Problem Set 6 ACSE-2 November 2019
Stress tensors

1) Given a stress tensor at a point in a body $\boldsymbol{\sigma} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 0 \end{bmatrix}$ MPa
   a. Find the normal stresses on the coordinate planes through the point with normal in $\hat{\mathbf{e}}_1$and $\hat{\mathbf{e}}_3$ direction.
   b. Find the total shear stresses on the two planes from a.
   c. Find the traction on a plane through the point with normal in the direction of $2\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3$.

2) Given that at a point in a continuum the stress state is such that σ_{11} =1 MPa and σ_{22} =-1 MPa and all other stress components σ_{ij}=0.
   a. Show that the only plane on which the stress vector is zero, is the plane with normal in the $\hat{\mathbf{e}}_3$ direction.
   b. Give three planes on which no normal stress is acting

3) For the following state of stress:
$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{MPa}$$
in Cartesian rectangular coordinate system $\{\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3\}$, find σ'_{11}, σ'_{21} and σ'_{33} on a new basis $\{\hat{\mathbf{e}}'_1, \hat{\mathbf{e}}'_2, \hat{\mathbf{e}}'_3\}$ obtained by rotating about the $\hat{\mathbf{e}}_3$ axis by 90°, such that $\hat{\mathbf{e}}'_1 = \hat{\mathbf{e}}_2$.

4) The stress state in which the only non-vanishing stresses are a single set of shear stresses is called simple shear. Take the case where $\sigma_{12} = \sigma_{21} = \tau$, and all other σ_{ij} =0.
   a. Find the principal stress values and the principal stress directions
   b. Find the maximum shear stress and the planes on which it acts.

5) Given the following stress distribution: $\boldsymbol{\sigma} = \begin{bmatrix} x_1 + x_2 & \sigma_{12}(x_1, x_2) & 0 \\ \sigma_{12}(x_1, x_2) & x_1 - 2x_2 & 0 \\ 0 & 0 & x_2 \end{bmatrix}$
   Find σ_{12} so that the stress distribution is in equilibrium with zero body force, and so that the stress vector on plane x_1=1 is given by:
$$\mathbf{t} = (1 + x_2)\hat{\mathbf{e}}_1 + (5 - x_2)\hat{\mathbf{e}}_2$$

6) For any stress state $\boldsymbol{\sigma}$ we can define a deviatoric stress $\boldsymbol{\sigma}'$ to be $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \frac{\sigma_{kk}}{3}\mathrm{I}$, where σ_{kk} is the first invariant (trace) of the stress tensor $\boldsymbol{\sigma}$.
   a. Show that the first invariant of $\boldsymbol{\sigma}'$ vanishes.
   b. Evaluate $\boldsymbol{\sigma}'$ for the stress tensor: $\boldsymbol{\sigma} = 100 \begin{bmatrix} 6 & 5 & -2 \\ 5 & 3 & 4 \\ -2 & 4 & 9 \end{bmatrix}$ kPa
   c. Show that the principal directions of $\boldsymbol{\sigma}$ coincide with those of the deviatoric stress tensor $\boldsymbol{\sigma}'$