MONASH University

REVOLUTION ANALYTICS

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Forecasting using

7. Transformations and adjustments

OTexts.com/fpp/2/4/

Outline

Exponential smoothing

`ets()` function

- Automatically chooses a model by default using the AIC, AICc or BIC.
- Can handle any combination of trend, seasonality and damping
- Produces prediction intervals for every model
- Ensures the parameters are admissible (equivalent to invertible)
- Produces an object of class `ets`.

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`ets` objects

- **Methods:** `coef()`, `plot()`, `summary()`, `residuals()`, `fitted()`, `simulate()` and `forecast()`
- `plot()` function shows time plots of the original time series along with the extracted components (level, growth and seasonal).

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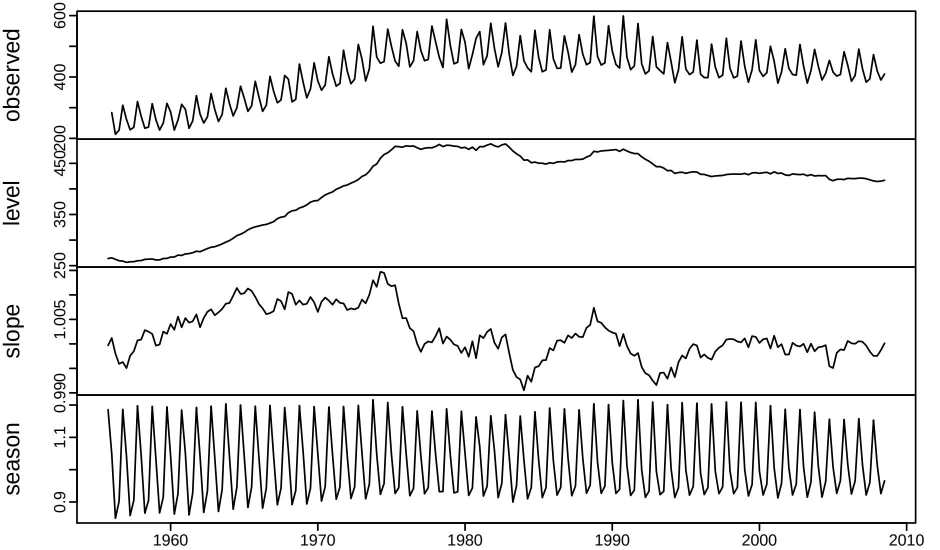
Exponential smoothing

`plot(fit)`

Decomposition by ETS(M,Md,M) method

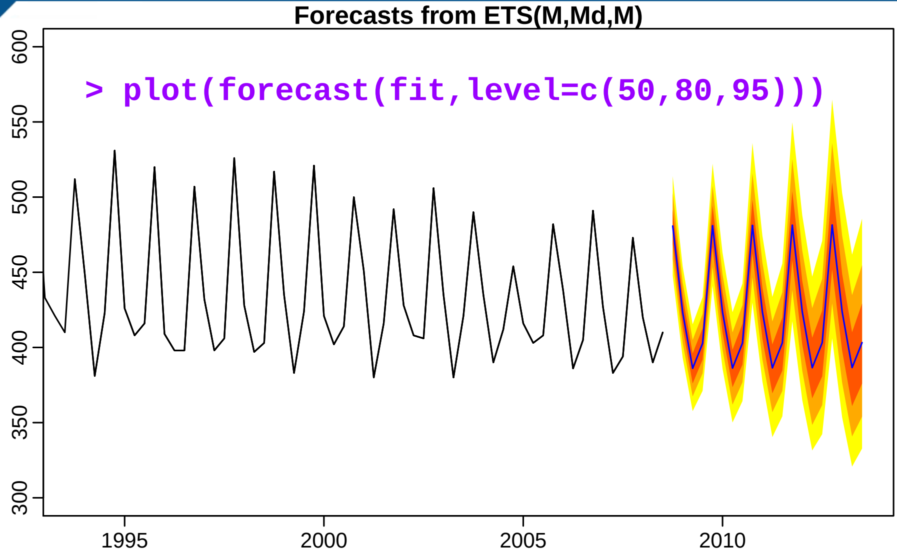


Goodness-of-fit

```
> accuracy(fit)
      ME     RMSE      MAE      MPE     MAPE     MASE
 0.17847 15.48781 11.77800  0.07204  2.81921  0.20705

> accuracy(fit2)
      ME     RMSE      MAE      MPE     MAPE     MASE
-0.11711 15.90526 12.18930 -0.03765  2.91255  0.21428
```

Forecast intervals



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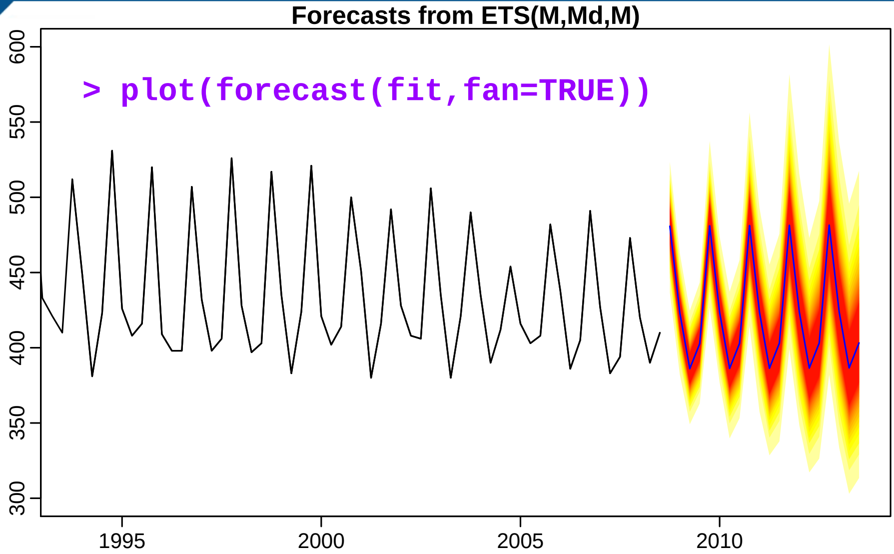

Exponential smoothing

`ets()` function also allows refitting model to new data set.

```
> usfit <- ets(usnetelec[1:45])
> test <- ets(usnetelec[46:55], model = usfit)

> accuracy(test)
      ME      RMSE       MAE       MPE      MAPE      MASE
-3.35419 58.02763 43.85545 -0.07624  1.18483  0.52452

> accuracy(forecast(usfit,10), usnetelec[46:55])
        ME        RMSE         MAE         MPE        MAPE         MASE
  40.7034   61.2075   46.3246    1.0980    1.2620    0.6776
```

Unstable models

- ETS(M,M,A)
- ETS(M,M_d,A)
- ETS(A,N,M)
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In practice, the models work fine for short- to medium-term forecasts provided the data are strictly positive.

Forecastability conditions

```
ets(y, model="ZZZ", damped=NULL, alpha=NULL,
    beta=NULL, gamma=NULL, phi=NULL,
    additive.only=FALSE,
    lower=c(rep(0.0001,3),0.80),
    upper=c(rep(0.9999,3),0.98),
    opt.crit=c("lik","amse","mse","sigma"),
    nmse=3,
    bounds=c("both","usual","admissible"),
    ic=c("aic","aicc","bic"), restrict=TRUE)
```

The magic `forecast()` function

- `forecast` returns forecasts when applied to an `ets` object (or the output from many other time series models).
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Transformations to stabilize the variance

If the data show different variation at different levels of the series, then a transformation can be useful.

Denote original observations as $y_1, \dots, y_n$ and transformed observations as $w_1, \dots, w_n$.

Mathematical transformations for stabilizing variation

Square root	$w_t = \sqrt{y_t}$	$\downarrow$
Cube root	$w_t = \sqrt[3]{y_t}$	Increasing
Logarithm	$w_t = \log(y_t)$	strength

Logarithms, in particular, are useful because they are more interpretable: changes in a log value are **relative (percent) changes on the original scale.**

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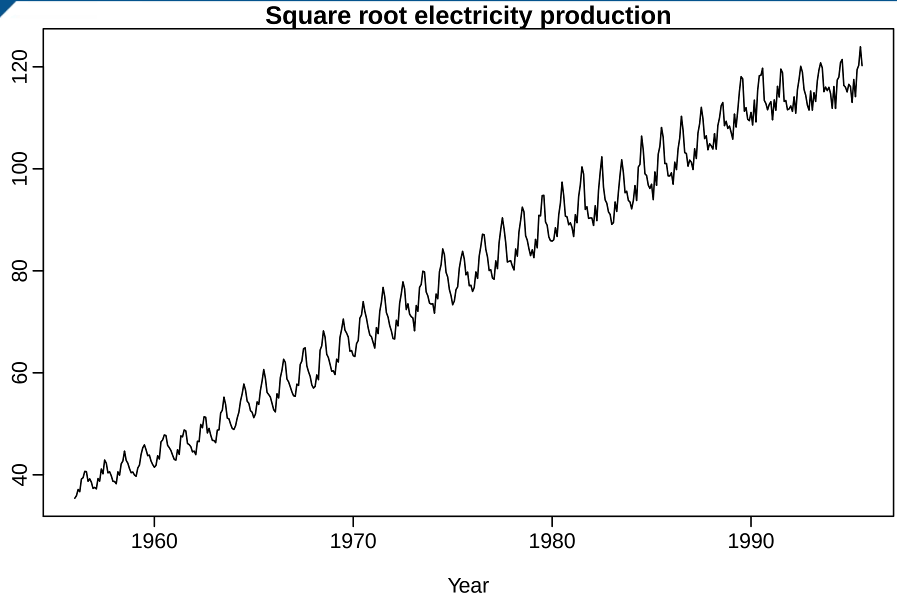

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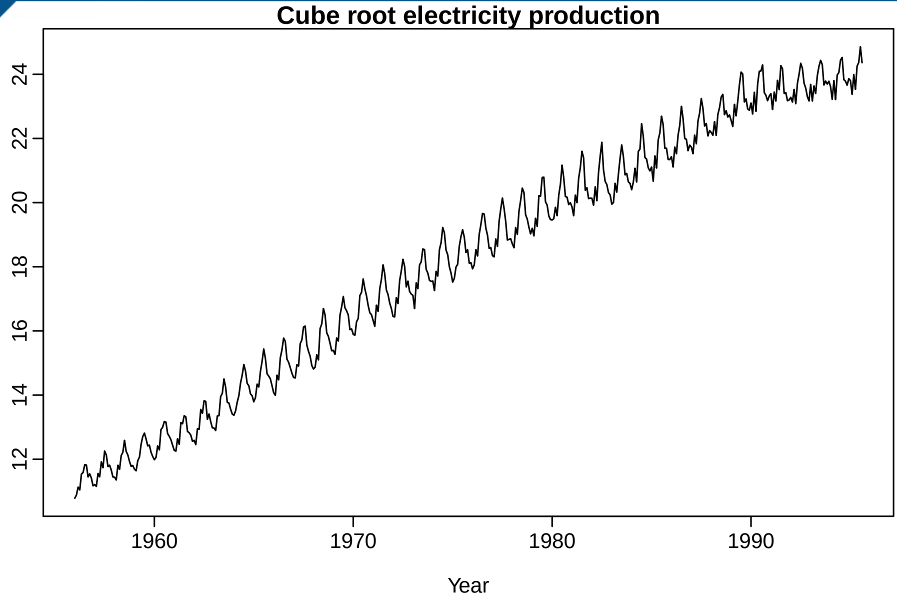

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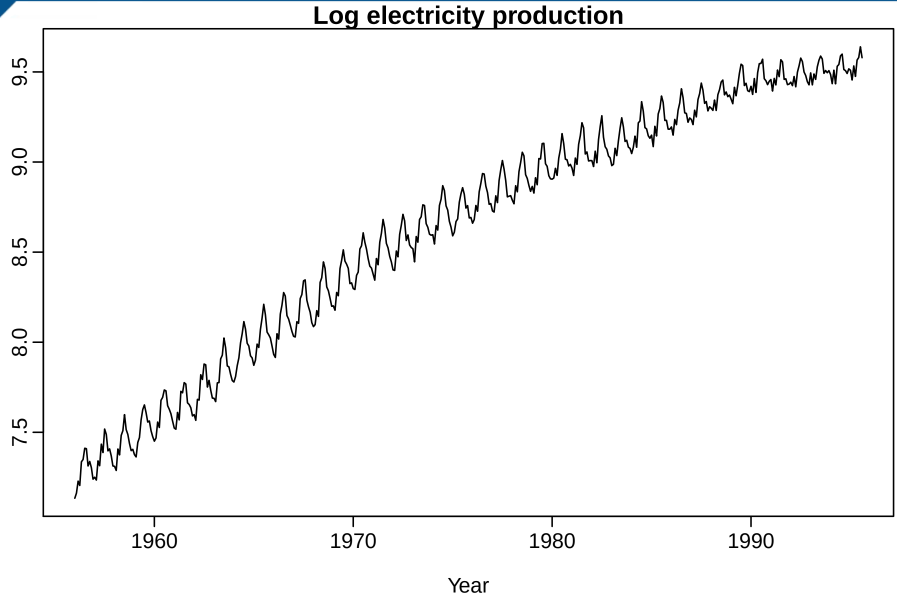

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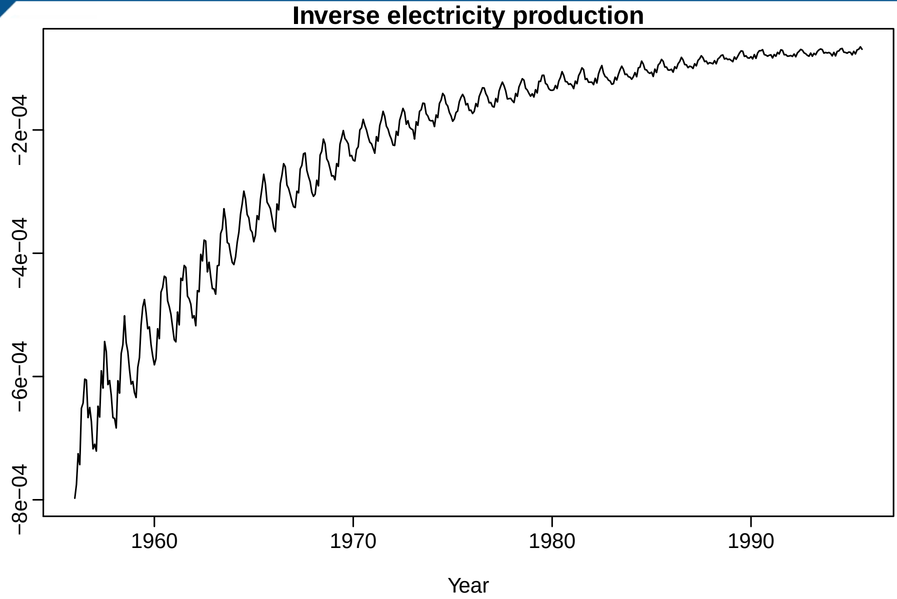

Box-Cox transformations

Each of these transformations is close to a member of the family of **Box-Cox transformations**:

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (y_t^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

- $\lambda = 1$: (No substantive transformation)
- $\lambda = \frac{1}{2}$: (Square root plus linear transformation)
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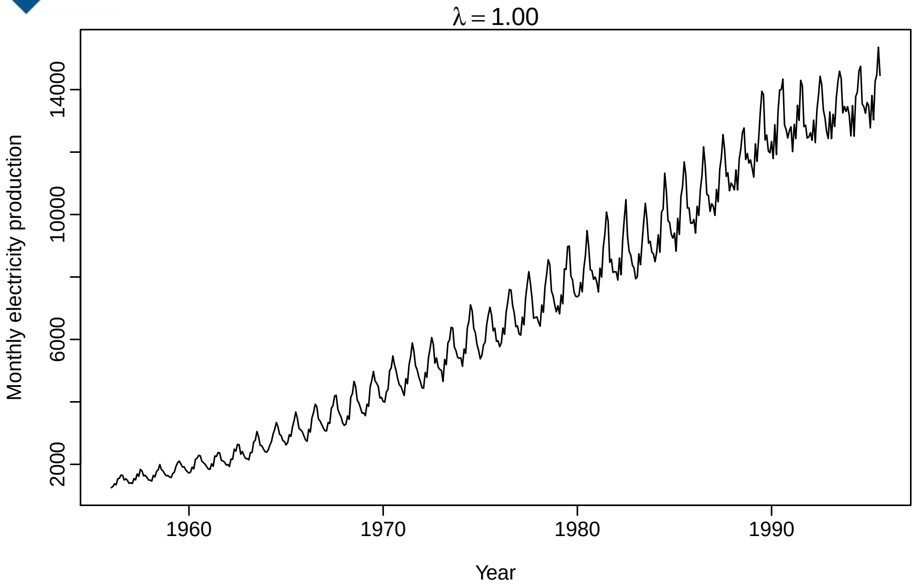

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- y_t^{λ} for λ close to zero behaves like logs.
- If some $y_t = 0$, then must have $\lambda > 0$
- if some $y_t < 0$, no power transformation is possible unless all y_t adjusted by **adding a constant to all values**.
- Choose a simple value of λ. It makes explanation easier.
- Results are relatively insensitive to value of λ
- Often no transformation ($\lambda = 1$) needed.
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Back-transformation

We must reverse the transformation (or *back-transform*) to obtain forecasts on the original scale. The reverse Box-Cox transformations are given by

$$y_t = \begin{cases} \exp(w_t), & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda}, & \lambda \neq 0. \end{cases}$$

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plot(BoxCox(elec,lambda=1/3))
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- This attempts to balance the seasonal fluctuations and random variation across the series.
- Always check the results.
- A low value of λ can give extremely large prediction intervals.

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ETS and transformations

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- A Box-Cox transformation followed by STL + ETS is often better than an ETS model without transformation.
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Calendar adjustments

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- Trading day

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Month length adjustment

If this is not removed, it shows up as a seasonal effect, which may not cause problems though it does make any seasonal pattern hard to interpret. It is easily adjusted for:

$$
\begin{aligned}
y_t^* &= y_t \times \frac{\text{no. of days in an average month}}{\text{no. of days in month } t} \\
&= y_t \times \frac{365.25/12}{\text{no. of days in month } t}
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where y_t has already been transformed if necessary.

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`monthdays` gives the number of days in each month or quarter.

Trading day adjustment

- occurs in monthly data when there is also a weekly cycle, since proportions of various days in given month vary from year to year.
- number of trading days is predictable, but effects of various days are unknown.
- **Simplest case:** All trading days assumed to have same effect.

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Explainable variation

Examples:

- Calendar variation
- Increasing population
- Inflation
- Strikes
- Changes in government
- Changes in law

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