LAB 1: PERIOD OF A PENDULUM - PART 1: MANUAL MEASUREMENTS

LAB OBJECTIVES

You will measure the period of a pendulum as a function of its length and compare your results to theoretical values. You will describe the accuracy of these measurements and attempt to understand the sources of error.

BACKGROUND

A pendulum is an example of a simple harmonic oscillator. Let's construct a coordinate system so that z is the direction of the pendulum string (so the bob moves in approximately the $x-y$ plane) and call the origin ($x = y = 0$) the position where the bob comes to rest. If you displace the bob a distance a along the x-axis and release it from rest at time $t = 0$, the resulting motion is

$$x(t) = a\cos(\omega t);\ \omega = \sqrt{\frac{g}{L}} \tag{1}$$

where g is the local acceleration due to gravity and L is the length of the pendulum - the distance from the pivot point to the center of mass.

The period (T) of the pendulum is the amount of time it takes for the pendulum to return to its initial position. Since $\cos(2\pi) = 1$,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \tag{2}$$

For small values of the amplitude, a ($a/L \ll 1$), the period of the pendulum is approximately independent of the amplitude. The first correction to the period due to amplitude is

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\frac{a^2}{L^2}\right) \tag{3}$$

PRE-LAB QUESTIONS

Please answer these in your lab book. It will help to make a sketch of your planned experiment first.

1. If $T = 2\pi\sqrt{\frac{L}{g}}$, then what is the relation between the fractional change in length $\Delta L/L$ and the fractional change in period $\Delta T/T$? (IE if $\Delta T/T = \alpha\Delta L/L$, find α). Hint: this is really easy if you take the logarithm of both sides of $T = 2\pi\sqrt{\frac{L}{g}}$, then differentiate.

2. Using the formula $T = 2\pi\sqrt{\frac{L}{g}}$ and ignoring the effects of amplitude will cause a systematic error. Show that this formula predicts a period that is too short by an relative amount ($\Delta T/T$) of $\frac{1}{16}\frac{a^2}{L^2}$ (Hint: start with Equation 3 and make a small manipulation). Find the approximate value of this error a 1 meter long pendulum and a 5 cm amplitude.

3. What change in length for a 1 meter pendulum will cause a change in period of $\frac{1}{6400}$?

4. For the experiments you plan to do today, do you think it's a good approximation to ignore the effects of amplitude on the period and just use $T = 2\pi\sqrt{\frac{L}{g}}$?

SETTING UP

Materials
Nylon fishing line or Kevlar Thread
Support PCB with LEDs, mounted
Assembled PCB with Pico, firmware flashed
speaker wire
micro usb cable
Assembled platform with both aluminum disks mounted
M6x1 eyenut
M6x1 hex nut
2" dia steel ball with M6 screw glued on
Ruler
Stopwatch

You pendulum will consist of a bob on a string. The string is a nylon fishing line, or Kevlar thread. The bob will be the larger steel ball. If you have time, you can play with the smaller steel ball, and we also have some wood balls in lab you can play with. For the steel balls, you will be able to adjust the length in fine increments by screwing in/out of the eyenut. After you adjust the length, tighten the M6 hex nut against the eyenut to lock it in place.

1. Insert the nylon line through a hole in the center of the PCB. The diameter of the nylon line is nominally .017 inch (17 thousandths of an inch). There are 9 holes in the center of the support PCB. The numbers in the 3x3 grid on the PCB silkscreen indicate the nominal diameter of the corresponding hole in thousandths of an inch. The PCB fab house has a tolerance of -.003/+.005 inch. The center hole is nominally .019 inch, but with the tolerances, this means that the PCB fab house only guarantees that it will be between .016 and .024 inch.

   Before you start, make sure you have a clean cut on the nylon line (you shouldn't feel a ball at the end). Try the .019" center hole first.

2. Pull the nylon line through until the end is at the floor/desk/lowest surface. Cut the line, leaving enough extra (1 - 2 feet) to tie it off. Tie a knot in the end of the line above the PCB to prevent it from getting pulled all the way through the hole.

3. Using an extra screw or some other method, secure the top of the line with the bottom near the floor.

4. When you change the weight on the string, you'll have to adjust the length. This is generally easier to do from the bottom. Use some beads or a button to make an adjustable connection (see diagram: TODO) around the eye nut without attaching the ball.

5. Attach the larger ball to the eye nut leaving as much length as possible on the screw (just tighten the screw starts to peek out into the eye hole), then adjust the string length so the ball is just hanging.

6. (For nylon line) Give a few firm tugs to stretch the line and adjust the height again.

7. (For nylon line) The nylon line will relax under tension and get longer over time. This creep is most pronounced in the first day or so. If you are working on your bob at home, ***if possible***, leave the heaviest ball hanging on the line to pre-stretch it. This will make your later measurements more consistent. In lab, please leave a heavy bob hanging on the line when you're done.

8. Adjust the height of the large steel bob to be about 2-3" above the table/floor/bottom surface.

9. Connect the LED lights to the control board. In this class, our convention will be that the gold wire is positive. Attach the speaker wire from the LED supports to the board using the spring terminals, with the gold wire connected to the "high" terminal. Connect the board to power (either through a computer USB port or wall charger). The LED light should turn on if your board is powered up and the software is installed on it.

TAKING DATA

1. Screw the large pendulum bob midway into the eyenut. Set the length of the line so that the bob is an inch or two above the floor/desk and just barely above a reference object of your choice (your assembled platform is a good choice, but you could also use e.g. a large flat book). You will know the bob just clears the object if it swings freely. Lock the nut in place once you have the object set.

2. Measure the vertical distance from the support bracket to the reference object. Try a couple of techniques - e.g. measuring directly from the bracket to the object, measuring from the bracket to the table and the table to the top of the object, hanging a string down and measuring the length of the string. Sketch the apparatus with lengths you measured in your lab book.

3. Write down the length in your lab book; estimate how accurate you think this measurement is. IE Would you bet money that you are right to within a cm? How about a mm?

4. The center of mass of the pendulum should be very near the center of the ball, which is 1 inch (2.54 cm) below the top of the ball. Calculate the expected period of your pendulum and write it down in your lab book. Given what you think is the accuracy of your length prediction (e.g. +/- 2 mm), how accurate do think this prediction should be (e.g. +/- 20ms)[1]? Write this range down in your lab book.

5. Remove the object and save it for next week.

6. Tape a piece of paper under the bob. Make sure the bob is still, then mark the right edge of the shadow.

7. Pull the bob back and release it gently. Try to have it go straight back and forth and not in an ellipse. Practice a few times. Once you are good at launching the bob, keep going.

8. Measure the period (method A). Wait for the bob shadow to cross the line you made on the paper from left to right (if the shadow is hard to see, find another way to set up a reference). Start a stopwatch when the bob crosses the line from left to right and stop the stopwatch when the bob crosses the line again ***heading in the same direction***. This is one period. Make sure that your measurement agrees fairly well with your prediction. (What does fairly well mean? Compare to the expected range of measurements you predicted earlier)

9. Measure the period 20 times using method A. Write the values down in your lab book. Note that you don't have to start and restart the bob each time you make a measurement. Just let the pendulum keep swinging between measurements.

10. Measure the period (method B). Start a stopwatch when the bob crosses the centerline from left to right. Count 10 left-right crossings and stop the stopwatch on the 10th crossing. This is 10 periods. Write down the raw data (the time for 10 periods) and the calculated period (the time you measured over 10). Repeat this 20 times, recording the values in your lab book.

11. Measure the period (method C). Like method B, but count 100 periods. Hint: using your measurements from A and B (especially B), work out the range of times that 100 periods could fall in. (IE if you think the period is between 3 and 3.1 seconds, 100 periods is between 300 and 310 seconds). If this range is less than 1 period, you don't need to count to 100 - you can just run the stopwatch until you get past the minimum time and then stop it on the next left-right crossing. If your measurements in B aren't precise enough to narrow the range down to less than 1 period, try taking additional measurements averaging 20 or 50 periods first. *You should also do one measurement where you count all the crossings to be sure you've gotten it right*

12. Repeat method C at least 10 times (20 seems like a lot if each one takes 5 minutes).

[1] note that I'm not saying that a 2mm error in length should give you 20 ms timing error - that's for you to figure out

13. Method D: Measure a single period (as in method A). Repeat this measurement 10 times, starting and stopping the watch each time; i.e. don't use the lap feature - each measurement you make should start and stop independently of every other measurement. Write down each single measurement, then calculate the *average* of these 10 measurements. Now repeat this process 10 times. So you should have 100 individual measurements of the period and 10 averages.

If time allows - optional extension: Measure the period of the small steel ball pendulum at the same length as the large steel ball

1. If you don't make any other changes to the apparatus, you should be able to replace the large steel ball with the small steel ball at almost exactly the same height. The center of the small steel ball is 0.75" above its bottom, while the center of the large steel ball is 1" above its bottom. So
   a) Start with the pendulum you've used in the previous experiments.
   b) Return the reference object to beneath the pendulum and make sure that the large bob just touches/clears the object.
   c) Remove the large ball and screw in the small ball. Adjust the height until the small ball just rests on the items you've stacked below. Because the nylon string stretches under tension, you will first need to adjust the length from the top by letting line out. Once you've got the height adjusted, remove the reference object.
   d) Mark the small ball so you can keep track of the rotation. Screw the small ball up into the eyenut by 0.25" = 6.35mm. This is $6\frac{1}{3}$ turns of the M6 screw.
2. If you screw the bob into (or out of) the eyenut by 1 full turn, the length will change by 1 mm. Calculate the change in period that results from this change in length.
3. Devise a protocol to measure the period of the pendulum with sufficient accuracy to resolve this change in period. This protocol should explain how you will do the measurement (i.e. if you are going to average 150 measurements, will you count 150 crossings, or will you use the timing trick?) and how you will verify accuracy (will you repeat the measurement? how many times? what will you do if repeated measurements don't agree?). **Write this protocol in your lab book before proceeding**
4. Using this protocol, measure the period of the small-bob pendulum.
5. Calculate how much you need to change the length (if at all) of the pendulum so that it has the same period as the previously measured large-ball pendulum.
6. Make this adjustment using the screw. One complete turn of the screw is 1 mm; you should aim to adjust to the nearest 1/4-turn or about 0.25 mm.
7. Measure the period and verify your predictions.

Note in your lab book any differences you've observed in using the large and small steel bobs. Is one easier to use than the other? (There might not be any differences you notice)

ANALYZING DATA

You will do the following twice, first by hand (in your lab book) then using Python (as part of lab Analysis 1). This part, which we'll grade as part of next week's lab book, should be done by hand.

1. Make a histogram of your measurements of the period of the pendulum with the large steel ball using Methods A-D. For Methods A-C, each count represents a single measurement. For method D, each count represents the average of the 10 measurements. Choose a bin size so that there are at least 5 elements in the largest bar (3 if you only have 10 counts) and so that there is more than one bar.

2. Calculate the mean and standard deviation of your measurements of the period for Methods A-D. Use $1/(N-1)$ as the denominator for the standard deviation calculation. Please do these calculations by hand (calculators are fine, of course! but don't use a 'mean' or 'standard deviation' function on a calculator, spreadsheet etc.) and show the intermediate results (IE for the mean, you have to calculate the sum of the data first) in your lab book.

3. Calculate the standard error (the standard deviation divided by the square root of the number of measurements) for each of A-D. Make a plot showing the mean value and error ranges for A-D. Are these consistent with each other?

Lab book checkoff

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