



Forecasting using R

Rob J Hyndman

3.3 Hierarchical forecasting

Outline

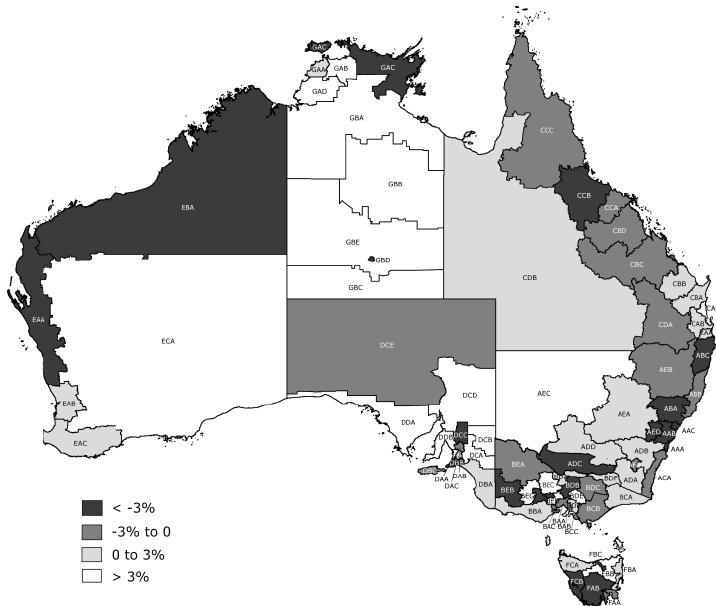
1 Hierarchical and grouped time series

2 Lab session 15

3 Temporal hierarchies

4 Lab session 16

Australian tourism demand



Australian tourism demand

- Quarterly data on visitor night from 1998:Q1 – 2013:Q4
- From: *National Visitor Survey*, based on annual interviews of 120,000 Australians aged 15+, collected by Tourism Research Australia.
- Split by 7 states, 27 zones and 76 regions (a geographical hierarchy)
- Also split by purpose of travel
 - Holiday
 - Visiting friends and relatives (VFR)
 - Business
 - Other
- 304 bottom-level series

☐ > 3%



Spectacle sales



- Monthly UK sales data from 2000 – 2014
- Provided by a large spectacle manufacturer
- Split by brand (26), gender (3), price range (6), materials (4), and stores (600)
- About 1 million bottom-level series

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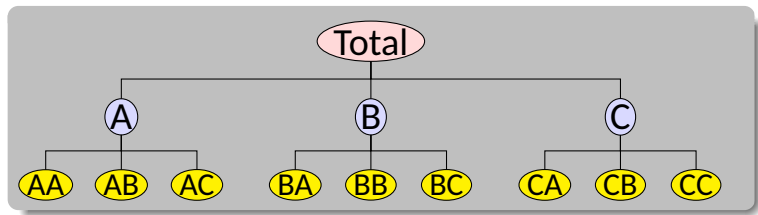
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Hierarchical time series

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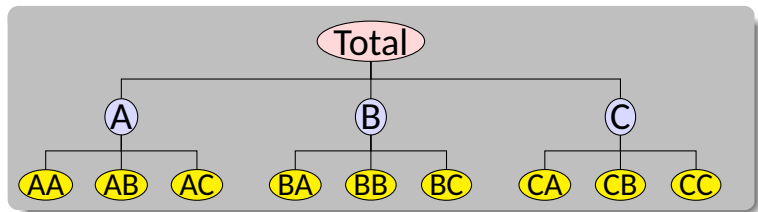


Examples

- Tourism by state and region

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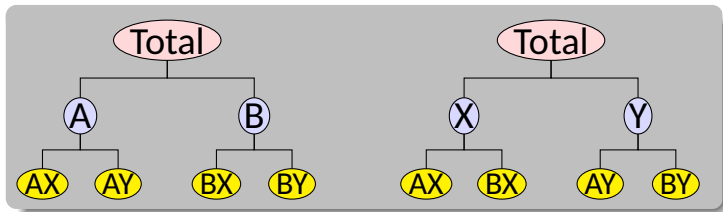


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- Tourism by state and region

Grouped time series

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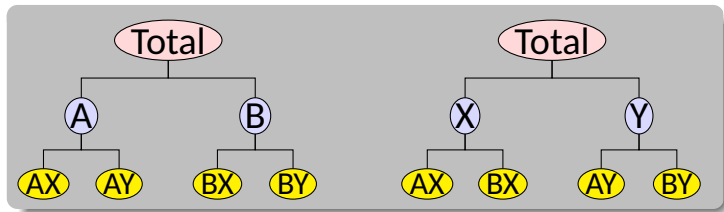


Examples

- Labour turnover by occupation and state
- Spectacle sales by brand, gender, stores, etc.

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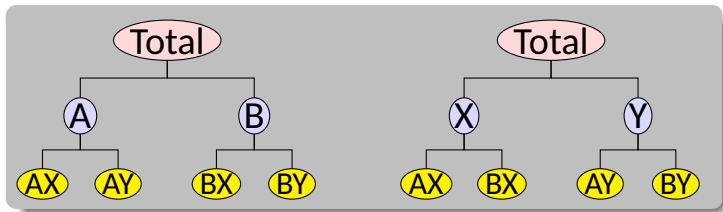


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The solution

- 1 Forecast all series at all levels of aggregation using an automatic forecasting algorithm (e.g., `ets`, `auto.arima`, ...)
- 2 Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
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hts package for R



hts: Hierarchical and Grouped Time Series

Methods for analysing and forecasting hierarchical and grouped time series

Version: 5.0

Depends: R ($\geq 3.0.2$), forecast (≥ 5.0), SparseM, Matrix, matrixcalc

Imports: parallel, utils, methods, graphics, grDevices, stats

LinkingTo: Rcpp ($\geq 0.11.0$), RcppEigen

Suggests: testthat

Published: 2016-04-06

Author: Rob J Hyndman, Earo Wang, Alan Lee, Shanika Wickramasuriya

Maintainer: Rob J Hyndman <Rob.Hyndman@monash.edu>

BugReports: <https://github.com/robjhyndman/hts/issues>

License: GPL (≥ 2)

Example using R

```
library(hts)
```

```
# bts is a matrix containing the bottom level time series
```

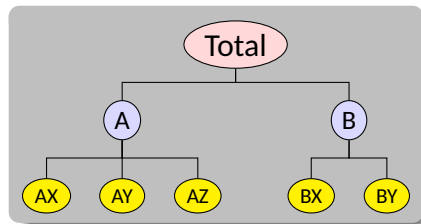
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# nodes describes the hierarchical structure
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```
y <- hts(bts, nodes=list(2, c(3,2)))
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Example using R

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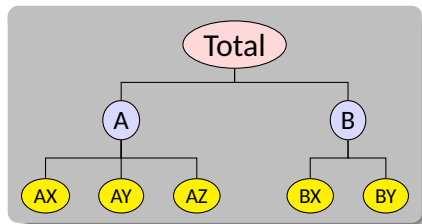
```
# nodes describes the hierarchical structure
```

```
y <- hts(bts, nodes=list(2, c(3,2)))
```

```
# Forecast 10-step-ahead using WLS combination method
```

```
# ETS used for each series by default
```

```
fc <- forecast(y, h=10)
```



gts function

Usage

```
gts(y, characters)
```

Arguments

- | | |
|------------|--|
| y | Multivariate time series containing the bottom level series |
| characters | Vector of integers, or list of vectors, showing how column names indicate group structure. |

Example

```
bnames <-  
  c("VIC1F", "VIC1M", "VIC2F", "VIC2M", "VIC3F", "VIC3M",  
    "NSW1F", "NSW1M", "NSW2F", "NSW2M", "NSW3F", "NSW3M")  
bts <- matrix(ts(rnorm(120))), ncol = 12)  
colnames(bts) <- bnames  
x <- gts(bts, characters = c(3, 1, 1))
```

gts function

Example 2

```
bnames <-  
  c("VICMelbAA", "VICMelbAB",  
    "VICGeelAA", "VICGeelAB",  
    "VICMelbBA", "VICMelbBB",  
    "VICGeelBA", "VICGeelBB",  
    "NSWSyndAA", "NSWSyndAB",  
    "NSWollAA", "NSWollAB",  
    "NSWSyndBA", "NSWSyndBB",  
    "NSWollBA", "NSWollBB")  
bts <- matrix(ts(rnorm(160)), ncol = 16)  
colnames(bts) <- bnames  
# Create a list of characters: list(c(2, 4), c(1, 1))
```


forecast.gts function

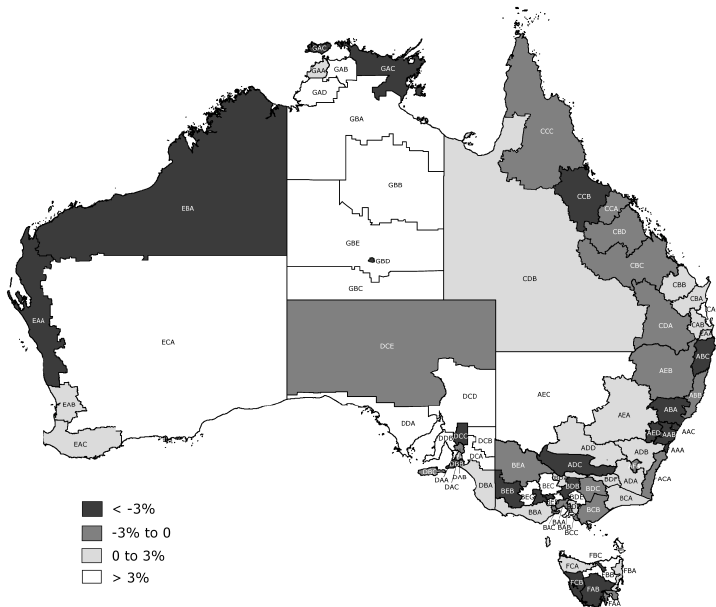
Usage

```
forecast(object, h,  
  method = c("comb", "bu", "mo", "tdgsa", "tdgsf", "tdfp"),  
  weights = c("wls", "ols", "mint", "nseries"),  
  fmethod = c("ets", "arima", "rw"),  
  algorithms = c("lu", "cg", "chol", "recursive", "slm"),  
  covariance = c("shr", "sam"),  
  positive = FALSE,  
  parallel = FALSE, num.cores = 2, ...)
```

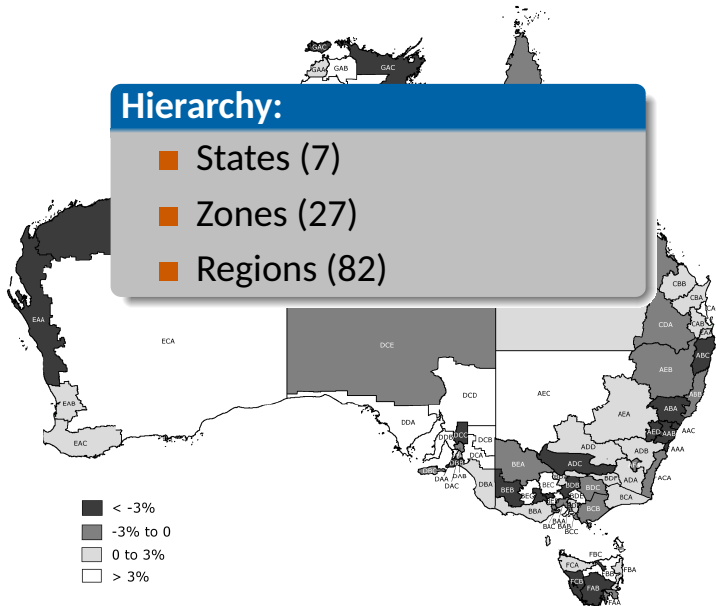
Arguments

object	Hierarchical time series object of class gts.
h	Forecast horizon
method	Method for distributing forecasts within the hierarchy.
weights	Weights used for "optimal combination" method. When weights = "sd", it takes account of the standard deviation of forecasts.
fmethod	Forecasting method to use
algorithm	Method for solving regression equations
positive	If TRUE, forecasts are forced to be strictly positive
parallel	If TRUE, allow parallel processing
num.cores	If parallel = TRUE, specify how many cores are going to be used

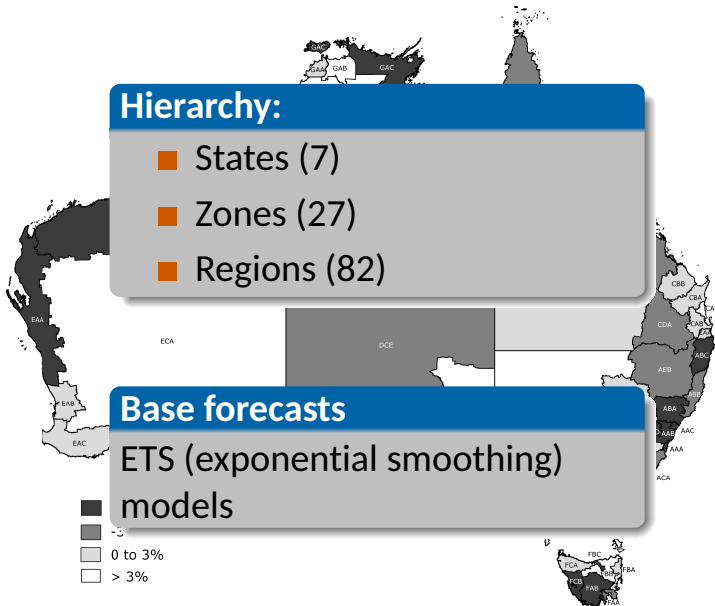
Example: Australian tourism



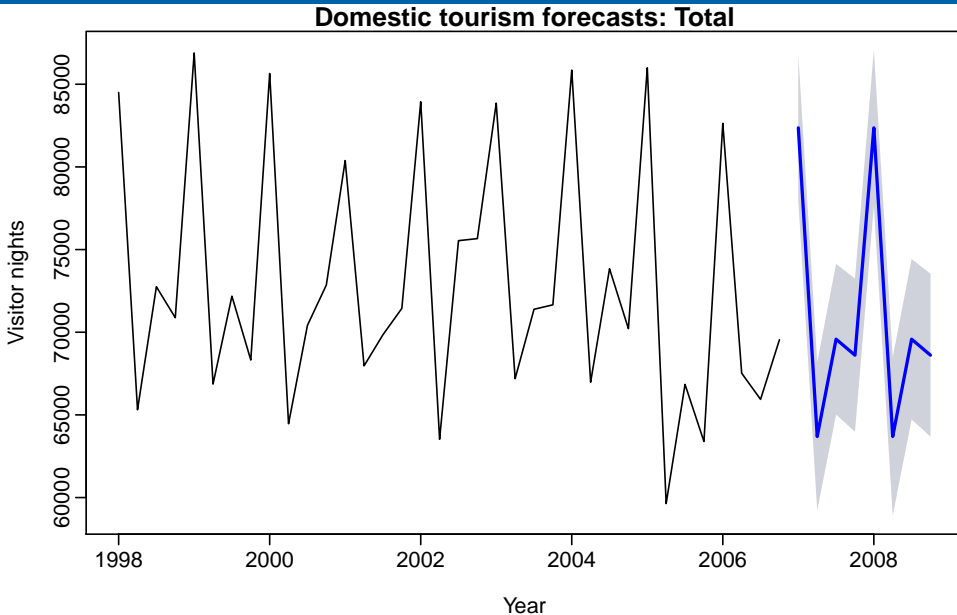
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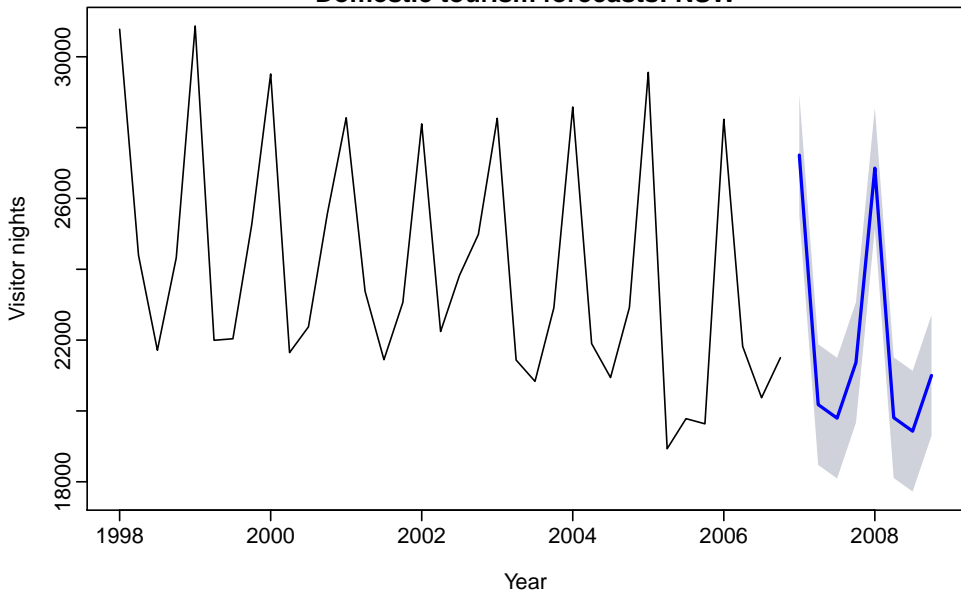


Base forecasts



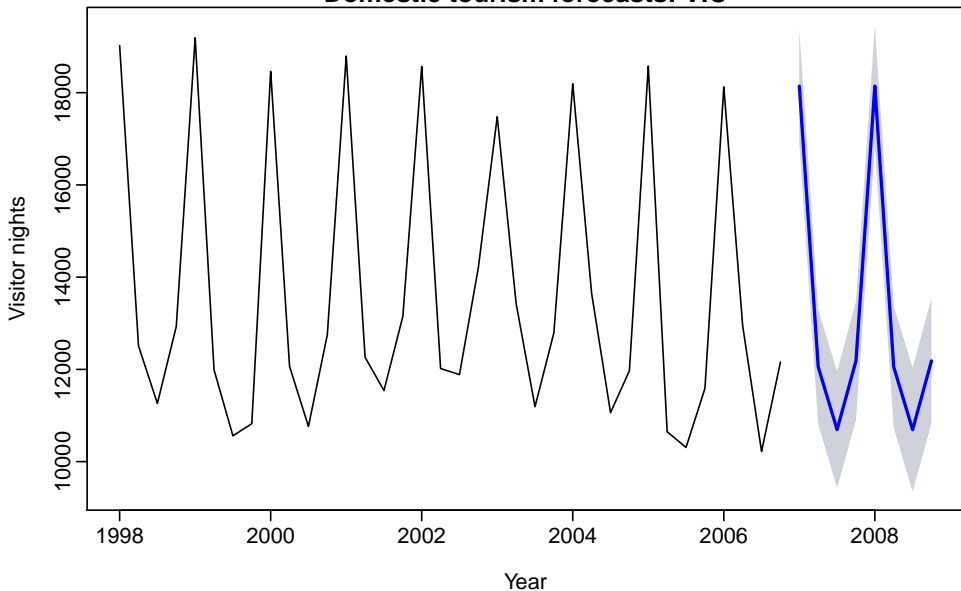
Base forecasts

Domestic tourism forecasts: NSW



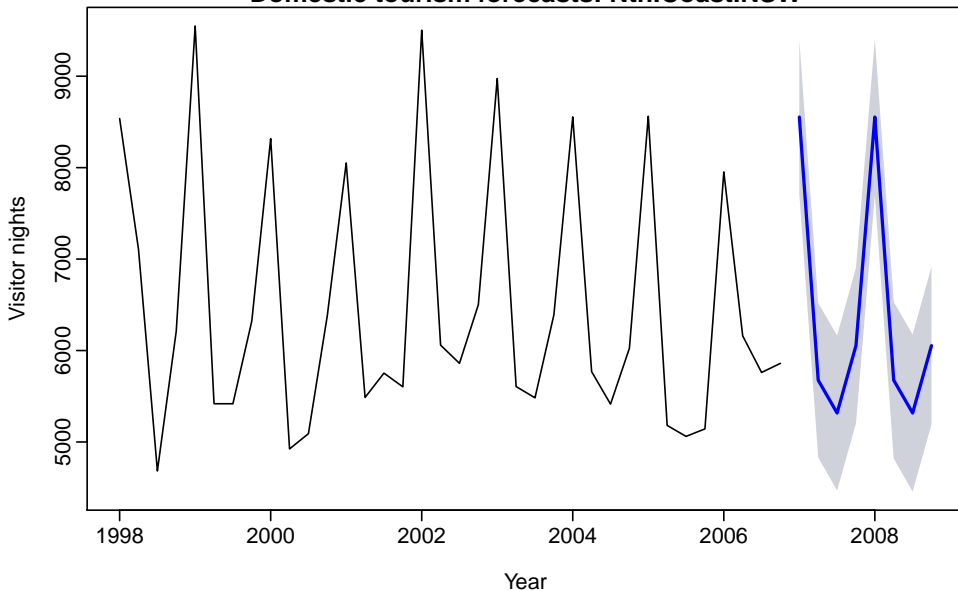
Base forecasts

Domestic tourism forecasts: VIC



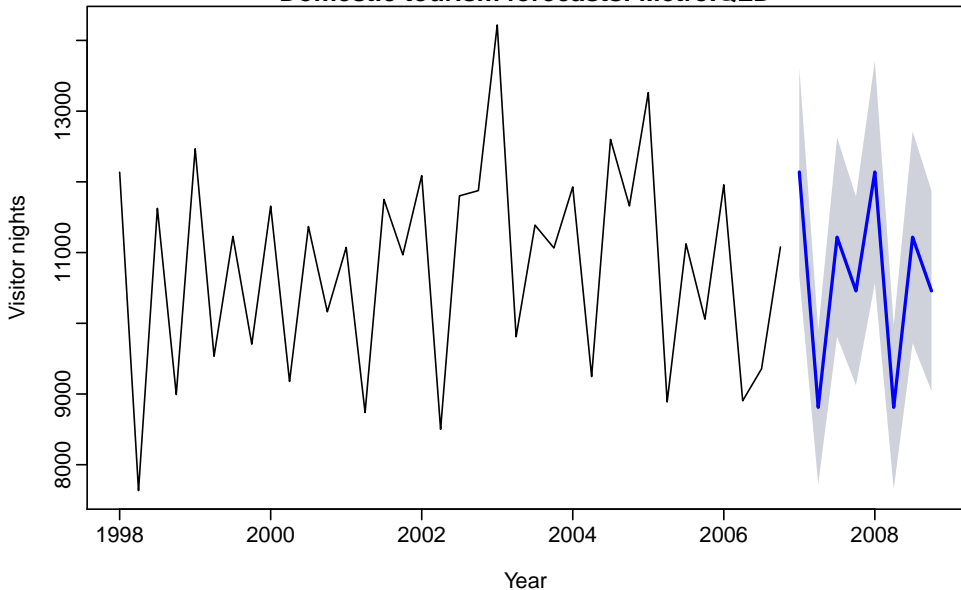
Base forecasts

Domestic tourism forecasts: Nth.Coast.NSW



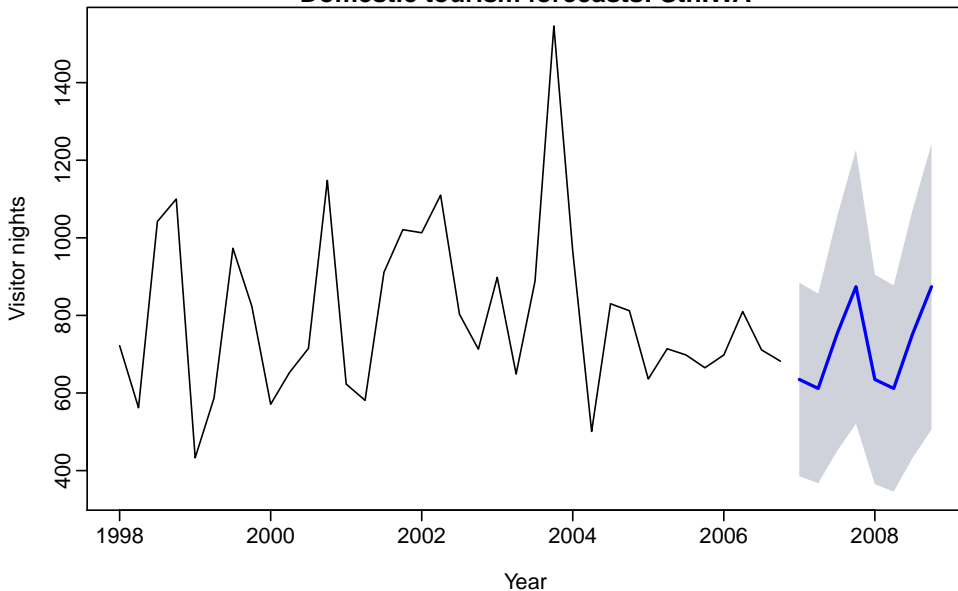
Base forecasts

Domestic tourism forecasts: Metro.QLD



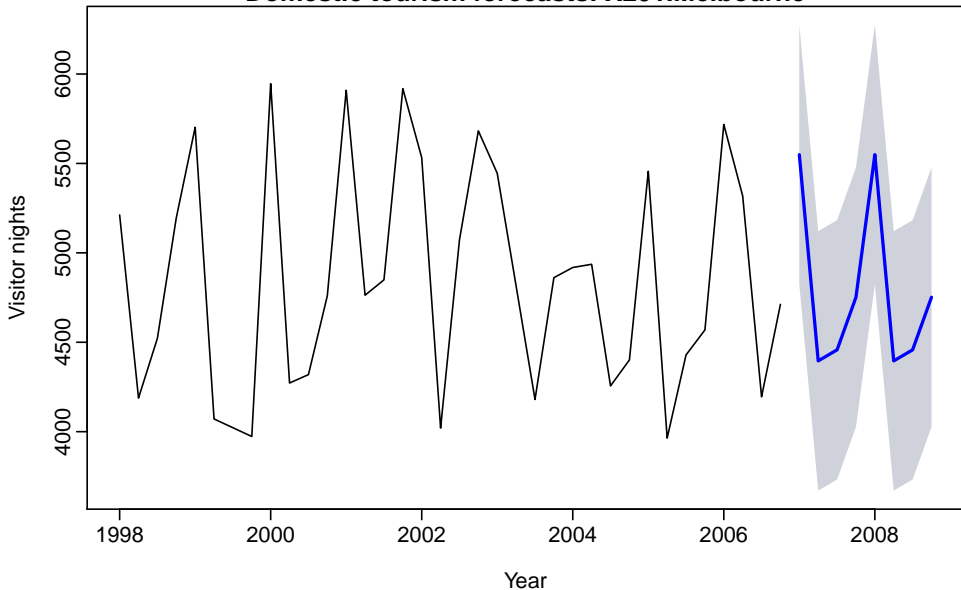
Base forecasts

Domestic tourism forecasts: Sth.WA



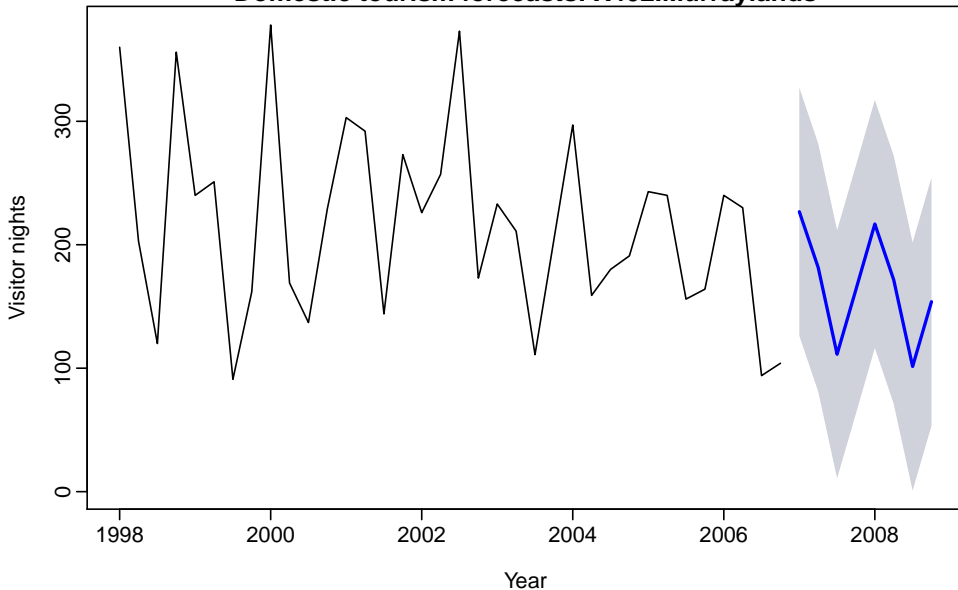
Base forecasts

Domestic tourism forecasts: X201.Melbourne



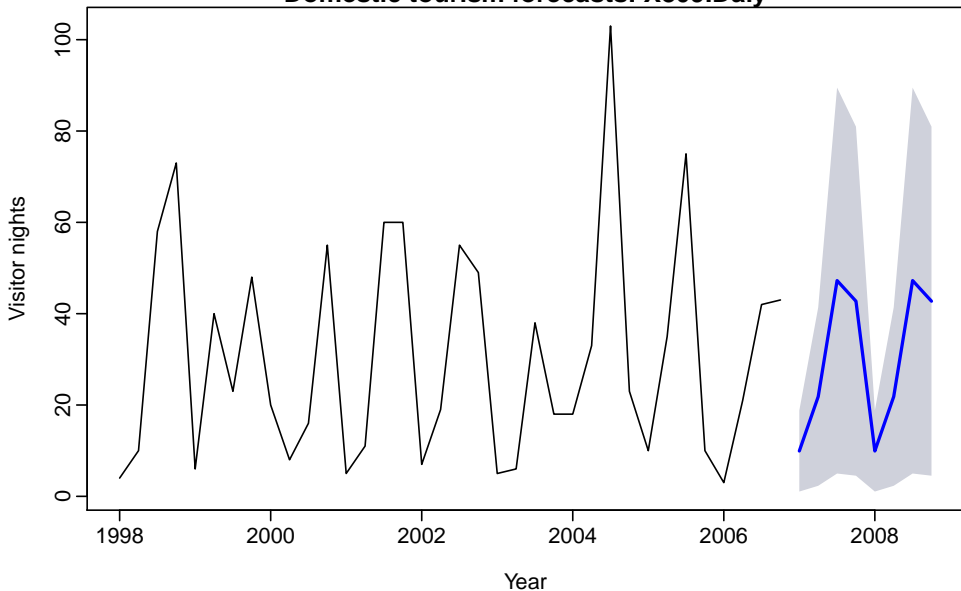
Base forecasts

Domestic tourism forecasts: X402.Murraylands



Base forecasts

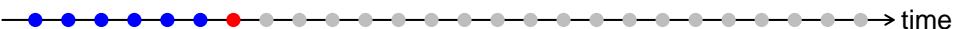
Domestic tourism forecasts: X809.Daly



Forecast evaluation

Training sets

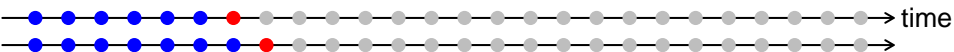
Test sets $h = 1$



Forecast evaluation

Training sets

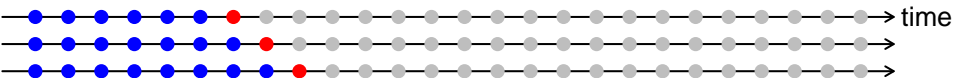
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Forecast evaluation

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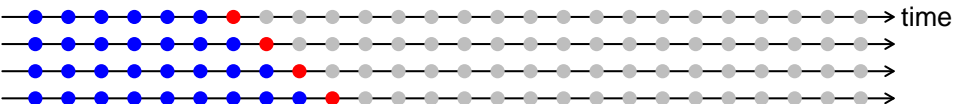
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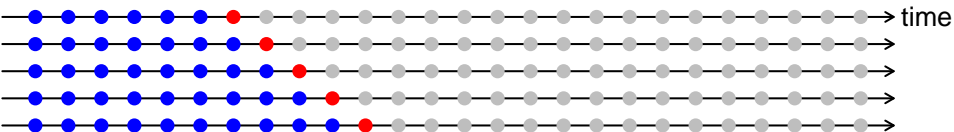
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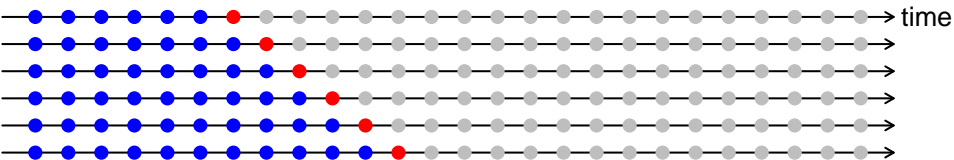
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Forecast evaluation

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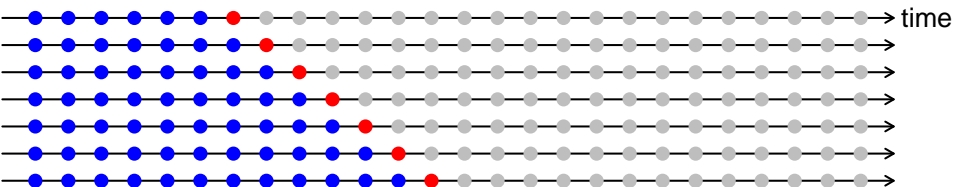
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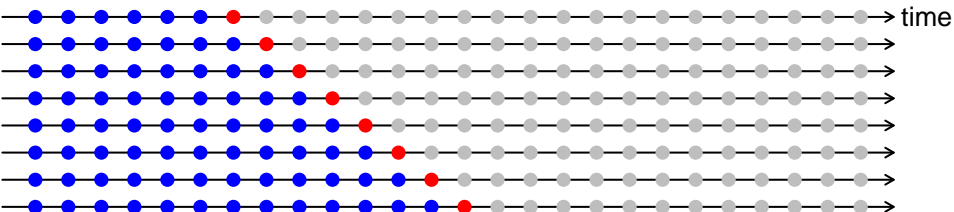
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Forecast evaluation

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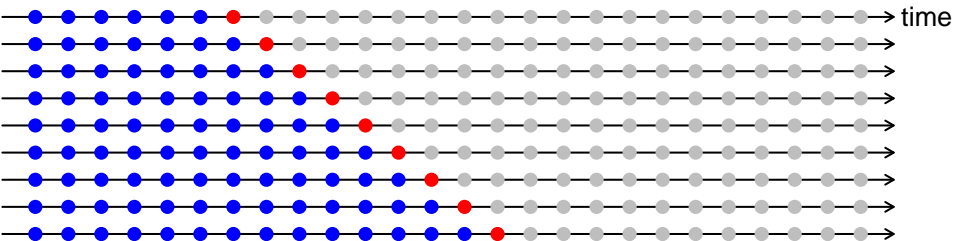
Test sets $h = 1$



Forecast evaluation

Training sets

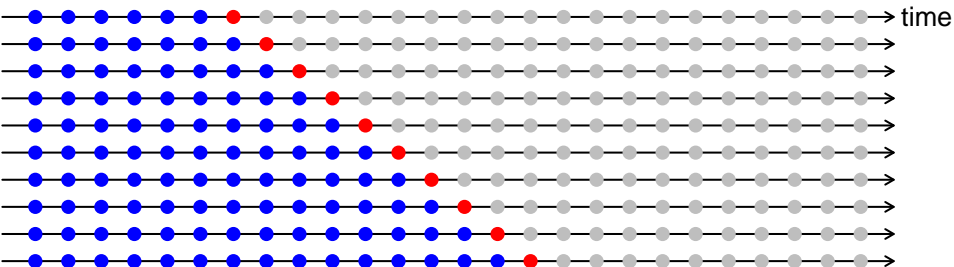
Test sets $h = 1$



Forecast evaluation

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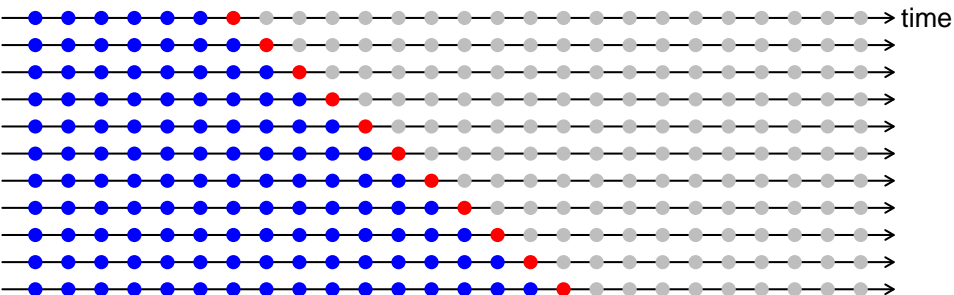
Test sets $h = 1$



Forecast evaluation

Training sets

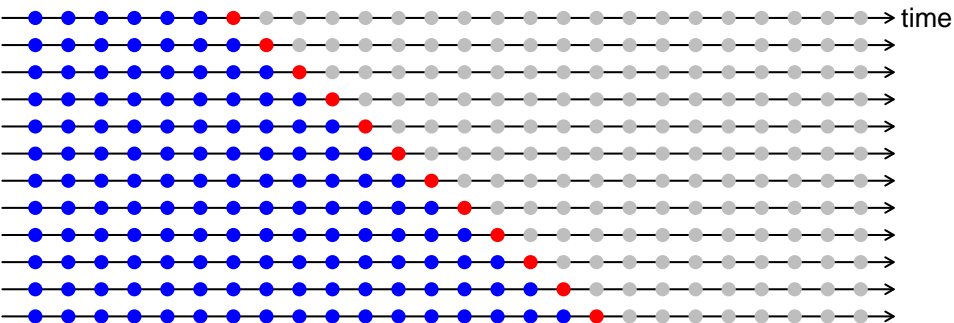
Test sets $h = 1$



Forecast evaluation

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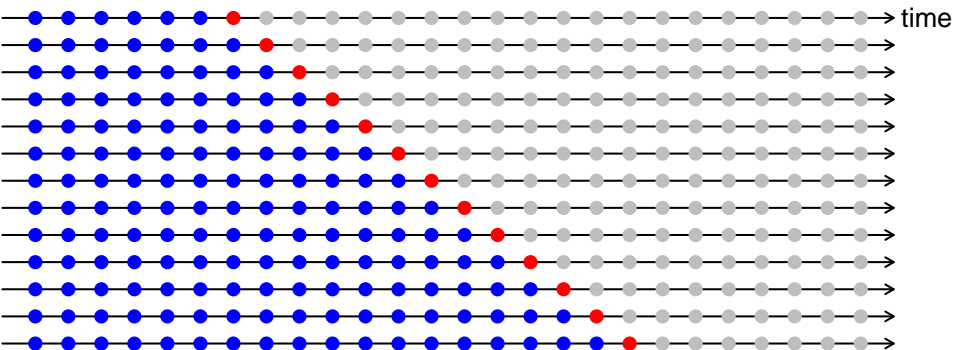
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Forecast evaluation

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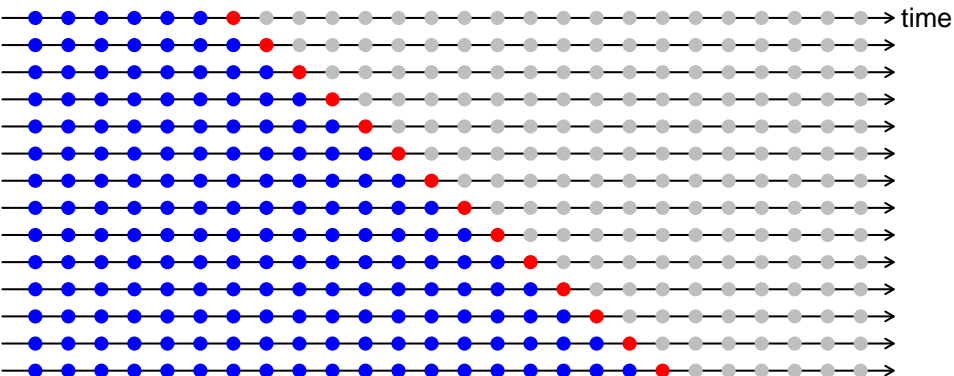
Test sets $h = 1$



Forecast evaluation

Training sets

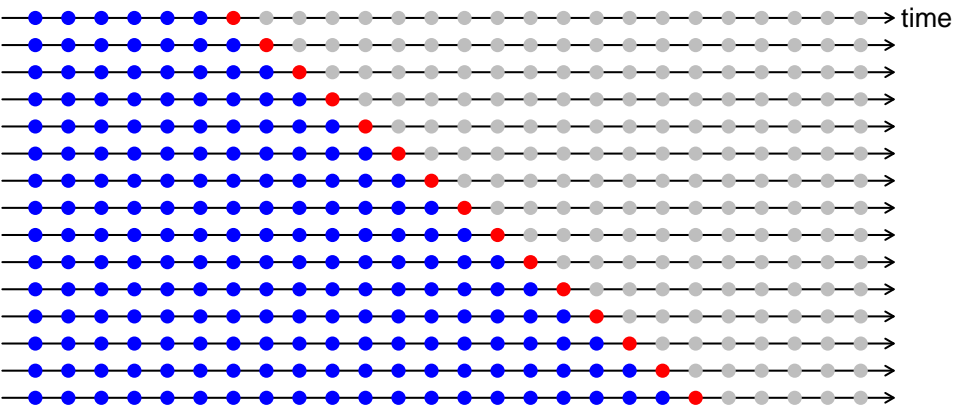
Test sets $h = 1$



Forecast evaluation

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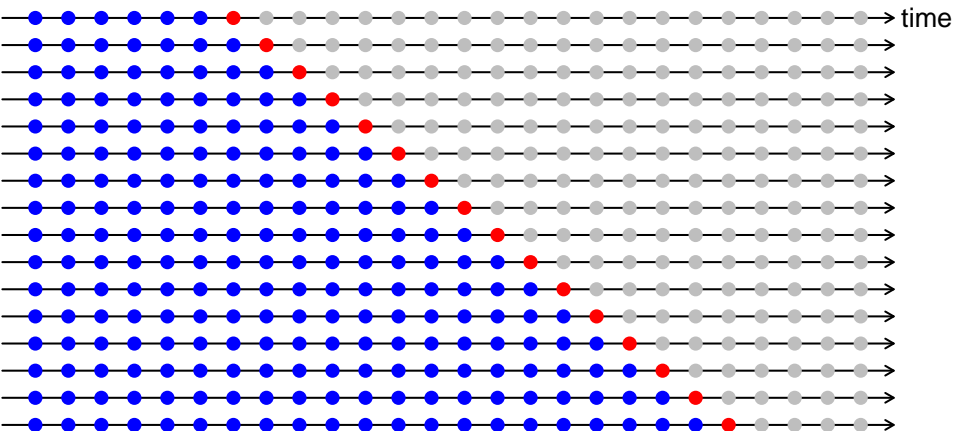
Test sets $h = 1$



Forecast evaluation

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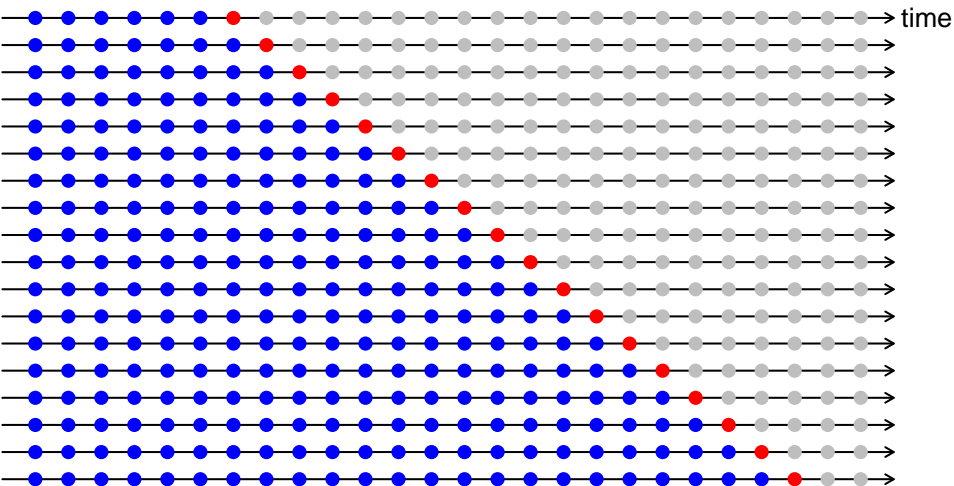
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Forecast evaluation

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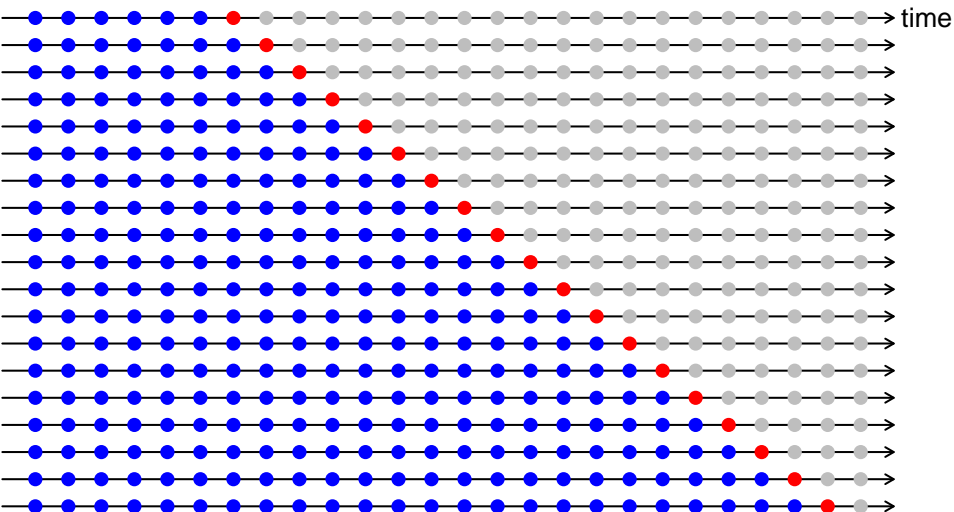
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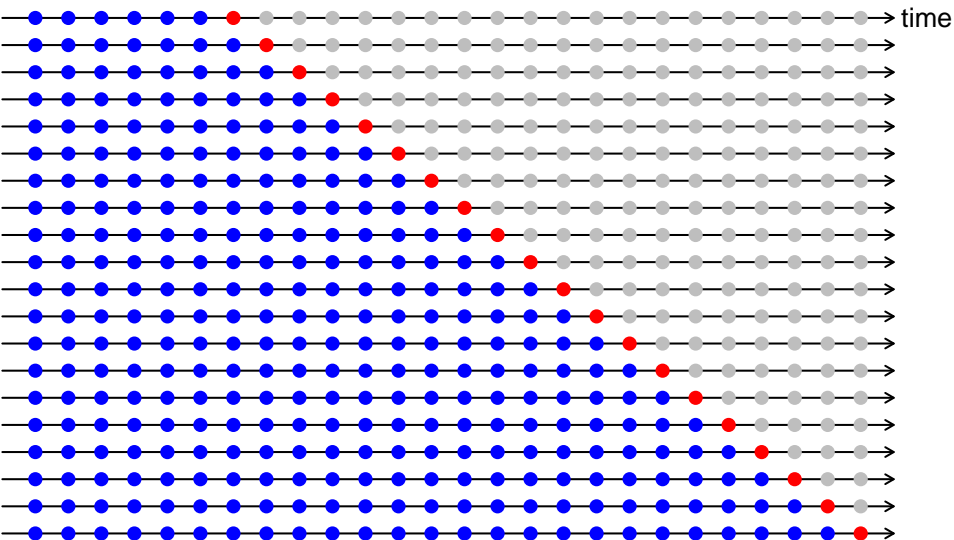
Test sets $h = 1$



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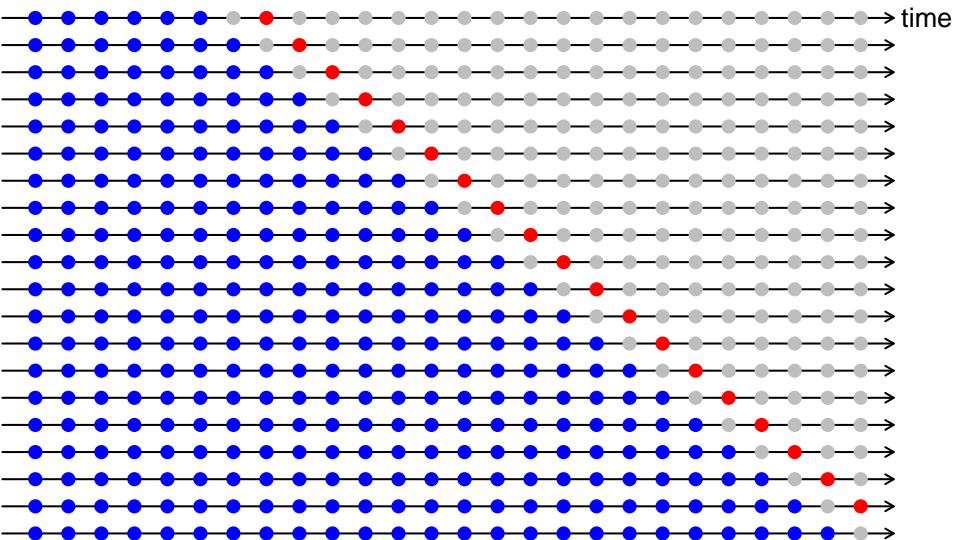
Test sets $h = 1$



Forecast evaluation

Training sets

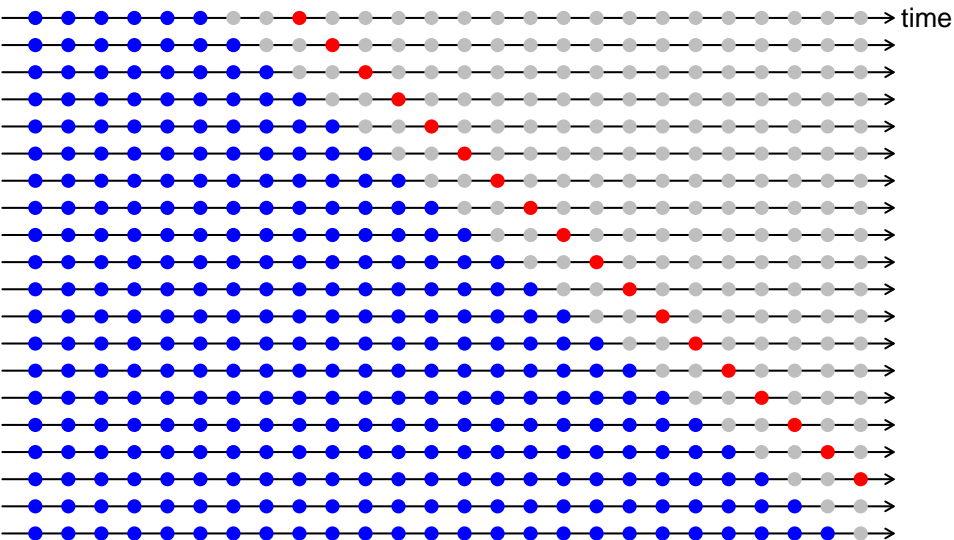
Test sets $h = 2$



Forecast evaluation

Training sets

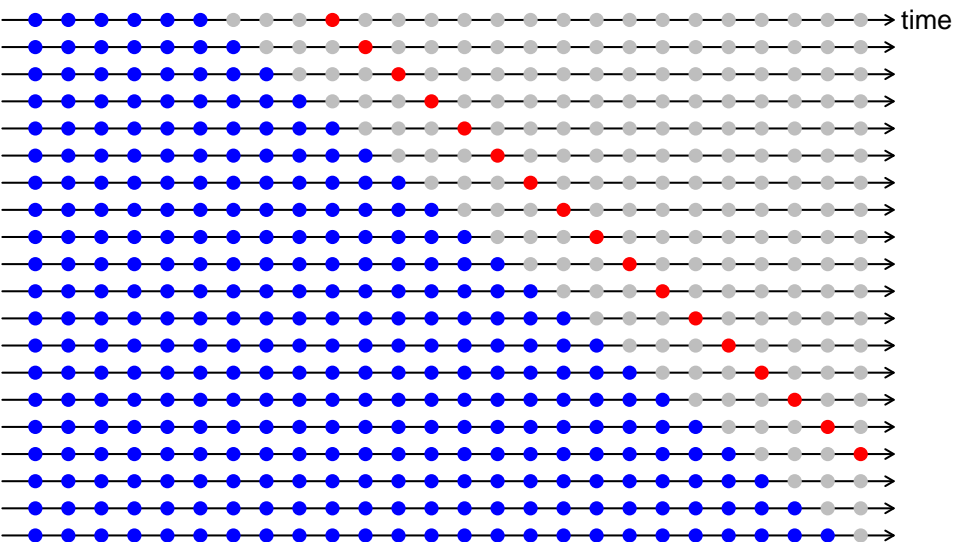
Test sets $h = 3$



Forecast evaluation

Training sets

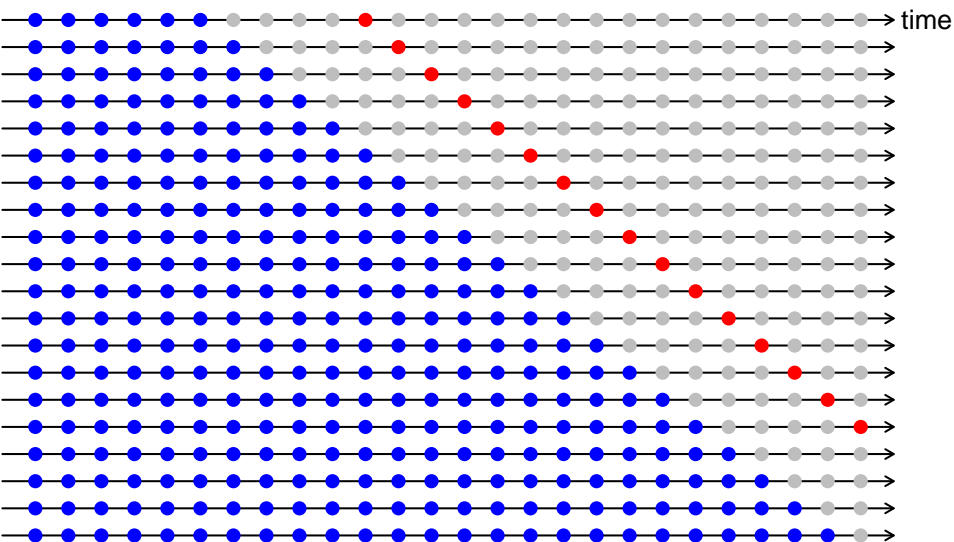
Test sets $h = 4$



Forecast evaluation

Training sets

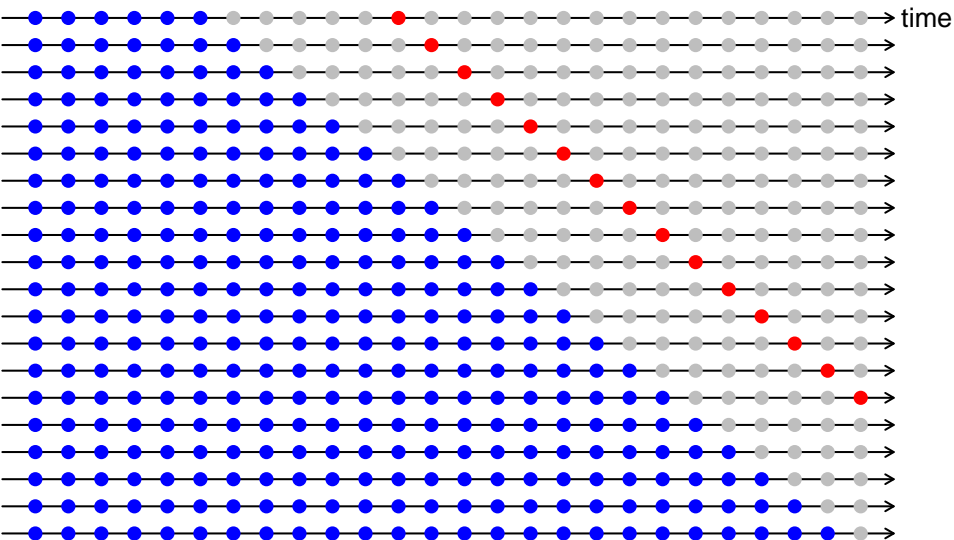
Test sets $h = 5$



Forecast evaluation

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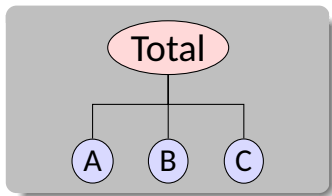
Test sets $h = 6$



Hierarchy: states, zones, regions

RMSE	Forecast horizon						Ave
	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	
Australia							
Base	1762.04	1770.29	1766.02	1818.82	1705.35	1721.17	1757.28
Bottom	1736.92	1742.69	1722.79	1752.74	1666.73	1687.43	1718.22
OLS	1747.60	1757.68	1751.77	1800.67	1686.00	1706.45	1741.69
WLS	1705.21	1715.87	1703.75	1729.56	1627.79	1661.24	1690.57
GLS	1704.64	1715.60	1705.31	1729.04	1626.36	1661.64	1690.43
States							
Base	399.77	404.16	401.92	407.26	395.38	401.17	401.61
Bottom	404.29	406.95	404.96	409.02	399.80	401.55	404.43
OLS	404.47	407.62	405.43	413.79	401.10	404.90	406.22
WLS	398.84	402.12	400.71	405.03	394.76	398.23	399.95
GLS	398.84	402.16	400.86	405.03	394.59	398.22	399.95
Regions							
Base	93.15	93.38	93.45	93.79	93.50	93.56	93.47
Bottom	93.15	93.38	93.45	93.79	93.50	93.56	93.47
OLS	93.28	93.53	93.64	94.17	93.78	93.88	93.71
WLS	93.02	93.32	93.38	93.72	93.39	93.53	93.39
GLS	92.98	93.27	93.34	93.66	93.34	93.46	93.34

Hierarchical time series

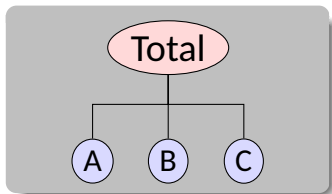


y_t : observed aggregate of all series at time t .

$y_{X,t}$: observation on series X at time t .

b_t : vector of all series at bottom level in time t .

Hierarchical time series

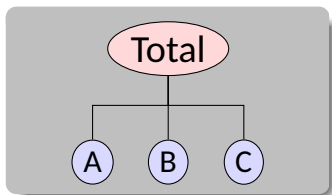


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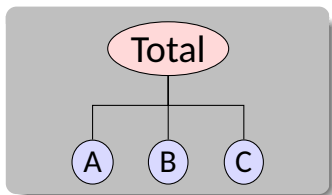
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$$\mathbf{y}_t = [y_t, y_{A,t}, y_{B,t}, y_{C,t}]' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

Hierarchical time series



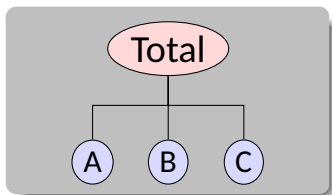
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Hierarchical time series



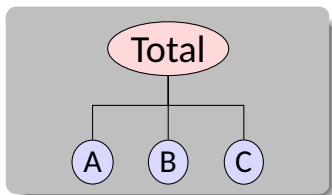
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$$y_t = [y_t, y_{A,t}, y_{B,t}, y_{C,t}]' = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_S \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{b_t}$$

Hierarchical time series



y_t : observed aggregate of all series at time t .

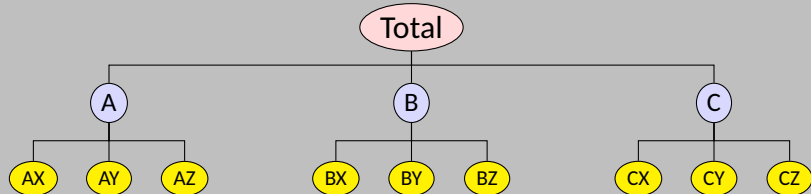
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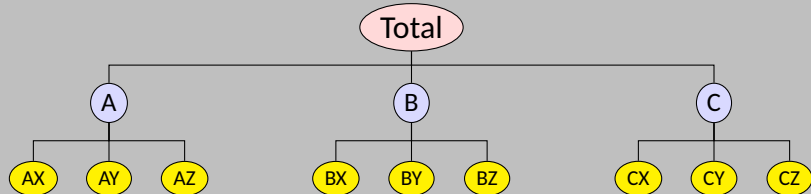
$$y_t = S b_t$$

Hierarchical time series



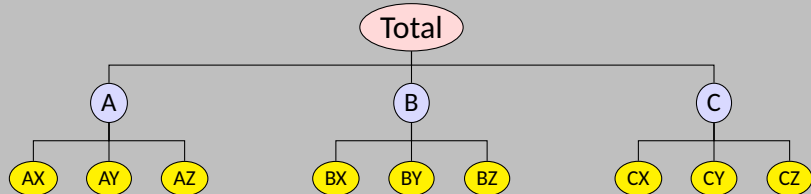
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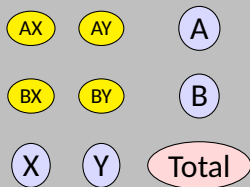
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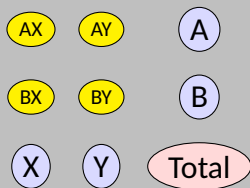
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Grouped data



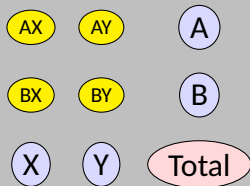
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Hierarchical and grouped time series

Every collection of time series with aggregation constraints can be written as

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

where

- \mathbf{y}_t is a vector of all series at time t
- \mathbf{b}_t is a vector of the most disaggregated series at time t
- \mathbf{S} is a “summing matrix” containing the aggregation constraints.

Forecasting notation

Let $\hat{\mathbf{y}}_n(h)$ be vector of initial h -step forecasts, made at time n , stacked in same order as \mathbf{y}_t .

(In general, they will not “add up”.)

Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}\mathbf{P}\hat{\mathbf{y}}_n(h)$$

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Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $\mathbf{P} = (\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}$.

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}(\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}\hat{\mathbf{y}}_n(h)$$

Reconciled forecasts

Base forecasts

Problem: Σ_h hard to estimate, especially for $h > 1$.

Solutions:

- Ignore Σ_h (OLS)
- Assume Σ_h diagonal (WLS) [Default in hts]
- Try to estimate Σ_h (GLS)

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1 Hierarchical and grouped time series

2 Lab session 15

3 Temporal hierarchies

4 Lab session 16

Lab Session 15

Outline

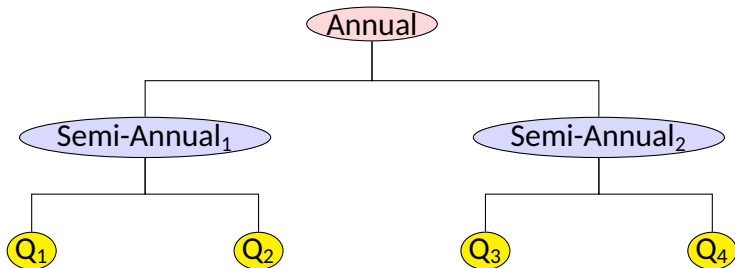
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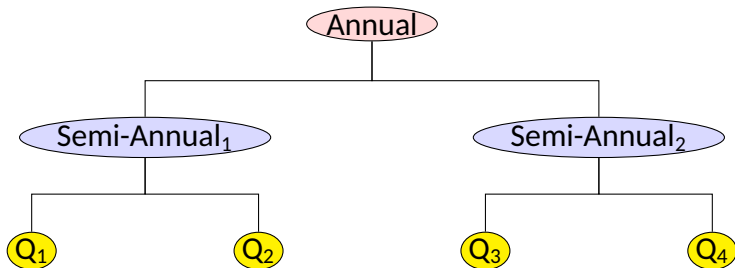
Temporal hierarchies



Basic idea:

- ➡ Forecast series at each available frequency.
- ➡ Optimally reconcile forecasts within the same year.

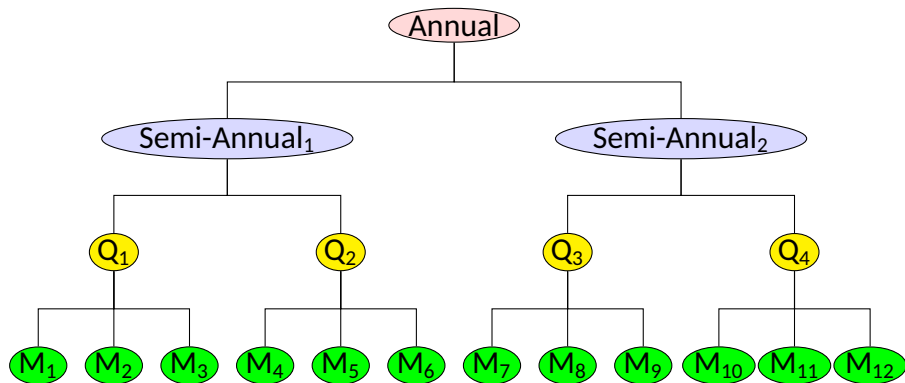
Temporal hierarchies



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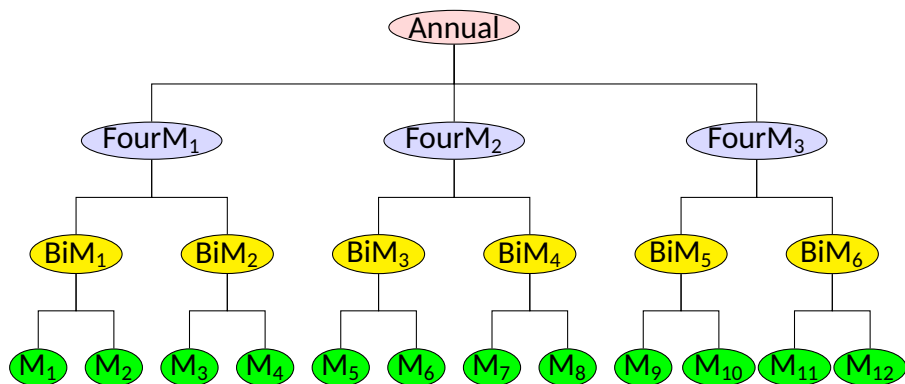
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Monthly series



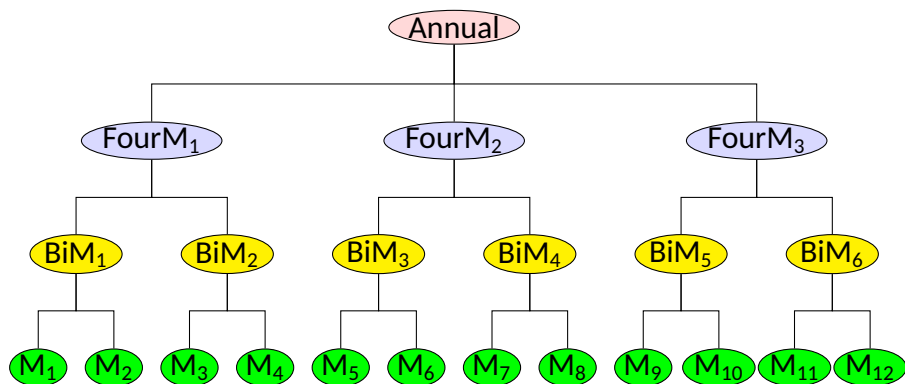
- $k = 2, 4, 12$ nodes
- $k = 3, 6, 12$ nodes
- Why not $k = 2, 3, 4, 6, 12$ nodes?

Monthly series



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Monthly series



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Monthly data

$$\underbrace{\begin{pmatrix} A \\ \text{Semi}A_1 \\ \text{Semi}A_2 \\ \text{Four}M_1 \\ \text{Four}M_2 \\ \text{Four}M_3 \\ Q_1 \\ \vdots \\ Q_4 \\ \text{Bi}M_1 \\ \vdots \\ \text{Bi}M_6 \\ M_1 \\ \vdots \\ M_{12} \end{pmatrix}}_{(28 \times 1)} = \underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}}_{\substack{I_{12} \\ S}} \underbrace{\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \\ M_{10} \\ M_{11} \\ M_{12} \end{pmatrix}}_{B_t}$$

In general

For a time series y_1, \dots, y_T , observed at frequency m , we generate aggregate series

$$y_j^{[k]} = \sum_{t=1+(j-1)k}^{jk} y_t, \quad \text{for } j = 1, \dots, \lfloor T/k \rfloor$$

- $k \in F(m) = \{\text{factors of } m\}$.
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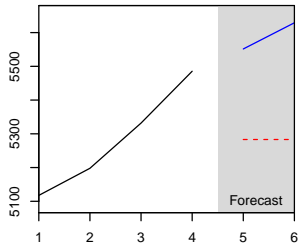
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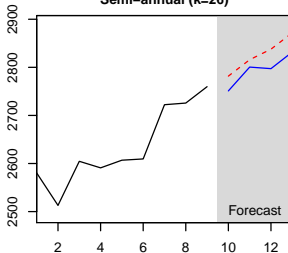
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UK Accidents and Emergency Demand

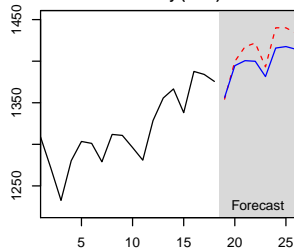
Annual (k=52)



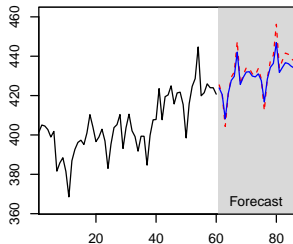
Semi-annual (k=26)



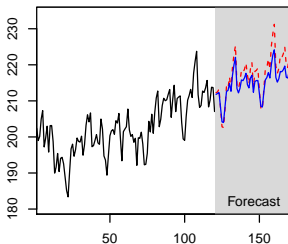
Quarterly (k=13)



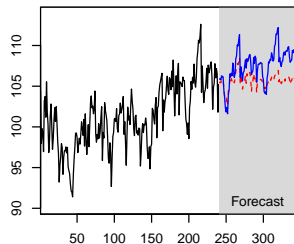
Monthly (k=4)



Bi-weekly (k=2)



Weekly (k=1)



--- base

— reconciled

UK Accidents and Emergency Demand

- 1 Type 1 Departments — Major A&E
- 2 Type 2 Departments — Single Specialty
- 3 Type 3 Departments — Other A&E/Minor Injury
- 4 Total Attendances
- 5 Type 1 Departments — Major A&E > 4 hrs
- 6 Type 2 Departments — Single Specialty > 4 hrs
- 7 Type 3 Departments — Other A&E/Minor Injury > 4 hrs
- 8 Total Attendances > 4 hrs
- 9 Emergency Admissions via Type 1 A&E
- 10 Total Emergency Admissions via A&E
- 11 Other Emergency Admissions (i.e., not via A&E)
- 12 Total Emergency Admissions
- 13 Number of patients spending > 4 hrs from decision to admission

UK Accidents and Emergency Demand

- **Minimum training set:** all data except the last year
- Base forecasts using `auto.arima()`.
- Mean Absolute Scaled Errors for 1, 4 and 13 weeks ahead using a rolling origin.

Aggr. Level	<i>h</i>	Base	Reconciled	Change
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thief package for R

thief: Temporal HIErarchical Forecasting

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