MONASH University

MONASH BUSINESS SCHOOL

Forecasting using R

Rob J Hyndman

3.4 Advanced methods

Outline

Vector autoregressions

Dynamic regression assumes a unidirectional relationship: forecast variable influenced by predictor variables, but not vice versa.
Vector AR allow for feedback relationships. All variables treated symmetrically.
i.e., all variables are now treated as "endogenous".

- Personal consumption may be affected by disposable income, and vice-versa.
- e.g., Govt stimulus package in Dec 2008 increased Christmas spending which increased incomes.

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Vector autoregressions

VAR(1)

$$y_{1,t} = c_1 + \phi_{11,1} y_{1,t-1} + \phi_{12,1} y_{2,t-1} + e_{1,t}$$
$$y_{2,t} = c_2 + \phi_{21,1} y_{1,t-1} + \phi_{22,1} y_{2,t-1} + e_{2,t}$$

Forecasts:

$$\hat{y}_{1,T+1|T} = \hat{c}_1 + \hat{\phi}_{11,1} y_{1,T} + \hat{\phi}_{12,1} y_{2,T}$$
$$\hat{y}_{2,T+1|T} = \hat{c}_2 + \hat{\phi}_{21,1} y_{1,T} + \hat{\phi}_{22,1} y_{2,T}.$$

$$\hat{y}_{1,T+2|T} = \hat{c}_1 + \hat{\phi}_{11,1} \hat{y}_{1,T+1} + \hat{\phi}_{12,1} \hat{y}_{2,T+1}$$
$$\hat{y}_{2,T+2|T} = \hat{c}_2 + \hat{\phi}_{21,1} \hat{y}_{1,T+1} + \hat{\phi}_{22,1} \hat{y}_{2,T+1}.$$

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VARs are useful when

- forecasting a collection of related variables where no explicit interpretation is required;
- testing whether one variable is useful in forecasting another (the basis of Granger causality tests);
- impulse response analysis, where the response of one variable to a sudden but temporary change in another variable is analysed;
- forecast error variance decomposition, where the proportion of the forecast variance of one variable is attributed to the effect of other variables.

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VAR example

```
> ar(usconsumption,order=3)
$ar
, , 1       consumption  income
consumption       0.222  0.0424
income            0.475 -0.2390

, , 2       consumption  income
consumption      0.2001 -0.0977
income           0.0288 -0.1097

, , 3       consumption  income
consumption       0.235 -0.0238
income            0.406 -0.0923

$var.pred
            consumption income
consumption       0.393  0.193
income            0.193  0.735
```

VAR example

```
> library(vars)
> VARselect(usconsumption, lag.max=8, type="const")$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     5      1      1      5
> var <- VAR(usconsumption, p=3, type="const")
> serial.test(var, lags.pt=10, type="PT.asymptotic")
Portmanteau Test (asymptotic)
data:  Residuals of VAR object var
Chi-squared = 33.3837, df = 28, p-value = 0.2219
```

VAR example

```
> summary(var)
VAR Estimation Results:
=========================
Endogenous variables: consumption, income
Deterministic variables: const
Sample size: 161

Estimation results for equation consumption:
============================================
               Estimate Std. Error t value Pr(>|t|)
consumption.l1  0.22280    0.08580   2.597 0.010326 *
income.l1       0.04037    0.06230   0.648 0.518003
consumption.l2  0.20142    0.09000   2.238 0.026650 *
income.l2      -0.09830    0.06411  -1.533 0.127267
consumption.l3  0.23512    0.08824   2.665 0.008530 **
income.l3      -0.02416    0.06139  -0.394 0.694427
const           0.31972    0.09119   3.506 0.000596 ***
```

VAR example

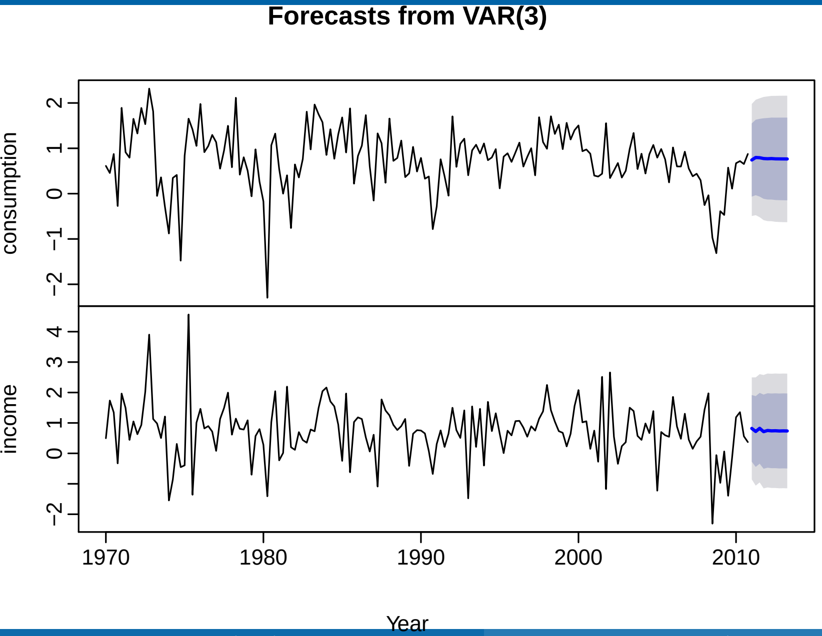
```
Estimation results for equation income:
=======================================
                Estimate Std. Error t value Pr(>|t|)
consumption.l1   0.48705    0.11637   4.186 4.77e-05 ***
income.l1       -0.24881    0.08450  -2.945 0.003736 **
consumption.l2   0.03222    0.12206   0.264 0.792135
income.l2       -0.11112    0.08695  -1.278 0.203170
consumption.l3   0.40297    0.11967   3.367 0.000959 ***
income.l3       -0.09150    0.08326  -1.099 0.273484
const            0.36280    0.12368   2.933 0.003865 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation matrix of residuals:
            consumption income
consumption      1.0000 0.3639
income           0.3639 1.0000
```

VAR example

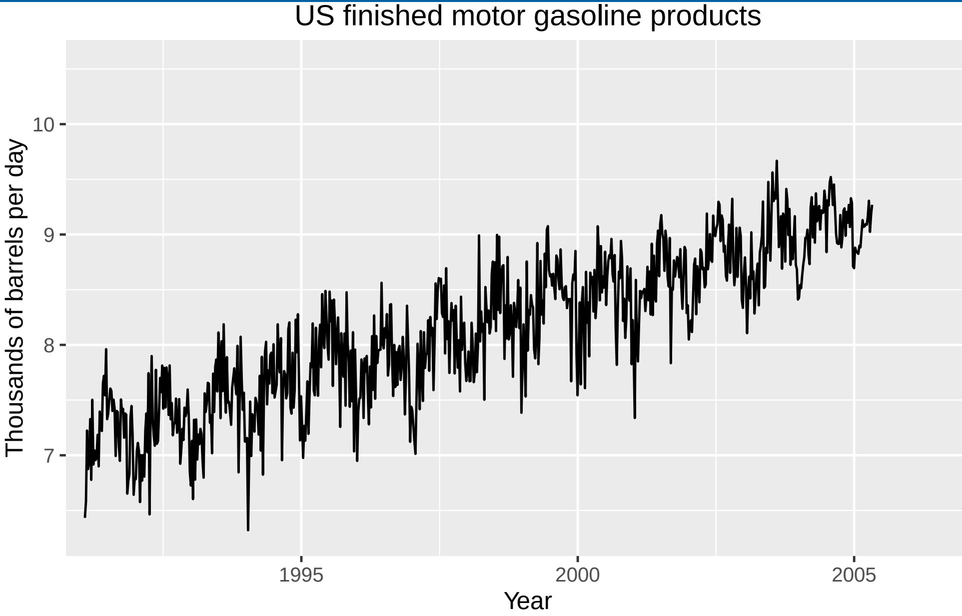
```r
fcst <- forecast(var)
plot(fcst, xlab="Year")
```

VAR example

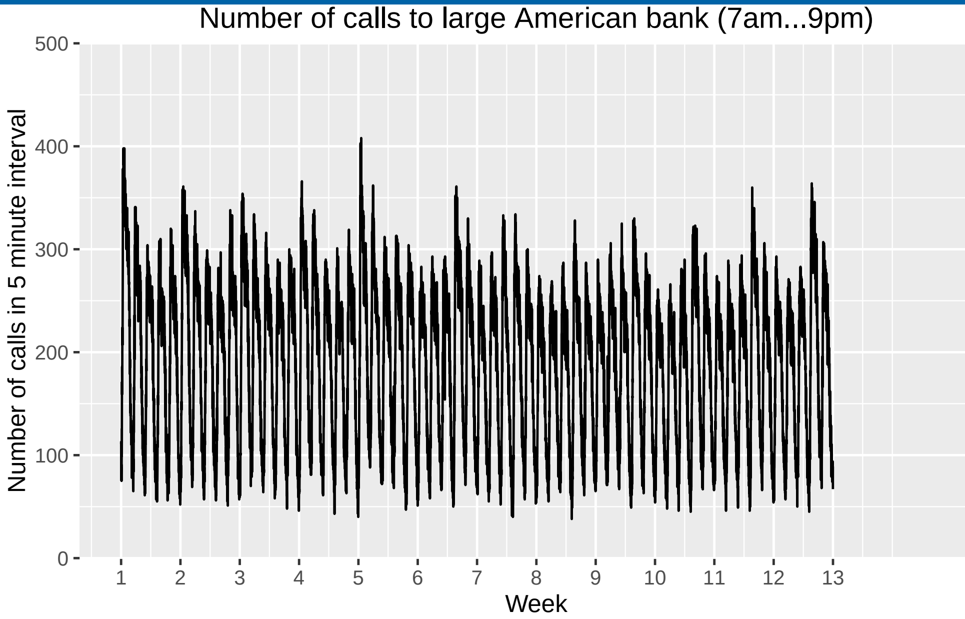

Forecasts from VAR(3)
consumption
2
1
0
−1
−2
income
4
3
2
1
0
−2
1970
1980
1990
2000
2010
Year

Outline

Examples

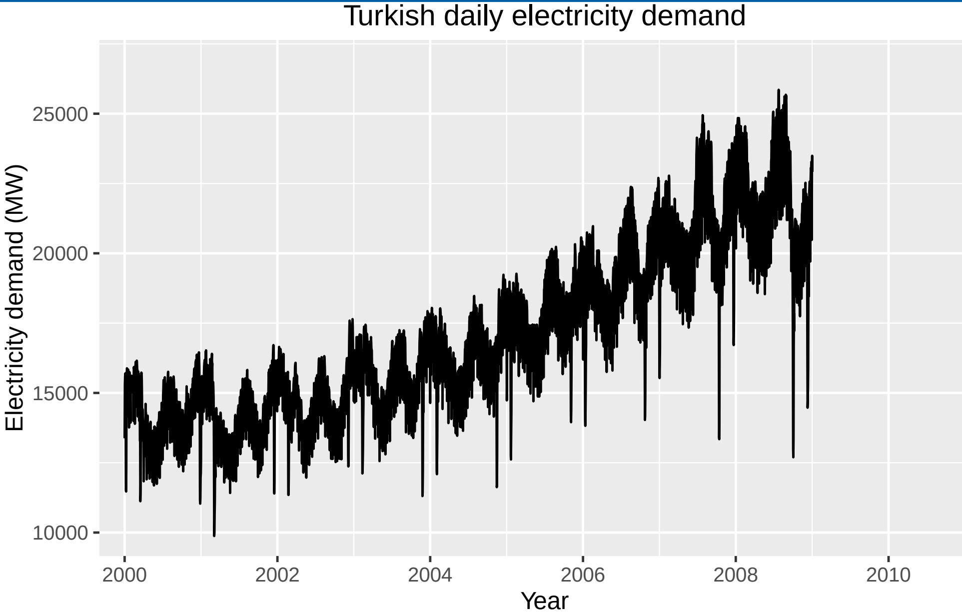

US finished motor gasoline products
Thousands of barrels per day
10
9
8
7
1995
2000
2005
Year

Examples



Number of calls to large American bank (7am...9pm)
Number of calls in 5 minute interval
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10
11
12
13
Week

Examples



Turkish daily electricity demand
Electricity demand (MW)
25000
20000
15000
10000
2000
2002
2004
2006
2008
2010
Year

TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$
\begin{aligned}
y_t &= \text{observation at time } t\\
y_t^{(\omega)} &= \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0;\\ \log y_t & \text{if } \omega = 0. \end{cases}\\
y_t^{(\omega)} &= \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t\\
\ell_t &= \ell_{t-1} + \phi b_{t-1} + \alpha d_t\\
b_t &= (1-\phi)b + \phi b_{t-1} + \beta d_t\\
d_t &= \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t
\end{aligned}
$$

$$
s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad
\begin{aligned}
s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t\\
s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t
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TBATS model

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Box-Cox transformation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

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global and local trend

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ARMA error

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ARMA error

Fourier-like seasonal terms

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$b_t = (1 -$

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TBATS

Trigonometric

Box-Cox

ARMA

Trend

Seasonal

Box-Cox transformation

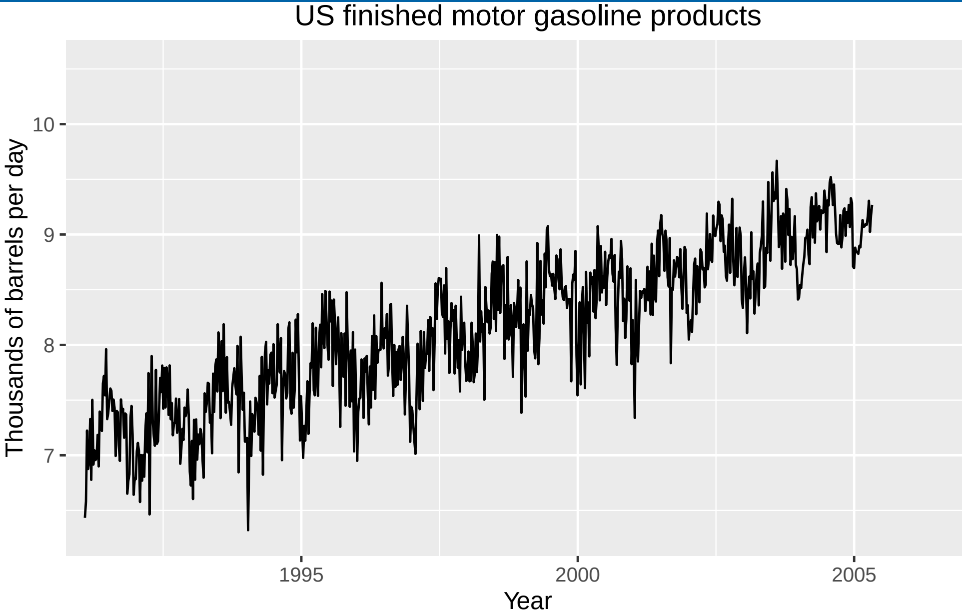
M seasonal periods

global and local trend

ARMA error

Fourier-like seasonal terms

Complex seasonality



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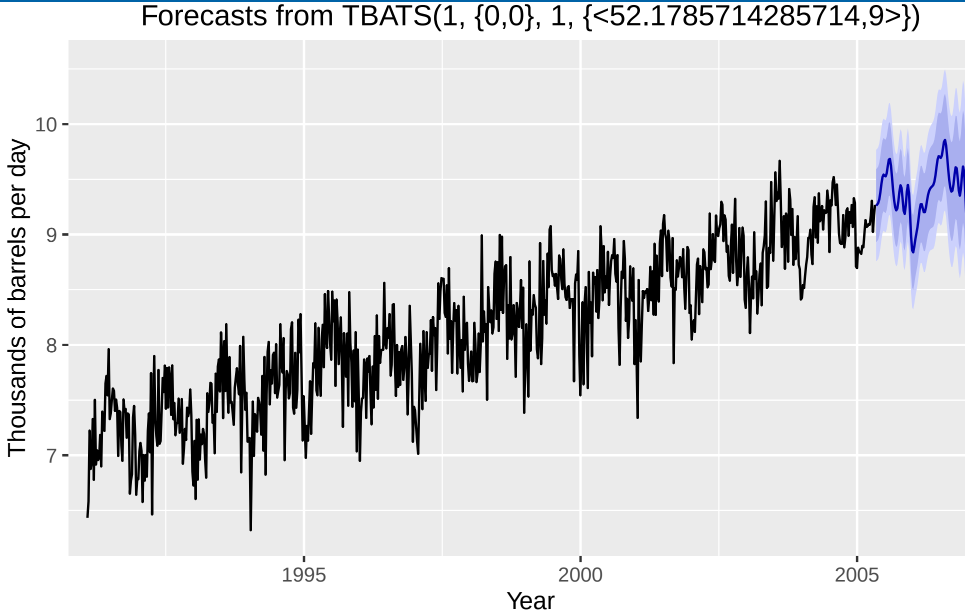

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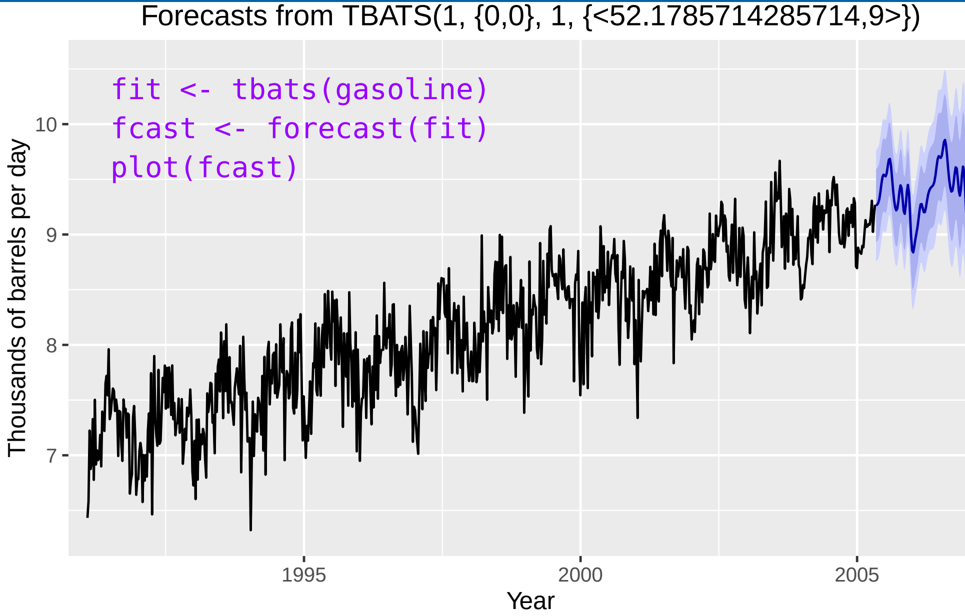
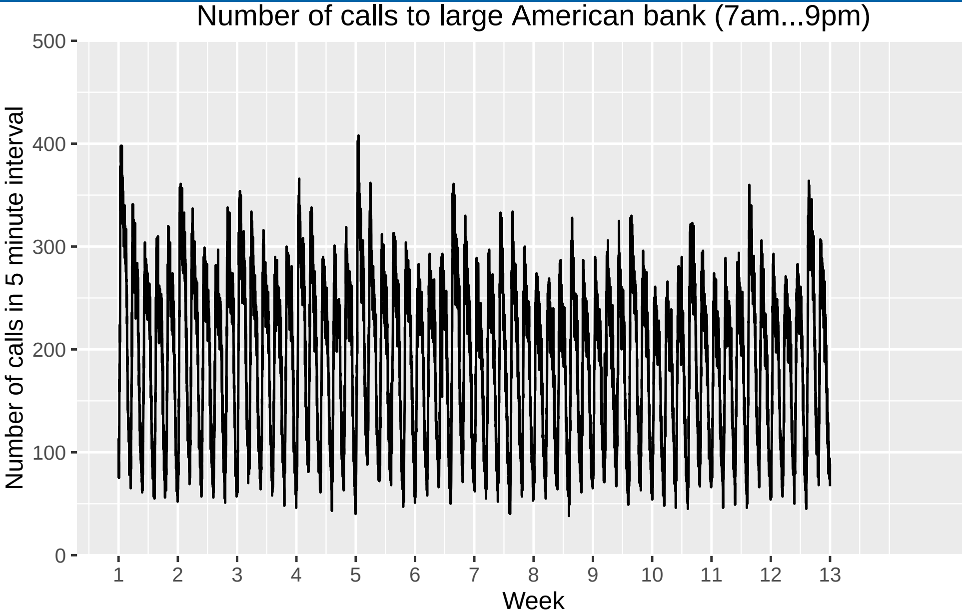



```r
fit <- tbats(gasoline)
fcast <- forecast(fit)
plot(fcast)
```

Complex seasonality



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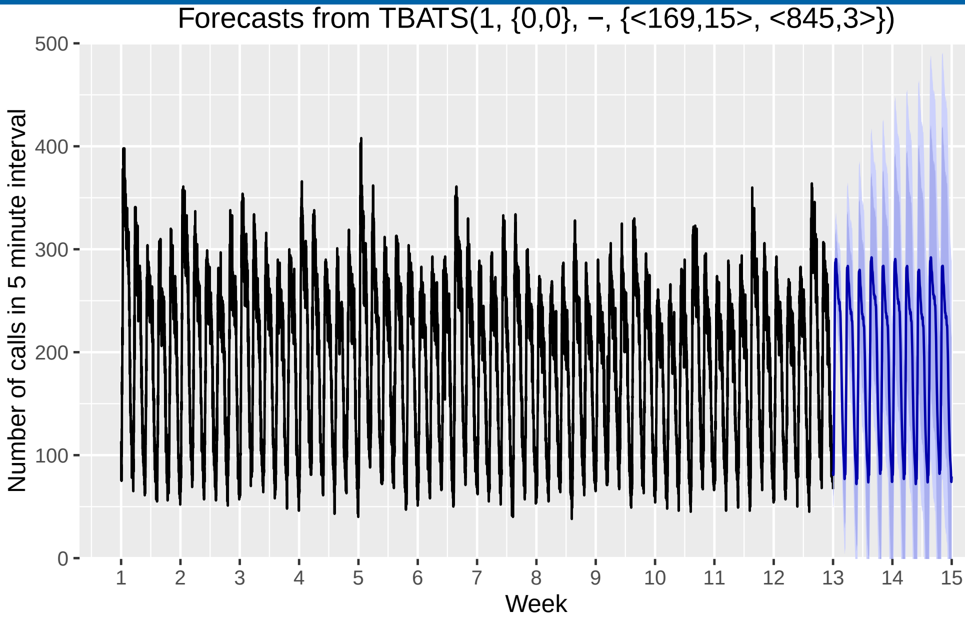

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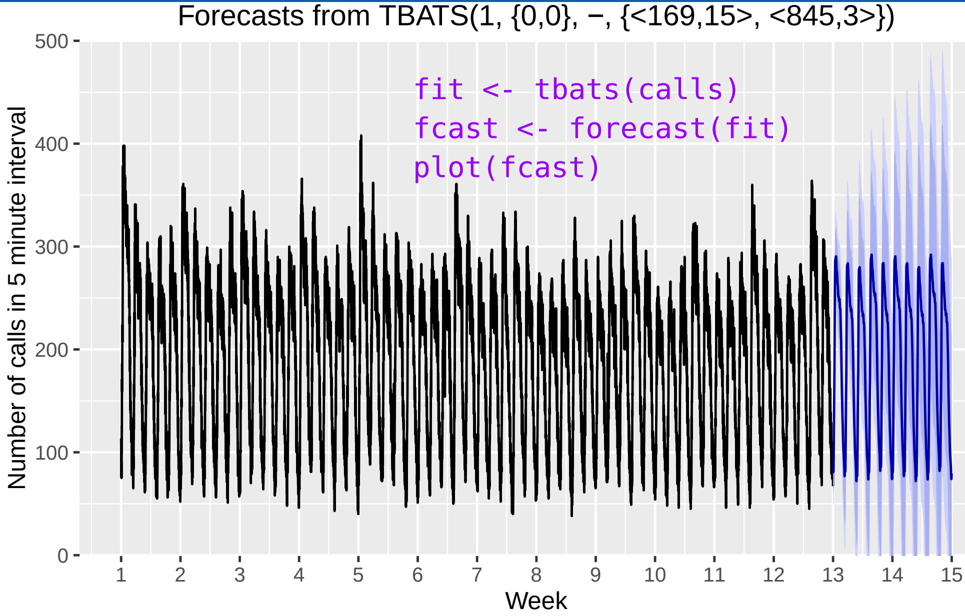

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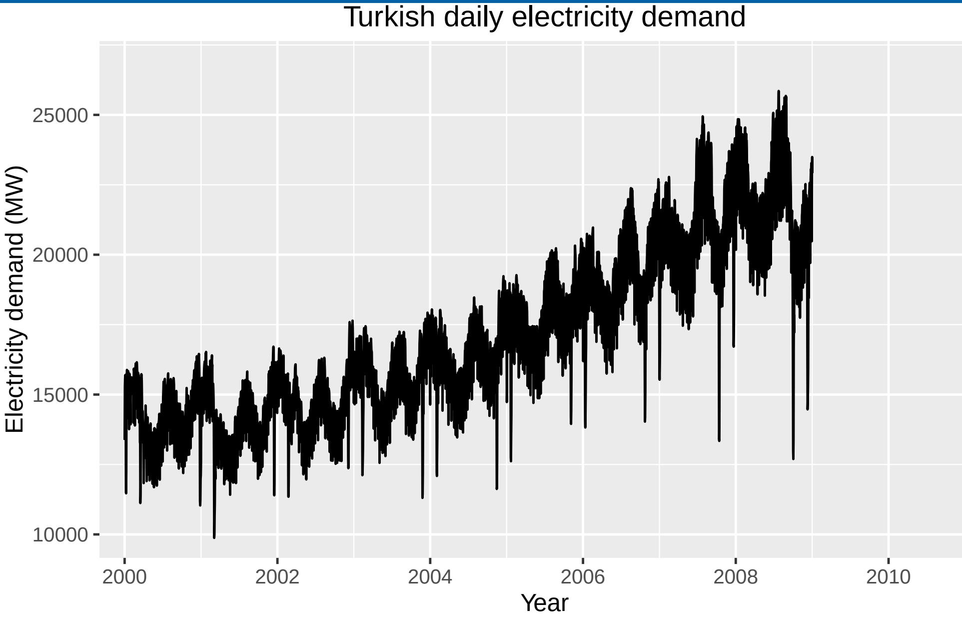

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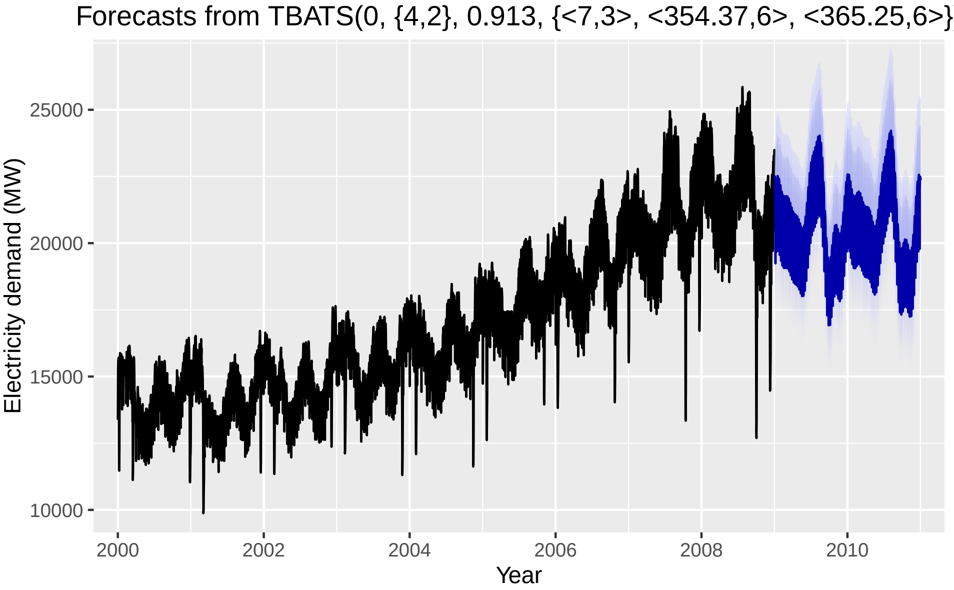
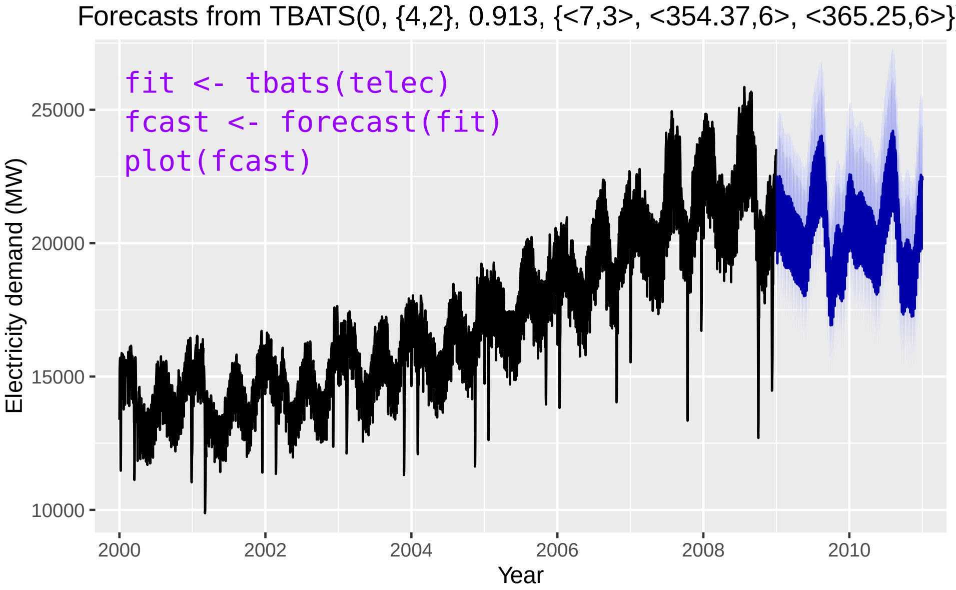


Complex seasonality

```
fit <- tbats(telec)
fcast <- forecast(fit)
plot(fcast)
```



TBATS model

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Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

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Outline

Lab Session 17

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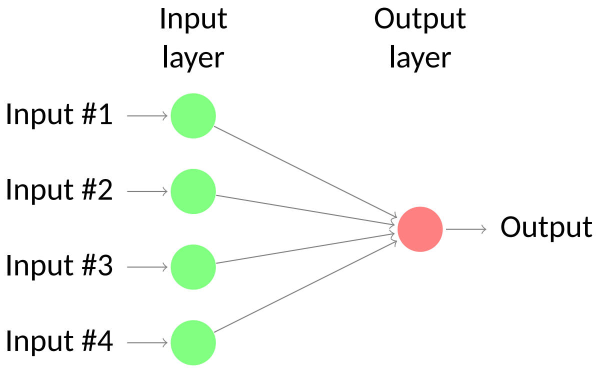
Neural network models

Simplest version: linear regression



- Coefficients attached to predictors are called "weights".
- Forecasts are obtained by a linear combination of inputs.
- Weights selected using a "learning algorithm" that minimises a "cost function".

Neural network models

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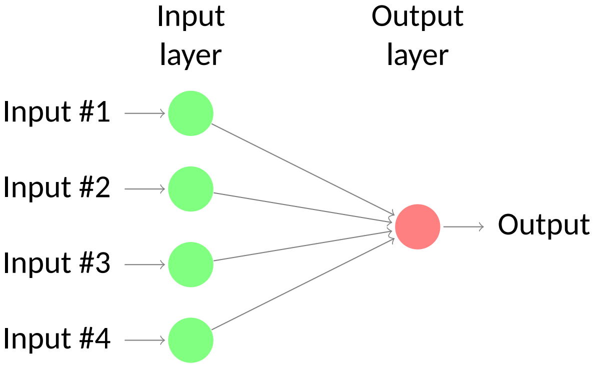

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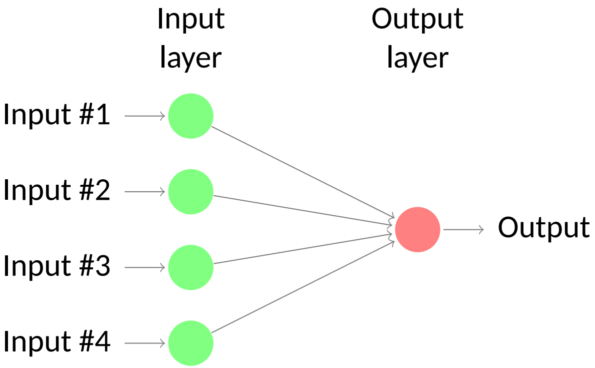

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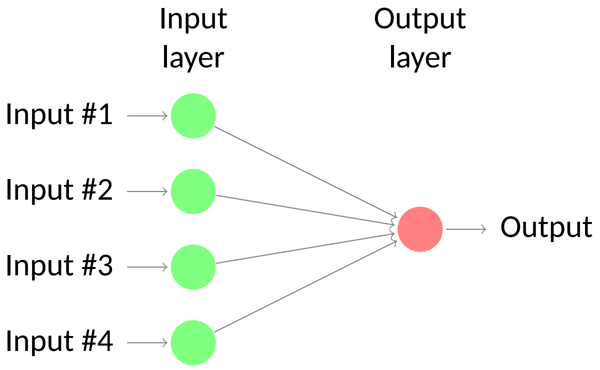

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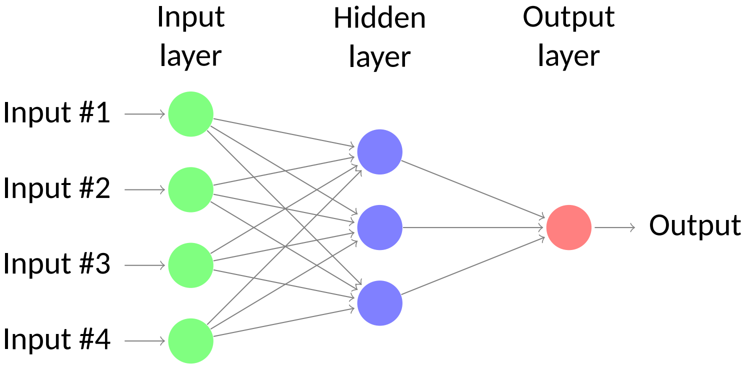
Neural network models

Nonlinear model with one hidden layer



- A **multilayer feed-forward network** where each layer of nodes receives inputs from the previous layers.
- Inputs to each node combined using linear combination.
- Result modified by nonlinear function before being output.

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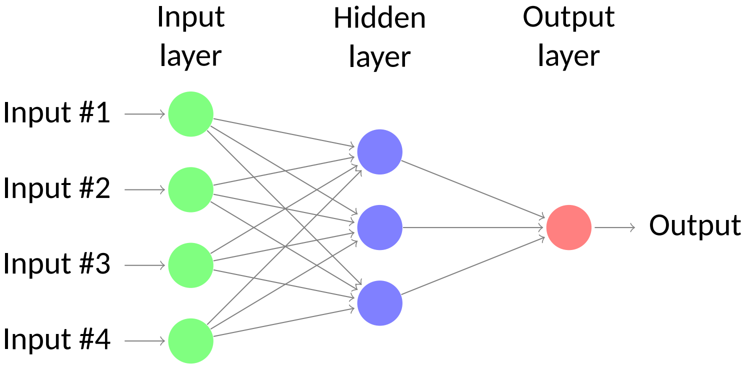

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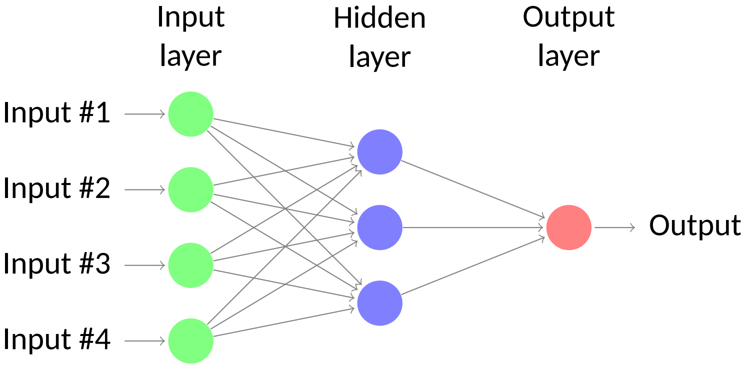

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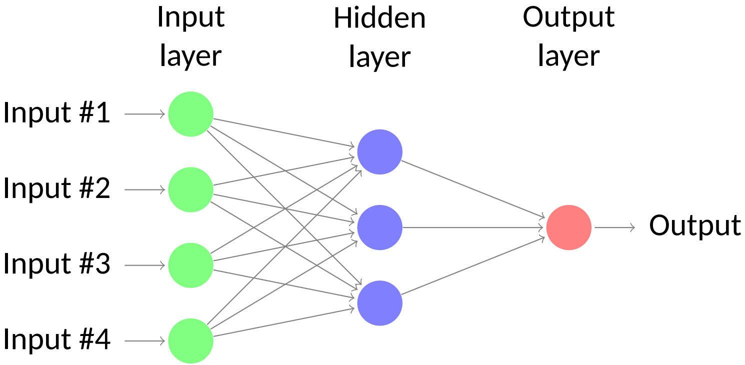

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Neural network models

Inputs to hidden neuron *j* linearly combined:

$$z_j = b_j + \sum_{i=1}^{4} w_{i,j} x_i.$$

Modified using nonlinear function such as a sigmoid:

$$s(z) = \frac{1}{1 + e^{-z}},$$

This tends to reduce the effect of extreme input values, thus making the network somewhat robust to outliers.

Neural network models

- Weights take random values to begin with, which are then updated using the observed data.
- There is an element of randomness in the predictions. So the network is usually trained several times using different random starting points, and the results are averaged.
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NNAR models

- Lagged values of the time series can be used as inputs to a neural network.
- NNAR(p, k): p lagged inputs and k nodes in the single hidden layer.
- NNAR(p, 0) model is equivalent to an ARIMA(p, 0, 0) model but without stationarity restrictions.
- Seasonal NNAR(p, P, k): inputs $(y_{t-1}, y_{t-2}, \dots, y_{t-p}, y_{t-m}, y_{t-2m}, y_{t-Pm})$ and k neurons in the hidden layer.
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NNAR models in R

- The `nnetar()` function fits an NNAR$(p, P, k)_m$ model.
- If p and P are not specified, they are automatically selected.
- For non-seasonal time series, default $p =$ optimal number of lags (according to the AIC) for a linear AR(p) model.
- For seasonal time series, defaults are $P = 1$ and p is chosen from the optimal linear model fitted to the seasonally adjusted data.
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Sunspots

- Surface of the sun contains magnetic regions that appear as dark spots.
- These affect the propagation of radio waves and so telecommunication companies like to predict sunspot activity in order to plan for any future difficulties.
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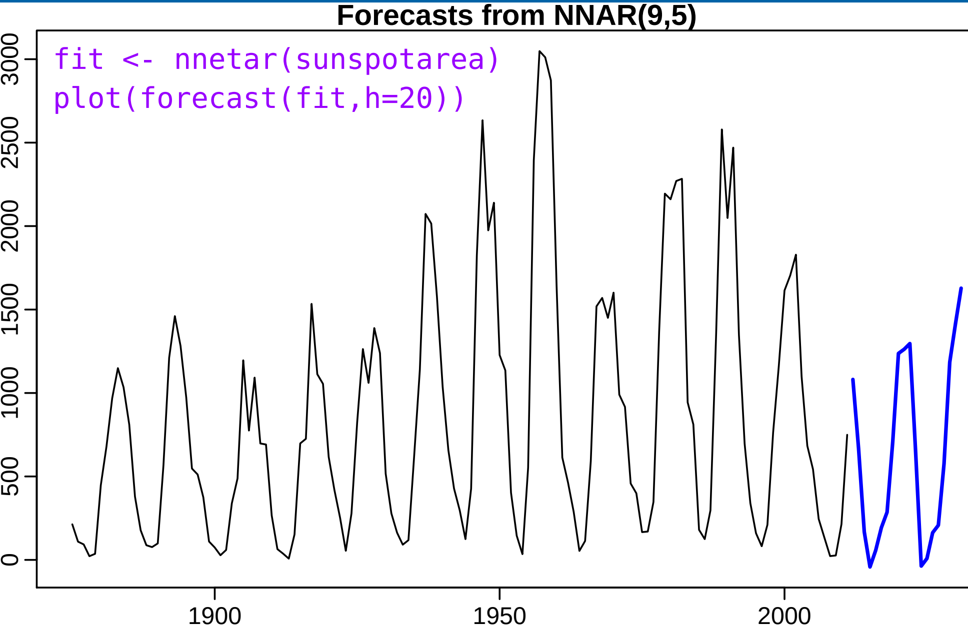
NNAR(9,5) model for sunspots




```r
fit <- nnetar(sunspotarea)
plot(forecast(fit,h=20))
```

Outline

Lab Session 18

Outline

Lab Session 19