Problem Set 5 – ACSE-2 – November 2019

(1) **Tensor maths:** Given vectors **a** and **b** and second order tensor **S** with the following components:

$$\mathbf{a} = \begin{pmatrix} \mathbf{2} \\ \mathbf{-1} \\ \mathbf{4} \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} \mathbf{8} \\ \mathbf{5} \\ \mathbf{-7} \end{pmatrix}, \ \mathbf{S} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{5} \\ \mathbf{3} & \mathbf{7} & \mathbf{4} \\ \mathbf{9} & \mathbf{8} & \mathbf{6} \end{pmatrix}$$

determine:

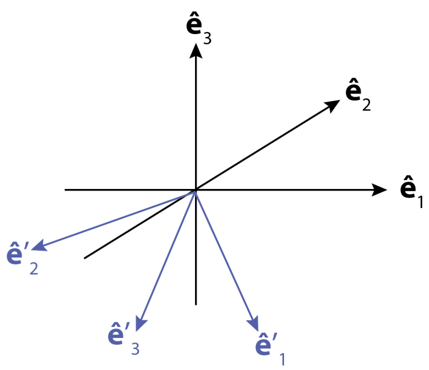
(a) tr(**S**)	(b) **S**:**S**	(c) **S**:$\mathbf{S}^{\mathrm{T}}$	(d) **a**·**S**
(e) **S**·**a**	(f) $\mathbf{S}^{\mathrm{T}}$·**a**	(g) **ab**	(g) **ba**

(2) **Coordinate transformation:** Let $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3$ be a set of orthonormal base vectors. Then define a new right-handed coordinate system by $\widehat{\mathbf{e}'}_1, \widehat{\mathbf{e}'}_2, \widehat{\mathbf{e}'}_3$ (see figure) where:

$$\hat{\mathbf{e}}'_1 = \tfrac{1}{3}(2\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3)$$

$$\hat{\mathbf{e}}'_2 = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_1 - \hat{\mathbf{e}}_2)$$

$$\hat{\mathbf{e}}'_3 = \frac{1}{3\sqrt{2}}(\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2 - 4\hat{\mathbf{e}}_3)$$



Check that $\hat{\mathbf{e}}'_1 \cdot \hat{\mathbf{e}}'_2 = 0$ and $\hat{\mathbf{e}}'_3 = \hat{\mathbf{e}}'_1 \times \hat{\mathbf{e}}'_2$. Is the new basis orthonormal? Determine the direction cosines of the transformation and write out the transformation matrix.

(3) **Special tensors, index notation**: Simplify the following expressions:

a) Simplify: $\delta_{ij}\delta_{jk}\delta_{kp}\delta_{pi}$

b) Show that: $\varepsilon_{ijk}\varepsilon_{lmk}=\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}$

c) Use the identity in b) and index notation to show that: **a**x(**b**x**c**)=(**a**·**c**)**b**-(**a**·**b**)**c**

d) If you manage question c, you could also try to show that the various scalar triple product identities, **a**·(**b**x**c**)= (**a**x**b**)·**c** and others in the lecture slides.

(4) **Vector derivatives:** Use index notation to establish the following identities, where F is a scalar function and **v** is a vector

a) $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}F) = \mathbf{0}$

b) $\boldsymbol{\nabla}^2\mathbf{v} = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{v}) - \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{v})$