Worksheet

Question 1

A fluid is flowing between two large parallel plates (density ρ and viscosity μ). The plates are a distance h apart and the pressure drop is $\Delta P/L$. The plates are inclined at an angle θ. Assuming that the flow is laminar, calculate the volumetric flowrate through the system per meter of width as a function of the inclination angle. Simplify the Navier-Stokes equation in order to carry out this calculation.

At what angle does the flow stop in terms of the other variables (you can also check this based on a force balance over the entire system)?

Question 2

A power law fluid is flowing in a cylindrical pipe driven by a pressure gradient of $\Delta P/L$. The flow is occurring in the *z* direction, with the shear stress thus being the result of a velocity gradient in the *r* direction:

$$\tau = k\frac{\partial u_y}{\partial x}\left|\frac{\partial u_y}{\partial x}\right|^{n-1}$$

Use a force/momentum balance to calculate the velocity profile as a function of the power law exponent *n*.

Plot these results in order to see how the velocity profile is influenced by changes in the rheology.

Question 3

Assume that a soil flow can be approximated as a Bingham plastic. Consider a layer of soil on a slope which can be assumed to be infinitely wide and long. The layer is 1 m thick and is at an angle of 30° from the horizontal. Assume that it is initially at its angle of repose, which implies that the yield stress is just able to balance the gravitational force down the slope. After heavy rain the yield stress of the material decreases by 20%.

Assuming that the $\mu_\infty = 100Pa.s$, what is the maximum velocity that the slide achieves down the slope?

Question 4

Calculating the volumetric flow through a pipe for a given pressure head is a common calculation to have to carry out. This is typically done using a friction factor which relates the pressure head to the liquid velocity in the pipe. For the Fanning friction factor, f:

$$\frac{\Delta P}{L} = f\,\frac{\rho u^2}{R}$$

Where $\frac{\Delta P}{L}$ is the pressure gradient, ρ is the liquid density, R is the pipe radius and u is the average liquid velocity. Doing this manually a graph relating the friction factor to the Reynolds number is typically used. This is not convenient if you wish to do this computationally and therefore a number of different approximations have been developed (note that the solution for Laminar flow is analytical and therefore not an approximation). The most general of these equations is the following:

Laminar flow: $Re < 2300$: $f = \frac{16}{Re}$

Turbulent flow: $Re > 2900$: $\frac{1}{\sqrt{f}} = -4log_{10}\left(\frac{\frac{\epsilon}{d}}{3.7} + \frac{1.26}{Re\sqrt{f}}\right)$

Where $\frac{\epsilon}{d}$ is the roughness relative to the pipe diameter. Also note that the equation in the turbulent region is implicit and needs to be solved using root finding.

You now need to write a program that returns the volumetric flowrate for given pressure drop, pipe length, pipe radius, pipe roughness, fluid density and fluid viscosity.

The tactic to use in solving this is to initially assume that the flow is laminar. You can then directly calculate a liquid velocity and therefore a Reynolds number. Check the Reynold number range. If it is laminar you have found your solution. If it is in the transition range return both the laminar and the turbulent results and if it is in the turbulent regime return the turbulent result.

The solution of the turbulent problem is highly implicit in that the friction factor depends on itself and the Reynolds number, while the Reynolds number also depends upon the friction factor. To solve this substitute the Reynolds number into the friction factor equation. You can now solve for the friction factor using a root finding algorithm (the solution lies between the laminar friction factor and about 0.02).