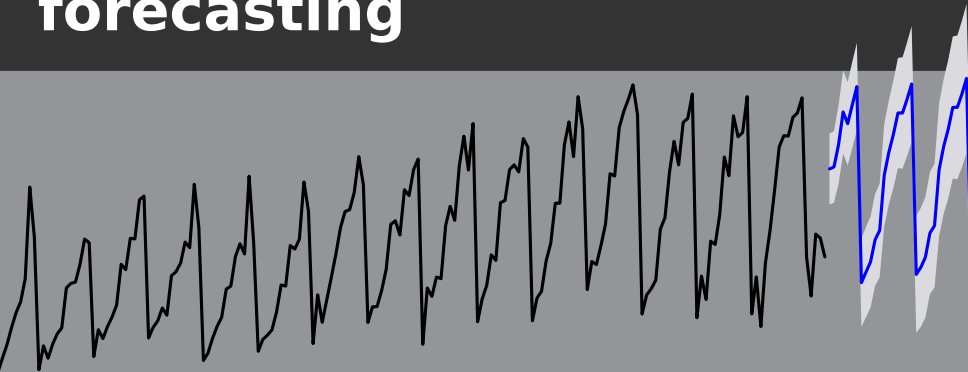




MONASH BUSINESS SCHOOL

Rob J Hyndman

# Automatic algorithms for time series forecasting



# Follow along using R



## Requirements

Install the fpp package and its dependencies.

# Motivation

- 1 Common in business to have over 1000 products that need forecasting at least monthly.
- 2 Forecasts are often required by people who are untrained in time series analysis.

## Specifications

Automatic forecasting algorithms must:

- ➡ determine an appropriate time series model;
- ➡ estimate the parameters;
- ➡ compute the forecasts with prediction intervals.

# Motivation

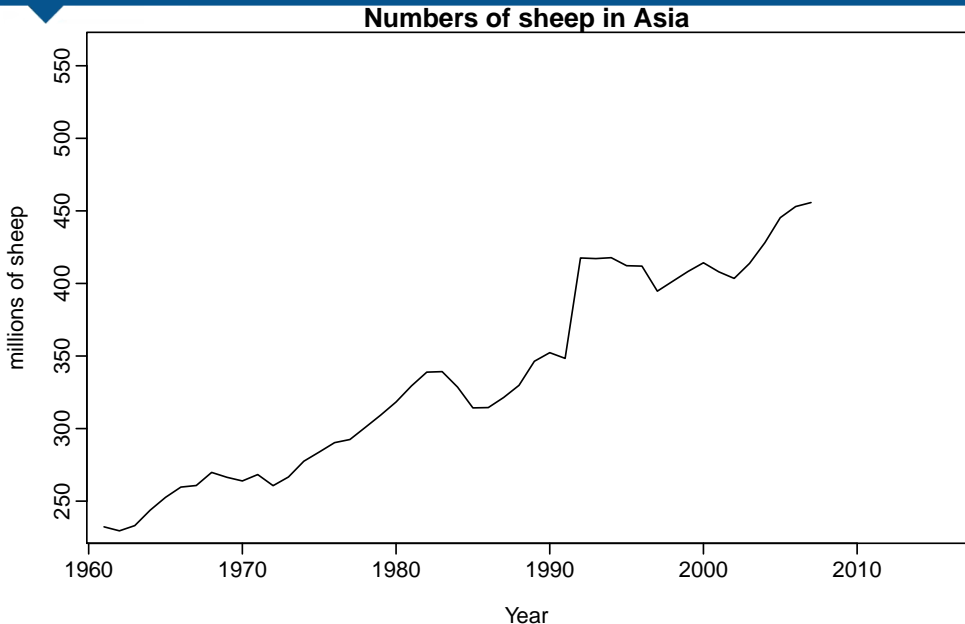
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## Specifications

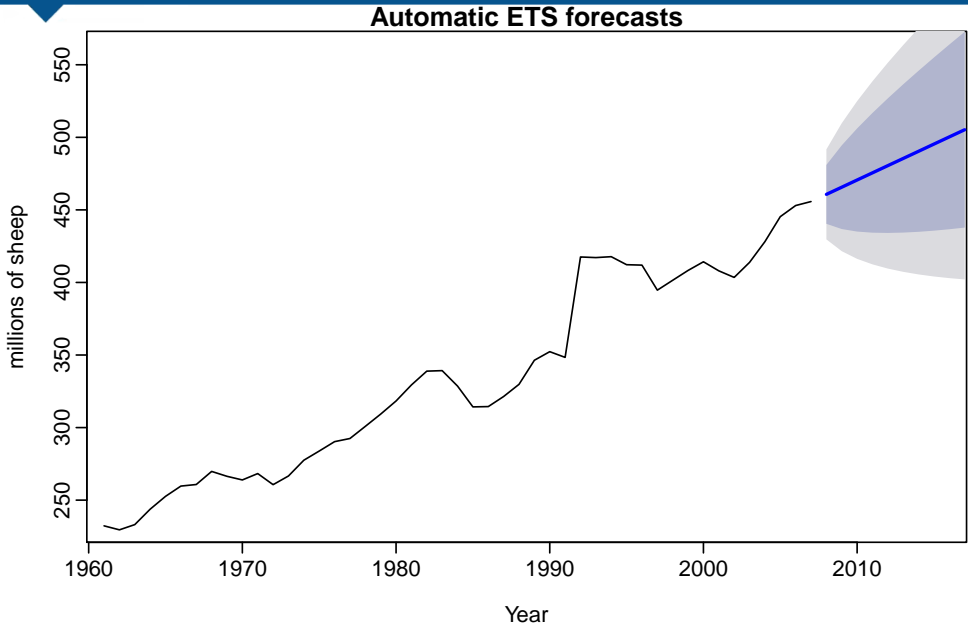
Automatic forecasting algorithms must:

- ➡ determine an appropriate time series model;
- ➡ estimate the parameters;
- ➡ compute the forecasts with prediction intervals.

# Example: Asian sheep

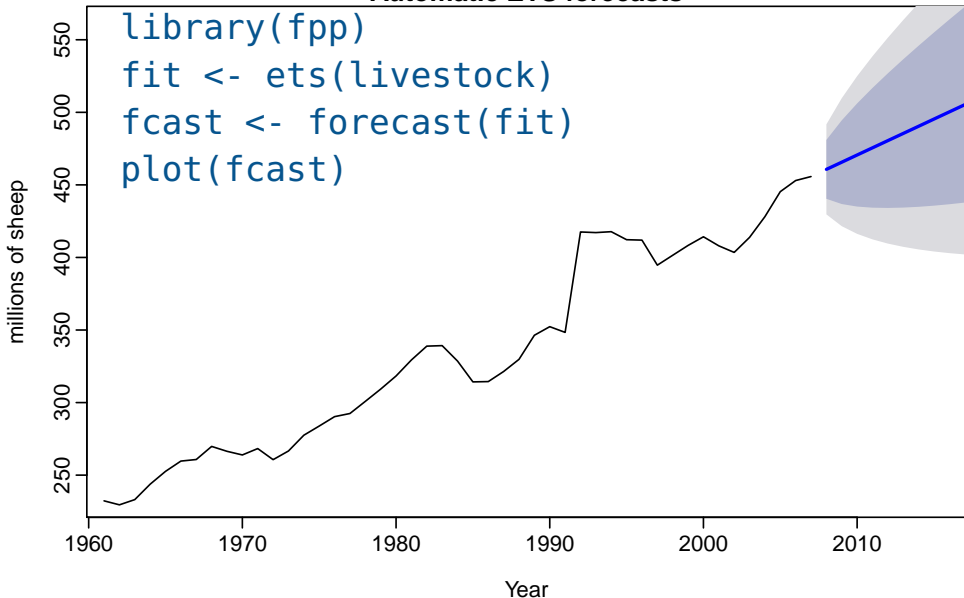


# Example: Asian sheep



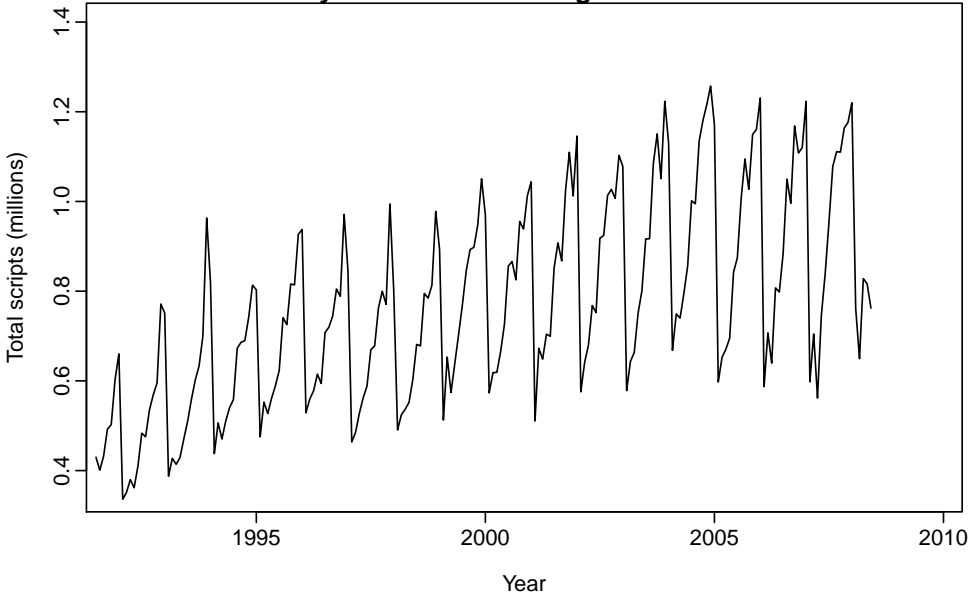
# Example: Asian sheep

Automatic ETS forecasts



# Example: Cortecosteroid sales

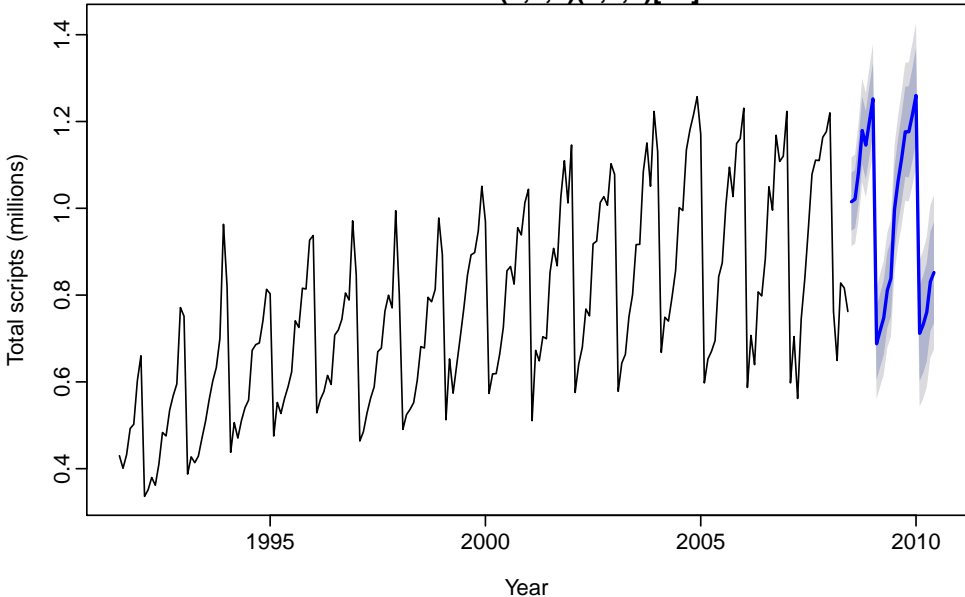
Monthly cortecosteroid drug sales in Australia





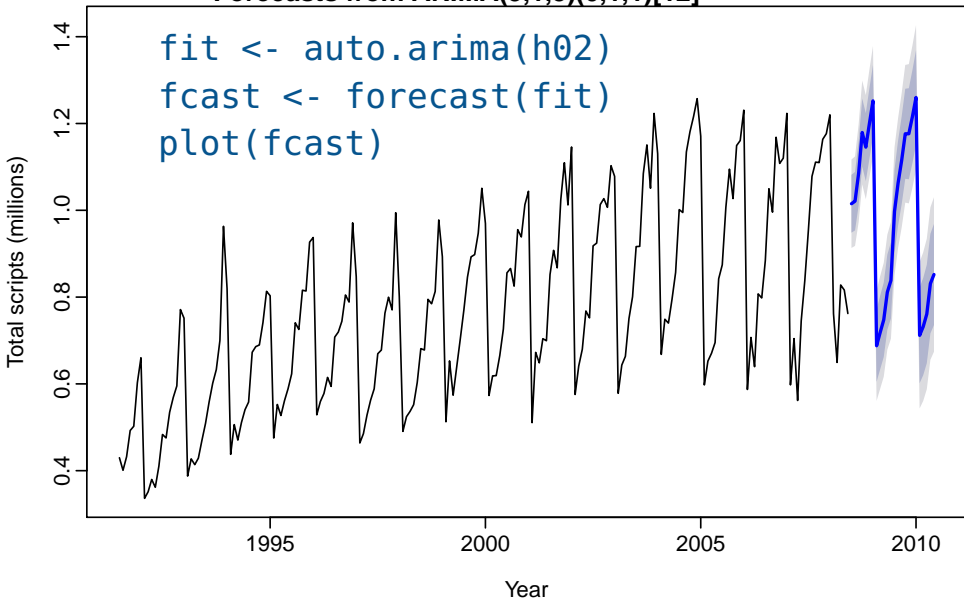
# Example: Cortecosteroid sales

Forecasts from  $ARIMA(3,1,3)(0,1,1)[12]$



# Auto ARIMA

Forecasts from ARIMA(3,1,3)(0,1,1)[12]



# Outline

- 1 Forecasting competitions**
- 2 Exponential smoothing
- 3 ARIMA modelling
- 4 Automatic nonlinear forecasting?
- 5 Time series with complex seasonality
- 6 Recent developments

# Makridakis and Hibon (1979)

*J. R. Statist. Soc. A* (1979),  
142, Part 2, pp. 97-145

## Accuracy of Forecasting: An Empirical Investigation

By SPYROS MAKRIDAKIS and MICHÈLE HIBON

*INSEAD—The European Institute of Business Administration*

[Read before the ROYAL STATISTICAL SOCIETY on Wednesday, December 13th, 1978,  
the President, SIR CLAUS MOSER in the Chair]

### SUMMARY

In this study, the authors used 111 time series to examine the accuracy of various forecasting methods, particularly time-series methods. The study shows, at least for time series, why some methods achieve greater accuracy than others for different types of data. The authors offer some explanation of the seemingly conflicting conclusions of past empirical research on the accuracy of forecasting. One novel contribution of the paper is the development of regression equations expressing accuracy as a function of factors such as randomness, seasonality, trend-cycle and the number of data points describing the series. Surprisingly, the study shows that for these 111 series simpler methods perform well in comparison to the more complex and statistically sophisticated ARMA models.

*Keywords:* FORECASTING; TIME SERIES; FORECASTING ACCURACY

### 0. INTRODUCTION

THE ultimate test of any forecast is whether or not it is capable of predicting future events

# Makridakis and Hibon (1979)

*J. R. Statist. Soc. A* (1979),  
142, Part 2, pp. 97–145

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**Keywords:** FORECASTING; TIME SERIES; FORECASTING ACCURACY

### 0. INTRODUCTION

THE ultimate test of any forecast is whether or not it is capable of predicting future events

# Makridakis and Hibon (1979)

This was the first large-scale empirical evaluation of time series forecasting methods.

- Highly controversial at the time.
- Difficulties:
  - ✗ How to measure forecast accuracy?
  - ✗ How to apply methods consistently and objectively?
  - ✗ How to explain unexpected results?
- Common thinking was that the more sophisticated mathematical models (ARIMA models at the time) were necessarily better.
- If results showed ARIMA models not best, it must be because analyst was unskilled.

# Consequences of M&H (1979)

As a result of this paper, researchers started to:

- ➔ consider how to automate forecasting methods;
- ➔ study what methods give the best forecasts;
- ➔ be aware of the dangers of over-fitting;
- ➔ treat forecasting as a different problem from time series analysis.

Makridakis & Hibon followed up with a new competition in 1982:

- 1001 series
- Anyone could submit forecasts (avoiding the charge of incompetence)
- Multiple forecast measures used.

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## **The Accuracy of Extrapolation (Time Series) Methods: Results of a Forecasting Competition**

S. MAKRIDAKIS

*INSEAD, Fontainebleau, France*

A. ANDERSEN

*University of Sydney, Australia*

R. CARBONE

*Université Laval, Quebec, Canada*

R. FILDES

*Manchester Business School, Manchester, England*

M. HIBON

*INSEAD, Fontainebleau, France*

R. LEWANDOWSKI

*Marketing Systems, Essen, Germany*

J. NEWTON

E. PARZEN

*Texas A & M University, Texas, U.S.A.*

R. WINKLER

*Indiana University, Bloomington, U.S.A.*

### **ABSTRACT**

In the last few decades many methods have become available for forecasting. As always, when alternatives exist, choices need to be made so that an appropriate forecasting method can be selected and used for the specific situation being considered. This paper reports the results of a forecasting competition that provides information to facilitate such choice. Seven experts in each of the 24 methods forecasted up to 1001 series for six up to eighteen time horizons. The results of the competition are presented in this paper whose purpose is to provide empirical evidence about *differences* found to exist among the various extrapolative (time series) methods used in the competition.

# M-competition

## Main findings

- 1 Statistically sophisticated or complex methods do not necessarily provide more accurate forecasts than simpler ones.
- 2 The relative ranking of the performance of the various methods varies according to the accuracy measure being used.
- 3 The accuracy when various methods are being combined outperforms, on average, the individual methods being combined and does very well in comparison to other methods.
- 4 The accuracy of the various methods depends upon the length of the forecasting horizon involved.

## The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon\*

*INSEAD, Boulevard de Constance, 77305 Fontainebleau, France*

---

### Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy

# Makridakis and Hibon (2000)

“The M3-Competition is a final attempt by the authors to settle the accuracy issue of various time series methods. . . The extension involves the inclusion of more methods/ researchers (in particular in the areas of neural networks and expert systems) and more series.”

- 3003 series
- All data from business, demography, finance and economics.
- Series length between 14 and 126.
- Either non-seasonal, monthly or quarterly.
- All time series positive.
- M&H claimed that the M3-competition supported the findings of their earlier work.
- However, best performing methods far from “simple”.

# Makridakis and Hibon (2000)

## Best methods:

### Theta

- A very confusing explanation.
- Shown by Hyndman and Billah (2003) to be average of linear regression and simple exponential smoothing with drift, applied to seasonally adjusted data.
- Later, the original authors claimed that their explanation was incorrect.

### Forecast Pro

- A commercial software package with an unknown algorithm.
- Known to fit either exponential smoothing or ARIMA models using BIC.

# M3 results (recalculated)

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
ForecastPro	18.00	13.06	1.47
ForecastX	17.35	13.09	1.42
Automatic ANN	17.18	13.98	1.53
B-J automatic	19.13	13.72	1.54

# M3 results (recalculated)

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
ForecastPro	18.00	13.06	1.47
ForecastX	17.35	13.09	1.42

- Calculations do not match published paper.
- Some contestants apparently submitted multiple entries but only best ones published.

# Outline

- 1 Forecasting competitions
- 2 Exponential smoothing**
- 3 ARIMA modelling
- 4 Automatic nonlinear forecasting?
- 5 Time series with complex seasonality
- 6 Recent developments



# Exponential smoothing methods

		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
N	(None)	N,N	N,A	N,M
A	(Additive)	A,N	A,A	A,M
A <sub>d</sub>	(Additive damped)	A <sub>d</sub> ,N	A <sub>d</sub> ,A	A <sub>d</sub> ,M
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N,N: Simple exponential smoothing

# Exponential smoothing methods

Trend Component		Seasonal Component		
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A,N: Holt's linear method

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A<sub>d</sub>,N: Additive damped trend method

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- There are 15 separate exp. smoothing methods.

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- Each can have an additive or multiplicative error, giving 30 separate models.

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- Only 19 models are numerically stable.

# Exponential smoothing methods

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- There are 15 separate exp. smoothing methods.
- Each can have an additive or multiplicative error, giving 30 separate models.
- Only 19 models are numerically stable.
- Multiplicative trend models give poor forecasts leaving 15 models.

# Exponential smoothing methods

Trend Component		Seasonal Component		
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N	(None)	N,N	N,A	N,M
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General notation    E T S : Exponential Smoothing

# Exponential smoothing methods

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**General notation**    E T S : **Exponent**Tial **S**moother

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**General notation**    **E T S : Exponential Smoothing**

↑  
**Trend**

## Examples:

A,N,N: Simple exponential smoothing with additive errors

A,A,N: Holt's linear method with additive errors

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**Trend    Seasonal**

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**General notation**    **E T S : Exponential Smoothing**


**Error   Trend   Seasonal**

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# Exponential smoothing methods

## Innovations state space models

- ➔ All ETS models can be written in innovations state space form (IJF, 2002).
- ➔ Additive and multiplicative versions give the same point forecasts but different prediction intervals.

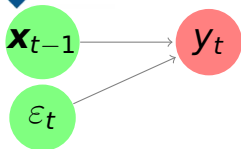
**General notation** **ETS** : **Exponential Smoothing**

  
**Error Trend Seasonal**

## Examples:

- A,N,N: Simple exponential smoothing with additive errors
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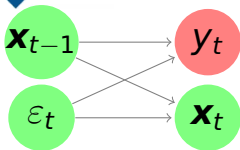
# ETS state space model



## State space model

$\mathbf{x}_t = (\text{level}, \text{slope}, \text{seasonal})$

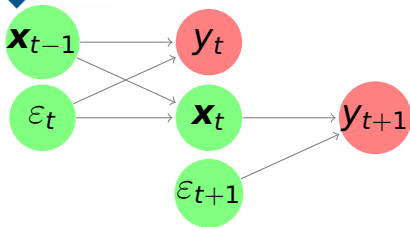
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## State space model

$\mathbf{x}_t = (\text{level, slope, seasonal})$

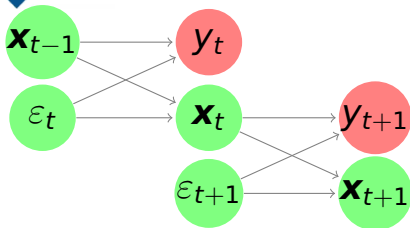
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## State space model

$\mathbf{x}_t = (\text{level, slope, seasonal})$

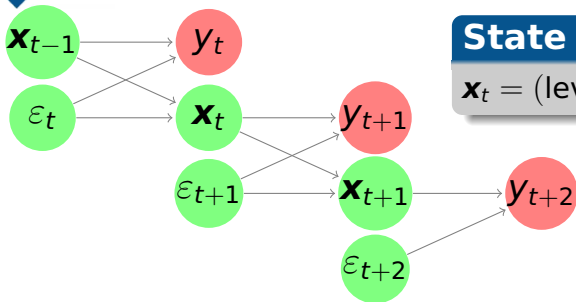
# ETS state space model



## State space model

$\mathbf{x}_t = (\text{level, slope, seasonal})$

# ETS state space model

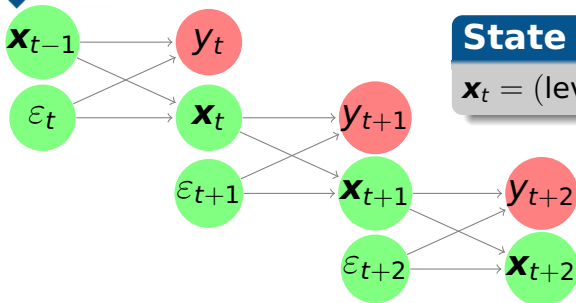


## State space model

$\mathbf{x}_t = (\text{level}, \text{slope}, \text{seasonal})$



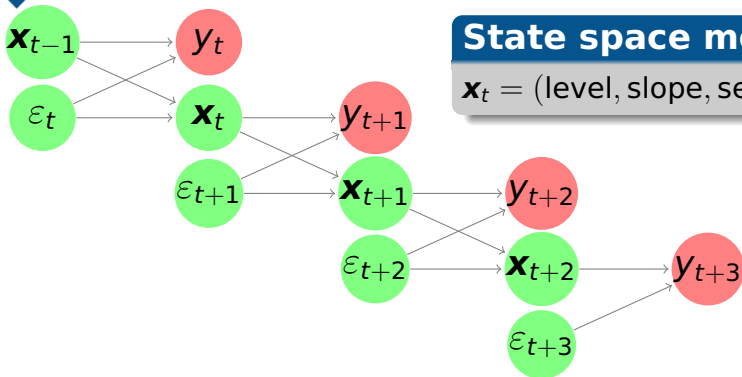
# ETS state space model



## State space model

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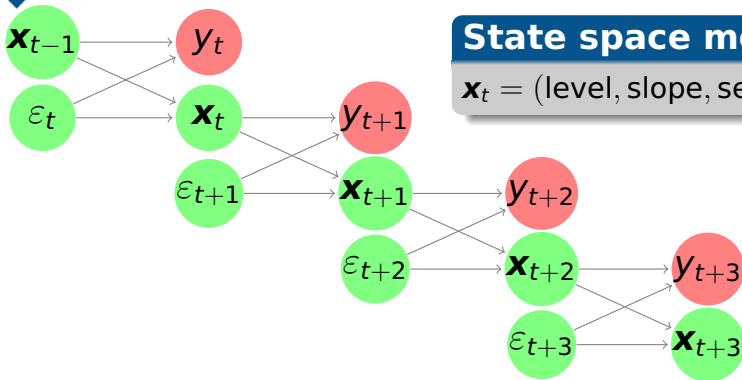
# ETS state space model



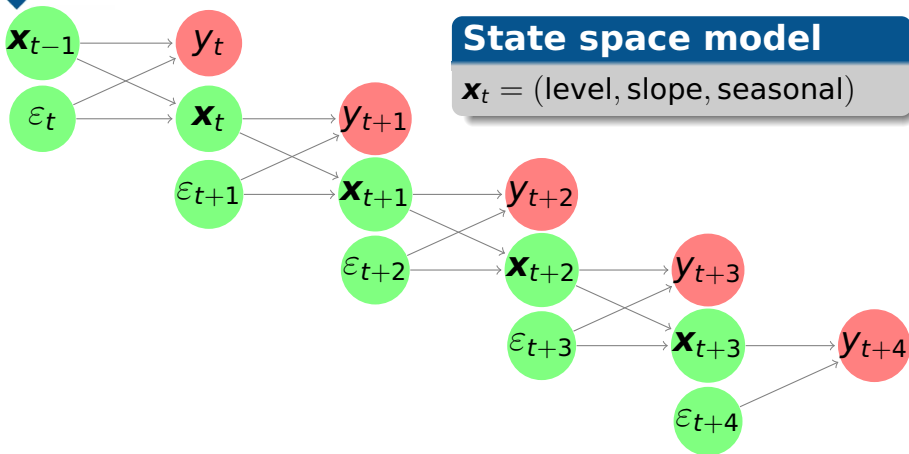
## State space model

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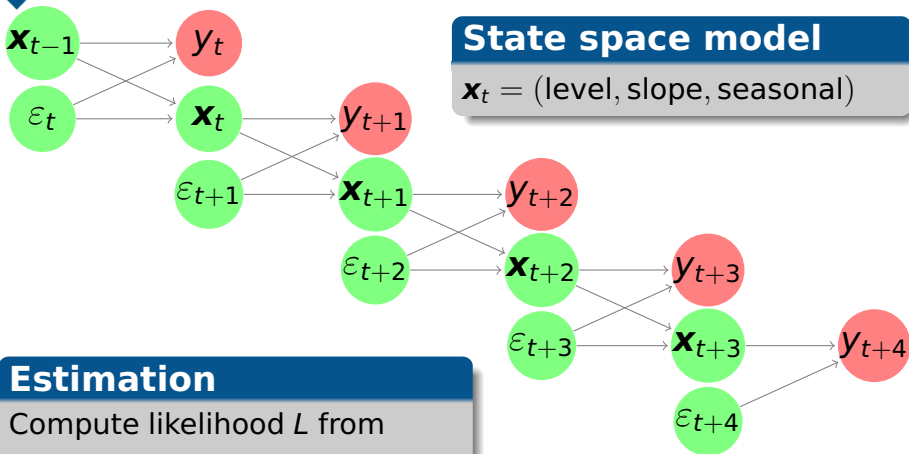
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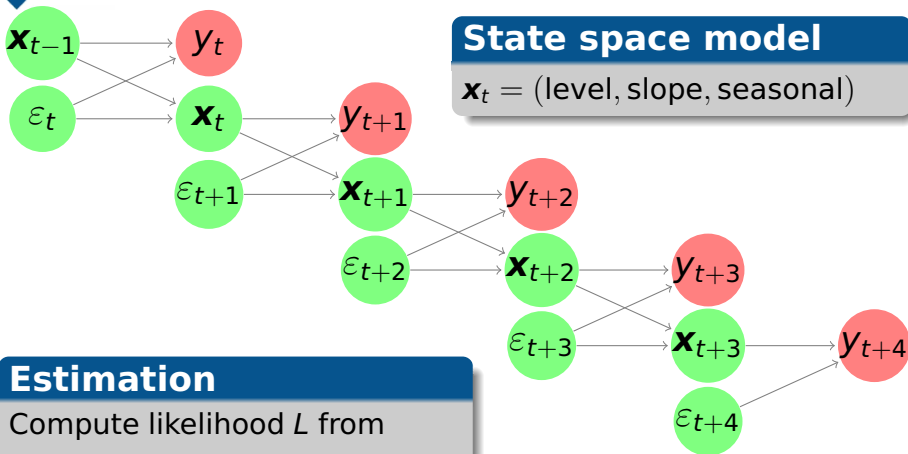
## Estimation

Compute likelihood  $L$  from

$\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T$ .

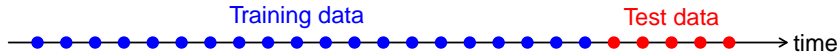
Optimize  $L$  wrt model parameters.

# ETS state space model



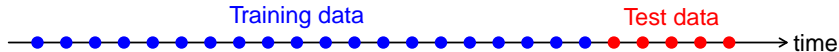
# Cross-validation

## Traditional evaluation

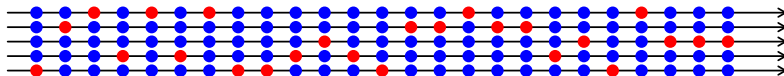


# Cross-validation

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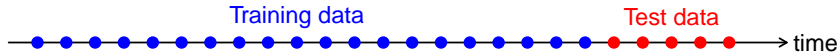
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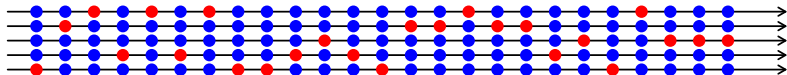


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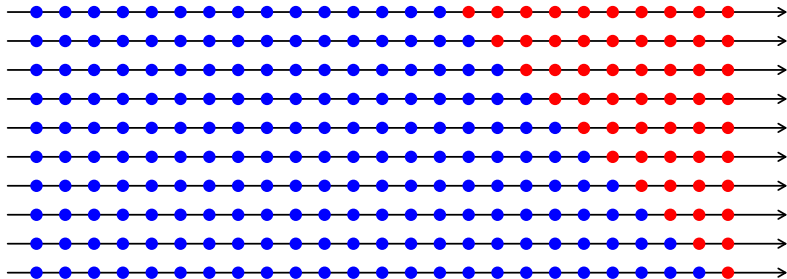
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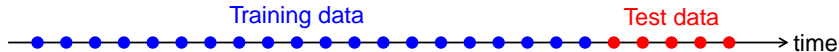


## Time series cross-validation

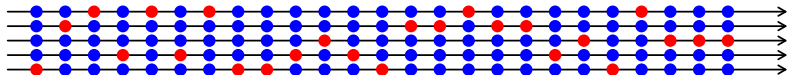


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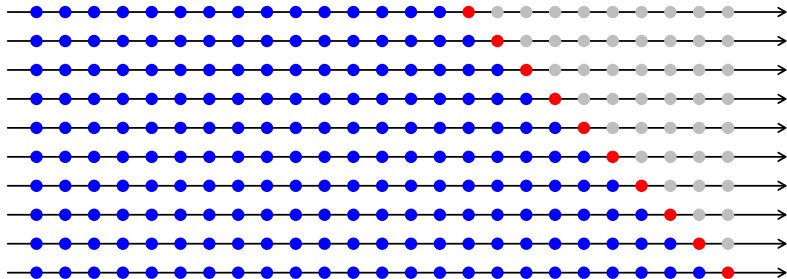
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## Standard cross-validation

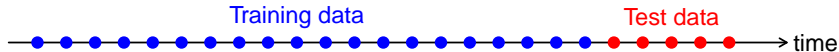


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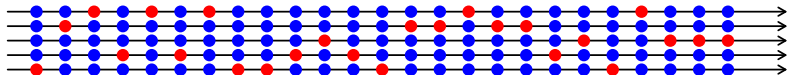


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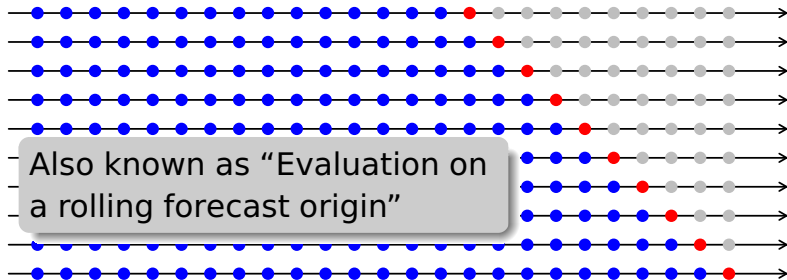
## Traditional evaluation



## Standard cross-validation



## Time series cross-validation



# Akaike's Information Criterion

$$\text{AIC} = -2 \log(L) + 2k$$

where  $L$  is the likelihood and  $k$  is the number of estimated parameters in the model.

- This is a *penalized likelihood* approach.
- If  $L$  is Gaussian, then  $\text{AIC} \approx c + T \log \text{MSE} + 2k$  where  $c$  is a constant, MSE is from one-step forecasts on **training set**, and  $T$  is the length of the series.

Minimizing the Gaussian AIC is asymptotically equivalent (as  $T \rightarrow \infty$ ) to minimizing MSE from one-step forecasts on **test set** via time series cross-validation.

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# Akaike's Information Criterion

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## Corrected AIC

For small  $T$ , AIC tends to over-fit. Bias-corrected version:

$$\text{AIC}_c = \text{AIC} + \frac{2(k+1)(k+2)}{T-k}$$

- CV-MSE too time consuming for most automatic forecasting purposes. Also requires large  $T$ .
- AICc asymptotically equivalent, can be used on small samples and is very fast to compute.

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# ets algorithm in R



**Based on Hyndman, Koehler, Snyder & Grose (IJF 2002):**

- Apply each of 15 models that are appropriate to the data. Optimize parameters and initial values using MLE.
- Select best method using AICc.
- Produce forecasts using best method.
- Obtain prediction intervals using underlying state space model.

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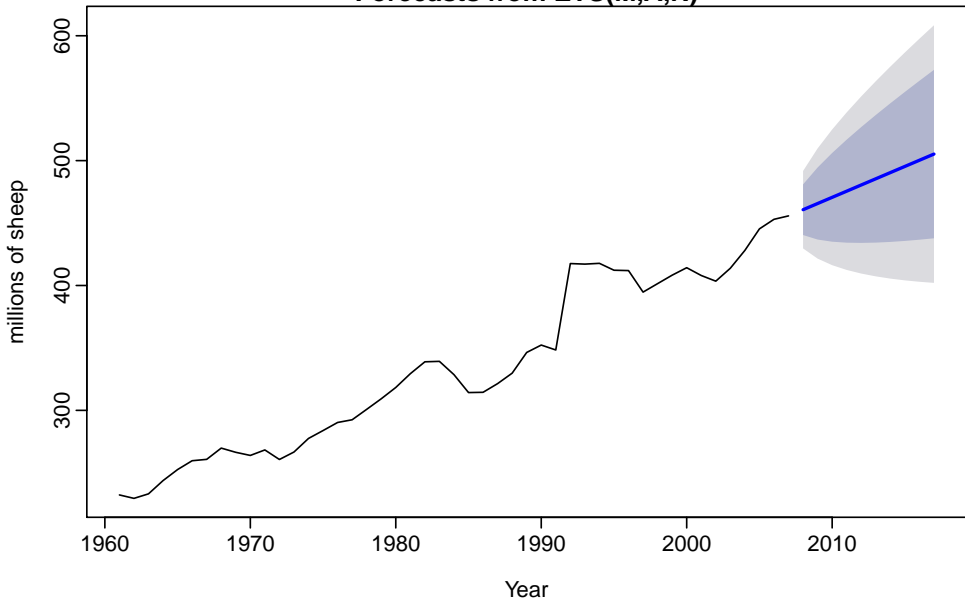


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# Exponential smoothing

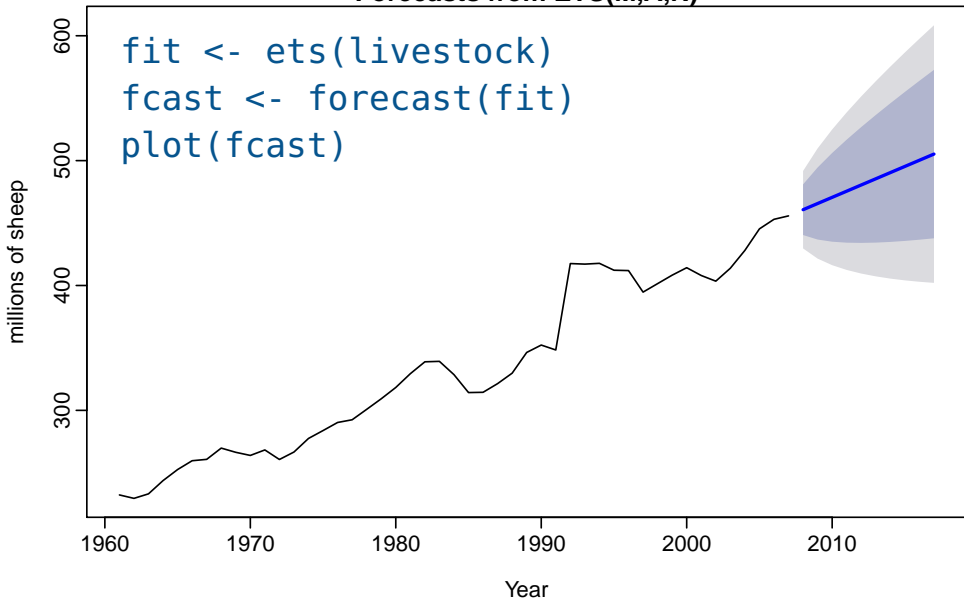
Forecasts from ETS(M,A,N)





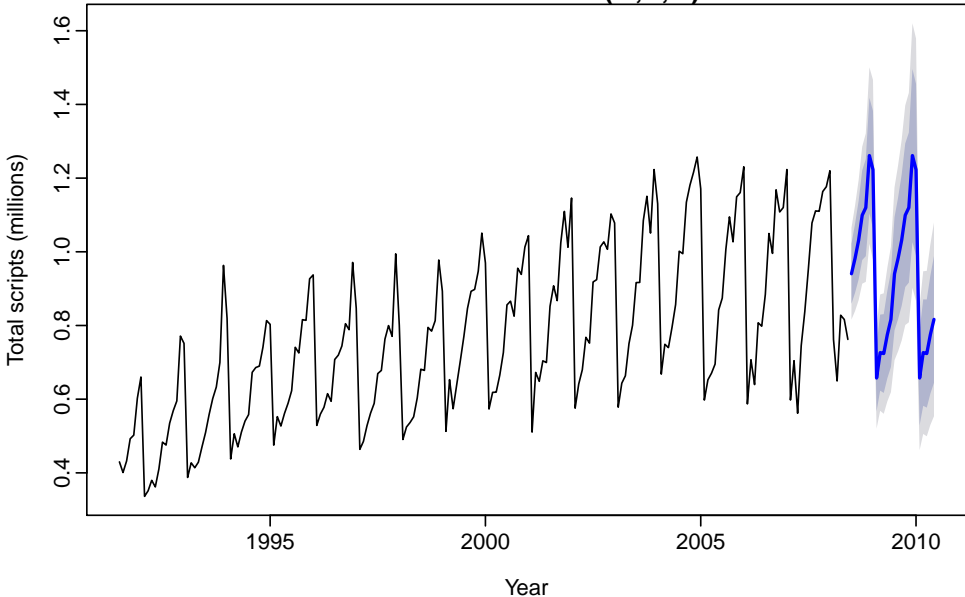
# Exponential smoothing

Forecasts from ETS(M,A,N)



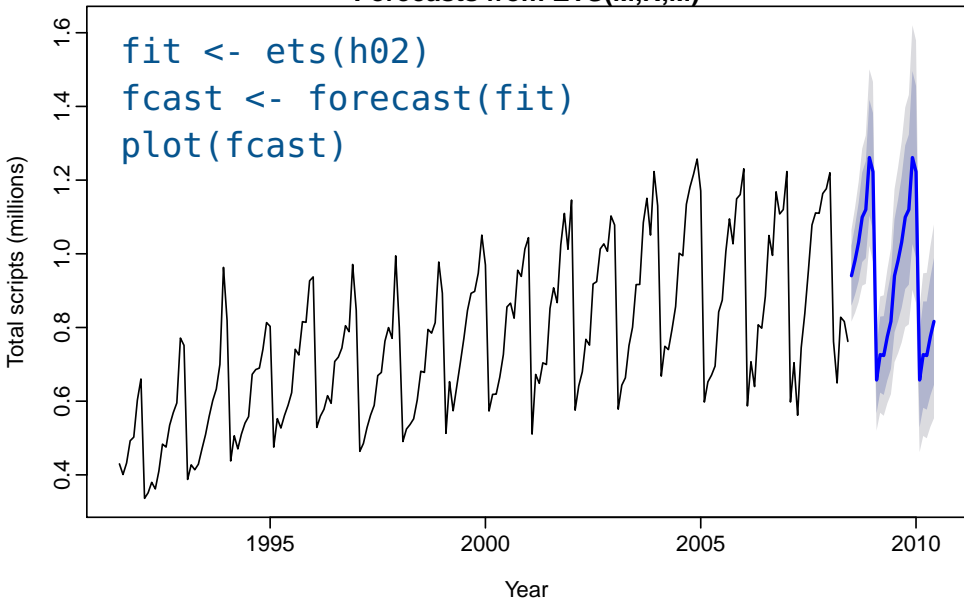
# Exponential smoothing

Forecasts from ETS(M,N,M)



# Exponential smoothing

Forecasts from ETS(M,N,M)



# Exponential smoothing

```
> fit  
ETS(M,N,M)
```

Smoothing parameters:

alpha = 0.4597

gamma = 1e-04

Initial states:

l = 0.4501

s = 0.8628 0.8193 0.7648 0.7675 0.6946 1.2921

1.3327 1.1833 1.1617 1.0899 1.0377 0.9937

sigma: 0.0675

AIC	AICc	BIC
-115.69960	-113.47738	-69.24592

# M3 comparisons

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
ForecastPro	18.00	13.06	1.47
ForecastX	17.35	13.09	1.42
Automatic ANN	17.18	13.98	1.53
B-J automatic	19.13	13.72	1.54
ETS	17.38	13.13	1.43

# Exponential smoothing

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## 7 Exponential smoothing

Exponential smoothing was proposed in the late 1950s (Brown 1959, Holt 1957 and Winters 1960 are key pioneering works) and has motivated some of the most successful forecasting methods. Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older. In other words, the more recent the observation the higher the associated weight. This framework generates reliable forecasts quickly and for a wide spectrum of time series which is a great advantage and of major importance to applications in industry.

This chapter is divided into two parts. In the first part we present in detail the mechanics of all exponential smoothing methods and their application in forecasting time series with various characteristics. This is key in understanding the intuition behind these methods. In this setting, selecting and using a forecasting method may appear to be somewhat ad-hoc. The

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# Exponential smoothing

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Springer Series in Statistics

Rob J. Hyndman · Anne B. Koehler  
J. Keith Ord · Ralph D. Snyder

## Forecasting with Exponential Smoothing

The State Space Approach

 Springer



# Exercise

- 1 Use `ets` to find the best ETS models for the following series: `ibmclose`, `eggs`, `bricksq`, `hsales`.
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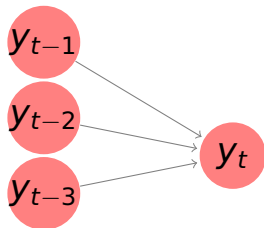
# Outline

- 1 Forecasting competitions
- 2 Exponential smoothing
- 3 ARIMA modelling**
- 4 Automatic nonlinear forecasting?
- 5 Time series with complex seasonality
- 6 Recent developments

# ARIMA models

Inputs

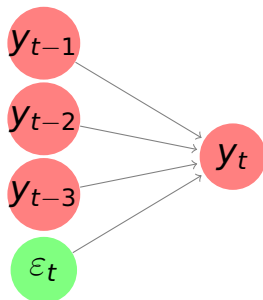
Output



# ARIMA models

Inputs

Output

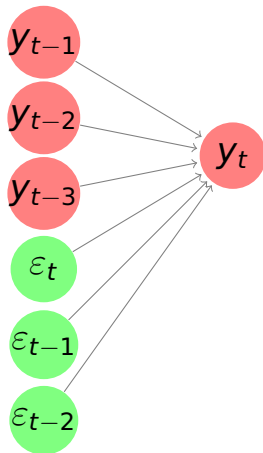


Autoregression (AR)  
model

# ARIMA models

Inputs

Output

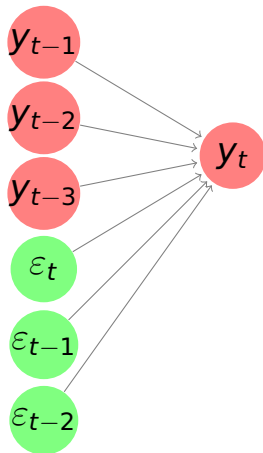


Autoregression moving average (ARMA) model

# ARIMA models

Inputs

Output



Autoregression moving average (ARMA) model

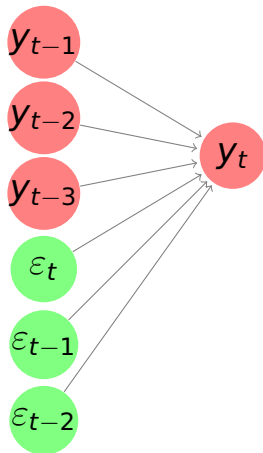
## Estimation

Compute likelihood  $L$  from  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T$ .  
Use optimization algorithm to maximize  $L$ .

# ARIMA models

Inputs

Output



Autoregression moving average (ARMA) model

## ARIMA model

Autoregression moving average (ARMA) model applied to differences.

## Estimation

Compute likelihood  $L$  from  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T$ .  
Use optimization algorithm to maximize  $L$ .



---

## *Journal of Statistical Software*

July 2008, Volume 26, Issue 3.

<http://www.jstatsoft.org/>

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### Automatic Time Series Forecasting: The forecast Package for R

Rob J. Hyndman  
Monash University

Yeasmin Khandakar  
Monash University

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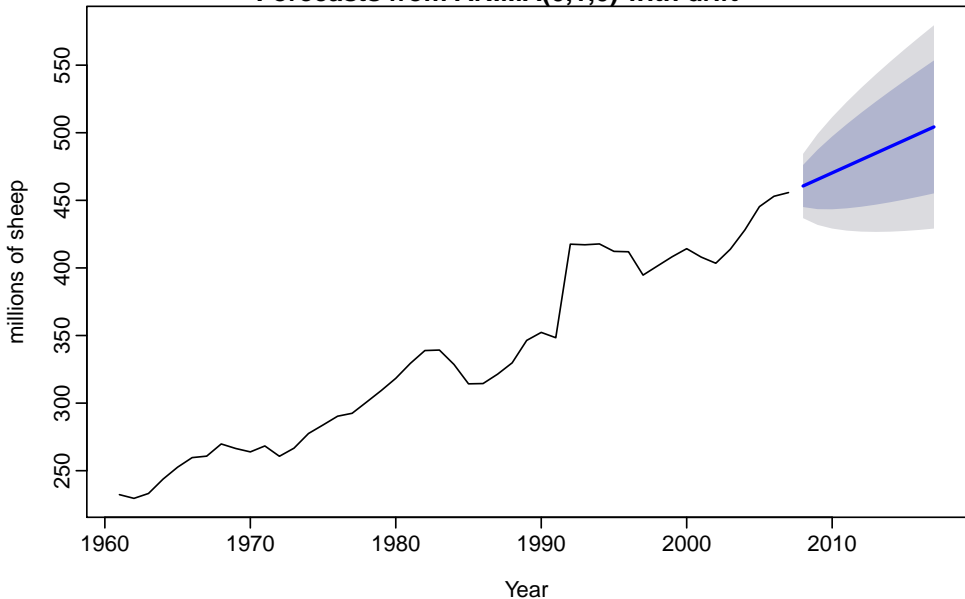
#### Abstract

Automatic forecasts of large numbers of univariate time series are often needed in



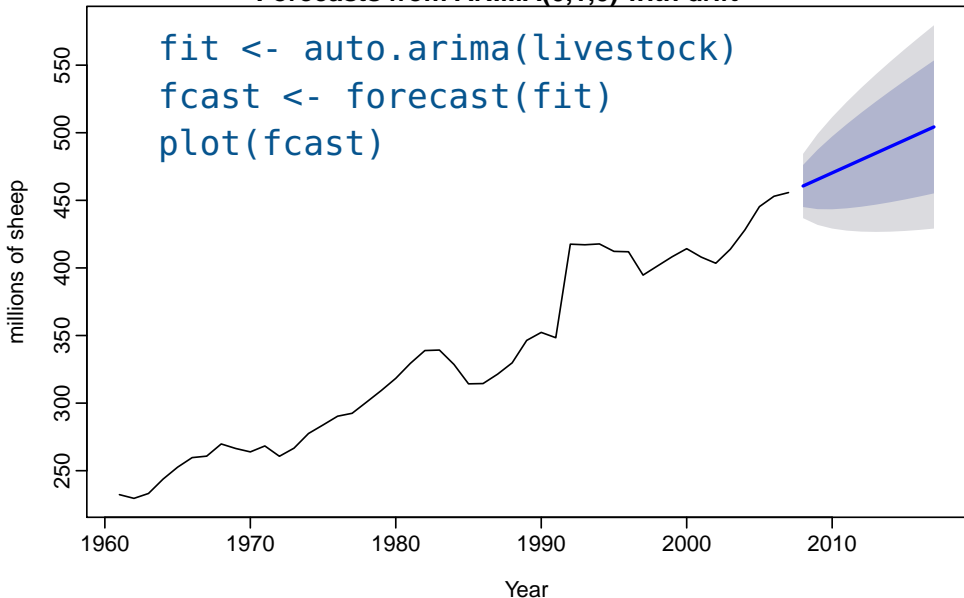
# Auto ARIMA

Forecasts from ARIMA(0,1,0) with drift



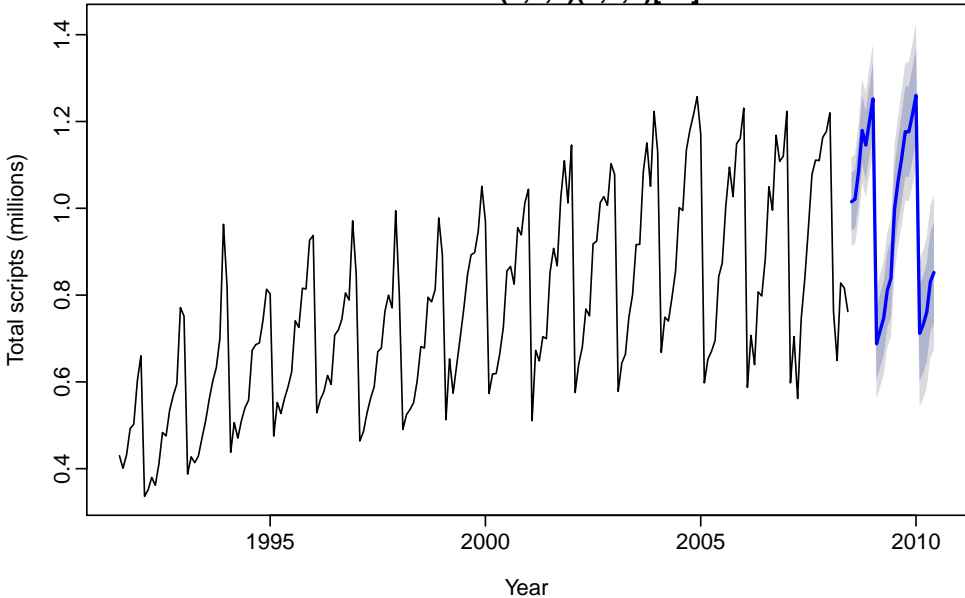
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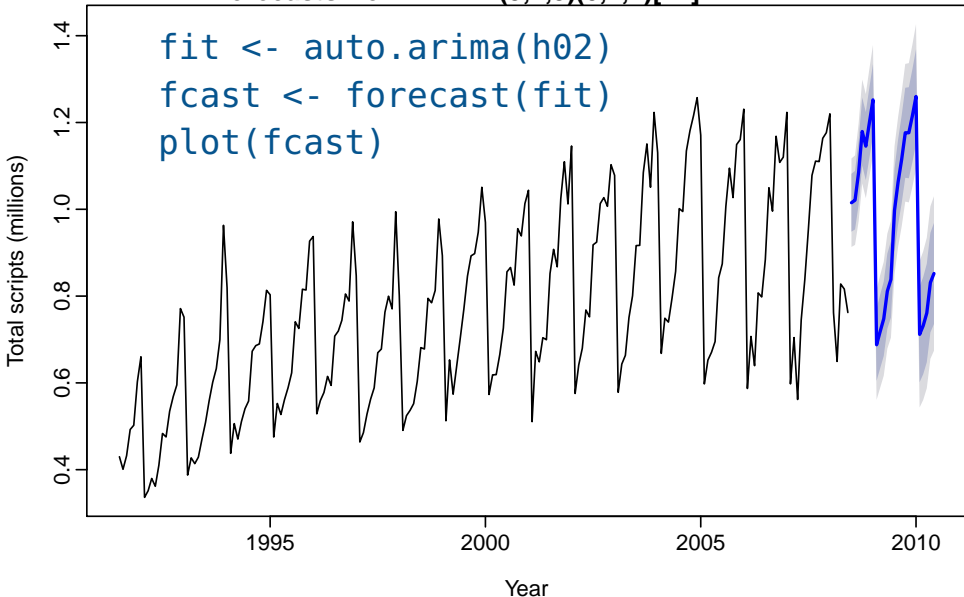
# Auto ARIMA

Forecasts from ARIMA(3,1,3)(0,1,1)[12]



# Auto ARIMA

Forecasts from ARIMA(3,1,3)(0,1,1)[12]



# Auto ARIMA

```
> fit
```

```
Series: h02
```

```
ARIMA(3,1,3)(0,1,1)[12]
```

```
Coefficients:
```

	ar1	ar2	ar3	ma1	ma2	ma3	sma1
	-0.3648	-0.0636	0.3568	-0.4850	0.0479	-0.353	-0.5931
s.e.	0.2198	0.3293	0.1268	0.2227	0.2755	0.212	0.0651

```
sigma^2 estimated as 0.002706: log likelihood=290.25
```

```
AIC=-564.5 AICc=-563.71 BIC=-538.48
```

# How does auto.arima() work?

## A non-seasonal ARIMA process

$$\phi(B)(1 - B)^d y_t = c + \theta(B)\varepsilon_t$$

Need to select appropriate orders  $p, q, d$ , and whether to include  $c$ .

Algorithm choices driven by forecast accuracy.

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### Hyndman & Khandakar (JSS, 2008) algorithm:

- Select no. differences  $d$  via KPSS unit root test.
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Algorithm choices driven by forecast accuracy.



# How does auto.arima() work?

## A seasonal ARIMA process

$$\Phi(B^m)\phi(B)(1-B)^d(1-B^m)^D y_t = c + \Theta(B^m)\theta(B)\varepsilon_t$$

Need to select appropriate orders  $p, q, d, P, Q, D$ , and whether to include  $c$ .

## Hyndman & Khandakar (JSS, 2008) algorithm:

- Select no. differences  $d$  via KPSS unit root test.
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# M3 comparisons

Method	MAPE	sMAPE	MASE
Theta	17.42	12.76	1.39
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B-J automatic	19.13	13.72	1.54
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# Exercise

- 1 Use `auto.arima` to find the best ARIMA models for the following series: `ibmclose`, `eggs`, `bricksq`, `hsales`.
- 2 Try `auto.arima` with `cangas` and `lynx`. What do you learn?
- 3 Can you find a series for which `auto.arima` gives bad forecasts?
- 4 How would you compare the ETS and ARIMA results?

# Outline

- 1 Forecasting competitions
- 2 Exponential smoothing
- 3 ARIMA modelling
- 4 Automatic nonlinear forecasting?**
- 5 Time series with complex seasonality
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# Automatic nonlinear forecasting

- Automatic ANN in M3 competition did poorly.
- Linear methods did best in the NN3 competition!
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- Some good recent work by Kourentzes and Crone on automated ANN for time series.
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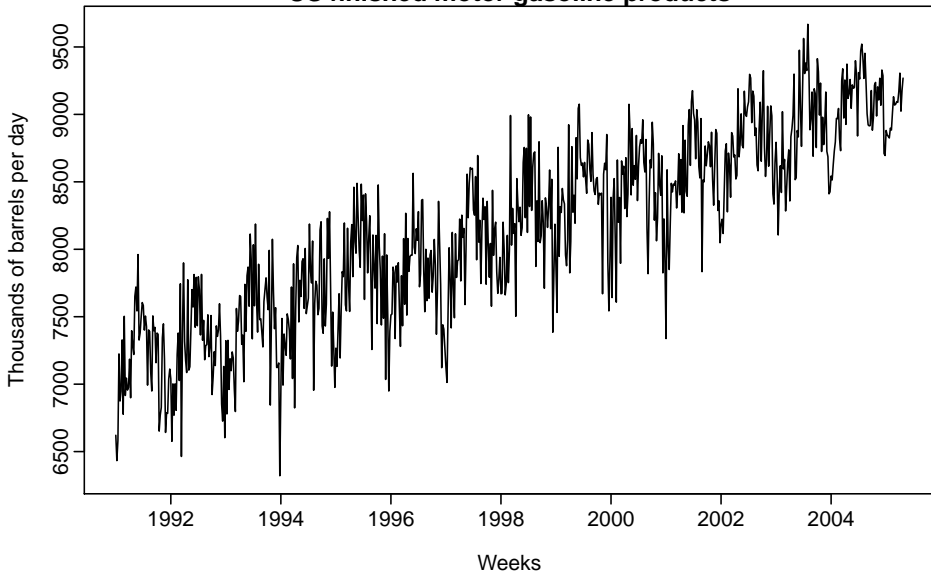
- Automatic ANN in M3 competition did poorly.
- Linear methods did best in the NN3 competition!
- Very few machine learning methods get published in the IJF because authors cannot demonstrate their methods give better forecasts than linear benchmark methods, even on supposedly nonlinear data.
- Some good recent work by Kourentzes and Crone on automated ANN for time series.
- **Watch this space!**

# Outline

- 1 Forecasting competitions
- 2 Exponential smoothing
- 3 ARIMA modelling
- 4 Automatic nonlinear forecasting?
- 5 Time series with complex seasonality**
- 6 Recent developments

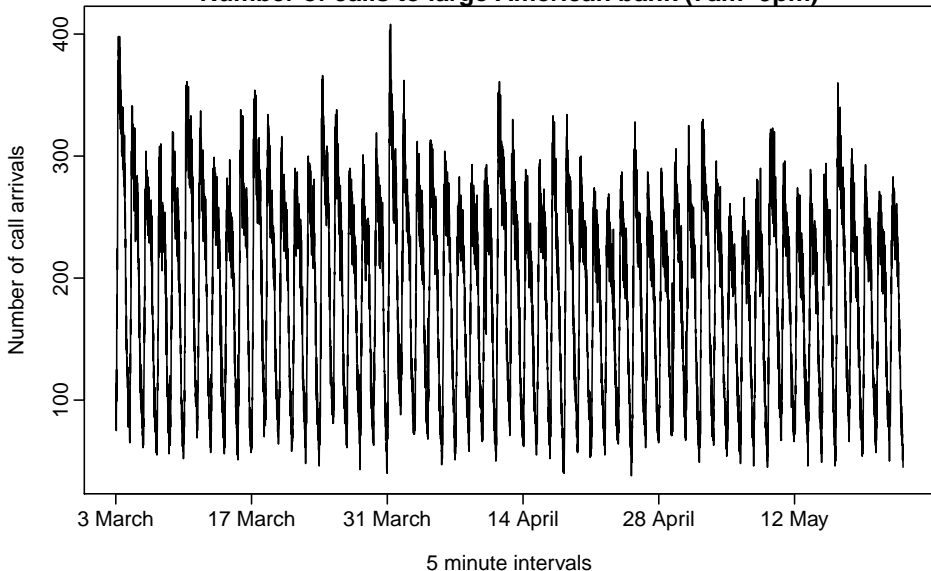
# Examples

US finished motor gasoline products



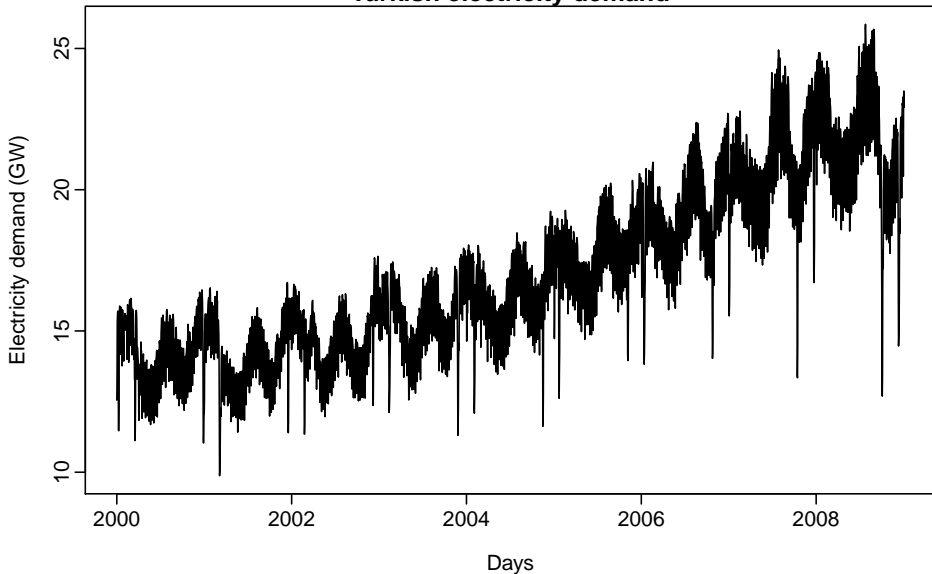
# Examples

Number of calls to large American bank (7am–9pm)



# Examples

Turkish electricity demand



# TBATS model

## TBATS

**T**rigonometric terms for seasonality

**B**ox-Cox transformations for heterogeneity

**A**RMA errors for short-term dynamics

**T**rend (possibly damped)

**S**easonal (including multiple and non-integer periods)



Automatic algorithm described in [AM De Livera, RJ Hyndman, and RD Snyder \(2011\)](#). “Forecasting time series with complex seasonal patterns using exponential smoothing”. *Journal of the American Statistical Association* **106**(496), 1513–1527.

# TBATS model

$y_t$  = observation at time  $t$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^M s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (\mathbf{1} - \phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^p \phi_i d_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \quad \begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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Box-Cox transformation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^M s_{t-m_i}^{(i)} + d_t$$

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ARMA error

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Fourier-like seasonal terms

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$$y_t^{(\omega)} = \ell_{t-1}$$

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**TBATS**

**Trigonometric**

**Box-Cox**

**ARMA**

**Trend**

**Seasonal**

Box-Cox transformation

$M$  seasonal periods

global and local trend

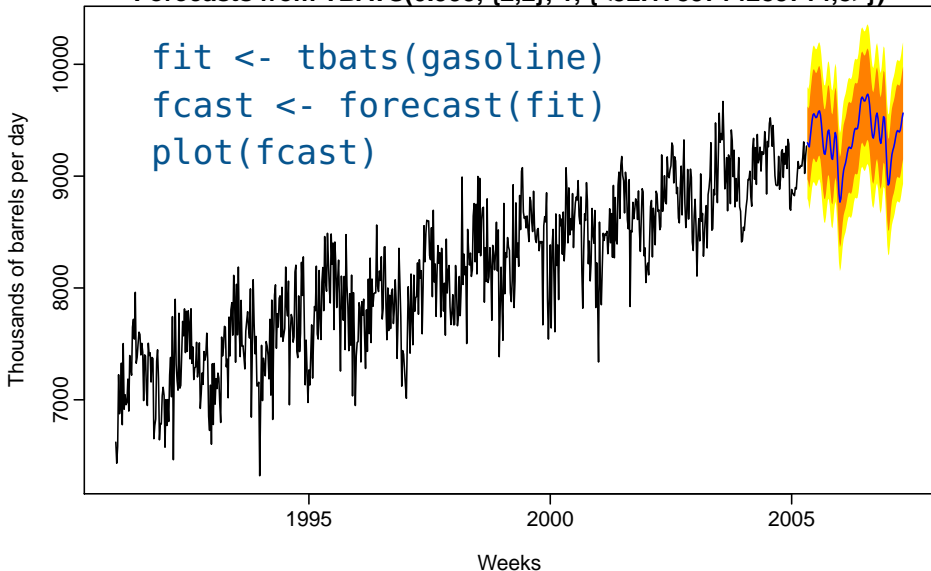
ARMA error

Fourier-like seasonal terms

# Examples

Forecasts from TBATS(0.999, {2,2}, 1, {<52.1785714285714,8>})

```
fit <- tbats(gasoline)
fcast <- forecast(fit)
plot(fcast)
```

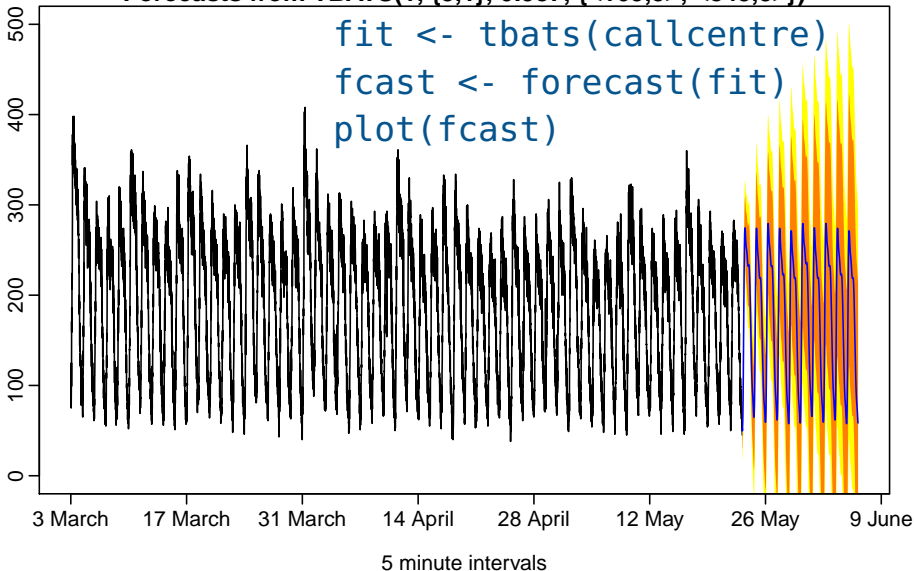


# Examples

Forecasts from TBATS(1, {3,1}, 0.987, {<169,5>, <845,3>})

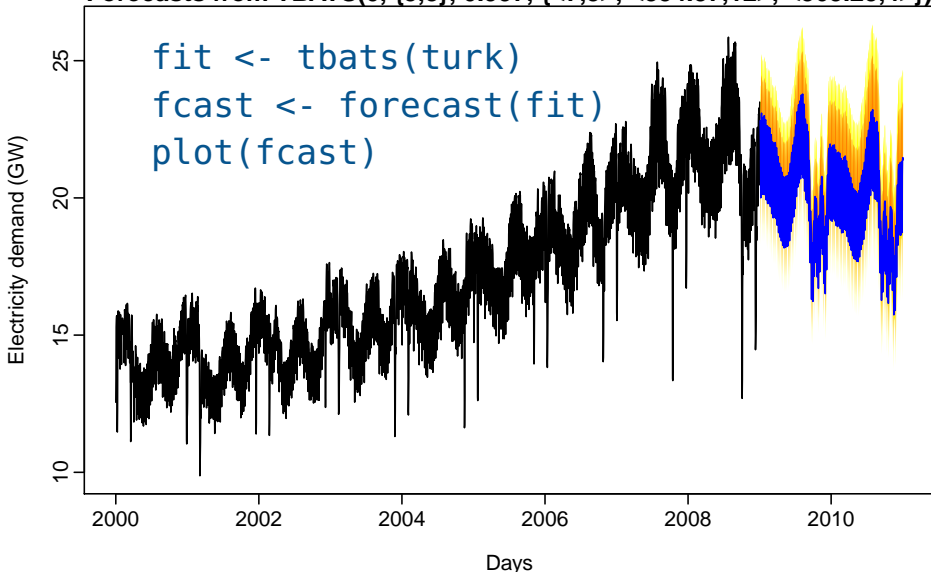
```
fit <- tbats(callcentre)  
fcast <- forecast(fit)  
plot(fcast)
```

Number of call arrivals



# Examples

Forecasts from TBATS(0, {5,3}, 0.997, {<7,3>, <354.37,12>, <365.25,4>})





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# Further competitions

- 1 2011 tourism forecasting competition.
- 2 Kaggle and other forecasting platforms.
- 3 GEFCom 2012: Point forecasting of electricity load and wind power.
- 4 GEFCom 2014: Probabilistic forecasting of electricity load, electricity price, wind energy and solar energy.

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- 1 Automatic algorithms will become more general — handling a wide variety of time series.
- 2 Model selection methods will take account of multi-step forecast accuracy as well as one-step forecast accuracy.
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