Worksheet

Question 1:

Electrical current is flowing through a square block. The top and left hand wall are insulated and current flows uniformly in through the bottom and out of the right hand wall. Calculate the values of the stream function along all the boundaries if the block is 1m by 1m and the total current is 100amps.

Question 2:

Calculate the stream function value for the electrical current paths at all the internal locations using the boundary conditions derived in the previous question. Note that the finite difference equation to be solved is the same one as derived in question 2a. It should be solved on a 101 by 101 grid using Simultaneous Over Relaxation (SOR). Iteratively update the values using checker-boarding. Stop the simulation when the average magnitude of the residual becomes sufficiently small.

Always let the simulation run for a few 100 iterations even if the average residual is small. This is because changes in the dependent variable will propagate inwards from the boundaries. This means that average residual can often start small, but will consist of very large residuals at the edges of the system and zero residuals in the middle of the system, resulting in an average that is small. An alternative is to stop the simulation when both the average and maximum residual drop below specified values (which should be different!).

You can use imshow() to plot your results, but note that it has (0, 0) in top left corner and is in row-column format.

Question 3

Calculate the *x* and *y* components of the current flux at every point in the system. Use the central difference for the derivative where you can, but use the forward or backwards derivative at the boundaries as appropriate.

Central Difference $\frac{\partial \psi}{\partial x} \approx \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\,\Delta x}$

Forward Difference $\frac{\partial \psi}{\partial x} \approx \frac{\psi_{i+1,j} - \psi_{i,j}}{\Delta x}$

Backward Difference $\frac{\partial \psi}{\partial x} \approx \frac{\psi_{i,,j} - \psi_{i-1\,,j}}{\Delta x}$

Remember the definitions for the flux when using a stream function formulation:

$$\boldsymbol{v} = \begin{pmatrix} \frac{\partial \psi}{\partial y} \\ -\frac{\partial \psi}{\partial x} \end{pmatrix}$$

Use the quiver() function to plot the results.

Question 4

Calculate the analytical solution to this problem. It will take the form of an infinite series solution with one set of terms for each of the boundaries. You should use sufficient terms in your approximation of the infinite series that your residual is less than a value that you choose (accuracy to 5 or 6 significant figures is likely to be sufficient). Note that the infinite series solution will not converge in the corner points as these will remain zero.

Compare your analytical results to the numerical solution obtained in question 3.