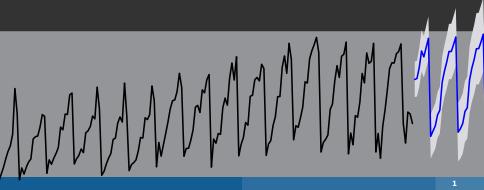


Rob J Hyndman

Automatic time series forecasting



Outline

- 1 Motivation
- **2** Forecasting competitions
- 3 Evaluating forecast accuracy
- 4 Exponential smoothing
- 5 ARIMA modelling
- 6 Automatic nonlinear forecasting?
- 7 Time series with complex seasonality
 - 8 Forecasts about automatic forecasting



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- Common in business to have over 1000 products that need forecasting at least monthly.
- Forecasts are often required by people who are untrained in time series analysis.

Specifications

Automatic forecasting algorithms must:

- determine an appropriate time series model;
- estimate the parameters;
- compute the forecasts with prediction intervals

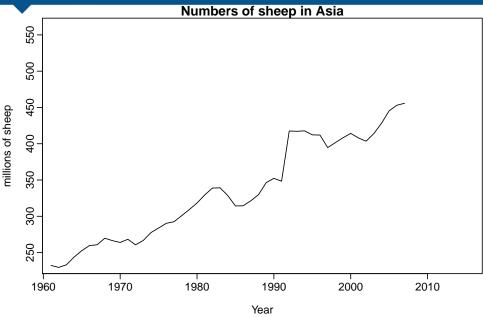
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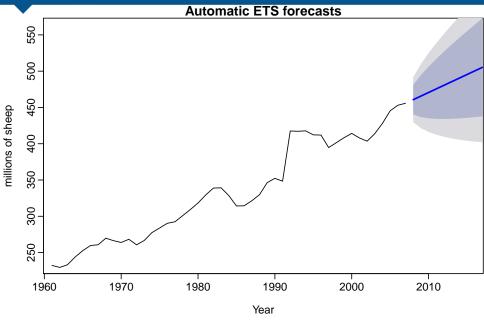
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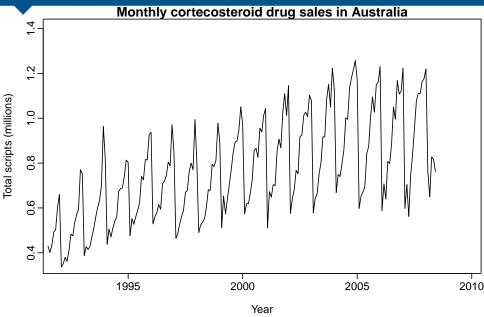
Example: Asian sheep



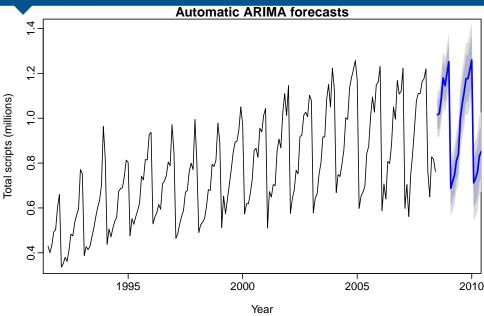
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Example: Cortecosteroid sales



Example: Cortecosteroid sales



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J. R. Statist. Soc. A (1979), 142, Part 2, pp. 97-145

Accuracy of Forecasting: An Empirical Investigation

By Spyros Makridakis and Michèle Hibon

INSEAD—The European Institute of Business Administration

[Read before the ROYAL STATISTICAL SOCIETY on Wednesday, December 13th, 1978, the President, SIR CLAUS MOSER in the Chair]

SUMMARY

In this study, the authors used 111 time series to examine the accuracy of various forecasting methods, particularly time-series methods. The study shows, at least for time series, why some methods achieve greater accuracy than others for different types of data. The authors offer some explanation of the seemingly conflicting conclusions of past empirical research on the accuracy of forecasting. One novel contribution of the paper is the development of regression equations expressing accuracy as a function of factors such as randomness, seasonality, trend-cycle and the number of data points describing the series. Surprisingly, the study shows that for these 111 series simpler methods perform well in comparison to the more complex and statistically sophisticated ARMA models.

Keywords: FORECASTING; TIME SERIES; FORECASTING ACCURACY

0. Introduction

THE ultimate test of any forecast is whether or not it is capable of predicting future events

J. R. Statist. Soc. A (1979), 142, Part 2, pp. 97-145

DMIF

Film

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Introduction

THE ultimate test of any forecast is whether or not it is capable of predicting future events

This was the first large-scale empirical evaluation of time series forecasting methods.

- Highly controversial at the time.
- Difficulties:
 - How to measure forecast accuracy?
 - How to apply methods consistently and objectively?
 - How to explain unexpected results?
- Common thinking was that the more sophisticated mathematical models (ARIMA models at the time) were necessarily better.
- If results showed ARIMA models not best, it must be because analyst was unskilled.

I do not believe that it is very fruitful to attempt to classify series according to which forecasting techniques perform "best". The performance of any particular technique when applied to a particular series depends essentially on (a) the model which the series obeys; (b) our ability to identify and fit this model correctly and (c) the criterion chosen to measure the forecasting accuracy.

— M.B. Priestley

... the paper suggests the application of normal scientific experimental design to forecasting, with measures of unbiased testing of forecasts against subsequent reality, for success or failure. A long overdue reform.

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Modern man is fascinated with the subject of forecasting — W.G. Gilchrist

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Why do empirical studies sometimes give different answers? It may depend on the selected sample of time series, but I suspect it is more likely to depend on the skill of the analyst and on their individual interpretations of what is meant by Method X.

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Consequences of M&H (1979)

As a result of this paper, researchers started to:

- consider how to automate forecasting methods;
- study what methods give the best forecasts;
- be aware of the dangers of over-fitting;
- treat forecasting as a different problem from time series analysis.

Makridakis & Hibon followed up with a new competition in 1982:

- **1001** series
- Anyone could submit forecasts (avoiding the charge of incompetence)
- Multiple forecast measures used.

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M-competition

Journal of Forecasting, Vol. 1, 111-153 (1982)

The Accuracy of Extrapolation (Time Series) Methods: Results of a Forecasting Competition

S. MAKRIDAKIS INSEAD, Fontainebleau, France A ANDERSEN University of Sydney, Australia R CARRONE Université Laval, Quebec, Canada R. FILDES Manchester Business School, Manchester, England M. HIBON INSEAD, Fontainebleau, France R. LEWANDOWSKI Marketing Systems, Essen, Germany J. NEWTON E. PARZEN Texas A & M University, Texas, U.S.A. R. WINKLER Indiana University, Bloomington, U.S.A.

ABSTRACT

In the last few decades many methods have become available for forecasting, As always, when alternatives exist, choices need to be made so that an appropriate forecasting method can be selected and used for the specific situation being considered. This paper reports the results of a forecasting competition that provides information to facilitate such choice. Seven experts in each of the 24 methods forecasted up to 1001 series for six up to eighteen time horizons. The results of the competition are presented in this paper whose purpose is to provide empirical evidence about differences found to exist among the various extrapolative (time series) methods used in the competition.

M-competition

Main findings (taken from Makridakis & Hibon, 2000)

- Statistically sophisticated or complex methods do not necessarily provide more accurate forecasts than simpler ones.
- The relative ranking of the performance of the various methods varies according to the accuracy measure being used.
- The accuracy when various methods are being combined outperforms, on average, the individual methods being combined and does very well in comparison to other methods.
- The accuracy of the various methods depends upon the length of the forecasting horizon involved.

M3 competition



International Journal of Forecasting 16 (2000) 451-476



www.elsevier.com/locate/ijforecast

The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon*

INSEAD, Boulevard de Constance, 77305 Fontainebleau. France

Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and

directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

*Keywords: Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy

Makridakis and Hibon (2000)

"The M3-Competition is a final attempt by the authors to settle the accuracy issue of various time series methods... The extension involves the inclusion of more methods/ researchers (in particular in the areas of neural networks and expert systems) and more series."

- 3003 series
- All data from business, demography, finance and economics.
- Series length between 14 and 126.
- Either non-seasonal, monthly or quarterly.
- All time series positive.
- M&H claimed that the M3-competition supported the findings of their earlier work.
- However, best performing methods far from "simple".

Makridakis and Hibon (2000)

Best methods:

Theta

- A very confusing explanation.
- Shown by Hyndman and Billah (2003) to be average of linear regression and simple exponential smoothing with drift, applied to seasonally adjusted data.
- Later, the original authors claimed that their explanation was incorrect.

Forecast Pro

- A commercial software package with an unknown algorithm.
- Known to fit either exponential smoothing or ARIMA models using BIC.

M3 results (recalculated)

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
ForecastPro	18.00	13.06	1.47
ForecastX	17.35	13.09	1.42
Automatic ANN	17.18	13.98	1.53
B-J automatic	19.13	13.72	1.54

M3 results (recalculated)

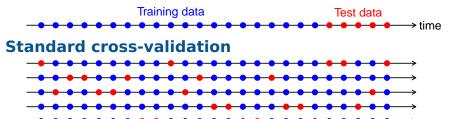
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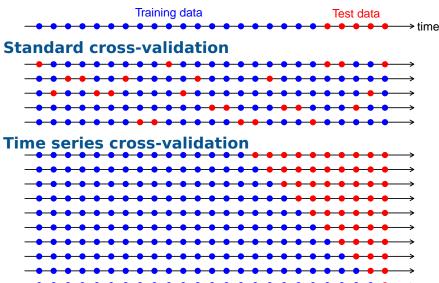
- Calculations do not match published paper.
- ➤ Some contestants apparently submitted multiple entries but only best ones published.

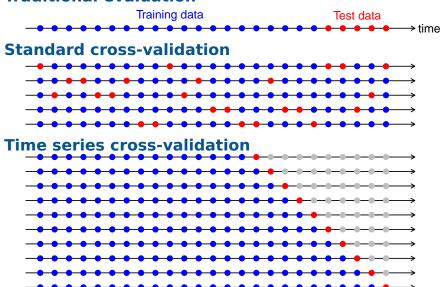
Outline

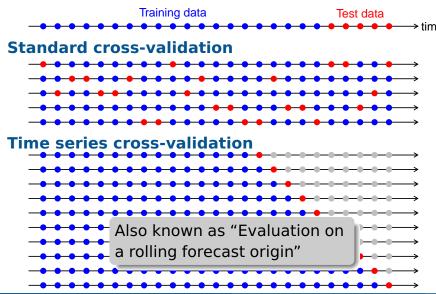
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where *L* is the likelihood and *k* is the number of estimated parameters in the model.

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- This is a *penalized likelihood* approach.
- If L is Gaussian, then AIC $\approx c + T \log MSE + 2k$ where c is a constant, MSE is from one-step forecasts on **training set**, and T is the length of the series.

Minimizing the Gaussian AIC is asymptotically equivalent (as $T \to \infty$) to minimizing MSE from one-step forecasts on **test set** via time series cross-validation.

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$$AIC = -2\log(L) + 2k$$

Corrected AIC

For small *T*, AIC tends to over-fit. Bias-corrected version:

$$AIC_C = AIC + \frac{2(k+1)(k+2)}{T-k}$$

Bayesian Information Criterion

$$BIC = AIC + k[\log(T) - 2]$$

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Exponential smoothing





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7 Exponential smoothing

Exponential smoothing was proposed in the late 1950s (Brown 1959, Holt 1957 and Winters 1960 are key pioneering works) and has motivated some of the most successful forecasting methods. Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older. In other words, the more recent the observation the higher the associated weight. This framework generates reliable forecasts quickly and for a wide spectrum of time series which is a great advantage and of major importance to applications in industry.

This chapter is divided into two parts. In the first part we present in detail the mechanics of all exponential smoothing methods and their application in forecasting time series with various characteristics. This is key in understanding the intuition behind these methods. In this setting, selecting and using a forecasting method may appear to be somewhat addoc. The

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Springer Series in Statistics

Rob J. Hyndman · Anne B. Koehler J. Keith Ord · Ralph D. Snyder

Forecasting with Exponential Smoothing

The State Space Approach



Springer

	Seasonal Component			mponent
	Trend	N	Α	M
	Component	(None)	(Additive)	(Multiplicative)
N	(None)	N,N	N,A	N,M
Α	(Additive)	A,N	A,A	A,M
A_d	(Additive damped)	A _d ,N	A_d , A	A _d ,M
М	(Multiplicative)	M,N	M,A	M,M
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N,N: Simple exponential smoothing

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A,N: Holt's linear method

A_d,N: Additive damped trend method

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M_d,N: Multiplicative damped trend method

A,A: Additive Holt-Winters' method

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N,N: Simple exponential smoothing

A,N: Holt's linear method

A_d,N: Additive damped trend method

M,N: Exponential trend method

M_d,N: Multiplicative damped trend method

A,A: Additive Holt-Winters' method

A,M: Multiplicative Holt-Winters' method

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■ There are 15 separate exponential smoothing methods.

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- There are 15 separate exponential smoothing methods.
- Each can have an additive or multiplicative error, giving 30 separate models.

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	Component	(None)	(Additive)	(Multiplicative)
N	(None)	N,N	N,A	N,M
Α	(Additive)	A,N	A,A	A,M
A_d	(Additive damped)	A _d ,N	A_d ,A	A_d ,M
М	(Multiplicative)	M,N	M,A	M,M
M_d	(Multiplicative damped)	M _d ,N	M_d ,A	M_d , M

- There are 15 separate exponential smoothing methods.
- Each can have an additive or multiplicative error, giving 30 separate models.
- Only 19 models are numerically stable.

	Seasonal Component			nponent
	Trend	N	Α	M
	Component	(None)	(Additive)	(Multiplicative)
N	(None)	N,N	N,A	N,M
Α	(Additive)	A,N	A,A	A,M
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General notation ETS: ExponenTial Smoothing

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General notation E T S : Exponen Tial Smoothing

Trend

Examples:

A,N,N: Simple exponential smoothing with additive errors

A,A,N: Holt's linear method with additive errors

M,A,M: Multiplicative Holt-Winters' method with multiplicative errors

		Seasonal Component		
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General notation $E T S_{\kappa}$: **E**xponen**T**ial **S**moothing

Trend Seasonal

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Exponential smoothing methods

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Error Trend Seasonal

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Error Trend Seasonal

Examples:

A,N,N: Simple exponential smoothing with additive errors

A,A,N: Holt's linear method with additive errors

M,A,M: Multiplicative Holt-Winters' method with multiplicative errors

Exponential smoothing methods

Innovations state space models

- → All ETS models can be written in innovations state space form (IJF, 2002).
- → Additive and multiplicative versions give the same point forecasts but different prediction intervals.

General notation ETS: ExponenTial Smoothing

Error Trend Seasonal

Examples:

A,N,N: Simple exponential smoothing with additive errors

A,A,N: Holt's linear method with additive errors

M,A,M: Multiplicative Holt-Winters' method with multiplicative errors

Let
$$\mathbf{x}_t = (\ell_t, b_t, s_t, s_{t-1}, \dots, s_{t-m+1})$$
 and $\varepsilon_t \stackrel{\text{iid}}{\sim} \mathsf{N}(0, \sigma^2)$.

$$y_t = \underbrace{h(\mathbf{x}_{t-1})}_{\mu_t} + \underbrace{k(\mathbf{x}_{t-1})\varepsilon_t}_{e_t}$$
 Observation equation

$$\mathbf{x}_t = f(\mathbf{x}_{t-1}) + g(\mathbf{x}_{t-1})\varepsilon_t$$
 State equation

Additive errors:

$$k(\mathbf{x}_{t-1}) = 1.$$
 $y_t = \mu_t + \varepsilon_t.$

Multiplicative errors:

$$k(\mathbf{x}_{t-1}) = \mu_t.$$
 $\mathbf{y}_t = \mu_t(\mathbf{1} + \varepsilon_t).$ $\varepsilon_t = (\mathbf{y}_t - \mu_t)/\mu_t$ is relative error.

- All models can be written in state space form.
- Additive and multiplicative versions give same point forecasts but different prediction intervals.

Estimation

$$L^*(\boldsymbol{\theta}, \mathbf{x}_0) = n \log \left(\sum_{t=1}^n \varepsilon_t^2 / k^2(\mathbf{x}_{t-1}) \right) + 2 \sum_{t=1}^n \log |k(\mathbf{x}_{t-1})|$$

$$= -2 \log(\text{Likelihood}) + \text{constant}$$

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= -2 log(Likelihood) + constant

Minimize wrt $\boldsymbol{\theta} = (\alpha, \beta, \gamma, \phi)$ and initial states $\mathbf{x}_0 = (\ell_0, b_0, s_0, s_{-1}, \dots, s_{-m+1}).$

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Estimation

$$\begin{aligned} L^*(\boldsymbol{\theta}, \boldsymbol{x}_0) &= n \log \bigg(\sum_{t=1}^n \varepsilon_t^2 / k^2(\boldsymbol{x}_{t-1}) \bigg) + 2 \sum_{t=1}^n \log |k(\boldsymbol{x}_{t-1})| \\ &= -2 \log(\text{Likelihood}) + \text{constant} \end{aligned}$$

■ Minimize wrt $\theta = (\alpha, \beta, \gamma, \phi)$ and initial states $\mathbf{x}_0 = (\ell_0, b_0, s_0, s_{-1}, \dots, s_{-m+1}).$



- Apply each of 19 models that are appropriate to the data. Optimize parameters and initial values using MLE.
- Select best method using AICc.
- Produce forecasts using best method.
- Obtain prediction intervals using underlying state space model.



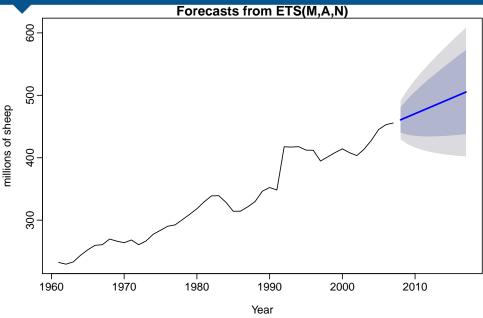
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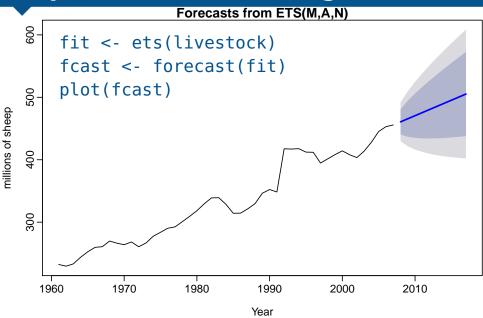


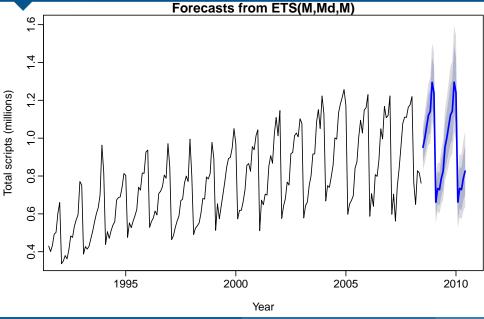
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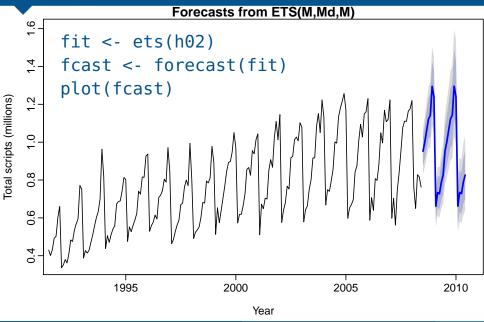


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M3 comparisons

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
ForecastPro	18.00	13.06	1.47
ForecastX	17.35	13.09	1.42
Automatic ANN	17.18	13.98	1.53
B-J automatic	19.13	13.72	1.54
ETS	18.06	13.38	1.52

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A non-seasonal ARIMA process

$$\phi(B)(1-B)^d y_t = c + \theta(B)\varepsilon_t$$

B is backshift operator and ε_t is (Gaussian) iid noise.

A seasonal ARIMA process (period m)

$$\Phi(B^m)\phi(B)(1-B)^d(1-B^m)^Dy_t=c+\Theta(B^m)\theta(B)\varepsilon_t$$

$-2 \times$ Log likelihood function

$$L^*(\beta) = f(\beta) + T \log \sigma^2 + \frac{1}{\sigma^2} \sum_{t=1}^{r} (y_t - \hat{y}_{t|t-1})^2 / r_t(\beta)$$

where β contains all parameters to be estimated and σ^2 is variance of $\{\varepsilon_t\}$.



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www.elsevier.com/locate/ijforecast

Automatic ARIMA modeling including interventions, using time series expert software

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Abstract

This article has three objectives: (a) to describe the method of automatic ARIMA modeling (AAM), with and without intervention analysis, that has been used in the analysis; (b) to comment on the results; and (c) to comment on the M3 Competition in general. Starting with a computer program for fitting an ARIMA model and a methodology for building univariate ARIMA models, an expert system has been built, while trying to avoid the pitfalls of most existing software packages. A software package called Time Series Expert TSE-AX is used to build a univariate ARIMA model with or without an intervention analysis. The characteristics of TSE-AX are summarized and, more especially, its automatic ARIMA modeling method. The motivation to take part in the M3-Competition is also outlined. The methodology is described mainly

A Course in Time Series Analysis
Edited by Daniel Peña, George C. Tiao and Ruey S. Tsay
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CHAPTER 7

Automatic Modeling Methods for Univariate Series

Víctor Gómez Ministerio de Hacienda

Agustín Maravall Banco de España



Journal of Statistical Software

July 2008, Volume 26, Issue 3.

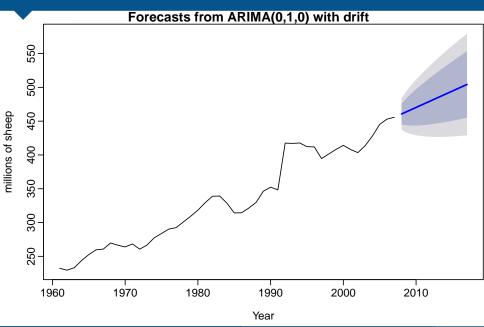
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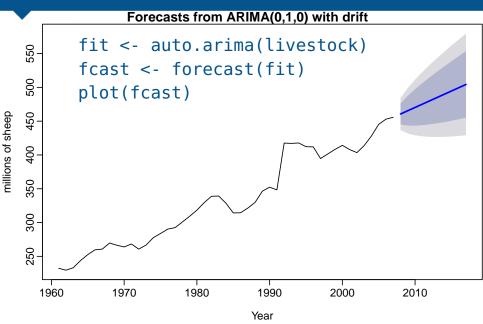
Automatic Time Series Forecasting: The forecast Package for R

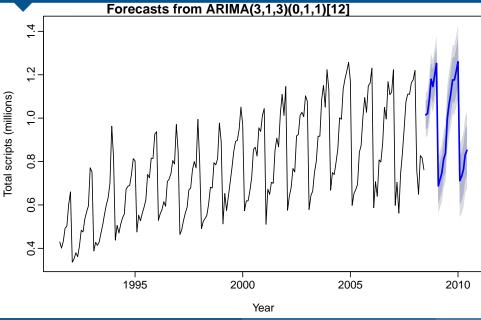
Rob J. Hyndman Monash University Yeasmin Khandakar Monash University

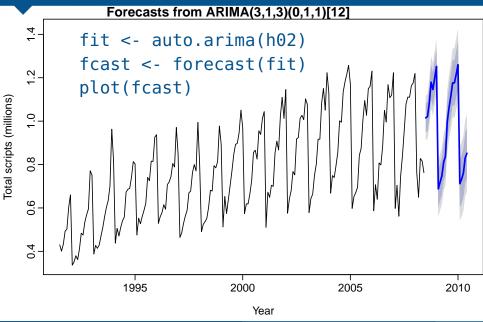
Abstract

Automatic forecasts of large numbers of univariate time series are often needed in









A non-seasonal ARIMA process

$$\phi(B)(1-B)^d y_t = c + \theta(B)\varepsilon_t$$

Need to select appropriate orders p, q, d, and whether to include c.

Algorithm choices driven by forecast accuracy

A non-seasonal ARIMA process

$$\phi(B)(1-B)^d y_t = c + \theta(B)\varepsilon_t$$

Need to select appropriate orders p, q, d, and whether to include c.

Hyndman & Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS unit root test.
- Select p, q, c by minimising AICc.
- Use stepwise search to traverse model space, starting with a simple model and considering nearby variants.

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$$\Phi(B^m)\phi(B)(1-B)^d(1-B^m)^Dy_t=c+\Theta(B^m)\theta(B)\varepsilon_t$$

Need to select appropriate orders p, q, d, P, Q, D, and whether to include c.

Hyndman & Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS unit root test.
- Select D using OCSB unit root test.
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M3 comparisons

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
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B-J automatic	19.13	13.72	1.54
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ETS-ARIMA	17.92	13.02	1.44

M3 conclusions

MYTHS

- Simple methods do better.
- Exponential smoothing is better than ARIMA.

FACTS

The best methods are hybrid approaches

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- Automatic ANN in M3 competition did poorly.
- Linear methods did best in the NN3 competition!
- Very few machine learning methods get published in the IJF because authors cannot demonstrate their methods give better forecasts than linear benchmark methods, even on supposedly nonlinear data.
- Some good recent work by Kourentzes and Crone (Neurocomputing, 2010) on automated ANN for time series.
- Watch this space!

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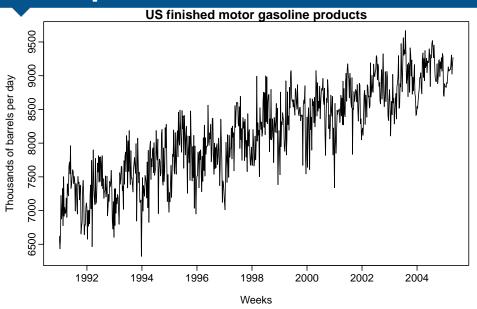
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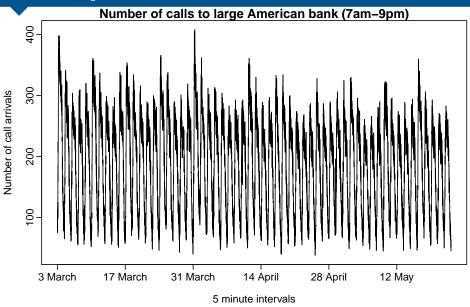
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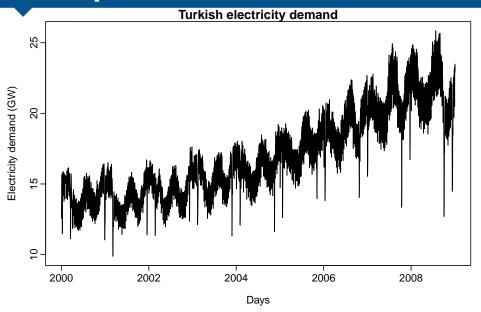
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TBATS model

TBATS

Trigonometric terms for seasonality

Box-Cox transformations for heterogeneity

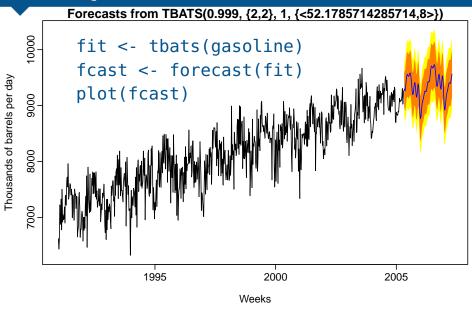
ARMA errors for short-term dynamics

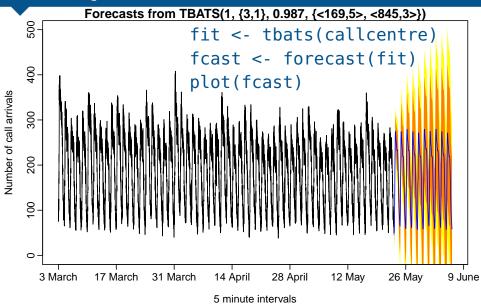
Trend (possibly damped)

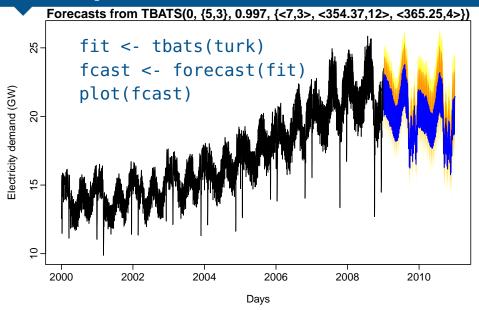
Seasonal (including multiple and non-integer periods)



Automatic algorithm described in AM De Livera, RJ Hyndman, and RD Snyder (2011). "Forecasting time series with complex seasonal patterns using exponential smoothing". *Journal of the American Statistical Association* **106**(496), 1513–1527.







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For further information

robjhyndman.com

- Slides and references for this talk.
- Links to all papers and books.
- Links to R packages.
- A blog about forecasting research.