The Wave Equation

An introduction

November 2019

What is a wave?

What is a wave?

- Not easy to define rigorously...
- Typically: transfer through space of oscillatory energy (vibrations in time)
- Obvious examples:
  - Light (E-M)
  - Sound

Why are waves important?

Why are waves important?

Almost everything we experience comes to us through waves...

- Throughout all areas of the physical world:
  - Light (E-M)
  - Sound
  - Gravity
  - Quantum

Applications for wave-modelling?

Applications for wave-modelling?

- Imaging, Exploration, NDT, Interferometry...
  - Seismology
    (e.g. earthquakes, volcanoes, petroleum, helioseismics)
  - Ultrasound
    (e.g. medical imaging, pipeline testing)
  - Electric / magnetic / E-M
    (e.g. pipeline testing, body-scanners, fibre-optic signals)
  - Gravitational
    (e.g. binary mergers, supernovas, primordial cosmology)

Characteristics of waves

Characteristics of waves

- Longitudinal
  - oscillating in same direction as propagation
  - e.g. sound (acoustic pressure), P-waves
- Transverse
  - oscillating perpendicular to propagation
  - e.g. E-M, S-waves (shear waves)

Characteristics of waves

- Frequency, f (also use angular freq: $\omega = 2\pi f$)
  - Rate of oscillation in time
  - S.I. unit: Hertz (Hz) = s^{-1} (or rad.s^{-1} for ω)
- Propagation speed, c
  - Distance per unit time
  - S.I. units: Metres per second (ms^{-1})
- Amplitude (various symbols)

Wavelength: $\lambda = c / f$ distance (m)

Also, (angular) wavenumber: $k = \omega / c = 2\pi / \lambda$ (rad.) m^{-1}

The 1D Wave Equation

The 1D Wave Equation

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}}$$

The 1D Wave Equation

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}}$$

- Second order P.D.E.
  - twice differentiated (in time & space)
  - domain over more than one variable (x & t)
- Hyperbolic (rather than elliptic/parabolic)
  - signal travels at finite speed