x', y'). Without considering the atmosphere refraction and earth's curvature, the MDD between transmitters and target can be given as

$$\Delta R_k \approx 2h_k h_0 / d(tx, ty, x_0+\gamma, y_0+\beta) \quad (1)$$

and $\gamma \in [-\Delta x/2, \Delta x/2]$, $\beta \in [-\Delta y/2, \Delta y/2]$.

From (1), ΔR_k will change with the point (γ, β) and the mean of ΔR_k can be described as [25]

$$E(\Delta R_k) = \frac{1}{\Delta x \Delta y} \int_{-\Delta x/2}^{\Delta x/2} \int_{-\Delta y/2}^{\Delta y/2} \frac{2h_k h_0}{d(tx, ty, x_0+\gamma, y_0+\beta)} d\gamma d\beta \quad (2)$$

For (2), it can be further simplified by using the approximation that $\gamma^2+\beta^2 \ll (tx-x_0)^2 + (ty-y_0)^2$ and $h_k / d(tx, ty, x_0, y_0) \approx 0$ in (3).

$$\begin{aligned} E(\Delta R_k) &= p_k \\ &\approx \frac{1}{\Delta x \Delta y} \cdot \int_{-\Delta x/2}^{\Delta x/2} \int_{-\Delta y/2}^{\Delta y/2} \frac{2h_k h_0}{\sqrt{a^2 - 2a\gamma + b^2 - 2b\beta}} d\gamma d\beta \\ &= \frac{2h_k h_0}{\sqrt{a^2+b^2}} \approx \frac{2h_k (h_0 - h_k)}{d(tx, ty, x_0, y_0)} = 2h_k \tan\theta \end{aligned} \quad (3)$$

where $a = tx - x_0$, $b = ty - y_0$, and θ is the direction of

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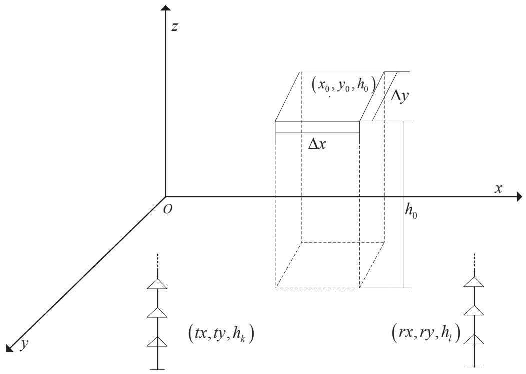

Fig. 1. Location for target and antennas

departure (DOD) at the transmitter.

Similarly, the mean of MDD between receivers and target can be described as

$$E\left(\Delta R_l\right) = q_l = \frac{2h_l h_0}{d\left(rx, ry, x_0, y_0\right)} \approx 2h_l \tan\varphi \tag{4}$$

where φ is the direction of arrival (DOA) at the receiver.

B. Received Signal

It is assumed that each transmitter is driven by a specific signal on the carrier frequency f_c or with wavelength λ and complex envelope $s_k(t)$. Denote by $r_l(t)$ the received signal and by $n_l(t)$ the additive noise and then $r_l(t)$ by the following model

$$r_l(t) = \sqrt{E/M} \cdot \sum_{k=1}^{M} \alpha_{lk}\alpha_{tk}\alpha_{rl}s_k(t-\tau) + n_l(t) \tag{5}$$

where E is the total average received energy, $\alpha_{lk} \sim CN(0,1)$ is the scattering coefficient, α_{tk} is the phase shift of the kth transmitter antenna, α_{rl} is the phase shift of the lth receiver antenna, τ is the time delay.

In the presence of multipath, radar echo from the target include four parts: directly-directly path, directly- reflected path, reflected-directly path, and reflected-reflected path. As a result, the received signal of a pair of transmit-receive antennas is

$$\begin{aligned} r_{lk} &= r_{lk}^{dd} + r_{lk}^{dr} + r_{lk}^{rd} + r_{lk}^{rr} + n_l(t) \\ &= \sqrt{E/M}\,\alpha_{lk}\left[1 + \rho e^{j\gamma_k} + \tilde{\rho} e^{j\beta_l} + \rho\tilde{\rho} e^{j(\gamma_k+\beta_l)}\right] \cdot \\ &\quad \alpha_{rk}\alpha_{tl}s_k(t-\tau) + n_l(t) \end{aligned} \tag{6}$$

where $\rho e^{j\gamma_k}$ is the multipath reflection coefficient for the kth transmitter, $\gamma_k = \gamma_k^1 + \gamma_k^2$, γ_k^1 is caused by ground reflection and $\gamma_k^2 = (2\pi/\lambda)\cdot\Delta R_k$ by the distance difference between directly path and reflected path; $\rho e^{j\beta_l}$ is the multipath reflection coefficient for the lth receiver, $\beta_l = \beta_l^1 + \beta_l^2$, β_l^1 is caused by ground reflection and $\beta_l^1 = (2\pi/\lambda)\cdot\Delta R_l$ by the distance difference between directly path and reflected path; λ is the wavelength in free space. Here, we assume $\rho = \tilde{\rho} = 1$, $\gamma_k^1 = \beta_l^1 = \pi$, $\mu = 2\pi/\lambda$.

From (6), it should be noted that the multipath phase shift is a random variable which is also an important factor of scattering channel. For the upcoming analysis of radar detection performance, we can define the new multipath scattering coefficient as

$$h_{lk} = \alpha_{lk}\left[1 + e^{j(\pi+\mu\Delta R_l)} + e^{j(\pi+\mu\Delta R_k)} + e^{j\mu(\Delta R_l+\Delta R_k)}\right] = \alpha_{lk}d_{lk} \tag{7}$$

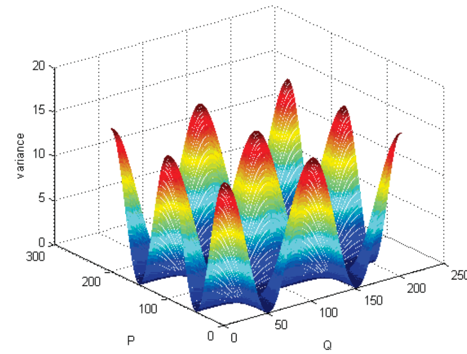

Fig 2. The variance of h_{lk}

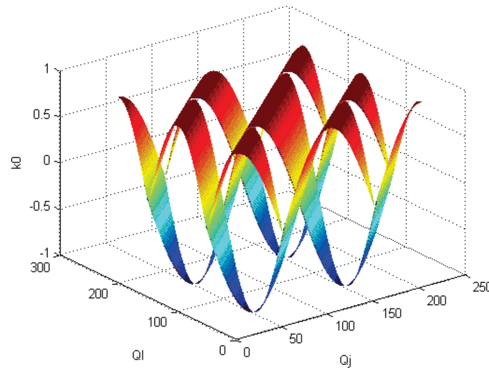

Fig. 3. The parameter k_0

1) Consider the variance of h_{lk}. As the multipath phase shift d_{lk} and the coefficient α_{lk} are uncorrelated, the variance of h_{lk} can be given as follows

$$\begin{aligned} &E\left(\left|h_{lk}\right|^2\right) = E\left(\alpha_{lk}\cdot\alpha_{lk}^*\right)E\left[d_{lk}d_{lk}^*\right] \\ &= 4 + 4\cos(\mu q_l) + 4\cos(\mu p_k) + 4\cos(\mu q_l)\cos(\mu p_k) \end{aligned} \tag{8}$$

According to (8), the variance h_{lk} has a dramatic change along with MDD. As the variance is a cosine function, it will have great change in a small scale. Fig. 2 provides the $E(|h_{lk}|^2)$ as a function of Q_l with P_l, where $Q_l = q_l/\lambda$ and $P_k = p_k/\lambda$. The grid is set as $0.01m$ in the x axis and y axis within two cycles, respectively.

It is obvious that the variance has a cyclical change along with MDD. For MIMO radar system, equivalent to MN independent radar systems, the distance among transmitter or receiver is very little relative to target range. The MDD among them will have a little distinguish which easily causes the diversity and uncertainty distributions of different independent radar systems (described in Fig. 2). Therefore, the traditional detection algorithm is limited in multipath environment. Meanwhile, the great changes in amplitude in a certain extent will also strengthen or relief the energy intensity of target echo signals.

2) Consider h_{lk} *and* h_{jk} $(j,l=1,\cdots N)$. The correlation between them can be written as

$$E\left(h_{lk}h_{jk}^{*}\right)=E\left(\alpha_{lk}\cdot\alpha_{jk}^{*}\right)\cdot k_{0} \tag{9}$$

and

$$\begin{aligned}k_{0}&=E\left(d_{lk}d_{jk}^{*}\right)\Big/\left[\sqrt{E\left(\left|h_{lk}\right|^{2}\right)}\cdot\sqrt{E\left(\left|h_{jk}\right|^{2}\right)}\right]\\&=\frac{(1/2)\operatorname{Re}\left(d_{jk}\right)\left[1+\cos\left(\mu p_{k}\right)\right]}{\sqrt{\left[1+\cos\left(\mu q_{l}\right)\right]\left[1+\cos\left(\mu p_{k}\right)\right]\left[1+\cos\left(\mu q_{j}\right)\right]\left[1+\cos\left(\mu p_{k}\right)\right]}}\\&=\frac{1+\cos\left(\mu q_{j}\right)+\cos\left(\mu q_{l}\right)+\cos\left[\mu\left(q_{l}-q_{j}\right)\right]}{2\sqrt{\left[1+\cos\left(\mu q_{l}\right)\right]\left[1+\cos\left(\mu q_{j}\right)\right]}}\end{aligned} \tag{10}$$

From (9), the multipath correlation coefficient is a function of carrier frequency, target configuration and MDD. In order to decide the influence of multipath on the correlation coefficient, we need to analyze the parameter k_0.

Fig.3 provides the k_0 as a function of Q_l with Q_j, where $Q_l=q_l/\lambda$ and $Q_j=q_j/\lambda$. The grid is set as $0.01m$ in the x axis and y axis within two cycles, respectively.

The parameter k_0 has a cyclical change for the different receiver antennas within [-1, 1]. In the case of $k_0=0$, the correlation coefficient is zero and all the receiver elements are uncorrelated. In the case of $k_0>0$, the received signal is strengthened due to the angle between directly signal and reflected signal which is within [-120°, 120°]. In the case of $k_0<0$, the received signals are weakened and the angle between directly signal and reflected signal is within [120°, 240°] . To sum up, multipath echo has an essential effect on the received signal of the MIMO radar.

III. Detection Model in Multipath Environment

In multipath environment, the change of multipath scattering coefficient h_{lk} will certainly increase the complexity of echo signal greatly, which make the traditional detection model for MIMO radar lose efficacy or detection performance poor. Consequently, using the multipath echo power is very important to improve the detection performance. Here, a new algorithm based on the entropy theory is proposed and the basic principle is provided as follows:

Step 1: Define

$$A_{lk}=\left|E\left(\left|h_{lk}\right|^{2}\right)-E\left(\alpha_{lk}\cdot\alpha_{lk}^{*}\right)\right|$$

which indicates the distance of two indicators in multipath environment and non-multipath environment, respectively.

Step 2: In order to evaluate the letter A_{lk}, it is better to normalize it as

$$B_{lk}=\left(\max_{k,l}A_{lk}-A_{lk}\right)\Big/\left(\max_{k,l}A_{lk}-\min_{k,l}A_{lk}\right) \tag{11}$$

Step 3: According to the theory of the information entropy, the self-information quantity denoted by $-\ln B_{lk}$ can be used to measure the uncertainty of B_{lk}. Since information quantity is additive, all the uncertainty of each radar system can be given as:

$$H=-\sum_{l=1}^{N}\sum_{k=1}^{M}\omega_{lk}\ln B_{lk},\sum_{l=1}^{N}\sum_{k=1}^{M}\omega_{lk}=1 \tag{12}$$

where ω_{lk} represents the weight.

Similarly, all the information quantity of the MIMO radar system, defined as the letter P, can be calculated as

$$P=\exp\left(-H\right) \tag{13}$$

and the final variance of the multipath scattering coefficient can be defined as $Y=E\left(\alpha_{lk}\cdot\alpha_{lk}^{*}\right)+P$.

Step 4: According to the principle of the binary hypothesis test, the optimal test statistic of MIMO radar is given by

$$T=\sum_{l=1}^{N}\sum_{k=1}^{M}\left|x_{lk}\right|^{2}\underset{H_0}{\overset{H_1}{\gtrless}}\delta \tag{14}$$

where $x_{lk}=\int r_l(t)s_k^{*}(t-\tau)dt$, δ is set to ensure the required probability of false alarm, and $\int_T s_k(t)s_k^{*}(t)dt=1$.

As a result, its distribution can be given as follows

$$T\sim\begin{cases}\dfrac{\sigma_n^2}{2}\chi_{2MN}^{2} & H_0\\ \left(\dfrac{E}{2M}\cdot Y+\dfrac{\sigma_n^2}{2}\right)\chi_{2MN}^{2} & H_1\end{cases} \tag{15}$$

where σ_n^2 is the variance of noise and χ_d^2 denotes a chi-square random variable with d degrees of freedom.

It is well known that the probability of false alarm P_{FA}, the probability of detection P_D, and the threshold δ by a series of one-to-one relations. They can be expressed as

$$\begin{aligned}P_{FA}&=P\left(T>\delta\left|H_0\right.\right)=P\left(\frac{\sigma_n^2\chi_{2MN}^2}{2}>\delta\right)=P\left(\chi_{2MN}^2>\frac{\delta}{2\sigma_n^2}\right)\\P_D&=P\left(T>\delta\left|H_1\right.\right)\\&=P\left[\left(\frac{E}{2M}Y+\frac{\sigma_n^2}{2}\right)\chi_{2MN}^2>\delta\right]=1-F_{\chi_{2MN}^2}\left(\frac{2\delta}{E/M\cdot Y+\sigma_n^2}\right)\end{aligned} \tag{16}$$

IV. Simulation Results

In this section, several numerical simulations about the detection performance are considered. To describe the signal detection model, we need the following assumptions: $h_k=5\bullet k$ (m), $h_l=5\bullet l$ (m), the DOD and DOA for the target are 20° and 25°; $\lambda=10m$, $\omega_{lk}=1/(MN)$, $SNR=E/\sigma_n^2$ and the noise satisfies Gaussian distribution.

Example 1: In this example, we examine the effectiveness of the new algorithm based on the entropy theory. The final variance in Fig. 4 has great improvement and keeps stable for different kinds of antennas. With the increase of the transmitter or receiver antennas, the variance remains stable about 1.5 (still increasing but very slowly), when the condition is $M>10$ and $N>10$. And it is noting that the new algorithm can effectively solve the turbulence characteristics of multipath scattering coefficient. Fig. 5 describes the detection perfor

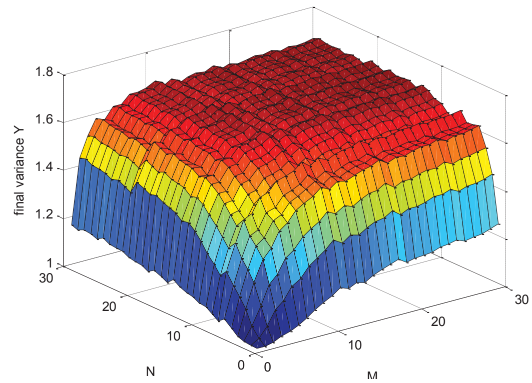

Fig. 4. The final variance Y

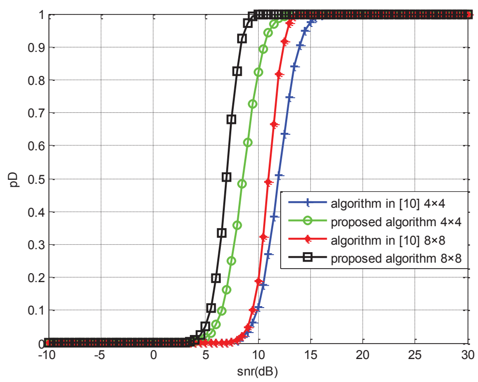

Fig. 5. Detection performance with different algorithms

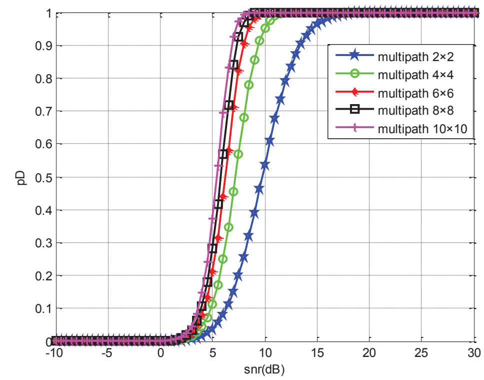

Fig. 6. Detection performance with different numbers of antennas

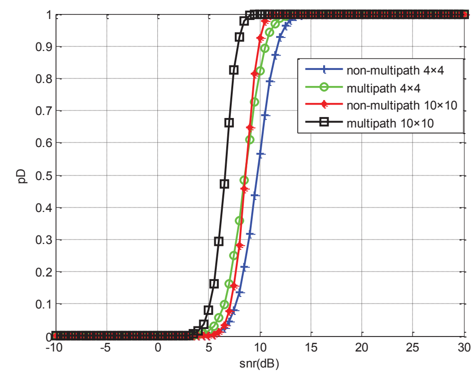

Fig. 7. Detection performance with or without multipath

mance between the proposed algorithm and the algorithm in [10], which demonstrates that our algorithm is better.

Example 2: In this example, we examine the detection probability with different numbers of antennas as a function of *SNR* in fig.6. The probability of false alarm is set by 10^{-6}. The more the number of transmitter antennas and receiver antennas has, the better the detection performance will be gotten for MIMO radar system. What's more, with the increase of antenna numbers, the change for detection performance becomes slowly, and the reason is that the final variance becomes more stable with the increase of antennas as described in fig. 3.

Example 3: In this example, we examine the detection performance by a comparison with and without multipath effect. In Fig 7, the probability of false alarm is set by 10^{-9}. In the case of multipath effect, the SNR should be more than 3dB to detect target; while for non-multipath effect, the SNR is much higher, reaching 6dB. It is easily seen that the new algorithm can use the multipath echo power to improve radar detection performance.

V. Conclusion

In this paper, the concept of Multipath Distance Differences (MDD) for MIMO is proposed, which is closely related to the location of targets and antennas. The multipath received signal model is given under four paths in multipath environment, and we analyze the effect of MDD for the multipath scattering coefficient, also with the multipath correlation coefficient. In order to improve the detection performance in multipath environment, a new method based on entropy theory is given in detail. The simulation has shown the importance of the new algorithm in solving the turbulence characteristics of multipath scattering coefficient. The target tracking and positioning will be considered for MIMO radar in near future.

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