Coding Exercise 5: Solving the Aiyagari Model

ECON 202A

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In this problem set, you will numerically solve the partial equilibrium, general equilibrium, and transition dynamics of the Aiyagari Model discussed in the analytical homework. Your task is to produce plots that illustrate key results of the model and to provide the code used to generate these plots.

Refer to the analytical homework for the model details. The utility function follows a constant relative risk aversion (CRRA) form, and a short-sale constraint on capital $k \geq 0$ is imposed.

The model parameters are as follows:

- Relative risk aversion coefficient: $\sigma = 2$
- Idiosyncratic productivity: $z = [z_L, z_H] = [1, 2]$
- Transition rates: $\lambda = [\lambda_L, \lambda_H] = [1/3, 1/3]$
- Discount rate: $\rho = 0.05$
- Depreciation rate: $\delta = 0.05$
- Capital share in production: $\alpha = 1/3$
- TFP: $A = 0.1$
- Maximum capital level: $k_{\max} = 20$

Please complete the following tasks and submit both your written answers and the code used. Ensure that your submission includes plots and a brief explanation of your results. Refer to the section syllabus for detailed code submission guidelines.

Partial Equilibrium

Assume the interest rate is fixed at $r = 0.035$.

1. Solve the Hamilton-Jacobi-Bellman (HJB) equation for this interest rate. Plot the value function $V(k, z)$, consumption policy function $c(k, z)$, and savings policy function $s(k, z)$ across the capital grid k for each productivity state z.

2. Solve the Kolmogorov Forward (KF) equation for this interest rate. Plot the stationary distribution $g(k, z)$ across the capital grid k for each productivity state z.

3. Assume the interest rate increases to $r = 0.04$. Plot the stationary distribution $g(k, z)$ across the capital grid k for each productivity state z and interpret how the distribution of households shifts with the interest rate change.

General Equilibrium

Now, the interest rate is no longer exogenous given, an it is determined endogenously at which the capital market is cleared.

4. Derive the steady-state employment and unemployment rates and calculate the equilibrium aggregate labor expressed in efficiency units.

5. Plot the aggregate excess savings as a function of the interest rate, $S(r)$. Identify the stationary equilibrium interest rate r^*, where $S(r^*) = 0$.

6. Solve for the stationary equilibrium using Newton's method. Plot the consumption policy function $c(k, z)$, savings policy function $s(k, z)$, and stationary distribution $g(k, z)$ across the capital grid k for each productivity state z. Discuss any notable features and their economic interpretations.

Transition Dynamics

In this section, we analyze the impulse response of the economy to a negative aggregate productivity shock that mean-reverts over time. This shock is modeled as an "MIT" shock, where total factor productivity (TFP) evolves according to a deterministic Ornstein-Uhlenbeck process:

$$dA_t = \nu(A^{\text{ss}} - A_t)dt,$$

where ν controls the speed of mean reversion, and A^{ss} is the steady-state level of productivity. We assume $\nu = 0.2$. The economy experiences a negative productivity shock, reducing TFP by 3%. Over time, TFP gradually converges back to A^{ss}, reflecting the mean-reversion dynamics. Agents are assumed to have perfect foresight regarding the evolution of this aggregate shock.

7. Plot the TFP sequence over time.

8. Solve for the transition paths of r_t, w_t, and K_t. Plot these paths and discuss their economic implications in response to negative productivity shock.