

## Fairness

[NIPS 2010: Discriminative Clustering by Regularized Information Maximization (RIM)]

Problem: learn a probabilistic discriminative classifier from an unlabeled dataset

$$X = (x_1, \dots, x_N), \text{ where } x_i = (x_{i1}, \dots, x_{iD})^T \in \mathbb{R}^D \xrightarrow{\text{learn}} p(y|x, W)$$

RIM:  $F(p(y|x, W); X; \lambda) \Rightarrow$  evaluate the suitability of  $p(y|x, W)$

① cluster assumption (decision boundaries  $\times$  dense)  $\rightarrow$  confidence  
conditional entropy  $\frac{1}{N} \sum_i H\{p(y|x_i, W)\}$  level

(On unsupervised assumption, it can be reduced by removing decision boundaries)

② class balance (avoid degenerate solutions)  $\rightarrow$  distribution level  
empirical label distribution:  $\hat{p}(y; W) = \frac{1}{N} \sum_i p(y|x_i, W)$   
entropy  $H\{\hat{p}(y; W)\}$

Combine ① + ②  $I_{Wf} f(y; x) = H\{\hat{p}(y; W)\} - \frac{1}{N} \sum_i H\{p(y|x_i, W)\}$   
mutual information

( $I_{Wf} f(y; x)$  may be trivially maximized by a conditional model that classifies each data point  $x_i$  into its own category  $y_i$ )

③ classifier complexity (penalty)

$$F(W; X, \lambda) = I_{Wf} f(y; x) - R(W; \lambda)$$

Learning a conditional distributional distribution for  $y$  that preserves information from the data set, subject to a complexity penalty.

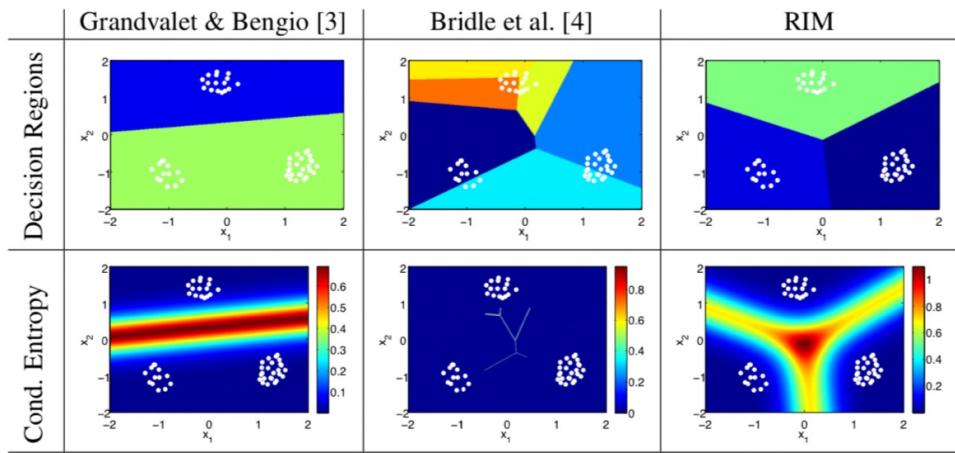


Figure 1: Example unsupervised multilogit regression solutions on a simple dataset with three clusters. The top and bottom rows show the category label  $\arg \max_y p(y|\mathbf{x}, \mathbf{W})$  and conditional entropy  $H\{p(y|\mathbf{x}, \mathbf{W})\}$  at each point  $\mathbf{x}$ , respectively. We find that both class balance and regularization terms are necessary to learn unsupervised classifiers suitable for multi-class clustering.

Since  $H\{\hat{p}(y; \mathbf{W})\} = \log K - KL\{\hat{p}(y; \mathbf{W}) \| V\}$ , then

$$F(\mathbf{W}; \mathbf{X}, \lambda) = -\frac{1}{N} \sum_i H\{\hat{p}(y|x_i, \mathbf{W})\} - KL\{\hat{p}(y; \mathbf{W}) \| V\} - R(\mathbf{W}; \lambda)$$

class balance  $\rightarrow D(y; r)$

Others

$$F(\mathbf{W}; \mathbf{X}, \lambda) = I_{\mathbf{W}}(\mathbf{x}; y) - H\{\hat{p}(y; \mathbf{W})\} - D(y; r) - R(\mathbf{W}; \lambda)$$

$$\text{In SSL, } S(\mathbf{W}; \tau, \lambda) = \underbrace{\tau I_{\mathbf{W}}(y; \mathbf{x}) - R(\mathbf{W}; \lambda)}_{D_V} + \underbrace{\sum_i \log(p(y_i | \mathbf{x}_i^L, \mathbf{W}))}_{D_L}$$

[IJCNN 2020: Pseudo-Labeling and Confirmation Bias in Deep SSL]

Two Reg' to improve convergence.

$$\textcircled{1} \quad R_A = \sum_{c=1}^C p_c \log\left(\frac{p_c}{\bar{p}_c}\right) \quad \begin{array}{l} \text{distribution level} \\ \downarrow \\ \text{prior distribution of } c \end{array} \quad \begin{array}{l} D_{KL}(V \| \bar{V}_{\mathbf{W}}) \\ \text{mean softmax probability of model for } c \end{array}$$

$$\textcircled{2} R_H = -\frac{1}{N} \sum_{i=1}^N \sum_{c=1}^C h_\theta^c(x_i) \log(h_\theta^c(x_i)) \quad \text{confidence level (entropy regularization)}$$

$$f = f^* + \lambda_A R_A + \lambda_H R_H$$

↑ mix up loss

[ ICLR 2023 : FreeMatch ]

Self-adaptive fairness (distribution level)

$\hat{p}_t(c) \rightarrow$  estimate of the expectation of prediction distribution over  $D_V$ .

In RIM, we use  $H(\hat{p}(y; w))$  to realize class balance. ( $H(\mathbb{E}_u[p_m(y|u)]) \max$ )

We optimize CE of  $\hat{p}_t$  and  $\bar{p} = \mathbb{E}_{p_B} [p_m(y|D_t(u_p))]$  as an estimate.

(We expect  $D_{KL}(\hat{p}_t \| \bar{p}) \approx 0$ , that is  $H(\hat{p}_t, \bar{p}) = H(\hat{p}_t) \approx H(\bar{p})$ )

? The underlying pseudo-label distribution may not be uniform ( $\hat{p}_t, \bar{p} \neq U$ )

( $\max H(\hat{p}_t, \bar{p}) \Rightarrow$  tend to be uniform distribution)

\* modulate the fairness objective in a self-adaptive way (normalize)

$$\bar{p} = \frac{1}{\mu_B} \sum_{b=1}^{\mu_B} \mathbf{1}(\max(q_b) \geq T_t(\text{argmax}(q_b))) Q_b$$

$$T_t = \text{Hist}_{p_B} (\mathbf{1}(\max(q_b) \geq T_t(\text{argmax}(q_b))) \hat{Q}_b)$$

$$\hat{h}_t = \gamma \hat{h}_{t-1} + (1-\gamma) \text{Hist}_{p_B}(\hat{q}_b)$$

The self-adaptive fairness (SAF)  $L_f$  at the  $t$ -th iteration is:

$$L_f = -H(\underbrace{\text{SumNorm}\left(\frac{\hat{p}_t}{\hat{h}_t}\right)}, \underbrace{\text{SumNorm}\left(\frac{\bar{p}}{\hat{h}}\right)})$$

↓                      ↓

Nearly Uniform