Coding Exercise 2: Solving Steady States and Dynamic Paths in the Ramsey-Cass-Koopmans Model

ECON 202A

October 28, 2025

In this problem set, you will analytically and numerically solve for the steady states and simulate dynamic paths in the continuous-time Ramsey-Cass-Koopmans model. The model equations are provided below.

Consumption Euler Equation:

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - \rho - \theta g}{\theta}$$

Capital Accumulation Equation:

$$\dot{k}_t = f(k_t) - c_t - (n+g)k_t$$

where:

- $c(t)$: consumption at time t,
- $k(t)$: capital stock at time t,
- $f(k) = Ak^{\alpha}$: production function.

The boundary conditions include the initial condition $k_0 = 10$ and a transversality condition as the terminal condition.

The model parameters are defined as follows:

- Discount rate, $\rho = 0.03$
- Inverse of intertemporal elasticity of substitution, $\theta = 1$
- Technology growth rate, $g = 0.02$
- Population growth rate, $n = 0.02$
- Capital share, $\alpha = \frac{1}{3}$
- Total factor productivity, $A = 1$

Please complete the following tasks and submit both your written answers and the code you used. Ensure that your submission includes plots and a brief explanation of your results. Refer to the course syllabus for detailed code submission guidelines.

1. Solve analytically for the steady-state values of capital and consumption.

2. Solve for the steady-state values of capital and consumption using `fsolve`.

3. Solve numerically for the steady-state values of capital and consumption by implementing Newton's Method without using `fsolve`.

4. Compare the analytical and numerical steady-state solutions for k^*, c^*, and r^* calculated in (1)-(3). Discuss any discrepancies observed between the methods.

5. Use the shooting algorithm to simulate dynamic paths for capital $k(t)$ and consumption $c(t)$. Then, calculate the implied rate of return on capital $r(t) = f'(k)$ and plot the dynamic paths of these three variables over time.

6. **Optional:** Assuming a competitive capital market, calculate the implied market-clearing interest rate $r_{\text{market}}(t)$ from the capital path $k(t)$ using Newton's Method. The implied market-clearing interest rate should satisfy: $k_d(r_{\text{market}}(t)) = k_s$.

7. **Optional:** Briefly explain the implications of the implied rate of return and the market-clearing interest rate in the context of the social planner's solution and competitive equilibrium.