NOTE: Zero-inflated Poisson regression using **proc countreg** or **proc genmod** is only available in SAS version 9.2 or higher.

This page shows an example of zero-inflated Poisson regression analysis with footnotes explaining the output in Stata. We have data on 250 groups that went to a park for a weekend, fish.sas7bdat (/stat/sas/code/fish.sas7bdat). Each group was questioned about how many fish they caught (**count**), how many children were in the group (**child**), how many people were in the group (**persons**), and whether or not they brought a camper to the park (**camper**). We explore the relationship of **count** with with **child**, **camper**, and **persons**.

For a Poisson model, we assume our response variable is a count variable, each subject has the same length of observation time, and the variance of the response variable is relatively close to the mean of the response variable. In a dataset in which the response variable is a count, the number of zeroes may seem excessive. With the example dataset in mind, consider the processes that could lead to a response variable value of zero. A group may have spent the entire weekend fishing, but failed to catch a fish. Another group may have not done any fishing over the weekend and, not surprisingly, caught zero fish. The first group *could* have caught one or more fish, but did not. The second group was certain to catch zero fish. The second group will be referred to from this point forward as a "certain zero". Thus, the number of zeroes may be inflated and the number of groups catching zero fish cannot be explained in the same manner as the groups that caught more than zero fish.

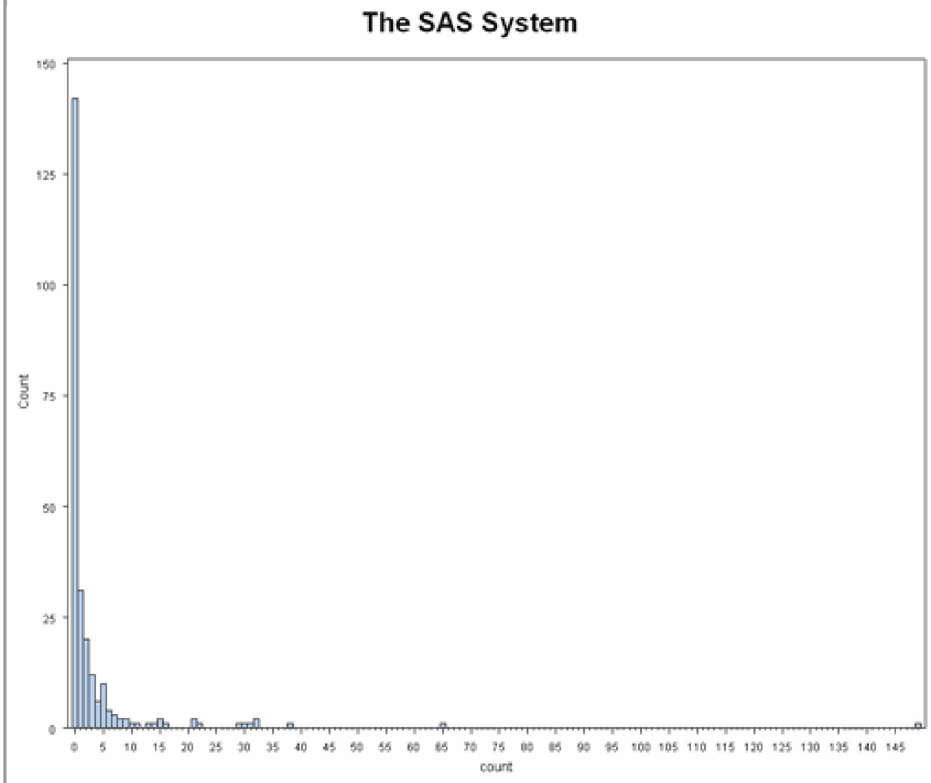
A standard Poisson model would not distinguish between the two processes causing an excessive number of zeroes, but a zero-inflated model allows for and accommodates this complication. When analyzing a dataset with an excessive number of outcome zeros and two possible processes that arrive at a zero outcome, a zero-inflated model should be considered. We can look at a histogram of the response variable to try to gauge if the number of zeros is excessive. (If two processes generated the zeroes in the response variable but there is not an excessive number of zeroes, a zero-inflated model may or may not be used.)

```
proc means data = fish mean std min max var;
  var count child persons;
run;
```

```
The MEANS Procedure

Variable             Mean         Std Dev         Minimum         Maximum        Variance
-----------------------------------------------------------------------------------------
count           3.2960000      11.6350281               0     149.0000000     135.3738795
child           0.6840000       0.8503153               0       3.0000000       0.7230361
persons         2.5280000       1.1127303       1.0000000       4.0000000       1.2381687
-----------------------------------------------------------------------------------------
```

```
proc univariate data = fish noprint;
  histogram count / midpoints = 0 to 50 by 1 vscale = count ;
run;
```



The zero-inflated Poisson regression generates two separate models and then combines them. First, a logit model is generated for the "certain zero" cases described above, predicting whether or not a student would be in this group. Then, a Poisson model is generated to predict the counts for those students who are not certain zeros. Finally, the two models are combined. There are two SAS procedures that can easily run a zero-inflated Poisson regression: **proc genmod** and **proc countreg**. Both will be shown on this page, starting with **proc genmod**.

When running a zero-inflated Poisson model in **proc genmod**, you must specify both models: first the count model in the **model** line, then the model predicting the certain zeros in the **zeromodel** line. In this example, we are predicting count with **child** and **camper** and predicting the certain zeros with **persons**.

```
proc genmod data = fish;
  model count = child camper /dist=zip;
  zeromodel persons /link = logit ;
run;

The GENMOD Procedure

             Model Information
Data Set                         WORK.FISH
Distribution          Zero Inflated Poisson
Link Function                          Log
Dependent Variable                   count

Number of Observations Read         250
Number of Observations Used         250

             Criteria For Assessing Goodness Of Fit
Criterion                    DF           Value        Value/DF
Deviance                              2063.2168
Scaled Deviance                       2063.2168
Pearson Chi-Square          245       1543.4597          6.2998
Scaled Pearson X2           245       1543.4597          6.2998
Log Likelihood                         774.8999
Full Log Likelihood                  -1031.6084
AIC (smaller is better)               2073.2168
AICC (smaller is better)              2073.4627
BIC (smaller is better)               2090.8241

Algorithm converged.


                    Analysis Of Maximum Likelihood Parameter Estimates
                                   Standard     Wald 95% Confidence          Wald
Parameter    DF     Estimate        Error           Limits              Chi-Square    Pr > ChiSq
Intercept     1       1.5979       0.0855      1.4302      1.7655         348.96        <.0001
```

```
child         1     -1.0428      0.1000     -1.2388     -0.8469         108.78        <.0001
camper        1      0.8340      0.0936      0.6505      1.0175          79.35        <.0001
Scale         0      1.0000      0.0000      1.0000      1.0000

NOTE: The scale parameter was held fixed.

             Analysis Of Maximum Likelihood Zero Inflation Parameter Estimates
                                Standard     Wald 95% Confidence          Wald
Parameter    DF    Estimate        Error           Limits            Chi-Square    Pr > ChiSq
Intercept     1      1.2974      0.3739      0.5647      2.0302          12.04        0.0005
persons       1     -0.5643      0.1630     -0.8838     -0.2449          11.99        0.0005
```

Model Information and Goodness of Fit–Proc Genmod

```
             Model Information[a]
Data Set                        WORK.FISH
Distribution          Zero Inflated Poisson
Link Function                         Log
```

```
Dependent Variable                    count

Number of Observations Read         250
Number of Observations Used         250

             Criteria For Assessing Goodness Of Fit
Criterion                     DF           Value        Value/DF
Deviance[b]                                 2063.2168
Scaled Deviance                             2063.2168
Pearson Chi-Square           245       1543.4597          6.2998
Scaled Pearson X2            245       1543.4597          6.2998
Log Likelihood                               774.8999
Full Log Likelihood[c]                     -1031.6084
AIC (smaller is better)[d]                  2073.2168
AICC (smaller is better)                    2073.4627
BIC (smaller is better)[e]                  2090.8241

Algorithm converged.[f]
```

a. **Model Information –** This block of output includes the distribution assumed for the overall model, the function linking the model predictions to the outcome, the dependent variable, and the number of observation read and used (or not dropped due to missingness in the outcome or predictors).

b. **Deviance –** The deviance of the model is equal to -2*(full log likelihood). Here, we can see that -2*-1031.6084 = 2063.2168.

c. **Full Log Likelihood –** This is the log likelihood of the fitted full model. It is used in the Likelihood Ratio Chi-Square test of whether all predictors' regression coefficients in the model are simultaneously zero.

d. **AIC –** This is the Akaike Information Criterion. It is calculated as **AIC** = -2 Log Likelihood + 2(*s*), where *s* is the total number of predictors in the model. Here, *s* = 5 so **AIC** = -2*-1031.6084 + 5*2 = 2073.2168. **AIC** is used for the comparison of models from different samples or non-nested models. It penalizes for the number of predictors in the model. Ultimately, the model with the smallest **AIC** is considered the best.

e. **BIC –** This is the Bayesian information criterion. Like the **AIC**, it is based on the log likelihood and penalizes for the number of predictors in the model. The smallest **BIC** is most desirable.

f. **Convergence –** The fitting of this model is an iterative procedure. With each iteration of the model, the parameters are updated and the log-likelihood is calculated. This process continues until the log-likelihood is no longer improved and has been maximized. If this point is reached, then the model is said to have converged. If this point is not reached, then SAS will note the failure to converge.

Parameter Estimates–Proc Genmod

```
                    Analysis Of Maximum Likelihood Parameter Estimates


                                Standard     Wald 95% Confidence          Wald
Parameterg   DF    Estimateh       Errori          Limitsj             Chi-Squarek   Pr > ChiSql


Intercept     1      1.5979      0.0855      1.4302      1.7655         348.96        <.0001
child         1     -1.0428      0.1000     -1.2388     -0.8469         108.78        <.0001
camper        1      0.8340      0.0936      0.6505      1.0175          79.35        <.0001
Scale         0      1.0000      0.0000      1.0000      1.0000

NOTE: The scale parameter was held fixed.


             Analysis Of Maximum Likelihood Zero Inflation Parameter Estimates

                                Standard     Wald 95% Confidence          Wald
Parameterm   DF    Estimaten       Errori          Limitsj             Chi-Squarek   Pr > ChiSql


Intercept     1      1.2974      0.3739      0.5647      2.0302          12.04        0.0005
persons       1     -0.5643      0.1630     -0.8838     -0.2449          11.99        0.0005
```

g. **Parameter (count model)** – These are the independent variables, as well as the intercept, predicting the count. SAS presents a **Scale** parameter as well, but it is set to 1 and not estimated because we are assuming our Poisson distribution has a scale parameter of 1.

h. **Estimate (count model)** – These are the regression coefficients related to the count model. The coefficients for **Intercept**, **child**, and **camper** are interpreted as you would interpret coefficients from a standard Poisson model: the expected number of fish caught changes by a factor of exp(**Estimate**) for each unit increase in the corresponding predictor.

Predicting Number of Fish Caught for the Non-"Certain Zero" Groups

child – If a group were to increase its **child** count by one, the expected number of fish caught would decrease by a factor of exp(-1.0428) = 0.3524664 while holding all other variables in the model constant. Thus, the more children in a group, the fewer caught fish are predicted.

camper – The expected number of fish caught in a weekend for a group with a camper is exp(0.8340) =

camper – The expected number of fish caught in a weekend for a group with a camper is exp(0.8340) = 2.302510 times the expected number of fish caught in a weekend for a group without a camper while holding all other variables in the model constant. Thus, if a group with a camper and a group without a camper are not certain zeros and have identical numbers of children, the expected number fish caught for the group with a camper is 2.302510 times the expected number of fish caught by the group without a camper.

Intercept – If all of the predictor variables in the model are evaluated at zero, the predicted number of fish caught would be calculated as exp(**Intercept**) = exp(1.5979) = 4.942642. For groups without a camper or children (the variables **camper** and **child** evaluated at zero), the predicted number of fish caught would be 4.942642.

i. **Standard Error** – These are the standard errors of the individual regression coefficients. They are used in both the calculation of the **Wald Chi-Square** test statistic, superscript k.

j. **Wald 95% Confidence Limits** – This is the Wald Confidence Interval (CI) of an individual poisson regression coefficient, given the other predictors are in the model. For a given predictor variable with a level of 95% confidence, we'd say that we are 95% confident that upon repeated trials, 95% of the CI's would include the "true" population poisson regression coefficient. It is calculated as **Estimate** ($z_{\alpha/2}$)*(**Standard Error**), where $z_{\alpha/2}$ is a critical value on the standard normal distribution. The CI is equivalent to the **Chi-Square** test statistic: if the CI includes zero, we'd fail to reject the null hypothesis that a particular regression coefficient is zero, given the other predictors are in the model. An advantage of a CI is that it is illustrative; it provides information on where the "true" parameter may lie and the precision of the point estimate.

k. **Wald Chi-Square** – The **Chi-Square** test statistic is the squared ratio of the **Estimate** to the **Standard Error** of the respective predictor. The **Chi-Square** value follows a standard chi-square distribution with degrees of freedom given by **DF**, which is used to test against the alternative hypothesis that the **Estimate** is not equal to zero.

l. **Pr > ChiSq** – This is the probability the **Wald Chi-Square** test statistic (or a more extreme test statistic) would be observed under the null hypothesis that its associated coefficient is zero, given that the rest of the predictors are in the model. For a given alpha level, **Pr > ChiSq** determines whether or not the null hypothesis can be rejected. If **Pr > ChiSq** is less than alpha, then the null hypothesis can be rejected and the parameter estimate is considered significant at that alpha level.

Predicting Number of Fish Caught for the Non-"Certain Zero" Groups

child – The **Wald Chi-Square** test statistic for the predictor **child** is $(-1.0428/0.1000)^2 = 108.78$ with an associated p-value of <.0001. If we set our alpha level to 0.05, we would reject the null hypothesis and conclude that the regression coefficient for **child** has been found to be statistically different from zero given the other variables are in the model.

camper –The **Wald Chi-Square** test statistic for the predictor **camper** is $(0.8340/0.0936)^2 = 79.35$ with an associated p-value of <.0001. If we set our alpha level to 0.05, we would reject the null hypothesis and conclude that the regression coefficient for **camper** has been found to be statistically different from zero given the other variables are in the model.

Intercept – The **Wald Chi-Square** test statistic for the intercept, **Intercept,** is $(1.5979/0.0855)^2 = 348.96$ with an associated p-value of < 0.001. If we set our alpha level at 0.05, we would reject the null hypothesis and conclude that **Intercept** has been found to be statistically different from zero given the other variables are in the model and evaluated at zero.

Predicting Membership in the "Certain Zero" Group

persons- The **Wald Chi-Square** test statistic for the predictor **persons** is $(-0.5643/0.1630)^2 = 11.99$ with an associated p-value of 0.0005. If we again set our alpha level to 0.05, we would reject the null hypothesis and conclude that the regression coefficient for **persons** has been found to be statistically different from zero given the other variables are in the model.

Intercept -The **Wald Chi-Square** test statistic for the intercept, **Intercept,** is $(1.2974/0.3739)^2 = 12.04$ with an associated p-value of 0.0005. With an alpha level of 0.05, we would reject the null hypothesis and conclude that **Intercept** has been found to be statistically different from zero given the other variables are in the model.

m. **Parameter (inflation model) –** These are the independent variables, as well as the intercept, predicting the certain zeroes.

n. **Estimate (inflation model) –** These are the regression coefficients related to the inflation model. The coefficients for **Intercept** and **persons** are interpreted as you would interpret coefficients from a standard logistic model: the odds change by a factor of exp(**Estimate**) for each unit increase in the corresponding predictor.

Predicting Membership in the "Certain Zero" Group

persons- If a group were to increase its **persons** value by one, the odds that it would be in the "Certain Zero" group would decrease by a factor of exp(-0.5643) = 0.5687581. In other words, the more people in a group, the less likely the group is a certain zero.

Intercept – If all of the predictor variables in the model are evaluated at zero, the odds of being a "Certain Zero" is exp(1.2974) = 3.659769. This means that the predicted odds of a group with zero persons is 3.659769 (though remember that evaluating **persons** at zero is out of the range of plausible values–every group must have at least one person).

Code and output from Proc Countreg

Proc countreg can also be used to run a zero-inflated Poisson regression in SAS. The code and output can be found below. The superscripts in the output below corresponds to the equivalent portion of the **proc genmod** output. **Proc countreg** presents t values rather than Wald Chi-square test statistics. The t values seen below can be squared to get the Wald Chi-squares seen in the **proc genmod** output. The p-values are equivalent.

```
proc countreg data = fish method = qn;
  model count = child camper / dist= zip;
  zeromodel count ~ persons;
run;

The COUNTREG Procedure

              Model Fit Summary
Dependent Variable                          count
Number of Observations                        250
Data Set                                WORK.FISH
Model                                         ZIP
ZI Link Function                         Logistic
Log Likelihood                              -1032
Maximum Absolute Gradient              4.61991E-7
Number of Iterations                           15
Optimization Method             Dual Quasi-Newton
AIC[d]                                       2073
SBC[e]                                       2091

Algorithm converged.[f]

                           Parameter Estimates
                                                Standard                   Approx
Parameter[g,m]      DF        Estimate[h,n]       Error[i]    t Value     Pr > |t|[l]
Intercept           1         1.597889        0.085538      18.68        <.0001
child               1        -1.042838        0.099988     -10.43        <.0001
camper              1         0.834022        0.093627       8.91        <.0001
Inf_Intercept       1         1.297439        0.373850       3.47        0.0005
Inf_persons         1        -0.564347        0.162962      -3.46        0.0005
```

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