Rob J Hyndman

Forecasting:

Principles and Practice

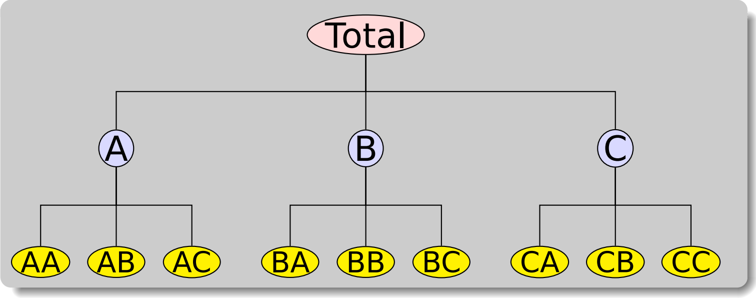
11. Hierarchical forecasting

OTexts.com/fpp/9/4/

Outline

1. **Hierarchical and grouped time series**
2. Forecasting framework
3. Optimal forecasts
4. Approximately optimal forecasts
5. Application: Australian tourism
6. Application: Australian labour market
7. hts package for R

Introduction



Examples

- Manufacturing product hierarchies
- Net labour turnover
- Pharmaceutical sales
- Tourism demand by region and purpose

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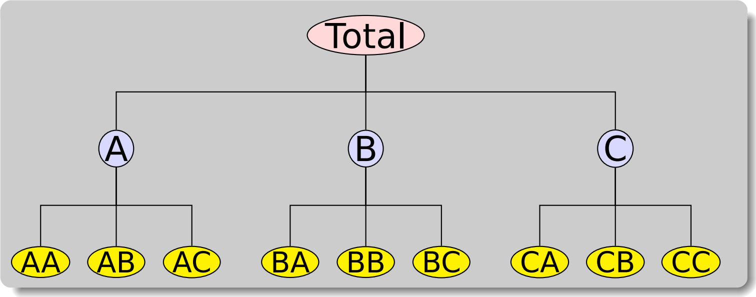

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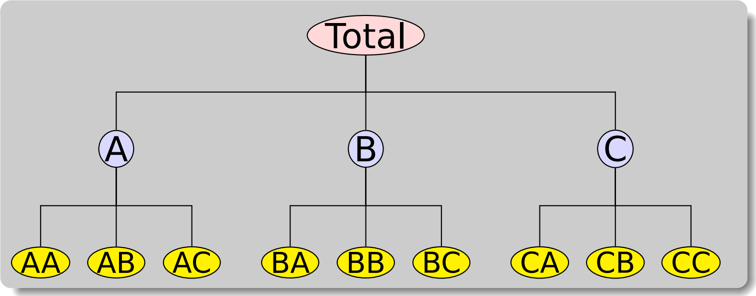

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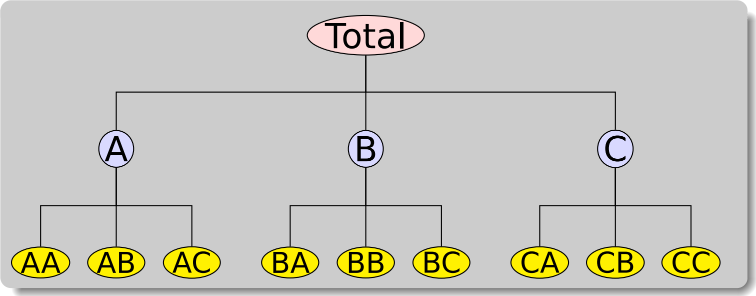

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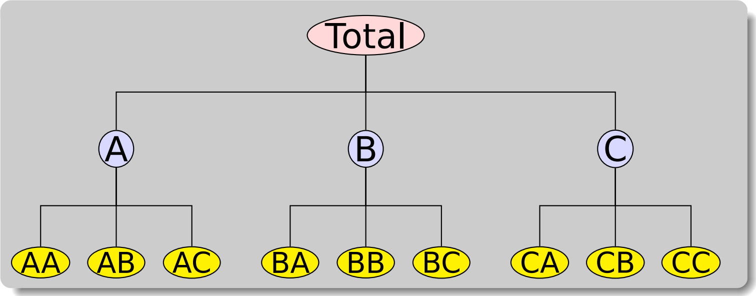

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Forecasting the PBS

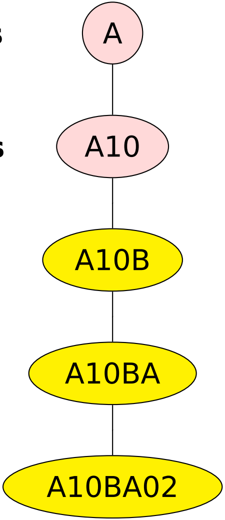
ATC drug classification

A	Alimentary tract and metabolism
B	Blood and blood forming organs
C	Cardiovascular system
D	Dermatologicals
G	Genito-urinary system and sex hormones
H	Systemic hormonal preparations, excluding sex hormones and insulins
J	Anti-infectives for systemic use
L	Antineoplastic and immunomodulating agents
M	Musculo-skeletal system
N	Nervous system
P	Antiparasitic products, insecticides and repellents
R	Respiratory system
S	Sensory organs
V	Various

ATC drug classification

14 classes — Alimentary tract and metabolism

84 classes — Drugs used in diabetes

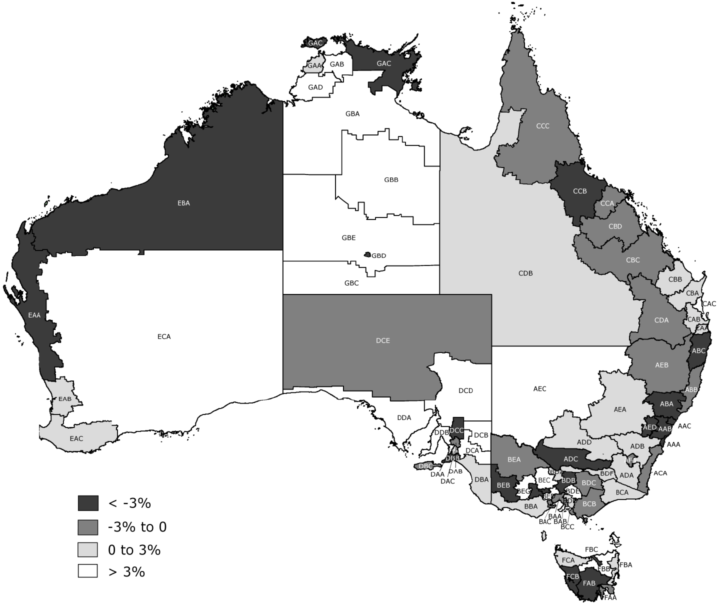


Blood glucose lowering drugs

Biguanides

Metformin

Australian tourism



Australian tourism

Also split by purpose of travel:

- Holiday
- Visits to friends and relatives
- Business
- Other

Hierarchical/grouped time series

- A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.

Example: Pharmaceutical products are organized in a hierarchy under the Anatomical Therapeutic Chemical (ATC) Classification System.

- A **grouped time series** is a collection of time series that are aggregated in a number of non-hierarchical ways.

Example: Australian tourism demand is grouped by region and purpose of travel.

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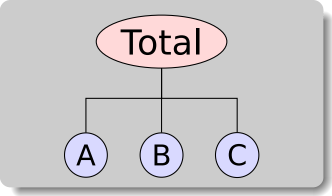

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$Y_{X,t}$: observation on series X at time t.

$\boldsymbol{B}_t$: vector of all series at bottom level in time t.

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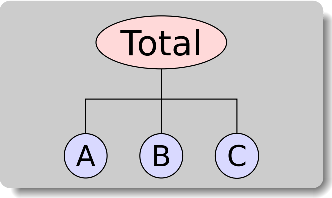

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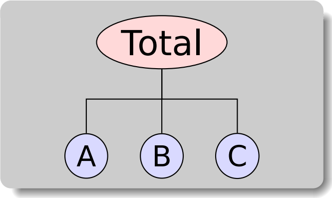

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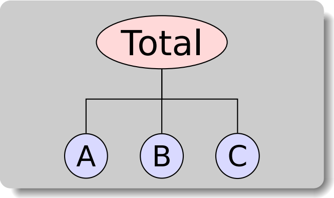

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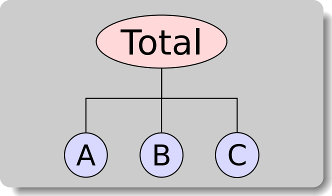

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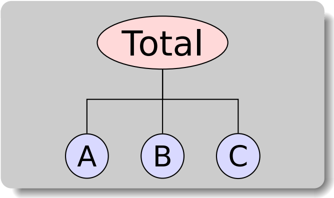

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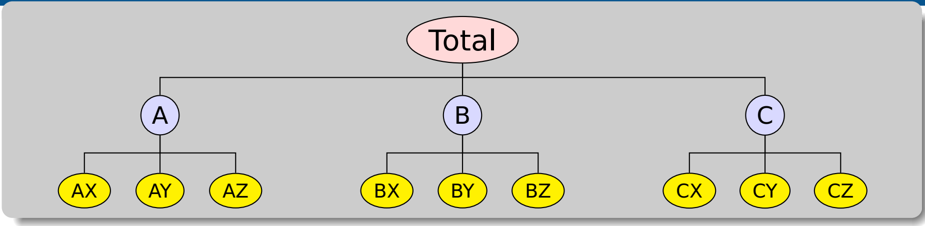

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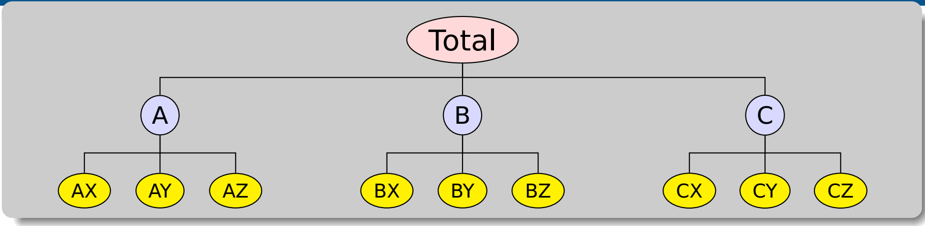

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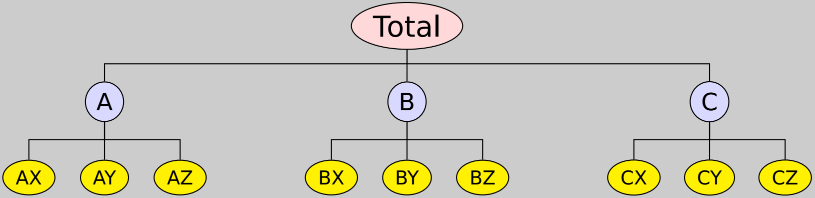

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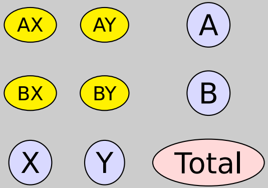

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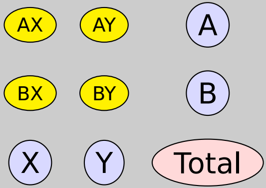

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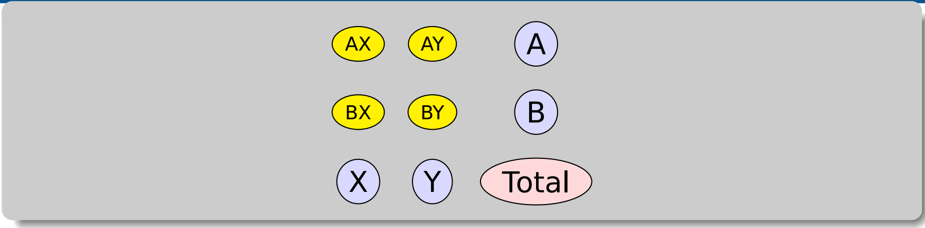

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Forecasting notation

Let $\hat{\boldsymbol{Y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as $\boldsymbol{Y}_t$. (They may not add up.)

Hierarchical forecasting methods of the form:

$$\tilde{\boldsymbol{Y}}_n(h) = \boldsymbol{S}\boldsymbol{P}\hat{\boldsymbol{Y}}_n(h)$$

for some matrix $\boldsymbol{P}$.

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Bottom-up forecasts

$$\tilde{\boldsymbol{Y}}_n(h) = \boldsymbol{S}\boldsymbol{P}\hat{\boldsymbol{Y}}_n(h)$$

Bottom-up forecasts are obtained using

$$\boldsymbol{P} = [\mathbf{0} \mid \mathbf{I}],$$

where $\mathbf{0}$ is null matrix and $\mathbf{I}$ is identity matrix.

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Top-down forecasts

$$\tilde{\boldsymbol{Y}}_n(h) = \boldsymbol{S}\boldsymbol{P}\hat{\boldsymbol{Y}}_n(h)$$

Top-down forecasts are obtained using

$$\boldsymbol{P} = [\boldsymbol{p} \mid \boldsymbol{0}]$$

where $\boldsymbol{p} = [p_1, p_2, \dots, p_{m_K}]'$ is a vector of proportions that sum to one.

- $\boldsymbol{P}$ distributes forecasts of the aggregate to the lowest level series.
- Different methods of top-down forecasting lead to different proportionality vectors $\boldsymbol{p}$.

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General properties: bias

$$\tilde{\boldsymbol{Y}}_n(h) = \boldsymbol{SP}\hat{\boldsymbol{Y}}_n(h)$$

Assume: base forecasts $\hat{\boldsymbol{Y}}_n(h)$ are unbiased:

$$\mathrm{E}[\hat{\boldsymbol{Y}}_n(h)|\boldsymbol{Y}_1,\dots,\boldsymbol{Y}_n] = \mathrm{E}[\boldsymbol{Y}_{n+h}|\boldsymbol{Y}_1,\dots,\boldsymbol{Y}_n]$$

- Let $\hat{\boldsymbol{B}}_n(h)$ be bottom level base forecasts with $\beta_n(h) = \mathrm{E}[\hat{\boldsymbol{B}}_n(h)|\boldsymbol{Y}_1,\dots,\boldsymbol{Y}_n]$.
- Then $\mathrm{E}[\hat{\boldsymbol{Y}}_n(h)] = \boldsymbol{S}\beta_n(h)$.
- We want the revised forecasts to be unbiased: $\mathrm{E}[\tilde{\boldsymbol{Y}}_n(h)] = \boldsymbol{SPS}\beta_n(h) = \boldsymbol{S}\beta_n(h)$.
- Result will hold provided $\boldsymbol{SPS} = \boldsymbol{S}$.
- True for bottom-up, but not for any top-down method or middle-out method

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$$\mathrm{Var}[\tilde{\boldsymbol{Y}}_n(h)|\boldsymbol{Y}_1, \dots, \boldsymbol{Y}_n] = \boldsymbol{S}\boldsymbol{P}\Sigma_h\boldsymbol{P}'\boldsymbol{S}'.$$

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Outline

Forecasts

Key idea: forecast reconciliation

- Ignore structural constraints and forecast every series of interest independently.
- Adjust forecasts to impose constraints.

Let $\hat{\boldsymbol{Y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as $\boldsymbol{Y}_t$.

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- Approximate $\Sigma_1^{\dagger}$ by $c\mathbf{I}$.
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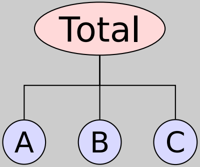

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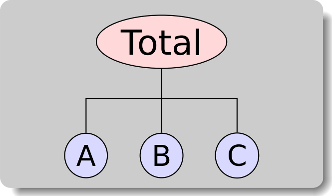

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Weights:

$$\boldsymbol{S}(\boldsymbol{S}'\boldsymbol{S})^{-1}\boldsymbol{S}' = \begin{bmatrix} 0.75 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.75 & -0.25 & -0.25 \\ 0.25 & -0.25 & 0.75 & -0.25 \\ 0.25 & -0.25 & -0.25 & 0.75 \end{bmatrix}$$

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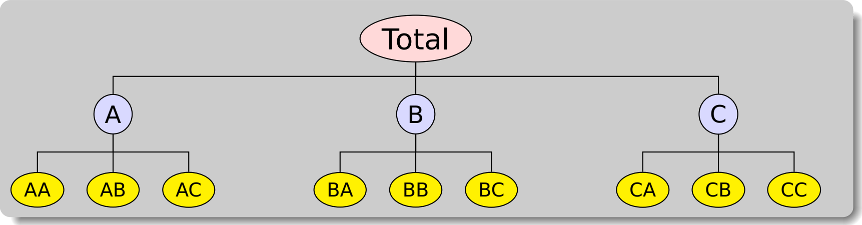

Weights: $\boldsymbol{S}(\boldsymbol{S}'\boldsymbol{S})^{-1}\boldsymbol{S}' =$

$$\begin{bmatrix}
0.69 & 0.23 & 0.23 & 0.23 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 \\
0.23 & 0.58 & -0.17 & -0.17 & 0.19 & 0.19 & 0.19 & -0.06 & -0.06 & -0.06 & -0.06 & -0.06 & -0.06 \\
0.23 & -0.17 & 0.58 & -0.17 & -0.06 & -0.06 & -0.06 & 0.19 & 0.19 & 0.19 & -0.06 & -0.06 & -0.06 \\
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0.08 & 0.19 & -0.06 & -0.06 & 0.73 & -0.27 & -0.27 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 \\
0.08 & 0.19 & -0.06 & -0.06 & -0.27 & 0.73 & -0.27 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 \\
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0.08 & -0.06 & 0.19 & -0.06 & -0.02 & -0.02 & -0.02 & -0.27 & 0.73 & -0.27 & -0.02 & -0.02 & -0.02 \\
0.08 & -0.06 & 0.19 & -0.06 & -0.02 & -0.02 & -0.02 & -0.27 & -0.27 & 0.73 & -0.02 & -0.02 & -0.02 \\
0.08 & -0.06 & -0.06 & 0.19 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & 0.73 & -0.27 & -0.27 \\
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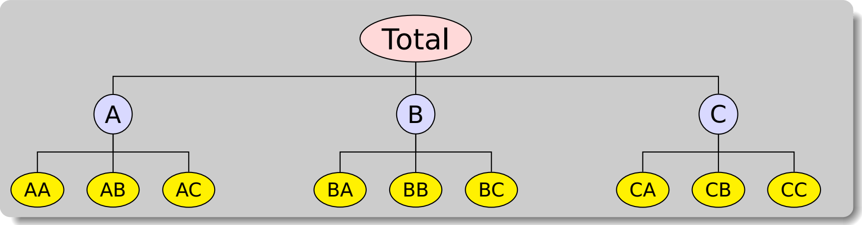
0.08 & -0.06 & -0.06 & 0.19 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.27 & -0.27 & 0.73
\end{bmatrix}$$

Optimal combination forecasts



Weights: $\boldsymbol{S}(\boldsymbol{S}'\boldsymbol{S})^{-1}\boldsymbol{S}' =$

$$
\begin{bmatrix}
0.69 & 0.23 & 0.23 & 0.23 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 \\
0.23 & 0.58 & -0.17 & -0.17 & 0.19 & 0.19 & 0.19 & -0.06 & -0.06 & -0.06 & -0.06 & -0.06 & -0.06 \\
0.23 & -0.17 & 0.58 & -0.17 & -0.06 & -0.06 & -0.06 & 0.19 & 0.19 & 0.19 & -0.06 & -0.06 & -0.06 \\
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$$

Features

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- Very simple and flexible method. Can work with *any* hierarchical or grouped time series.
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$$\tilde{\mathbf{Y}}_n(h) = \mathbf{S}(\mathbf{S}'\Sigma_1^{\dagger}\mathbf{S})^{-1}\mathbf{S}'\Sigma_1^{\dagger}\hat{\mathbf{Y}}_n(h)$$

Solution 1: OLS

- Approximate $\Sigma_1^{\dagger}$ by $c\mathbf{I}$.

Solution 2: Rescaling

- Suppose we approximate Σ_1 by its diagonal.
- Let $\Lambda = \left[\text{diagonal}\left(\Sigma_1\right)\right]^{-1}$ contain inverse one-step forecast variances.

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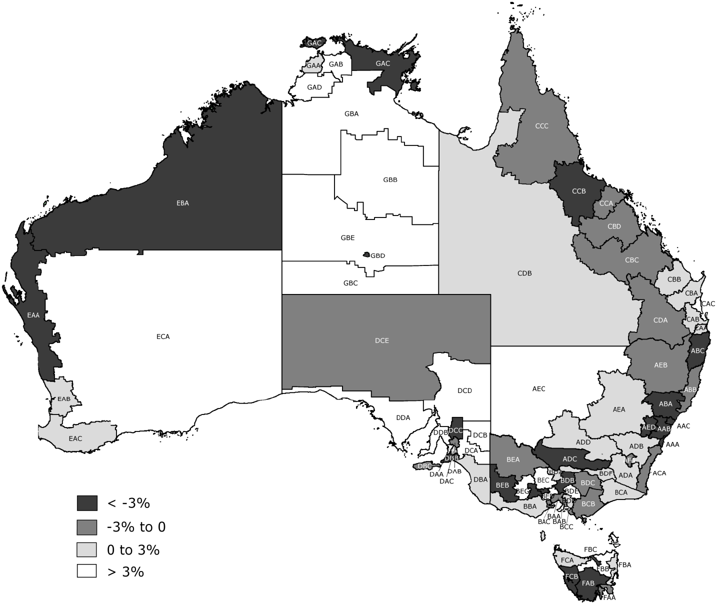
Outline

Australian tourism



Australian tourism

Domestic visitor nights

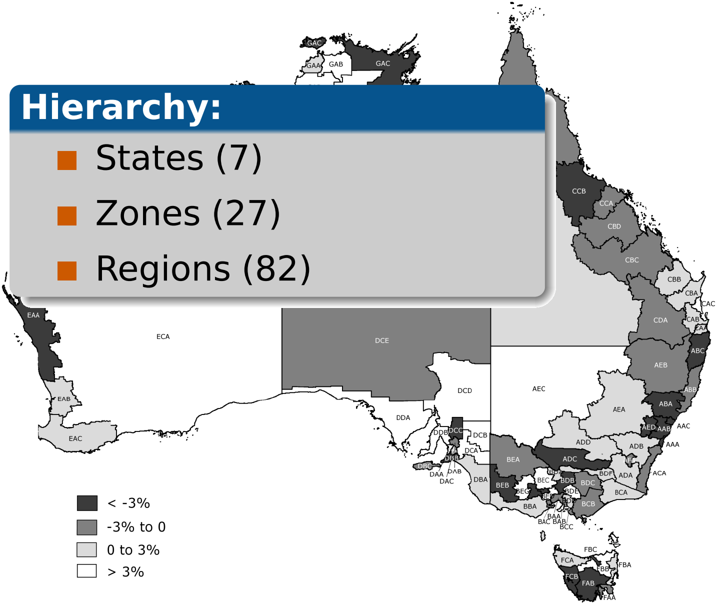
Quarterly data: 1998 – 2006.

From: *National Visitor Survey*, based on annual interviews of 120,000 Australians aged 15+, collected by Tourism Research Australia.

Australian tourism

Hierarchy:

- States (7)
- Zones (27)
- Regions (82)



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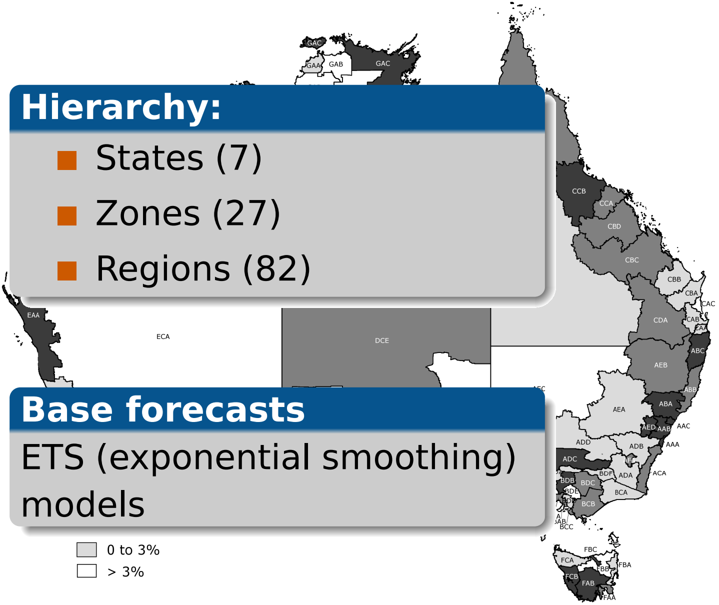

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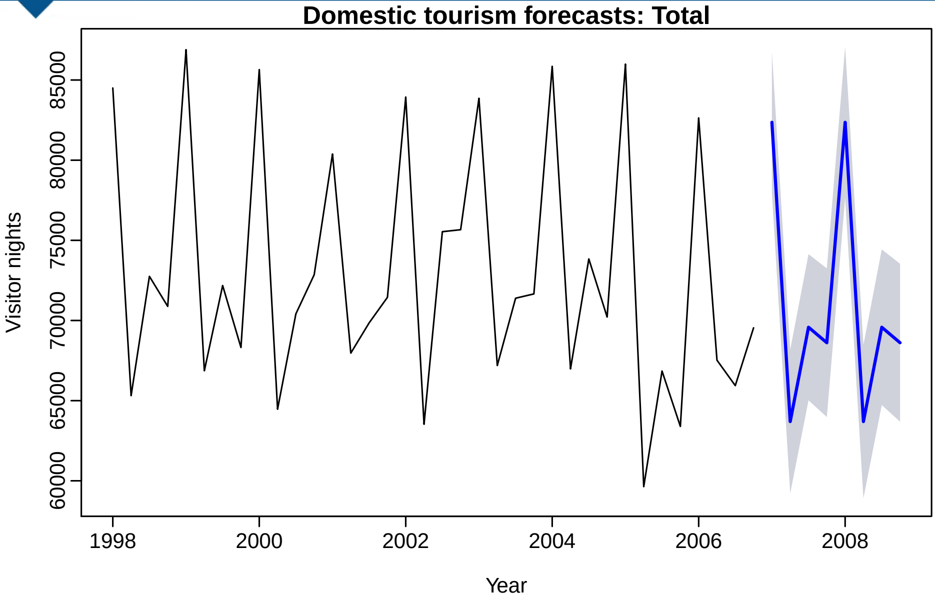
Base forecasts

ETS (exponential smoothing) models

Base forecasts



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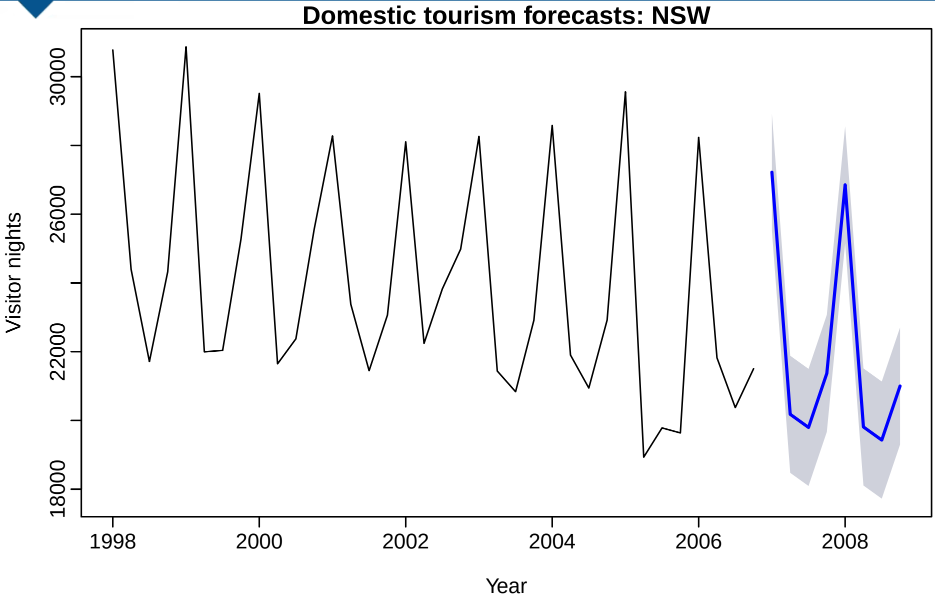

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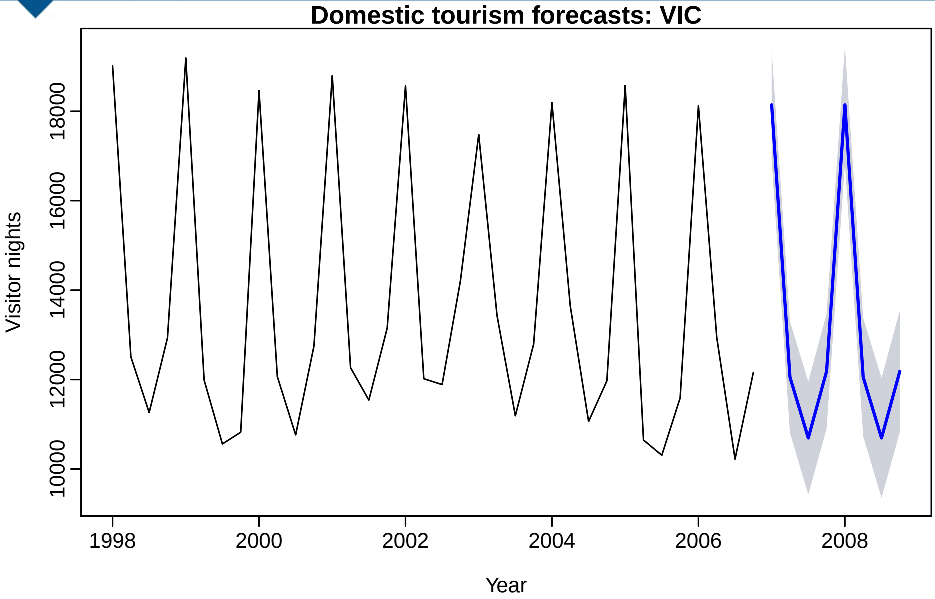

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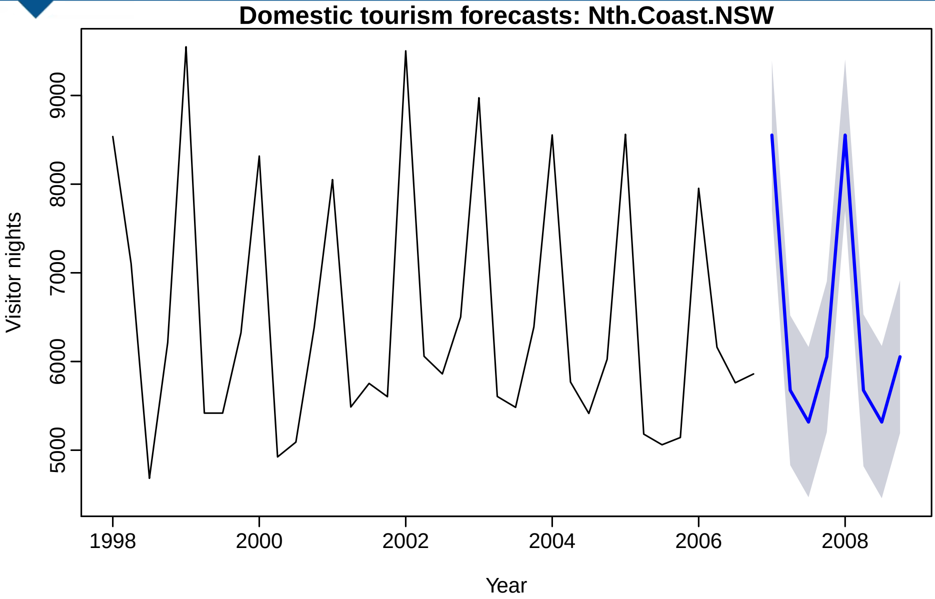

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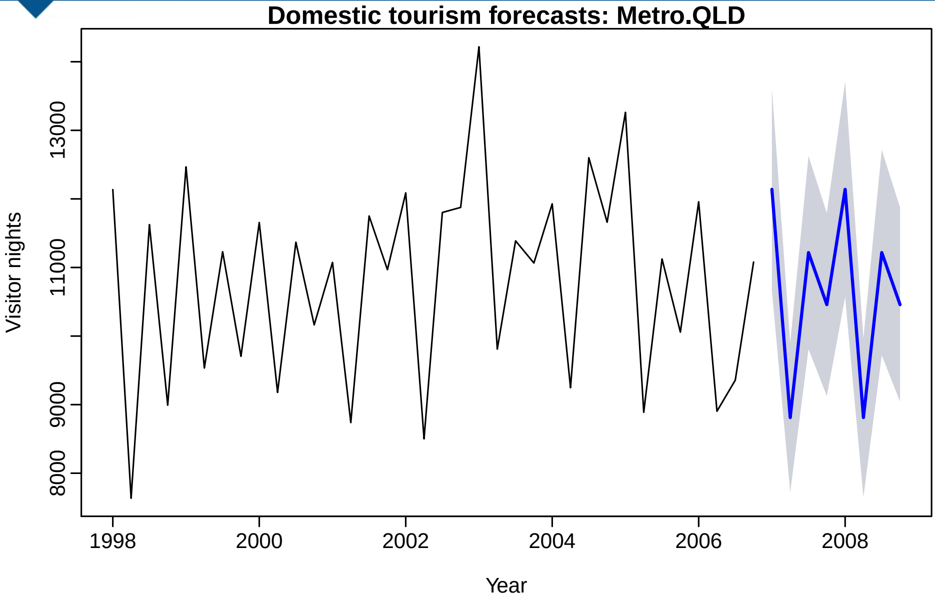

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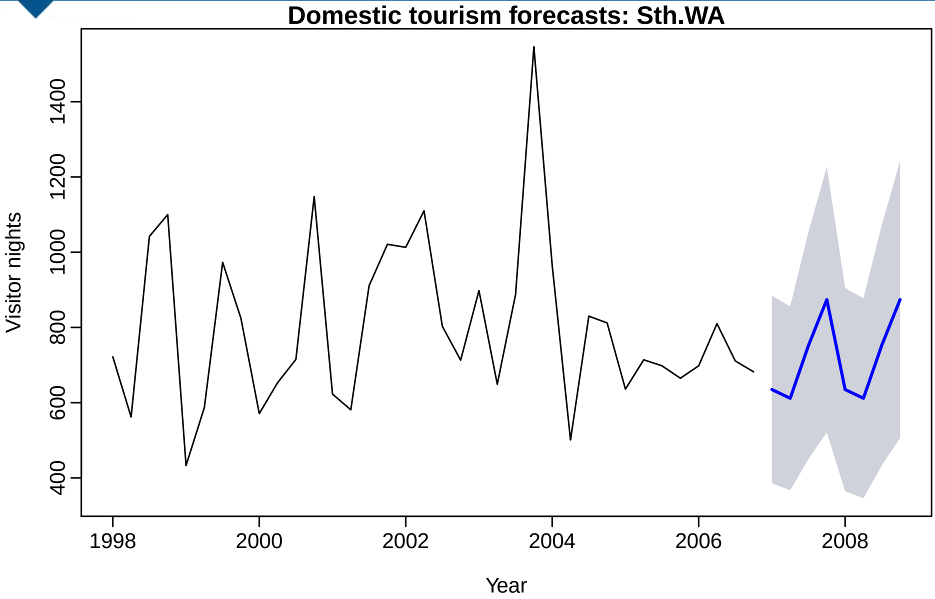

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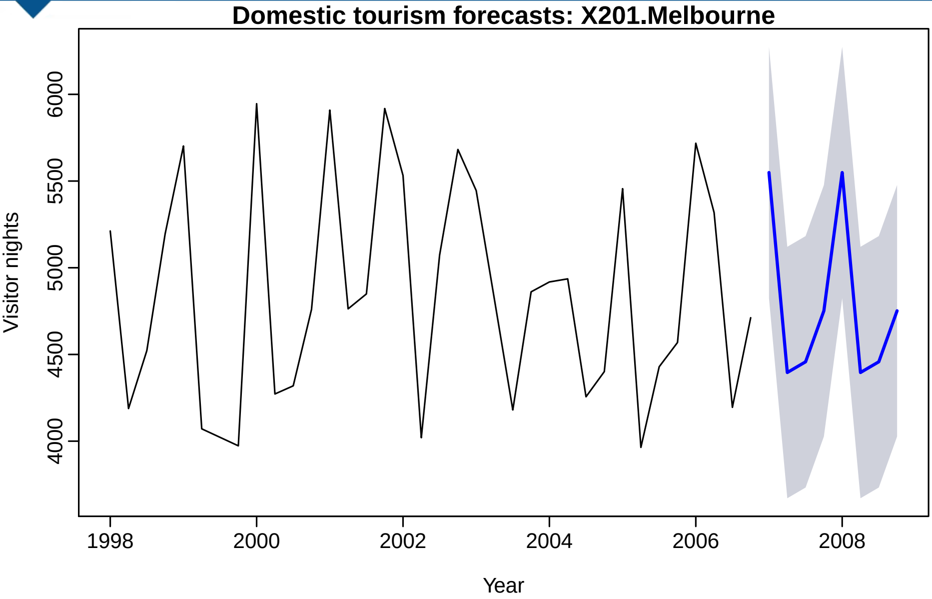

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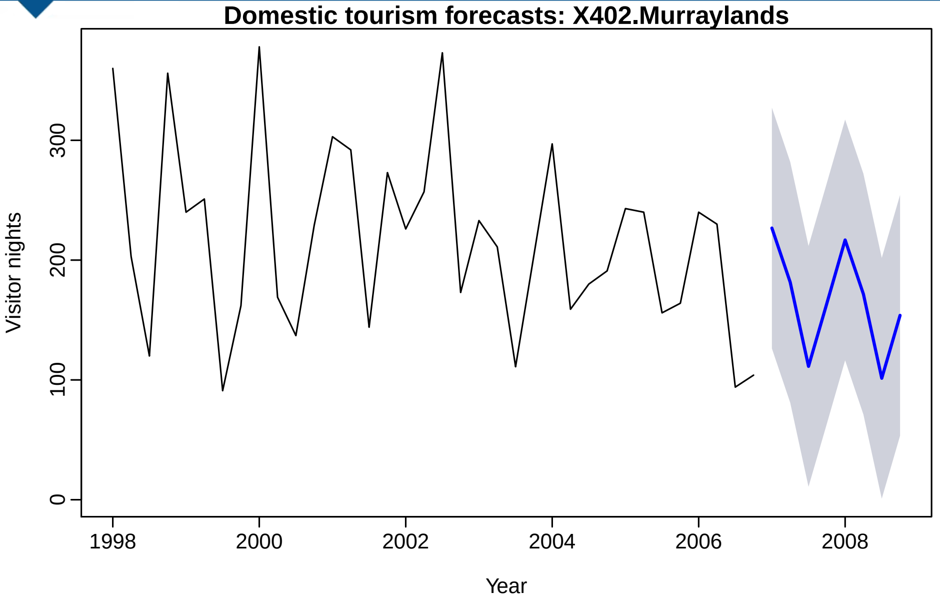

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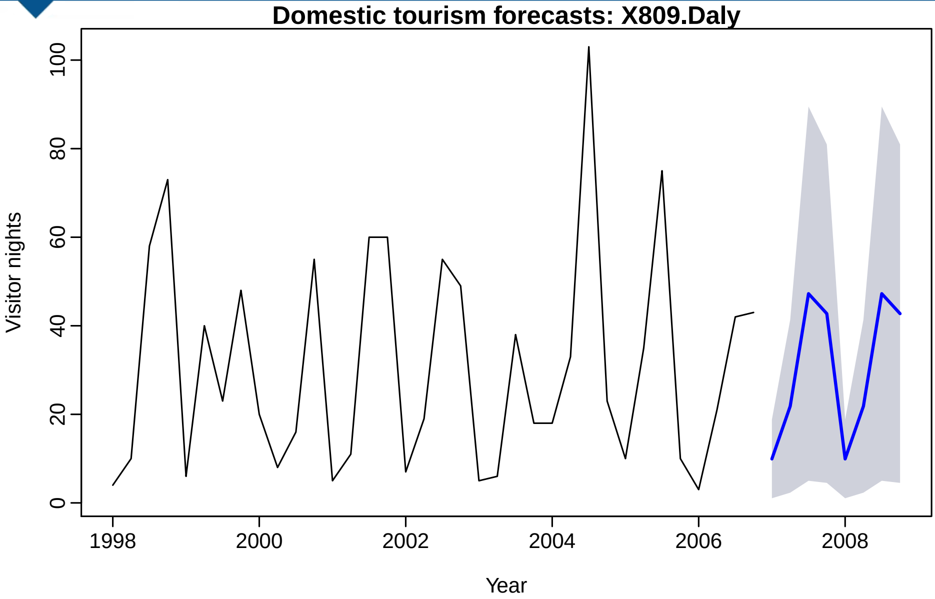
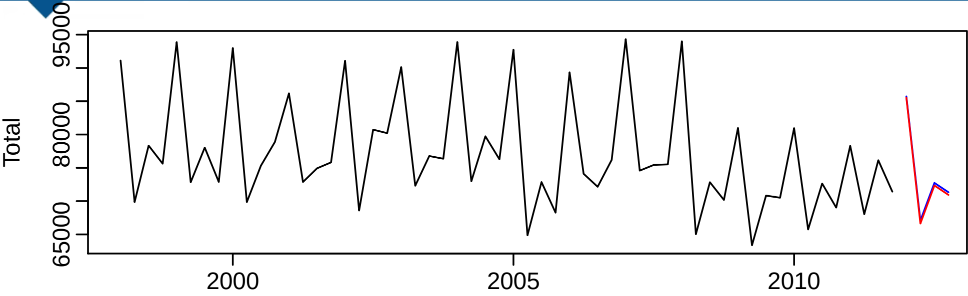


Reconciled forecasts



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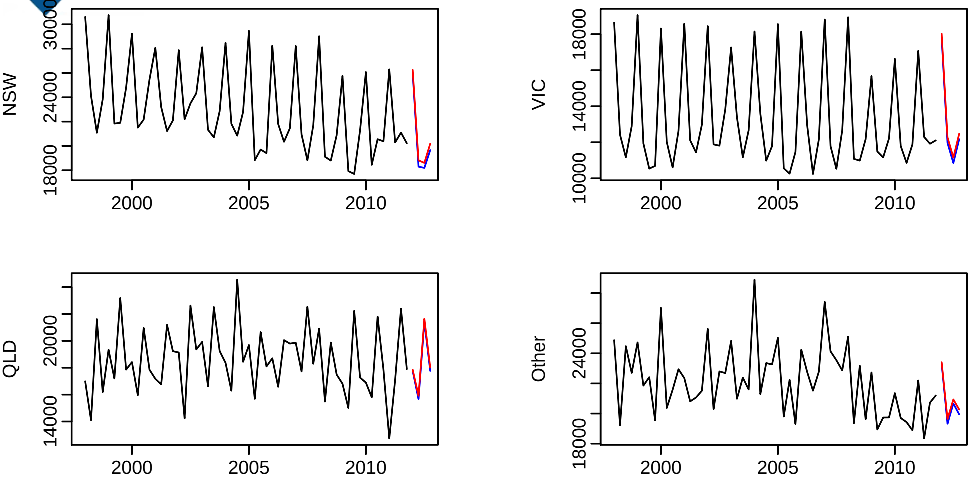

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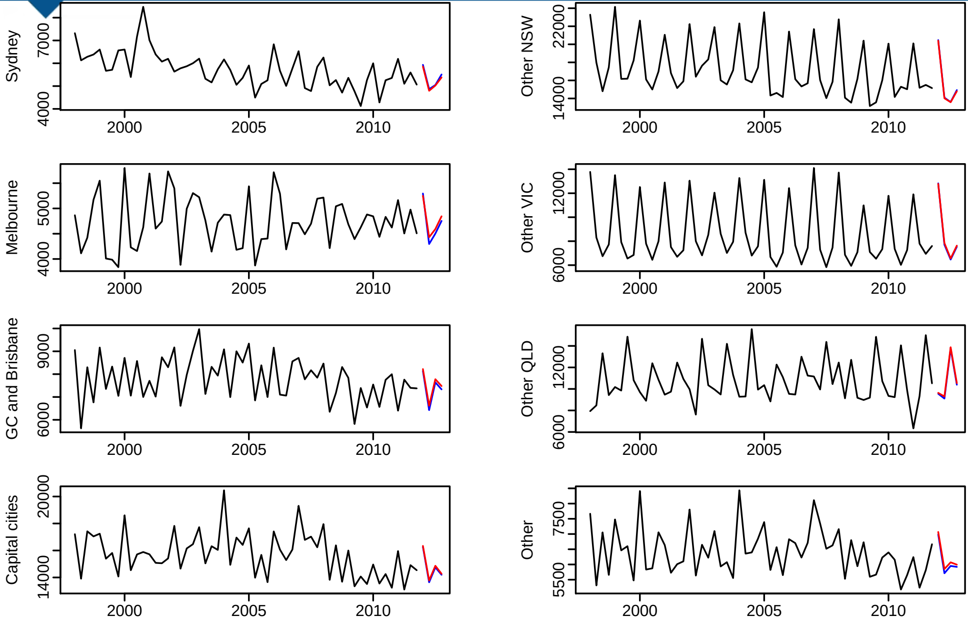

Forecast evaluation

- Select models using all observations;
- Re-estimate models using first 12 observations and generate 1- to 8-step-ahead forecasts;
- Increase sample size one observation at a time, re-estimate models, generate forecasts until the end of the sample;
- In total 24 1-step-ahead, 23 2-steps-ahead, up to 17 8-steps-ahead for forecast evaluation.

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Hierarchy: states, zones, regions

MAPE	$h=1$	$h=2$	$h=4$	$h=6$	$h=8$	Average
Top Level: Australia						
Bottom-up	3.79	3.58	4.01	4.55	4.24	4.06
OLS	3.83	3.66	**3.88**	**4.19**	4.25	**3.94**
Scaling (st. dev.)	**3.68**	**3.56**	3.97	4.57	4.25	4.04
Level: States						
Bottom-up	10.70	10.52	10.85	11.46	11.27	11.03
OLS	11.07	10.58	11.13	11.62	12.21	11.35
Scaling (st. dev.)	**10.44**	**10.17**	**10.47**	**10.97**	**10.98**	**10.67**
Level: Zones						
Bottom-up	14.99	14.97	14.98	15.69	15.65	15.32
OLS	15.16	15.06	15.27	15.74	16.15	15.48
Scaling (st. dev.)	**14.63**	**14.62**	**14.68**	**15.17**	**15.25**	**14.94**
Bottom Level: Regions						
Bottom-up	33.12	32.54	32.26	33.74	33.96	33.18
OLS	35.89	33.86	34.26	36.06	37.49	35.43
Scaling (st. dev.)	**31.68**	**31.22**	**31.08**	**32.41**	**32.77**	**31.89**

Outline

ANZSCO

Australia and New Zealand Standard Classification of Occupations

- 8 major groups
  - 43 sub-major groups
    - 97 minor groups
      - 359 unit groups
        - 1023 occupations

Example: statistician

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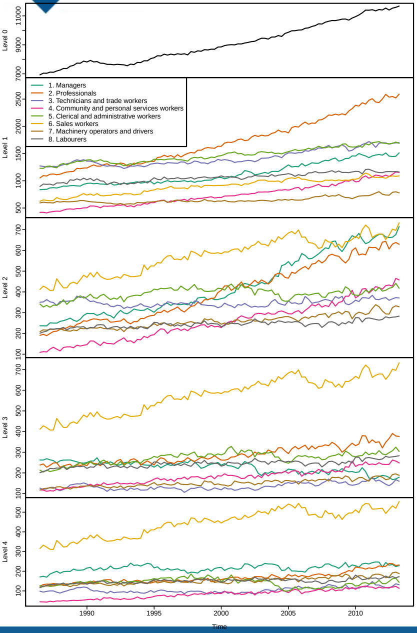
Australian Labour Market data



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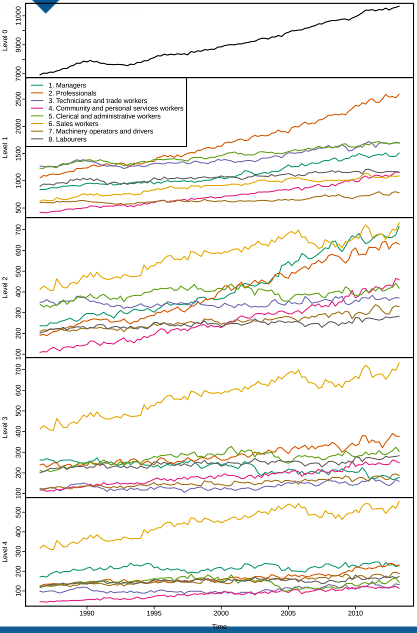

- Lower three panels show largest sub-groups at each level.

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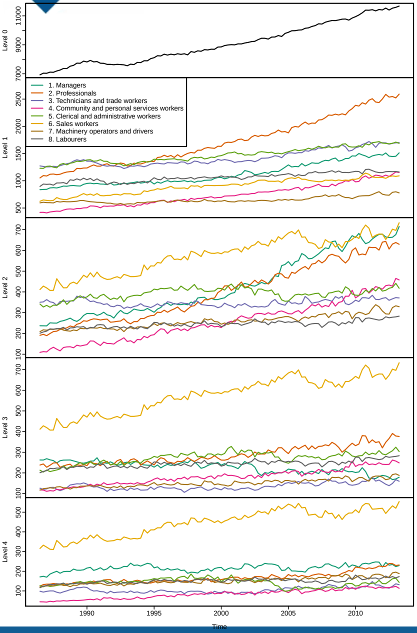

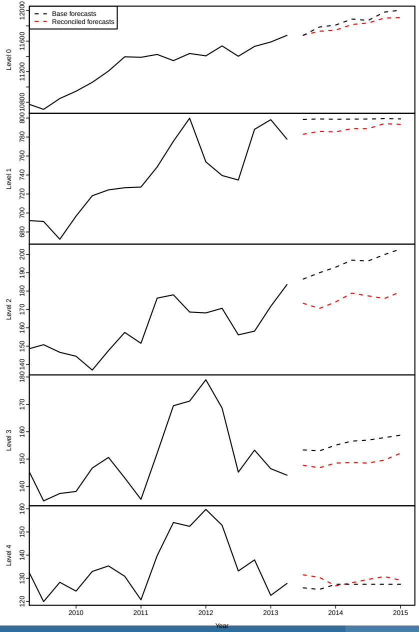

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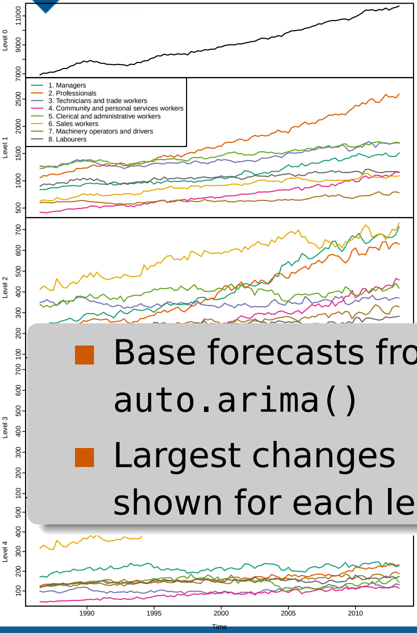
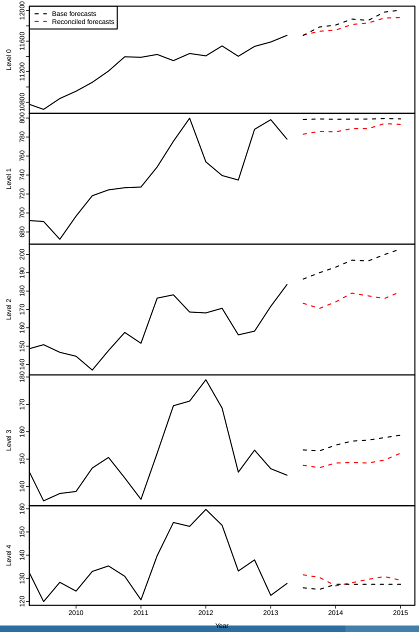


- Base forecasts from `auto.arima()`
- Largest changes shown for each level



Forecast evaluation (rolling origin)

RMSE	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$	Average
Top level									
Bottom-up	74.71	102.02	121.70	131.17	147.08	157.12	169.60	178.93	135.29
OLS	**52.20**	**77.77**	**101.50**	**119.03**	138.27	150.75	160.04	166.38	**120.74**
WLS	61.77	86.32	107.26	119.33	**137.01**	**146.88**	**156.71**	**162.38**	122.21
Level 1									
Bottom-up	21.59	27.33	30.81	32.94	35.45	37.10	39.00	40.51	33.09
OLS	21.89	28.55	32.74	35.58	38.82	41.24	43.34	45.49	35.96
WLS	**20.58**	**26.19**	**29.71**	**31.84**	**34.36**	**35.89**	**37.53**	**38.86**	**31.87**
Level 2									
Bottom-up	8.78	10.72	11.79	12.42	13.13	13.61	14.14	14.65	12.40
OLS	9.02	11.19	12.34	13.04	13.92	14.56	15.17	15.77	13.13
WLS	**8.58**	**10.48**	**11.54**	**12.15**	**12.88**	**13.36**	**13.87**	**14.36**	**12.15**
Level 3									
Bottom-up	5.44	6.57	7.17	7.53	7.94	8.27	8.60	8.89	7.55
OLS	5.55	6.78	7.42	7.81	8.29	8.68	9.04	9.37	7.87
WLS	**5.35**	**6.46**	**7.06**	**7.42**	**7.84**	**8.17**	**8.48**	**8.76**	**7.44**
Bottom Level									
Bottom-up	2.35	2.79	3.02	3.15	3.29	3.42	3.54	3.65	3.15
OLS	2.40	2.86	3.10	3.24	3.41	3.55	3.68	3.80	3.25
WLS	**2.34**	**2.77**	**2.99**	**3.12**	**3.27**	**3.40**	**3.52**	**3.63**	**3.13**

Outline

hts package for R

hts: Hierarchical and grouped time series

Methods for analysing and forecasting hierarchical and grouped time series

Version:	4.3
Depends:	forecast ($\geq$ 5.0)
Imports:	SparseM, parallel, utils
Published:	2014-06-10
Author:	Rob J Hyndman, Earo Wang and Alan Lee
Maintainer:	Rob J Hyndman <Rob.Hyndman at monash.edu>
BugReports:	`https://github.com/robjhyndman/hts/issues`
License:	GPL ($\geq$ 2)

Example using R

```
library(hts)

# bts is a matrix containing the bottom level time series
# nodes describes the hierarchical structure
y <- hts(bts, nodes=list(2, c(3,2)))
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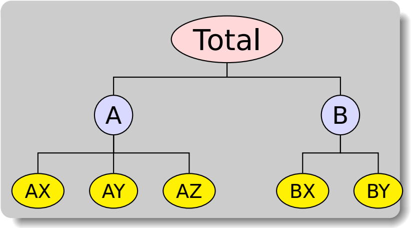

Example using R

```r
library(hts)

# bts is a matrix containing the bottom level time series
# nodes describes the hierarchical structure
y <- hts(bts, nodes=list(2, c(3,2)))

# Forecast 10-step-ahead using WLS combination method
# ETS used for each series by default
fc <- forecast(y, h=10)
```

forecast.gts function

Usage

```
forecast(object, h,
  method = c("comb", "bu", "mo", "tdgsf", "tdgsa", "tdfp"),
  fmethod = c("ets", "rw", "arima"),
  weights = c("sd", "none", "nseries"),
  positive = FALSE,
  parallel = FALSE, num.cores = 2, ...)
```

Arguments

`object`	Hierarchical time series object of class `gts`.
`h`	Forecast horizon
`method`	Method for distributing forecasts within the hierarchy.
`fmethod`	Forecasting method to use
`positive`	If TRUE, forecasts are forced to be strictly positive
`weights`	Weights used for `optimal combination` method. When weights = `sd`, it takes account of the standard deviation of forecasts.
`parallel`	If TRUE, allow parallel processing
`num.cores`	If parallel = TRUE, specify how many cores are going to be used