# Forecasting and the importance of being uncertain

**Rob J Hyndman**

MONASH University

# 2006 Belz Lecture

**Maurice H Belz (1897–1975)**

Professor of Statistics, University of Melbourne, 1955–1963

# Outline

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1. **Dangers and difficulties of forecasting**
2. A brief history of forecasting
3. Forecasting the PBS
4. Forecasting Australia's population
5. Conclusions

# Forecasters are to blame!

## News report on 16 August 2006

A Russian woman is suing weather forecasters for wrecking her holiday. A court in Uljanovsk heard that Alyona Gabitova had been promised 28 degrees and sunshine when she planned a camping trip to a local nature reserve, newspaper *Nowyje Iswestija* said.

But it did nothing but pour with rain the whole time, leaving her with a cold. Gabitova has asked the court to order the weather service to pay the cost of her travel.

# Reputations can be made & lost

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- “There is no reason anyone would want a computer in their home.” (President, DEC, 1977)
- “There are four ways economists can lose their reputation. Gambling is the quickest, sex is the most pleasurable and drink the slowest. But forecasting is the surest.” (Max Walsh, *The Age*, 1993)

# Those "unforeseen events"

Precautions should be taken against running into unforeseen occurrences or events. (Horoscope, *New York Times*)

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We are ready for any unforeseen event which may or may not occur.

(Dan Quayle)

# Outline

# What is it?

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**Clay model of sheep's liver**

Used by Babylonian forecasters approximately 600 B.C.

Now in British Museum.

# Delphic oracle

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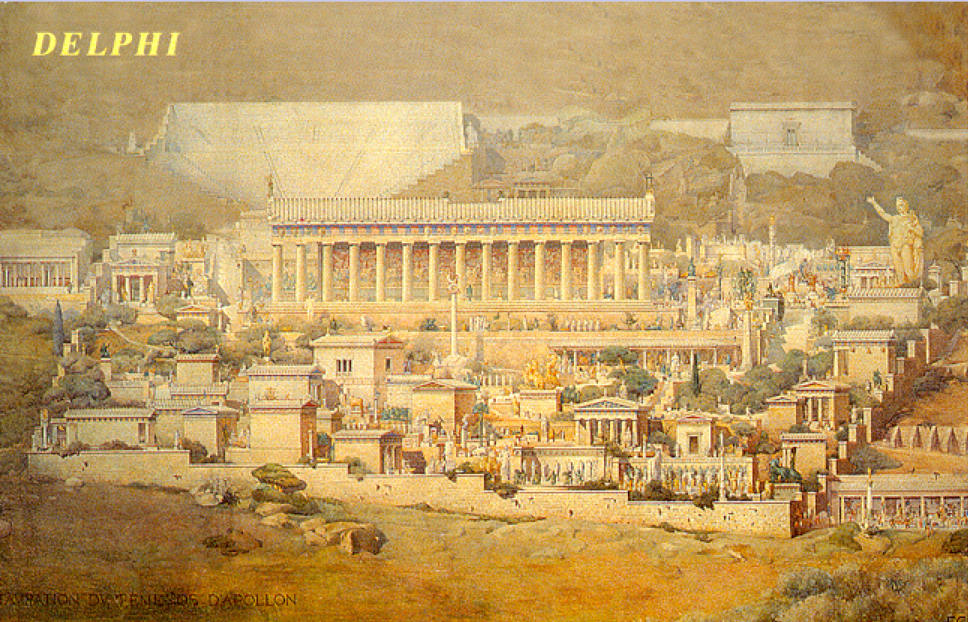
# Temple of Apollo



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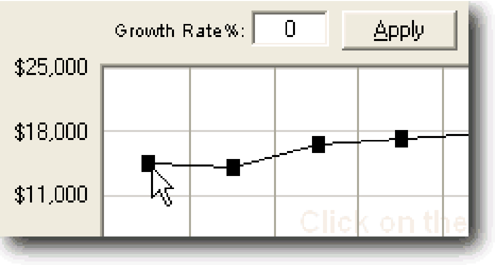
**Punishment:** a fine or three months' imprisonment with hard labour.

# Standard business practice today

## Graphic Forecaster

Create forecasts visually with a "drag and drop" graphic forecaster. The Graphic Forecaster is a simple and powerful tool to streamline the forecasting process. You can change your sales and expenses estimates by simply clicking your mouse button to move the line on your forecast chart or apply a specific growth rate to the whole year. Build forecasts using visual common sense.

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# Standard business practice today

## Crystal Xcelsius Showcase

Examples of what you can build with Crystal Xcelsius.

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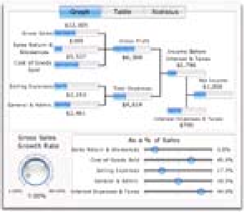
If you cannot open these demos, download the latest version of Macromedia's Flash Player.

### Featured Example: Profitability Analysis



**Profitability Analysis**

This profitability model allows you to create "what-if" scenarios by modifying sales growth rate and all other relevant accounts measured as a percentage of total sales. This example, built with fictitious data, depicts the most relevant accounts of a profit and loss statement, and shows the impact of changes on net income. The results change immediately, allowing you to create endless what-if scenarios.

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# Standard business practice today

## Budget Maestro by Centage

**Click here for a free demo** FREE!

**Application:** Business Intelligence and Analytics

**Price Range:** Solutions start at $5K

---

A huge advance over spreadsheet-based systems, Budget Maestro is a complete solution for budgeting, forecasting, what-if scenario planning, reporting and analysis. Budget Maestro takes the pain out of the budgeting process (no tedious data entry and formula verification) while providing you a tool to more accurately analyze and measure business performance and profitability. Budget Maestro's capabilities include:

**Budgeting and Forecasting:** Budget Maestro utilizes database technology for real-time data collection and reporting. A common interface for all users fosters collaboration and increases the accuracy of data entry. There are no formulas or macros to create, no tedious re-keying of data and no mystery links to chase down and fix. Budget Maestro's built-in "financial intelligence and business rules" builds the formulas for you ensuring 100% accuracy.

# Standard business practice today

- “What-if scenarios” based on assumed and fixed future conditions.

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**Is this any better than a sheep’s liver or hallucinogens?**

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- Classification and regression trees

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- Strong emphasis on out-of-sample predictive performance (the test data).
- Recognition that many problems are about prediction not p-values.

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- Limited interpretability of many models.
- Software can be up to 50 times the cost of comparable statistical software.
- Ignorance of basic statistical principles and methods.

# The rise of stochastic models

**1959** exponential smoothing (Brown)

**1970** ARIMA models (Box, Jenkins)

**1980** VAR models (Sims, Granger)

**1980** non-linear models (Granger, Tong, Hamilton, Teräsvirta, . . . )

**1982** ARCH/GARCH (Engle, Bollerslev)

**1986** neural networks (Rumelhart)

**1989** state space models (Harvey, West, Harrison)

**1994** nonparametric forecasting (Tjøstheim, Härdle, Tsay,. . . )

# Advantages of stochastic models

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- Based on empirical data
- Computable
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- Testable
- Objective measure of uncertainty
- Able to compute prediction intervals

# The skeptics

*It is utterly implausible that a mathematical formula should make the future known to us, and those who think it can would once have believed in witchcraft.*

(Bertrand de Jouvenel
The Art of Conjecture, 1967.)

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AUSTRALIAN BROADCASTING CORPORATION

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LISTEN: WMP | Real

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- ACT Election
- Middle East

POLITICS

**Opp demands drug price restriction after PBS budget blow-out**

The Federal Opposition has called for tighter controls on drug prices after the Pharmaceutical Benefits Scheme (PBS) budget blew out by almost $800 million.

The money was spent on two new drugs including the controversial anti-smoking aid Zyban, which dropped in price from $220 to $22 after it was listed on the PBS.

the Public Record
For full election coverage

FEATURES

Public Record
Federal Election 2001

For a fresh perspective on the federal election, reach into ABC Online's campaign weblog, The Poll Vault.

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ABC News Online Bulletin
VIDEO ON DEMAND

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Department of Health and Aging

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- All forecasts being done with the `FORECAST` function in MS-Excel applied to 10 year old data!

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- Forecast error now a few $million per year.

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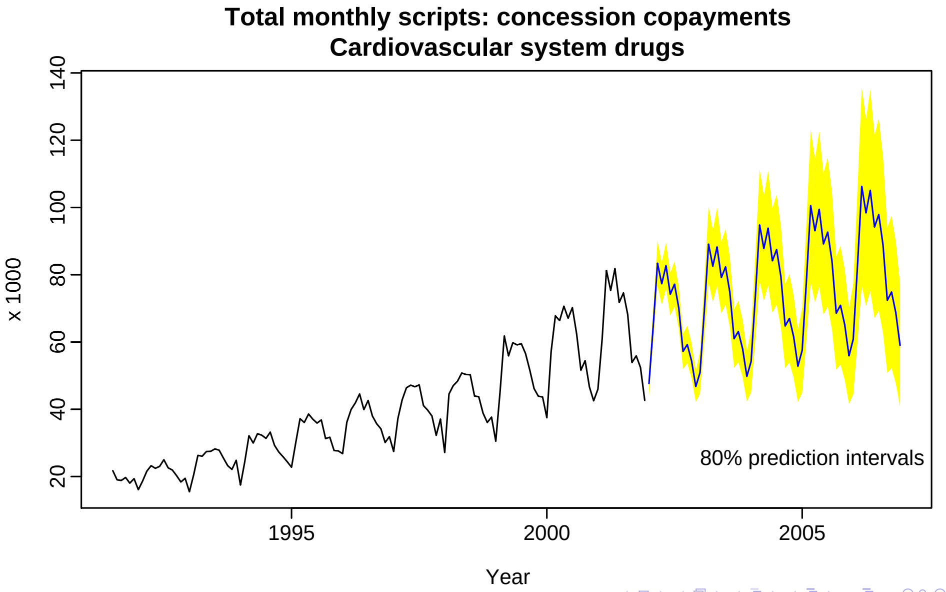

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- Stochastic models provide prediction intervals which give a sense of uncertainty.
- Class of models was based on exponential smoothing.
- At the time, exponential smoothing methods were not thought to be based on stochastic models.

# Exponential smoothing

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*"Unfortunately, exponential smoothing methods do not allow easy calculation of prediction intervals."*

Makridakis, Wheelwright and Hyndman, p.177.
(Wiley, 3rd ed., 1998)

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- an algorithm for automatic forecasting using the new class of models.
- new results on the admissible parameter space.

# Exponential smoothing state space models

All exponential smoothing methods can be written in the following form:

$$
\begin{aligned}
Y_t &= h(\boldsymbol{x}_{t-1}) + k(\boldsymbol{x}_{t-1})\varepsilon_t \\
\boldsymbol{x}_t &= f(\boldsymbol{x}_{t-1}) + g(\boldsymbol{x}_{t-1})\varepsilon_t
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where $\{\varepsilon_t\}$ is a Gaussian white noise process with mean zero and variance $\sigma^2$, and $\boldsymbol{x}_t$ is an unobserved state.

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Forecasts are given by $\mathrm{E}(Y_{n+h} \mid \boldsymbol{x}_n)$.

# Exponential smoothing state space models

## Holt-Winters' additive seasonal method

$$
\begin{aligned}
\hat{Y}_{t+h|t} &= \ell_t + hb_t + s_{t+h-m}. \\
\ell_t &= \alpha(Y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1}) \\
b_t &= \beta(\ell_t - \ell_{t-1}) + (1-\beta)b_{t-1} \\
s_t &= \gamma(Y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}
\end{aligned}
$$

where $0 \le \alpha \le 1$, $0 \le \beta \le 1$, $0 \le \gamma \le 1$ and $m$ is the period of seasonality.

# Exponential smoothing state space models

## Additive error state space model

Let $\boldsymbol{x}_t = (\ell_t, b_t, s_t, s_{t-1}, \dots, s_{t-m+1})'$. Then

$$\begin{aligned} Y_t &= h(\boldsymbol{x}_{t-1}) + \varepsilon_t \\ \boldsymbol{x}_t &= f(\boldsymbol{x}_{t-1}) + g(\boldsymbol{x}_{t-1})\varepsilon_t \end{aligned}$$

where $h(\boldsymbol{x}_{t-1}) = \ell_{t-1} + b_{t-1} + s_{t-m}$

$$f(\boldsymbol{x}_{t-1}) = \begin{bmatrix} \ell_{t-1} + b_{t-1} \\ b_{t-1} \\ s_{t-m} \\ s_{t-1} \\ \vdots \\ s_{t-m+1} \end{bmatrix}, \quad g(\boldsymbol{x}_{t-1}) = \begin{bmatrix} \alpha \\ \alpha\beta \\ \gamma \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

# Exponential smoothing state space models

Then $\hat{Y}_{n+h|n} \equiv \mathrm{E}[Y_{n+h} \mid \boldsymbol{x}_n] = \ell_n + hb_n + s_{n+h-m}.$

- So Holt-Winter forecasts are optimal for this state space model.

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- The likelihood can be optimized to obtain estimates of smoothing parameters and initial states.
- The model can be used to obtain prediction intervals.

# Exponential smoothing state space models

| Trend Component | Seasonal Component: **N** (none) | Seasonal Component: **A** (additive) | Seasonal Component: **M** (multiplicative) |
|---|---|---|---|
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**General notation**

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- provides facilities to compute (analytical) prediction intervals
- allows the state equations to be expressed in a form which coincides with the error-correction form of the usual smoothing equations.
- two possible state space models for each method (additive error and multiplicative error). Equivalent point forecasts, different prediction intervals.

# Automatic forecasting

For each method, we derive an equivalent state space formulation

- Apply each of the methods that are appropriate to the data. Optimize parameters using MLE (or some other criteria).

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This methodology is available as

- ➥ An R package (**forecast**)
- ➥ An Excel add-in (**PhiCast**)

# Exponential smoothing

## Key References

- Hyndman, R.J., Koehler, A.B., Snyder, R.D., & Grose, S. (2002) A state space framework for automatic forecasting using exponential smoothing methods. *International J. Forecasting*, **18**(3), 439–454.

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- Snyder, R.D., Koehler, A.B., Hyndman, R.J., & Ord, J.K. (2003) Exponential smoothing models: Means and variances for lead-time demand. *European Journal of Operational Research*, **158**(2) 444–455.

# Outline

# ABS population projections

The Australian Bureau of Statistics provide population "projections".

*"The projections are not intended as predictions or forecasts, but are illustrations of growth and change in the population that would occur if assumptions made about future demographic trends were to prevail over the projection period.*
*While the assumptions are formulated on the basis of an assessment of past demographic trends, both in Australia and overseas, there is no certainty that any of the assumptions will be realised. In addition, no assessment has been made of changes in non-demographic conditions."*

ABS 3222.0 - Population Projections, Australia, 2004 to 2101

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➥ No-one who uses the ABS projections appreciates the distinction between projections and forecasts.

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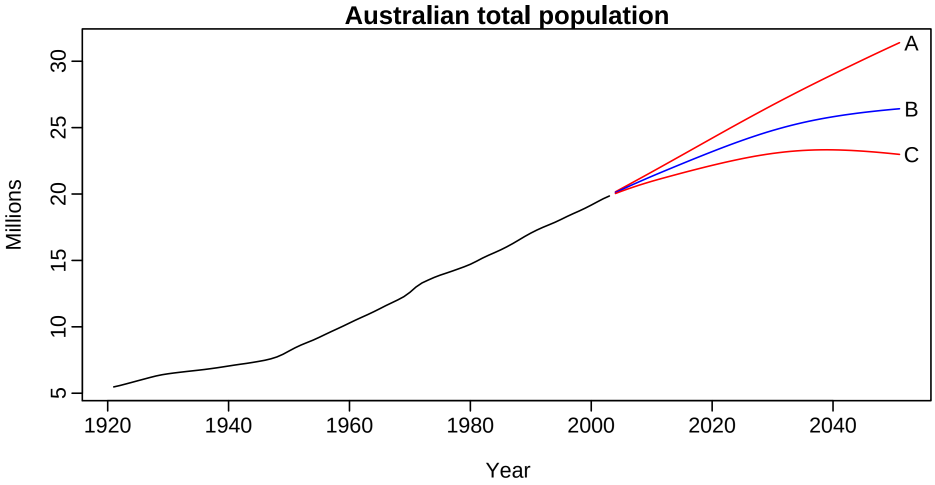
- Not prediction intervals.
- Most users use the “Medium” projection, but it is unrelated to the mean, median or mode of the future distribution.

# ABS population projections



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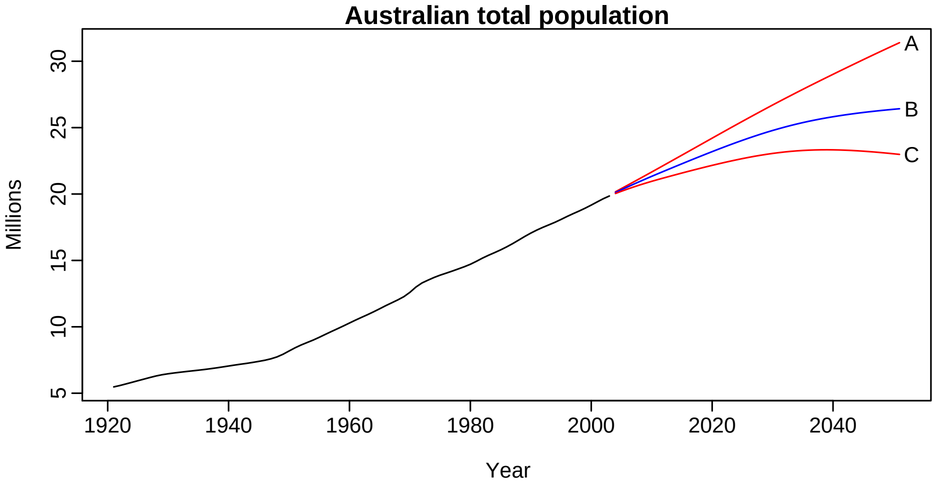


**What do these projections mean?**

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- Stochastic models allow true policy analysis to be carried out.

# Functional time series

# Functional time series model

$y_t(x)$ = log mortality rate, log fertility rate,
or net migration numbers for age $x$ in year $t$.

We observe

$$y_t(x) = f_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

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- We want to forecast **whole curve** $y_t(x)$ for $t = n + 1, \dots, n + h$.

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y_t(x) &= f_t(x) + \sigma_t(x)\varepsilon_{t,x} \\
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where $e_t(x) \sim \mathrm{N}(0, v(x))$.

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5. Put it all together to get forecasts of $y_t(x)$.

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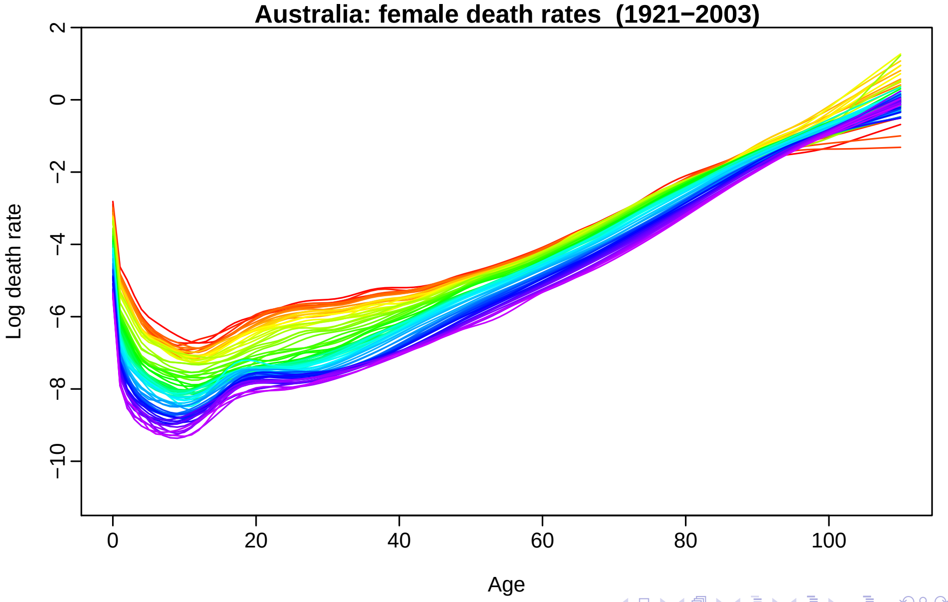

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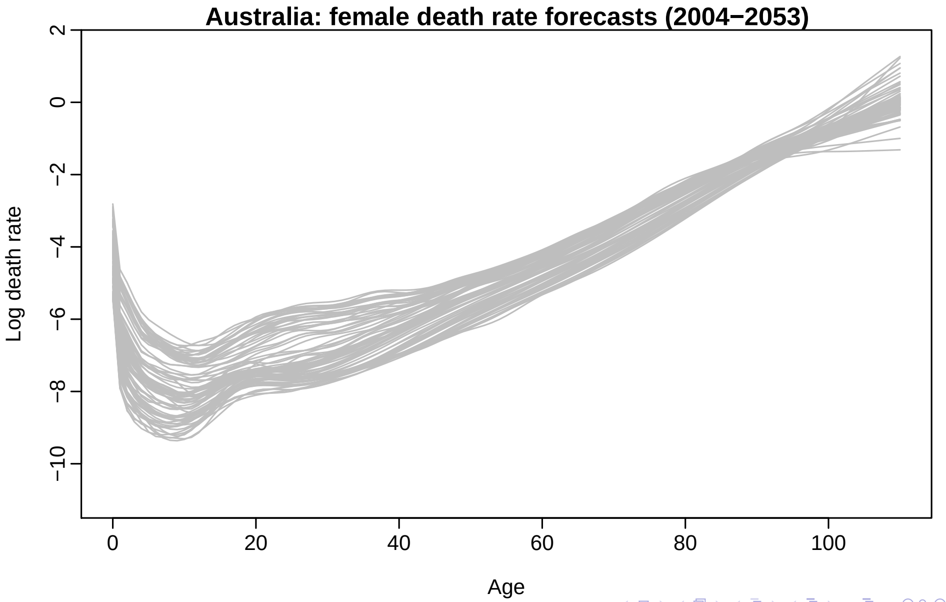

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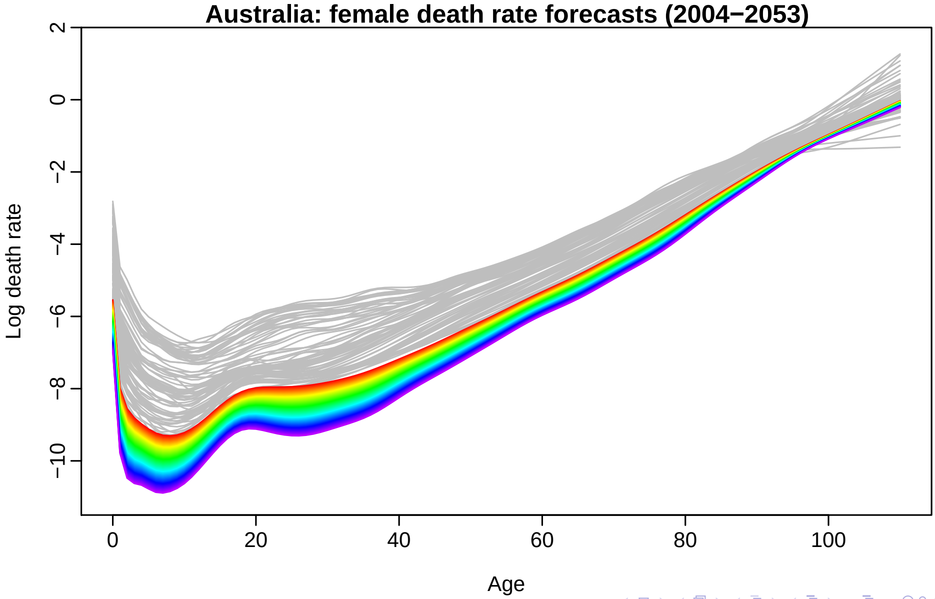

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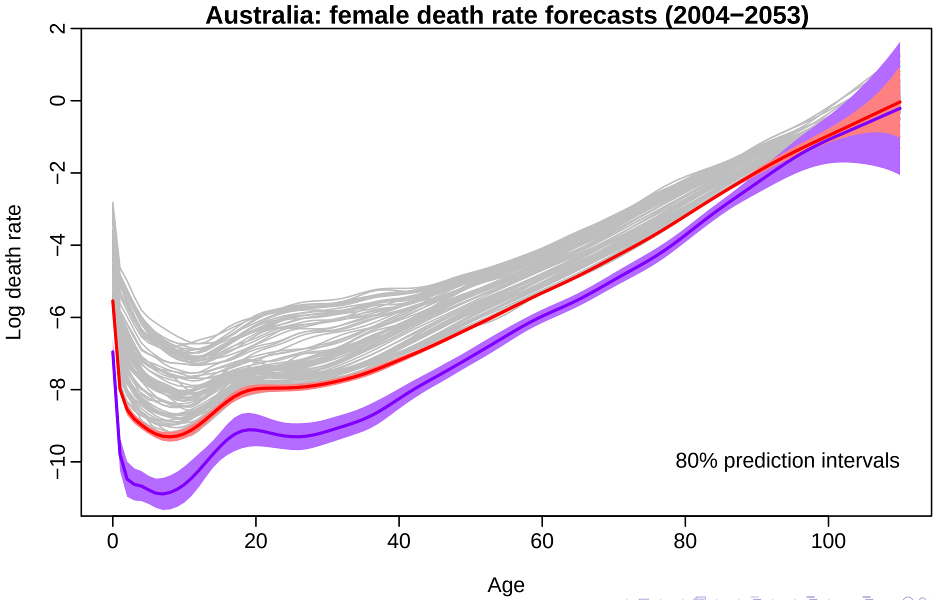

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- Approach depends on good data and good models for fertility, mortality, and net migration.

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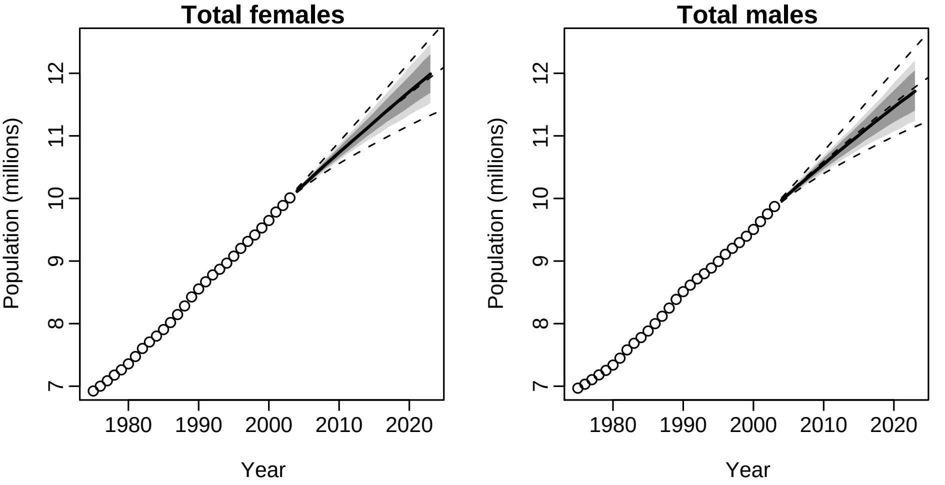

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## Some recent references

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- Booth (2004) On the importance of being uncertain: forecasting population futures for Australia. *People and Place*, **12**(2), 1–11.
- Hyndman and Booth (2006) Stochastic population forecasts using functional data models for mortality, fertility and migration. Monash working paper.

# Outline

# Conclusions

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- Data mining has been enthusiastically adopted in business, but statistical methods have not.

# Some further thoughts

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***Uncertainty limits: within 10 years $\pm$5 years.***

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# Final comments

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Slides available from
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