Problem Set 8 – ACSE-2 – November 2019

(1) Conservation of energy:

a. Write down the energy equation for a 1-D steady state problem without any contribution of strain or flow.

b. Solve this equation for a layer with constant material properties, with a fixed temperature on the bottom and an insulating top:

$$T(y=0)=T_0, \qquad \frac{dT}{dy}(y=h)=0$$

c. Plot the solution for the following parameter values: T_0 = 0°C, k = 80 $\mathrm{Wm^{-1}K^{-1}}$, A = 200 $\mathrm{W/m^3}$, h = 2 m.

(2) Hookean Elasticity

a. Show that for an isotropic Hookean solid, principal directions of stress and strain coincide.

b. Find a relation between the principal values of stress and strain using the two Lamé parameters.

(3) For the Hookean solid from (2) express the elastic **Young's modulus** E and **Poisson's ratio** ν, which are often used in engineering, in terms of the Lamé parameters λ and μ. The two engineering moduli are defined for a uniaxial state of stress, where only $\sigma_1 \neq 0$, a useful system for experimentally determining the elastic parameters. This stress leads to a maximum strain ε_1 in the direction of the applied stress and uniform strain in perpendicular direction, $\varepsilon_2 = \varepsilon_3$. The moduli are then defined as:

$$E=\sigma_1/\varepsilon_1 \text{ and } \nu=-\varepsilon_3/\varepsilon_1$$

(4) Navier-Stokes for Poiseuille flow – Consider the case of steady unidirectional flow of an incompressible fluid with constant viscosity η between two parallel fixed plates, both with normal in x_2 direction. Assume flow is in x_1 direction, and the plates extend infinitely in x_1 and x_3 direction (i.e. distance between the plates $\ll$ size of the plates). This type of flow is called plane Poiseuille flow. The velocity field for this flow has the form:

$$v_1=v(x_2),\ v_2=v_3=0.$$

a. Write down the Navier-Stokes equations for this problem.

b. Show that:

$$\frac{\partial^2 p}{\partial x_1^2}=0,$$

i.e., the pressure gradient is constant in the direction of flow.

c. With this information, find the solution for $v(x_2)$, assuming the plates are located at $x_2=-b$ and $x_2=b$.