ACSE-2 Coursework 1 – Potential Flow

November 2019

(100 marks)

A porous bed of activated carbon is used to remove impurities from water. We can assume that Darcy's law holds for the flow of liquid within the block, which means that it can be solved as a potential flow problem.

We will assume that there is no flow out of the front or back of the bed, resulting in a 2-dimensional flow pattern, with the bed being 1 m long in this direction. The bed is 2 m wide and 1 m deep. Liquid is forced into the bottom of the bed at a flowrate of 120 l/min. We can assume that the vertical flux is constant over the bottom of the bed. The flow then splits evenly out of the 2 sides of the bed. You can assume that on the open sides the liquid flows out of the entire height of bed and that the horizontal flux is constant over these boundaries.

You should use a stream function formulation to solve this problem

a) Calculate the values of the stream function along all the boundaries of the system. (10 marks)

b) Use Simultaneous Over Relaxation (SOR) to solve for the values of the stream function within the block. Use $\Delta x = \Delta y = 0.01\ m$. Plot the resultant values of the stream function. Note that this program will be very similar to the one that you wrote for the class worksheet. (30 marks)

c) Use the calculated stream function values to obtain the values of the liquid flux at all points within the bed. Use the following approximations (and the equivalent ones in the *y* direction) as appropriate:

   Central Difference $\quad \frac{\partial \psi}{\partial x} \approx \frac{\boldsymbol{\psi_{i+1,j} - \psi_{i-1,j}}}{\boldsymbol{2\ \Delta x}}$

   Forward Difference $\quad \frac{\partial \psi}{\partial x} \approx \frac{\boldsymbol{\psi_{i+1,j} - \psi_{i,j}}}{\boldsymbol{\Delta x}}$

   Backward Difference $\quad \frac{\partial \psi}{\partial x} \approx \frac{\boldsymbol{\psi_{i,,j} - \psi_{i-1\,,j}}}{\boldsymbol{\Delta x}}$

   Remember the definitions for the flux when using a stream function formulation:

$$\boldsymbol{v} = \begin{pmatrix} \dfrac{\partial \psi}{\partial y} \\ -\dfrac{\partial \psi}{\partial x} \end{pmatrix}$$

   Use the quiver() function to plot the results. (20 marks)

d) Derive the analytical solution to this problem in the form of an infinite series. Write a program to calculate this series out to an appropriate number of iterations. Compare the analytical solution to the numerical solution from part b). How does the discrepancy between the results change as the Δx in part b is changed (still assuming that $\Delta x = \Delta y$). Note that this is thus a convergence analysis. (40 marks)