MONASH University

Rob J Hyndman

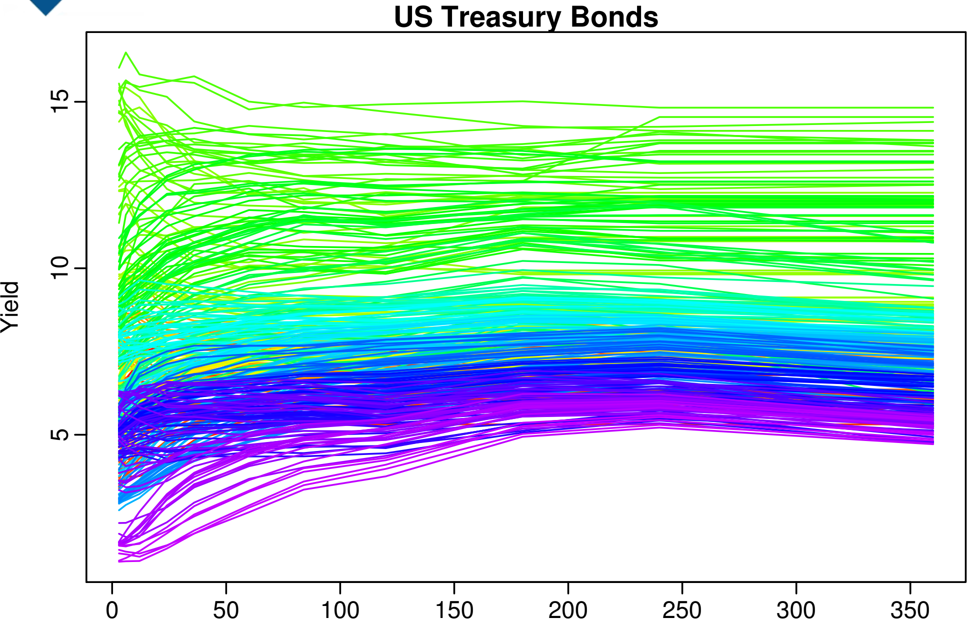
Functional time series

with applications in demography

4. Connections, extensions and applications

Outline

Example: Yield curves



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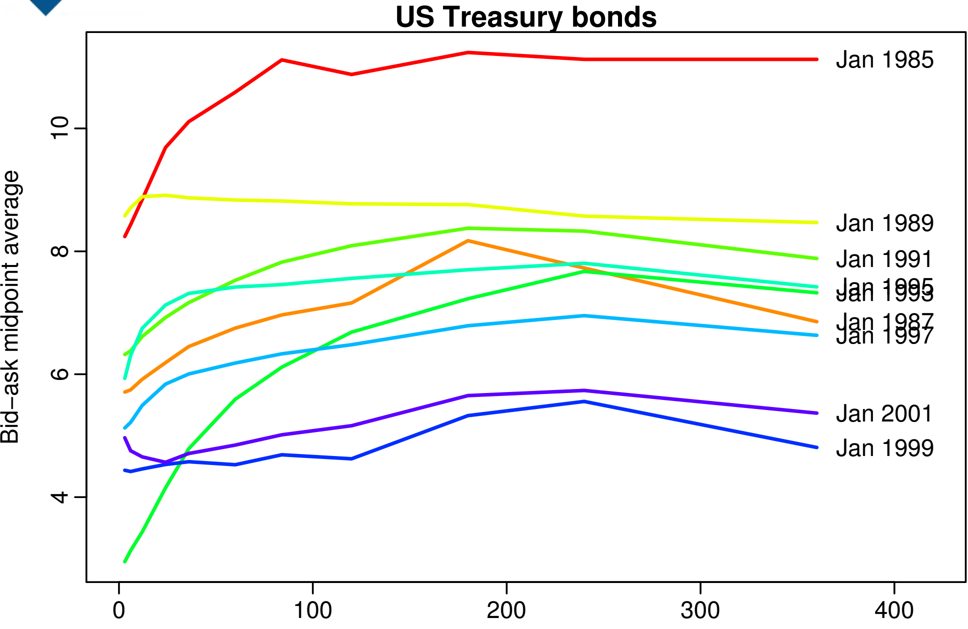

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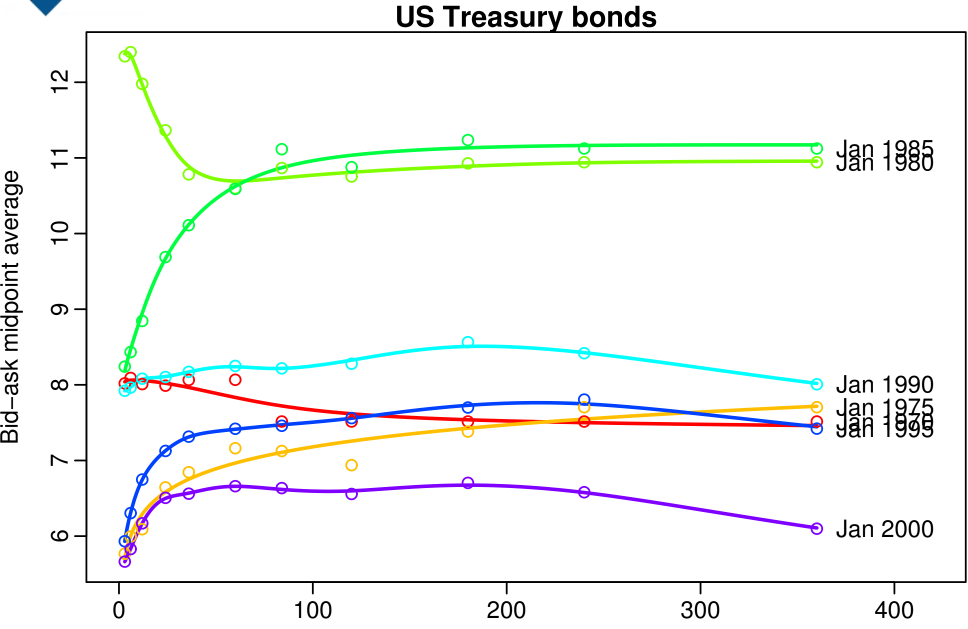
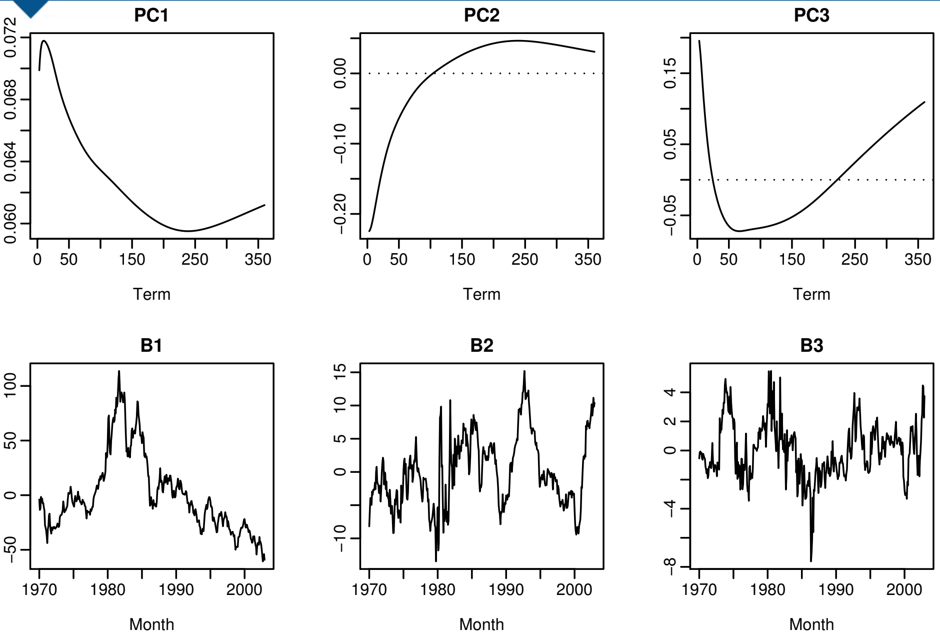
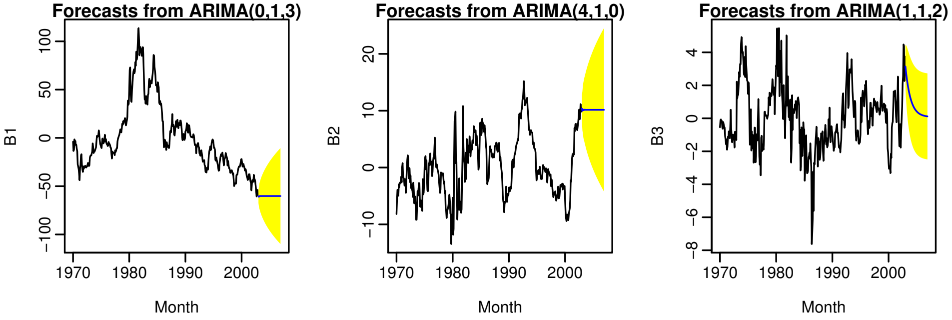


Model for yield curves

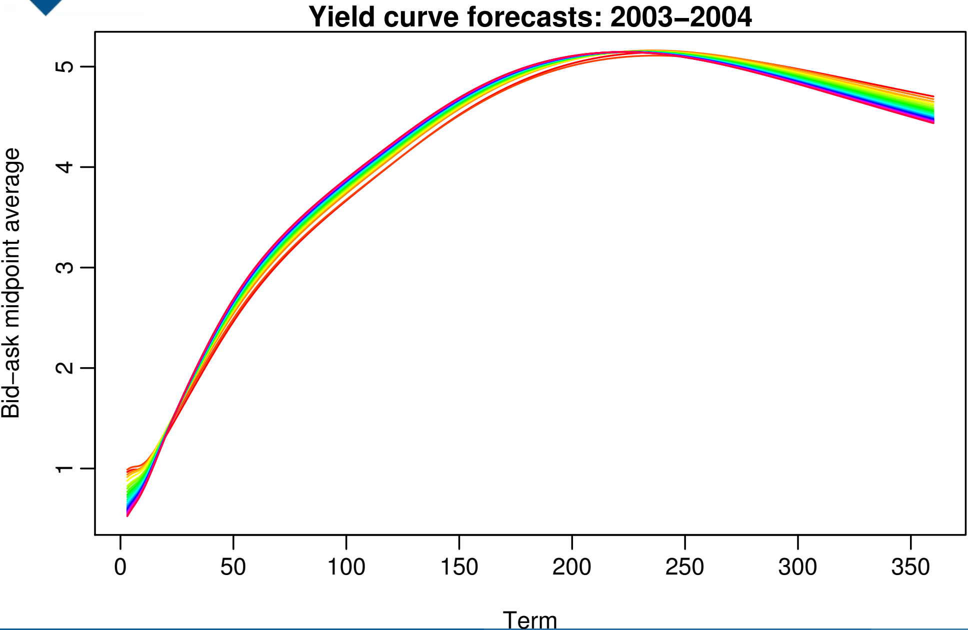


Forecasts of coefficients

ARIMA forecasts (with first differencing). 80% prediction intervals shown in yellow.



2 year forecasts for yield curves



Nelson-Siegel models

Functional time series model

$$y_{t,x} = \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\, \phi_k(x) + r_t(x) + \varepsilon_{t,x}$$

Nelson-Siegel model

$$y_{t,x} = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t x}}{\lambda_t x}\right) + \beta_{3,t} e^{-\lambda_t x} + r_t(x)$$

- Well-behaved at long maturities.
- λ_t is usually fixed and pre-specified.
- Useful for describing yield curves and predicting yield for unobserved x.
- Coefficients highly collinear making interpretation and forecasting difficult.

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Diebold-Li model

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- All coefficients usually forecast with univariate AR(1) models.

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F.X. Diebold, C. Li / Journal of Econometrics 130 (2006) 337–364 345

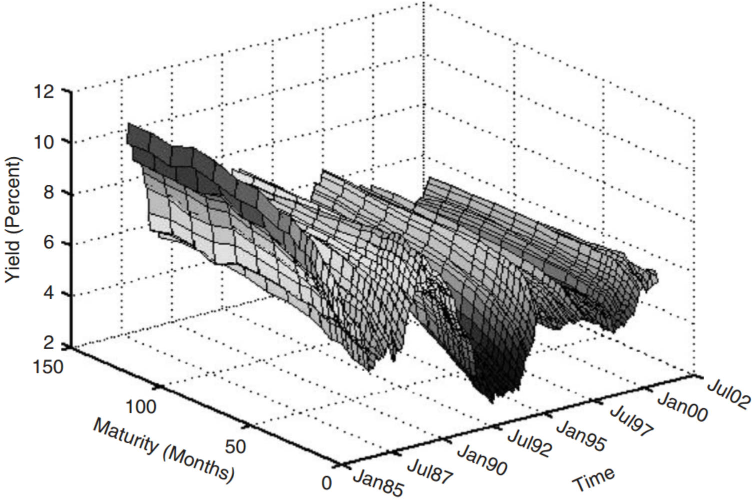

Fig. 2. Yield curves, 1985.01–2000.12. The sample consists of monthly yield data from January 1985 to December 2000 at maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months.

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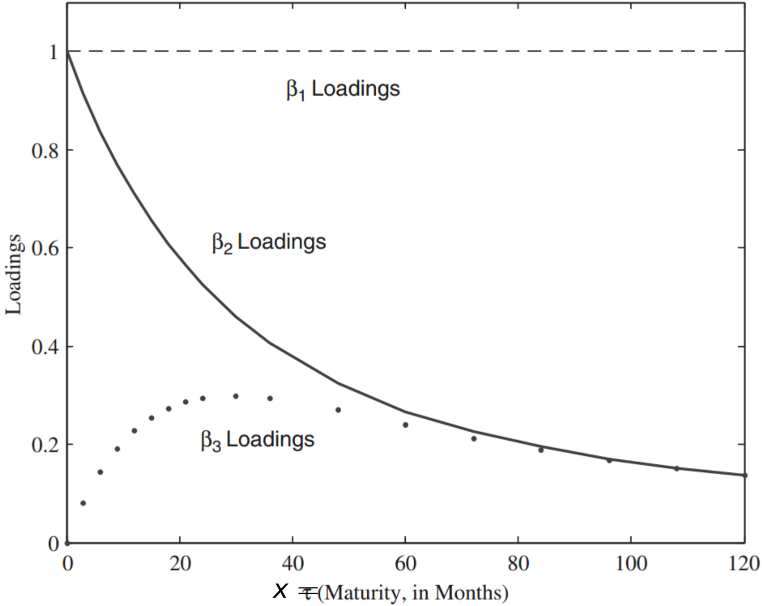

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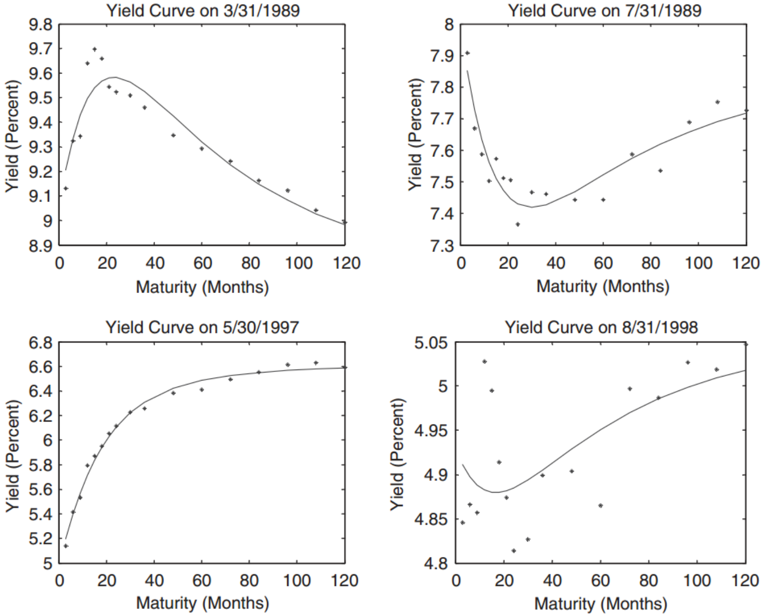

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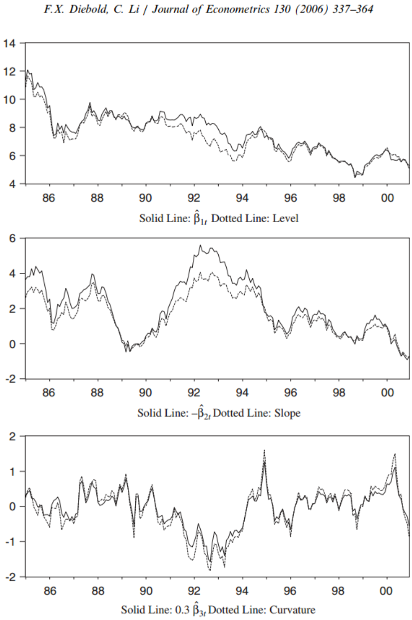

Bowsher-Meeks model

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$$y_{t,x} = \sum_{j=1}^{J} \gamma_{t,j}\,\xi_j(x) + \sigma_t(x)\varepsilon_{t,x}$$

- $\{\xi_j(x)\}$ are spline terms;
- $\Delta\boldsymbol{\gamma}_{t+1} = \boldsymbol{A}(\boldsymbol{B}\boldsymbol{\gamma}_t - \boldsymbol{\mu}) + \boldsymbol{\Psi}\Delta\boldsymbol{\gamma}_t + \boldsymbol{e}_t$ is a cointegrated VAR with $\boldsymbol{e}_t \sim N(\boldsymbol{0}, \boldsymbol{\Omega})$.

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$$s_t(x) = \sum_{k=1}^{K} \beta_{t,k}\,\phi_k(x) + r_t(x)$$

$$\beta_{t,k} - \mu_k = \sum_{r=1}^{p} \psi_{r,k}(\beta_{t-r,k} - \mu_k) + v_{t,k}$$

- Equations estimated simultaneously using penalized likelihood (analogous to PCA but taking account of autocorrelations).

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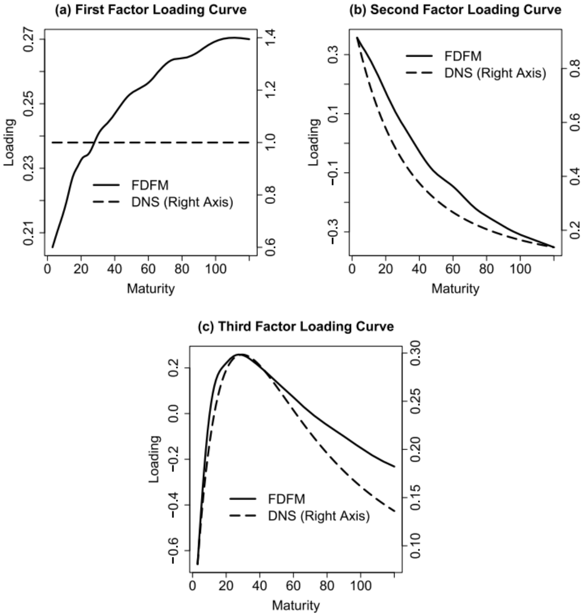
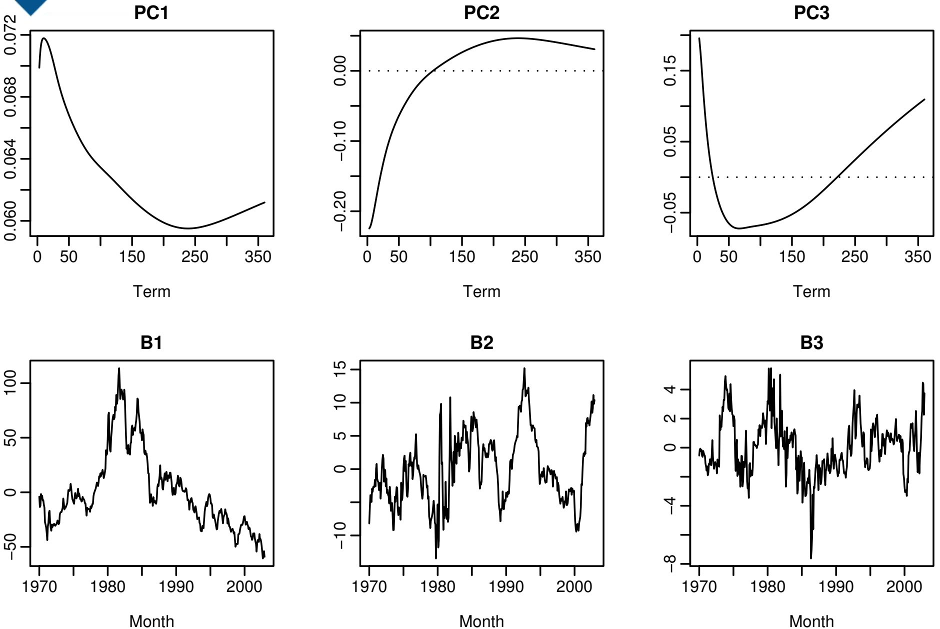
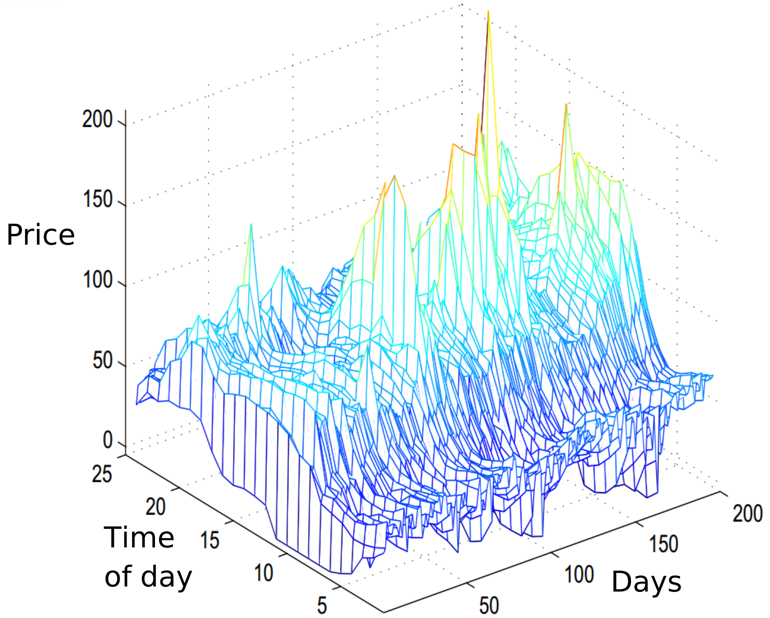


Compare FPCA



Outline

Electricity prices



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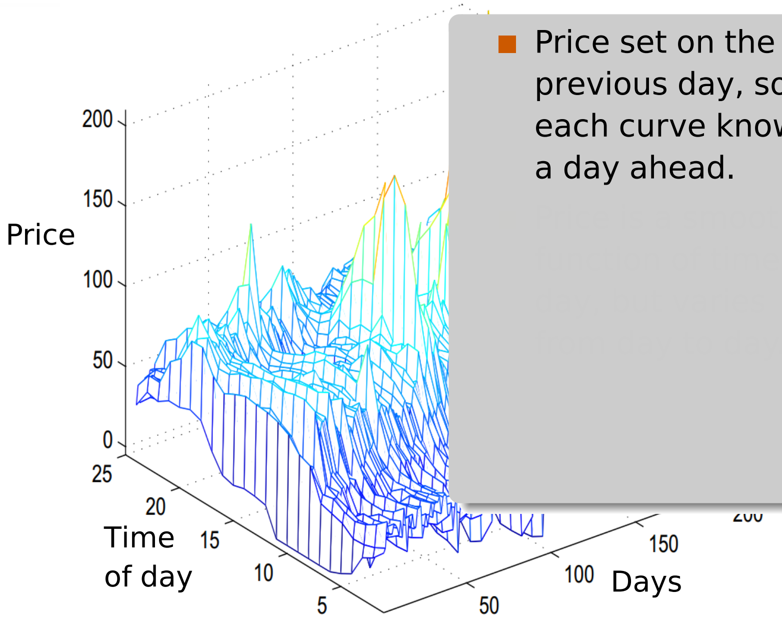

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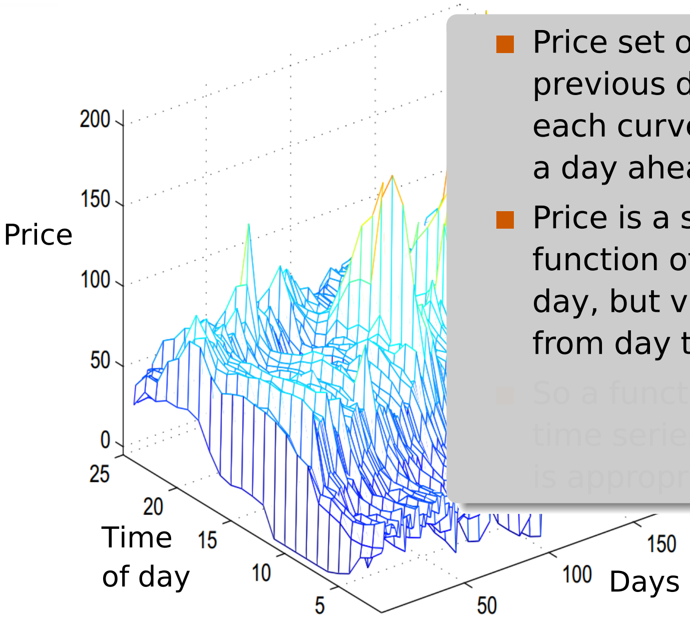

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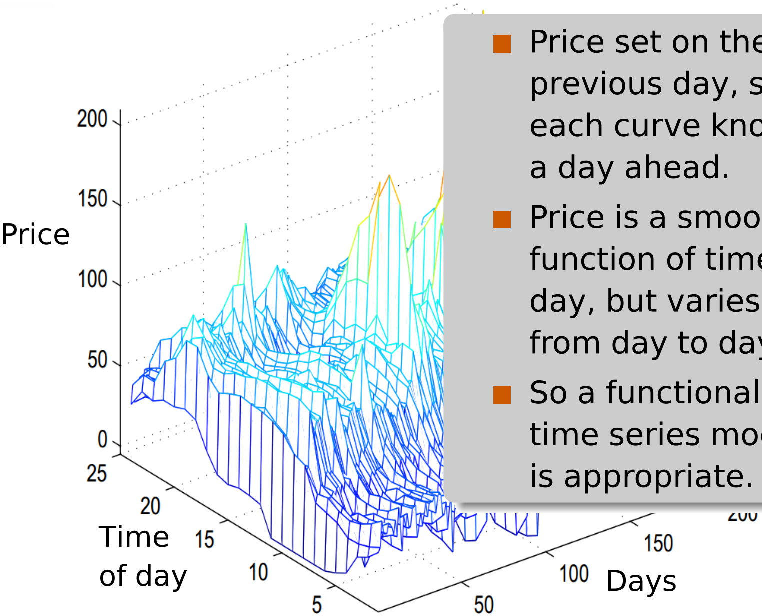

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- Functional time series models allows for multiple seasonality: time of day, day of week and time of year.
- Time of day is handled by the functions while day of week and time of year are handled via the PC scores.
- Different time of day patterns (e.g., weekdays and weekends) can be handled via different PCs.

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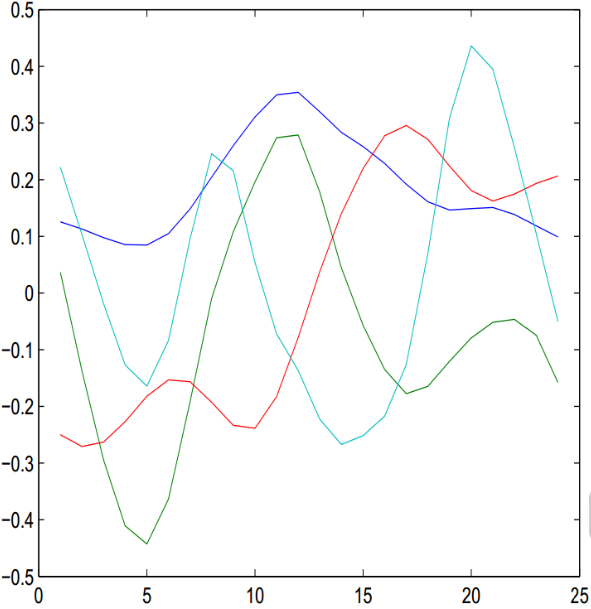

Variation explained

4%

74%

3%

10%

Eigenfunctions

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We split a seasonal univariate time series into sections of one year.

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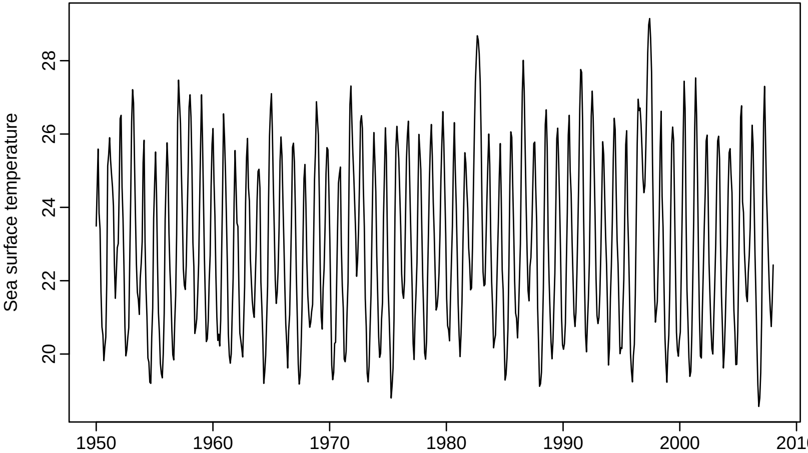

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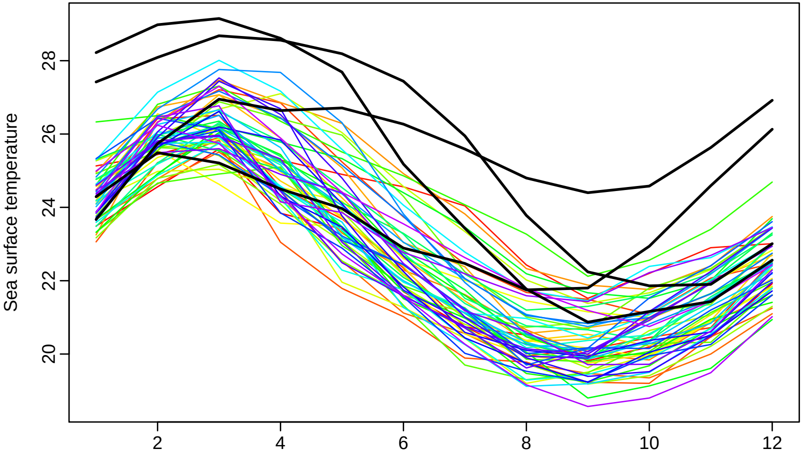
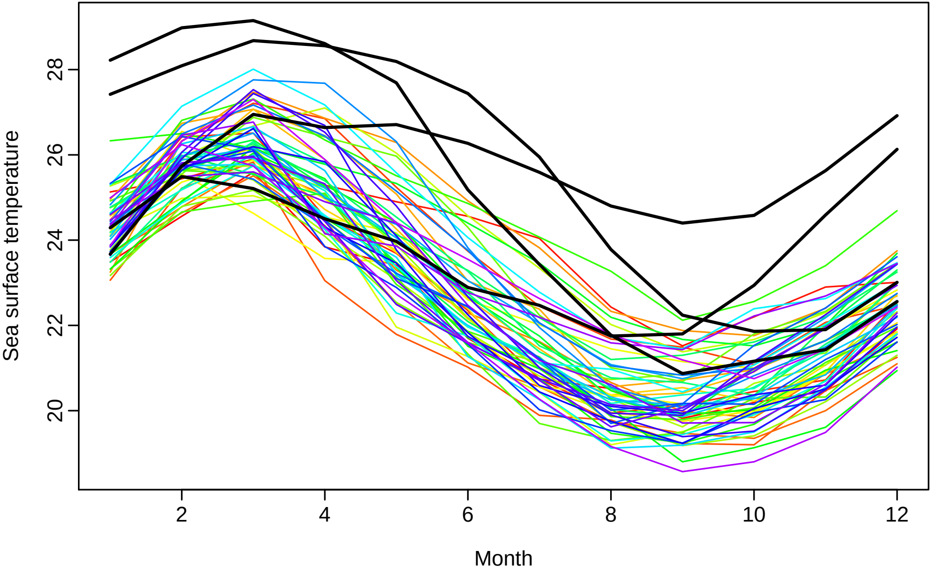


Sea surface temperatures



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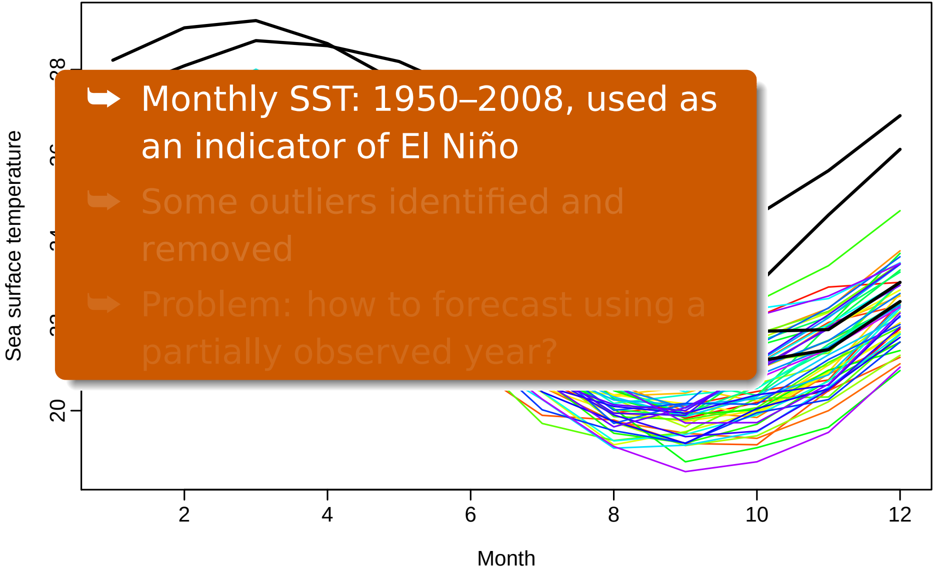

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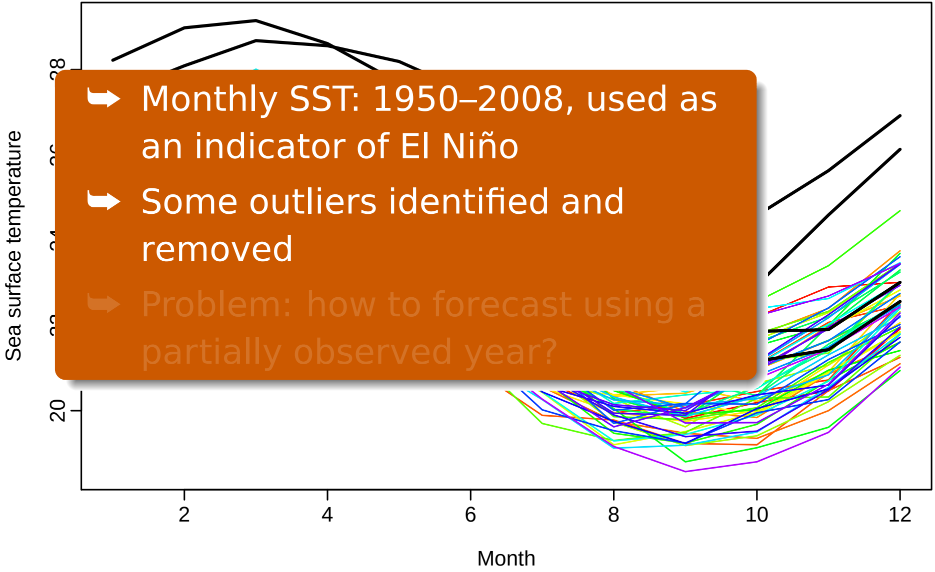

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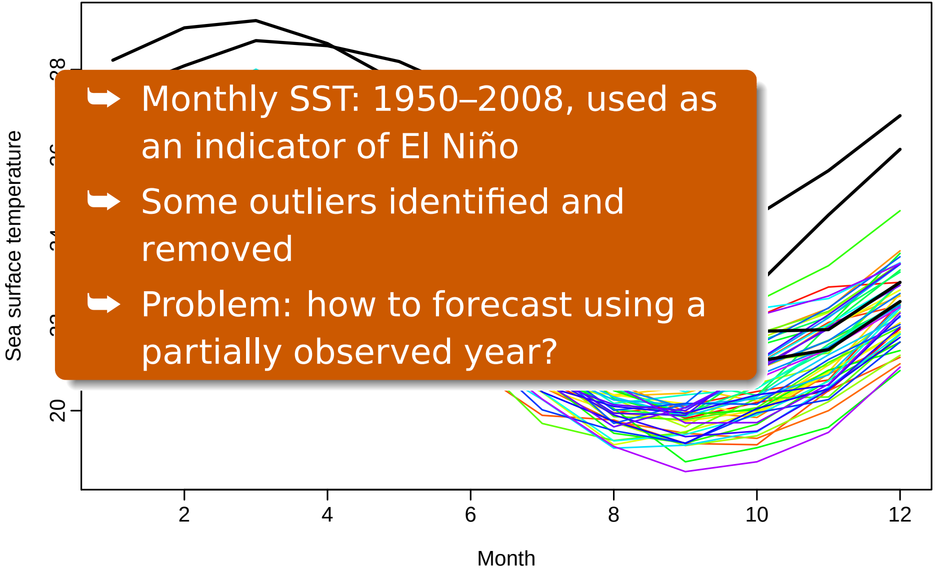

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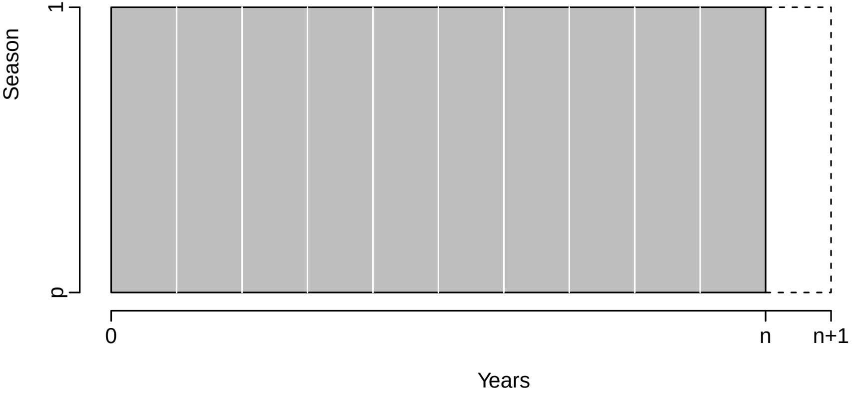
Redefine the year



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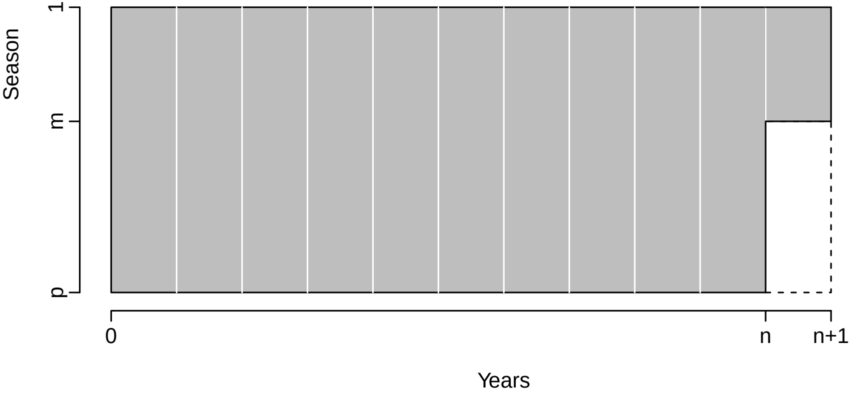

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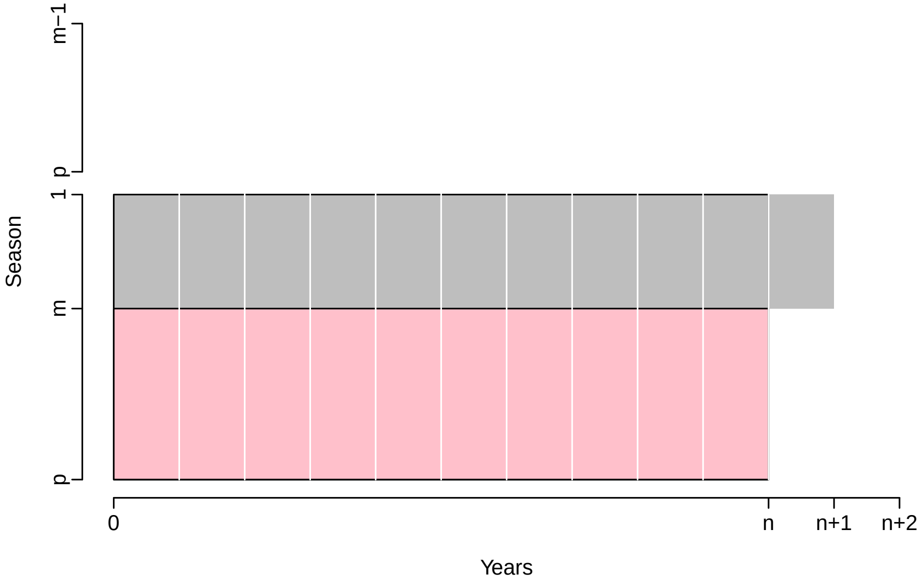

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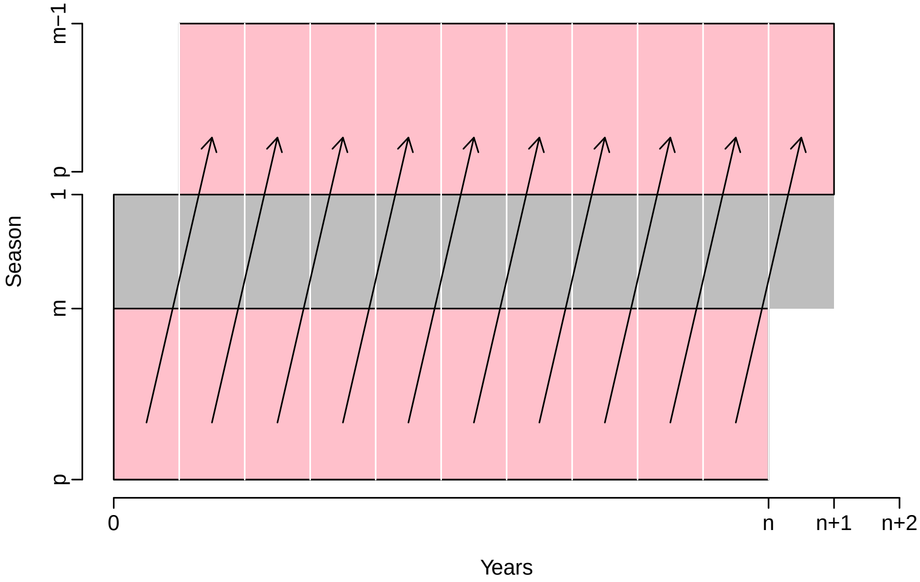

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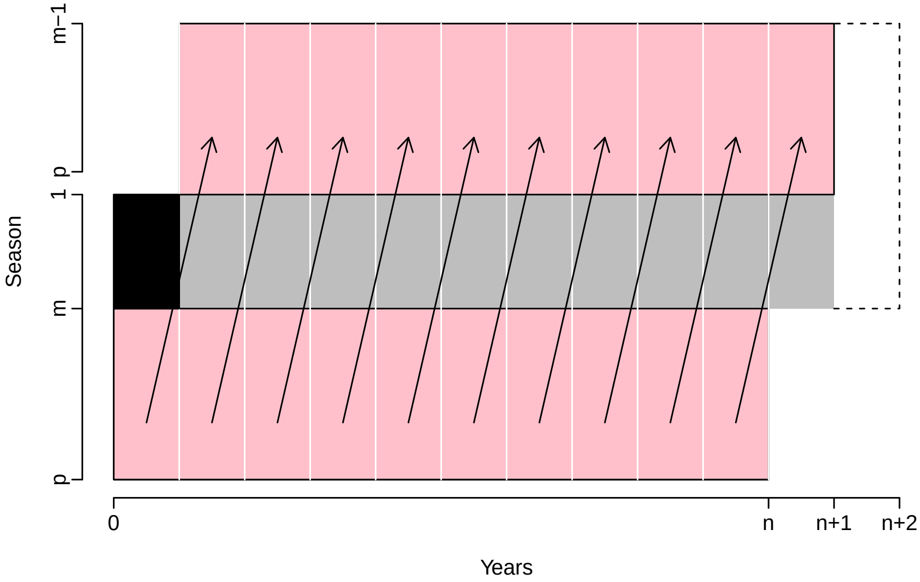

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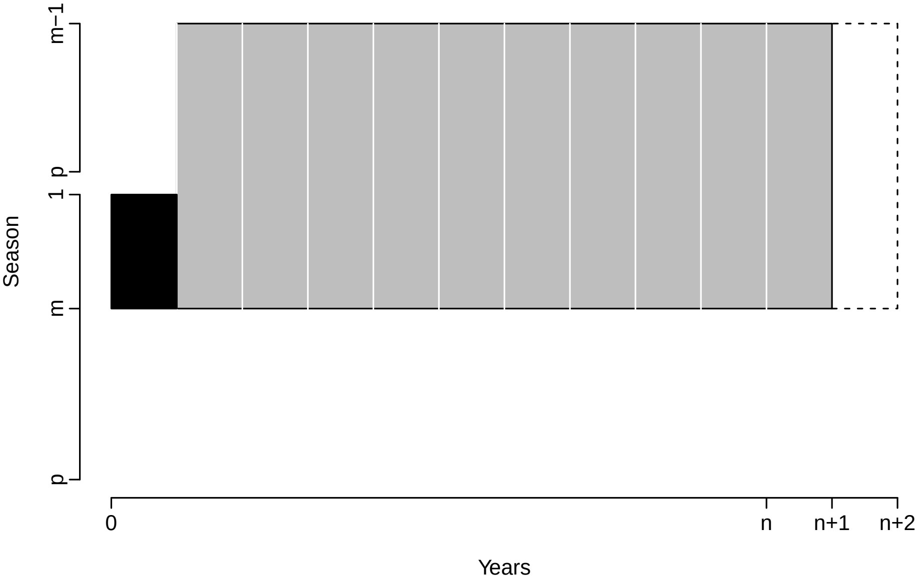
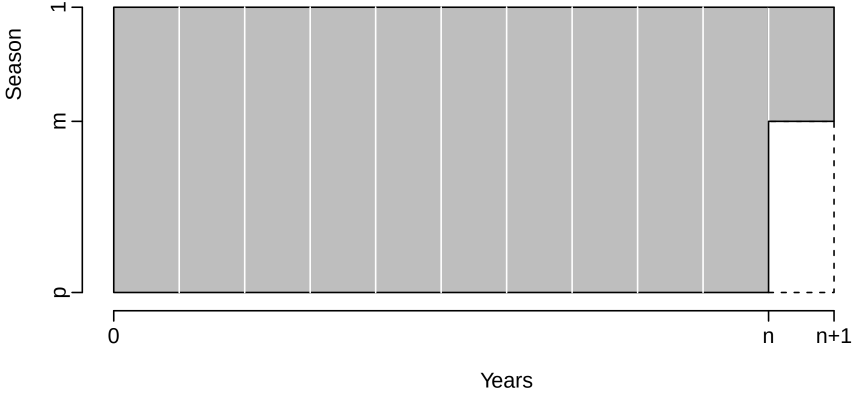


Dynamic updating: solution 2

OLS regression

- For each month, regress the month against the partial principal components.



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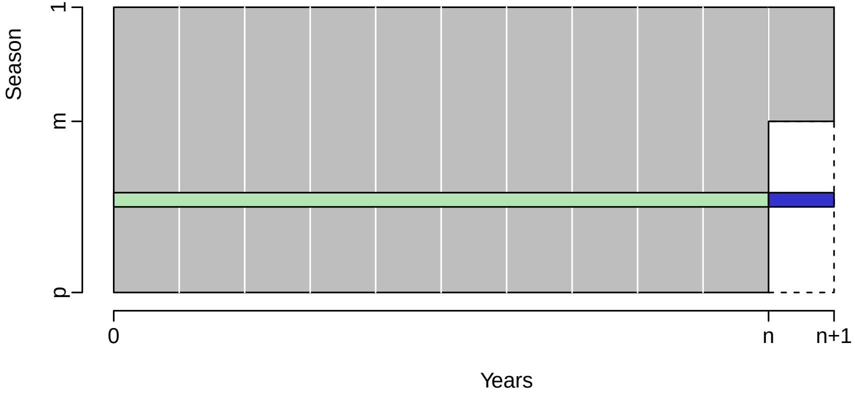
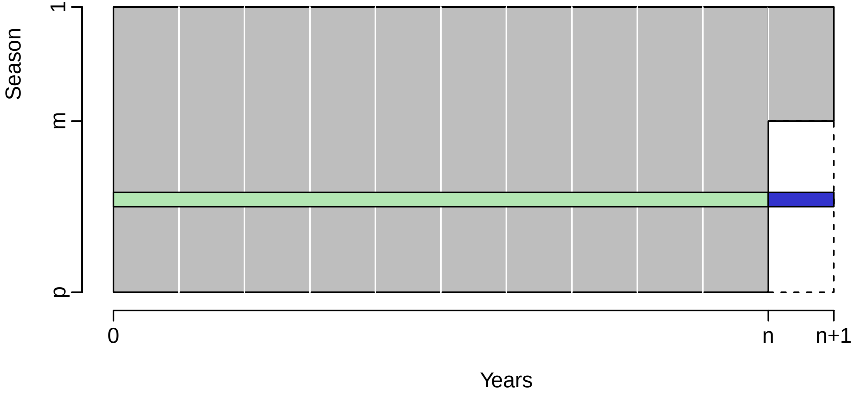


Dynamic updating: solution 3

Ridge regression

- For each month, regress the month against the partial principal components.



Dynamic updating

Mean squared error

Computed on a rolling forecast origin

Univariate methods				Dynamic updating		
MP	RW	SARIMA	FTS	BM	OLS	RR
0.69	1.45	0.98	0.74	0.69	1.04	0.48

- MP = mean predictor (mean of prior data)
- RW = random walk
- SARIMA = seasonal ARIMA model
- FTS = functional time series
- BM = block moved
- OLS = ordinary least squares
- RR = ridge regression

Outline

Functional ARH model

Autoregressive Hilbertian process of order 1

ARH(1) model

$$
\begin{aligned}
f_t(x) - \mu(x) &= \int [f_{t-1}(x) - \mu(x)]\theta(x,y)\,dy + e_t(x) \\
&= \theta[f_{t-1}(x) - \mu(x)] + e_t(x)
\end{aligned}
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- $\int\int \theta^2(x,y)\,dx\,dy < 1$ for stationarity.
- Bosq (2000) and Horváth & Kokoszka (2012) provide basic theory.
- Estimating $\theta(x,y)$ is very difficult to do well.

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ARH(p) model

$$f_t(x) - \mu(x) = \theta_1[f_{t-1}(x) - \mu(x)] + \cdots + \theta_p[f_{t-p}(x) - \mu(x)] + e_t(x)$$

- $\theta_j()$ are linear functions on a Hilbert space
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Functional ARH model

- I have never seen a convincing application of ARH models to real data.
- But the tools are nearly ready for real work.
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Outline

Selected references

- Nelson & Siegel (1987) Parsimonious modeling of yield curves. *J Business*.
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