# The Wave Equation

An introduction

November 2019

### What is a wave?

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Not easy to define rigorously...

 Typically: transfer through space of oscillatory energy (vibrations in time)

- Obvious examples:
  - Light (E-M)
  - Sound

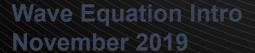
# Why are waves important?

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Almost everything we experience comes to us through waves...

- Throughout all areas of the physical world:
  - Light (E-M)
  - Sound
  - Gravity
  - Quantum

# Applications for wave-modelling?



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- Imaging, Exploration, NDT, Interferometry...
  - Seismology
    (e.g. earthquakes, volcanoes, petroleum, helioseismics)
  - Ultrasound
    (e.g. medical imaging, pipeline testing)
  - Electric / magnetic / E-M
    (e.g. pipeline testing, body-scanners, fibre-optic signals)
  - Gravitational
    (e.g. binary mergers, supernovas, primordial cosmology)

### Characteristics of waves

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#### Longitudinal

- oscillating in same direction as propagation.
- e.g. sound (acoustic pressure), P-waves

#### Transverse

- oscillating perpendicular to propagation
- e.g. E-M, S-waves (shear waves)

#### Characteristics of waves

- Frequency, f (also use angular freq:  $\omega = 2\pi f$ )
  - Rate of oscillation in time
  - S.I. unit: Hertz (Hz) =  $s^{-1}$  (or rad. $s^{-1}$  for  $\omega$ )
- Propagation speed, c
  - Distance per unit time
  - S.I. units: Metres per second (ms<sup>-1</sup>)
- Amplitude (various symbols)

Wavelength:  $\lambda = c / f$  distance (m)

Also, (angular) wavenumber:  $k = \omega/c = 2\pi/\lambda (\text{rad.}) \,\text{m}^{-1}$ 

## The 1D Wave Equation

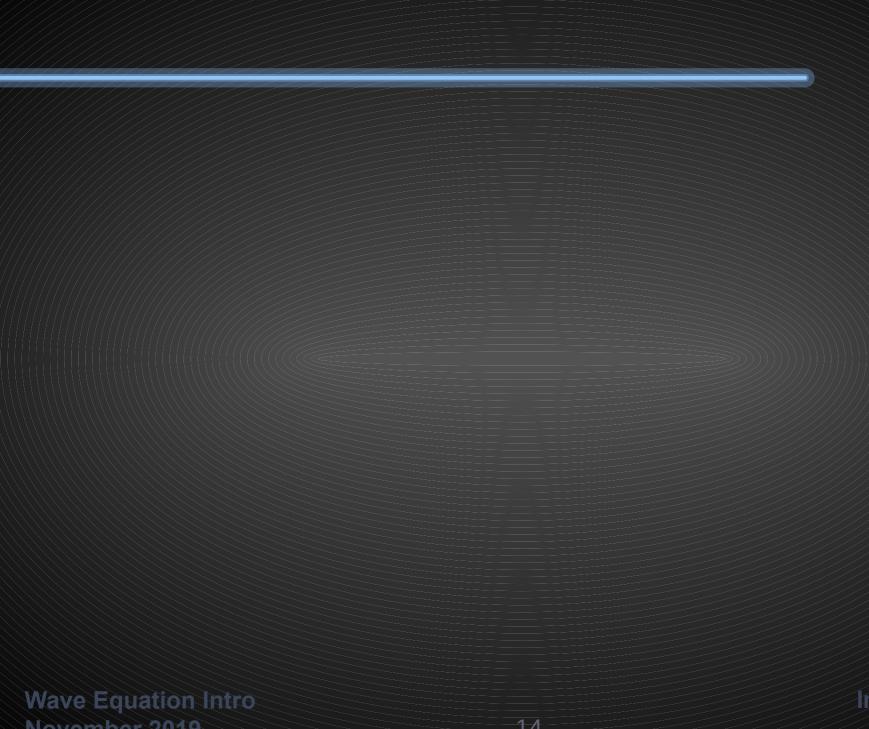
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- Second order P.D.E.
  - twice differentiated (in time & space)
  - domain over more than one variable (x & t)
- Hyperbolic (rather than elliptic/parabolic)
  - signal travels at finite speed



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