四. 多重线性回归（Python）

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
from sklearn.model_selection import train_test_split
```

1. 读取数据

```
data = pd.read_csv('./data.csv', encoding =  'UTF-8')
data
```

	x1	x2	x3	x4	x5	y
0	213432.8	7240	4.75	633.6	123.700000	622.33
1	253598.6	8755	4.75	712.6	121.400000	747.83
2	229495.5	8265	4.75	654.1	118.933333	646.92
3	219295.4	7780	4.75	717.0	121.333333	675.68
4	197920.0	7725	4.75	713.1	122.866667	745.90
...	...	...	...	...	...	...
60	34544.6	3210	5.49	133.3	95.333333	131.10
61	39767.4	3010	5.49	123.5	94.666667	122.80
62	35291.9	2920	5.49	113.6	92.333333	112.10
63	32537.3	3020	5.49	107.6	87.666667	110.00
64	29825.5	3020	5.49	105.0	97.700000	102.10

65 rows × 6 columns

提取因变量和自变量

```
X = data.iloc[:, 0:4]
Y = data.iloc[:, 5]
```

2. 判断多重共线性

ols法估计，R^2值高、F检验值高、且x1,x2，x3的t检验不显著

```
X = sm.add_constant(X) #加上一列常数1，这是回归模型中的常数项
reg = sm.OLS(Y, X) #生成回归模型
model = reg.fit() #拟合数据
model.summary()
```

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.962
Model:	OLS	**Adj. R-squared:**	0.959
Method:	Least Squares	**F-statistic:**	375.3
Date:	Mon, 13 Dec 2021	**Prob (F-statistic):**	1.04e-41
Time:	02:53:32	**Log-Likelihood:**	13.684
No. Observations:	65	**AIC:**	-17.37
Df Residuals:	60	**BIC:**	-6.495
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	3.469e-17	0.025	1.37e-15	1.000	-0.051	0.051
x1	0.0922	0.081	1.137	0.260	-0.070	0.254
x2	0.0454	0.041	1.116	0.269	-0.036	0.127
x3	0.0172	0.032	0.542	0.590	-0.046	0.081
x4	0.8669	0.081	10.637	0.000	0.704	1.030

Omnibus:	23.100	**Durbin-Watson:**	2.160
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	141.489
Skew:	-0.568	**Prob(JB):**	1.89e-31
Kurtosis:	10.138	**Cond. No.**	7.25

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

相关系数，对数据进行标准化处理（z-score标准化），可见有共线性

```
X = (X - X.mean())/np.std(X)
Y = (Y - Y.mean())/np.std(Y)
X.corr()
```

	x1	x2	x3	x4
x1	1.000000	0.703095	-0.464662	0.946787
x2	0.703095	1.000000	-0.059720	0.719972
x3	-0.464662	-0.059720	1.000000	-0.440459
x4	0.946787	0.719972	-0.440459	1.000000

分割数据

```
X_train, X_test, Y_train, Y_test = train_test_split(X,Y,train_size=0.7,
random_state=1)
```

3. 消除多重共线性（PCA法）

对模型进行训练，返回降维后数据

```
pca = PCA(n_components='mle')
pca.fit(X_train)
X_train = pca.transform(X_train)
Y_train= (Y_train - Y_train.mean())/np.std(Y)
X_train
```

```
array([[ 1.570424  , -0.93829933],
       [-2.30291629, -0.77854422],
       [ 1.08351187,  1.16012196],
       [ 0.32295353,  0.12070028],
       [-2.42997178, -1.28607699],
       [ 0.94438602,  1.02459601],
       [-2.32801688, -0.77829169],
       [-2.16486222, -0.67454388],
       [-1.93978149,  0.1724962 ],
       [ 0.01967437, -0.11280316],
       [-1.32073297,  0.75801798],
       [-1.97618471, -0.58105161],
       [-2.40838564, -1.22822222],
       [ 1.76706079, -0.97252849],
       [ 0.72709636,  1.02691268],
       [ 0.04243175,  0.80995503],
       [ 0.04285174,  1.64586828],
       [-0.56489392, -0.43367922],
       [ 0.76723663,  1.11295505],
       [ 2.05694611, -0.59451562],
       [ 1.09836063, -0.50078143],
       [ 0.36992529,  1.66065839],
       [ 0.06051041,  1.56617018],
       [-1.85288098,  1.97653715],
       [-2.34741203, -1.22598376],
       [-1.87922031,  0.17743686],
       [ 1.09024765,  1.25227001],
       [-2.57313034, -1.28851827],
       [ 0.86909685,  1.12346043],
       [ 0.81787884,  1.20212348],
       [ 2.39110855, -0.8672501 ],
```

```
       [-2.15215927,  1.44698343],
       [ 1.77843682, -0.8036404 ],
       [-2.52008806, -1.31471821],
       [ 2.8243233 , -0.28218367],
       [ 0.71865329,  0.46025756],
       [ 1.8817035 , -0.74220526],
       [ 0.83446377, -0.11817724],
       [ 2.28376973, -0.72526617],
       [ 1.29485017, -1.11822654],
       [ 2.23823353, -0.94888151],
       [ 1.79434283, -0.77141572],
       [ 1.21440562, -1.07899312],
       [-1.94424067,  1.69582372],
       [-0.20000637, -0.22854681]])
```

4. 重建线性回归

使用返回后的数据用线性回归模型建模，ols回归后R^2为0.933，p值小，说明模型拟合效果好

```
import statsmodels.api as sm
ols = sm.OLS(Y_train, X_train).fit()
ols.summary()
```

OLS Regression Results

Dep. Variable:	y	**R-squared (uncentered):**	0.933
Model:	OLS	**Adj. R-squared (uncentered):**	0.929
Method:	Least Squares	**F-statistic:**	297.6
Date:	Mon, 13 Dec 2021	**Prob (F-statistic):**	6.49e-26
Time:	02:27:45	**Log-Likelihood:**	-4.5617
No. Observations:	45	**AIC:**	13.12
Df Residuals:	43	**BIC:**	16.74
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
x1	0.5940	0.024	24.341	0.000	0.545	0.643
x2	0.0673	0.040	1.674	0.101	-0.014	0.148

Omnibus:	1.742	**Durbin-Watson:**	2.091
Prob(Omnibus):	0.419	**Jarque-Bera (JB):**	0.878
Skew:	-0.245	**Prob(JB):**	0.645
Kurtosis:	3.477	**Cond. No.**	1.65

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
pca.explained_variance_ratio_
```

```
array([0.70103682, 0.2586647 ])
```

```
pca.get_params()
```

```
{'copy': True,
 'iterated_power': 'auto',
 'n_components': 'mle',
 'random_state': None,
 'svd_solver': 'auto',
 'tol': 0.0,
 'whiten': False}
```

```
from sklearn.linear_model import LinearRegression
lr = LinearRegression()
lr.fit(X_train,Y_train)
lr.score(X_train, Y_train)
# X_test = data.iloc[64:, 0:4]
# y_test = data.iloc[64:, 5]
```

```
0.9326301840968643
```

5. 测试集验证

```
X_test = (X_test - X_test.mean())/np.std(X_test)
X_test = pca.transform(X_test)
X_test
```

```
array([[-1.51024245, -0.21962447],
       [-1.71293609, -0.7144579 ],
       [-1.73266676,  1.22094126],
       [ 3.12792355, -0.62282555],
       [-1.53383462,  0.70006592],
       [-1.52774474, -0.61751499],
       [ 1.95957121,  1.06265604],
       [ 1.02538936,  0.96167462],
       [-1.66458408,  1.77898767],
       [-0.59999531, -0.69743499],
       [ 0.46131224, -0.25488547],
       [-0.18712547, -0.6395096 ],
       [-2.119587  , -1.31631745],
       [ 0.55904671, -0.25298001],
       [-0.7278439 , -0.4330191 ],
       [-1.87791862, -1.18051858],
```

```
       [-1.62320808,  0.16471934],
       [ 3.11298947, -0.74480309],
       [ 0.52238133,  1.1746119 ],
       [ 1.25089457,  1.03010045]])
```

预测值

```
y_pred = lr.predict(X_test)
y_pred
```

```
array([-0.91181478, -1.06549217, -0.94703505,  1.81600913, -0.86396872,
       -0.94897312,  1.23540678,  0.67373625, -0.86906112, -0.40329085,
        0.25686302, -0.15416136, -1.34751326,  0.31504278, -0.46144457,
       -1.19483491, -0.95306198,  1.79893438,  0.3892856 ,  0.81228255])
```

真实值

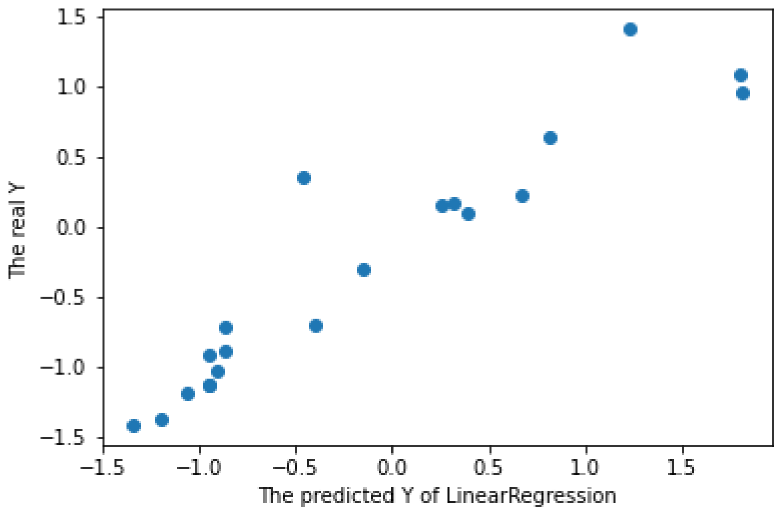
```
Y_test
```

```
51   -1.037059
55   -1.185173
46   -0.921221
2     0.957926
47   -0.884966
53   -1.133002
21    1.404832
26    0.226110
44   -0.715188
40   -0.704134
36    0.158906
39   -0.300026
63   -1.415966
35    0.169517
27    0.345928
58   -1.367332
50   -1.137865
3     1.085083
31    0.095239
24    0.640387
Name: y, dtype: float64
```

比较真实值与预测值

```
plt.scatter(y_pred, Y_test)
plt.xlabel('The predicted Y of LinearRegression')
plt.ylabel('The real Y')
```

```
Text(0, 0.5, 'The real Y')
```



R^2值为0.868，说明在测试集上回归效果较好，也说明PCA方法较好地消除了多重共线性

```
olsr = sm.OLS(y_pred, Y_test).fit()
olsr.summary()
```

OLS Regression Results

Dep. Variable:	y	**R-squared (uncentered):**	0.868
Model:	OLS	**Adj. R-squared (uncentered):**	0.862
Method:	Least Squares	**F-statistic:**	125.4
Date:	Mon, 13 Dec 2021	**Prob (F-statistic):**	8.25e-10
Time:	02:18:45	**Log-Likelihood:**	-7.7443
No. Observations:	20	**AIC:**	17.49
Df Residuals:	19	**BIC:**	18.48
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
y	1.0119	0.090	11.199	0.000	0.823	1.201

Omnibus:	7.487	**Durbin-Watson:**	1.953
Prob(Omnibus):	0.024	**Jarque-Bera (JB):**	6.318
Skew:	-0.601	**Prob(JB):**	0.0425
Kurtosis:	5.477	**Cond. No.**	1.00

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.