CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz I** (31 Aug 2022)
Name	MELBO			**20 marks**
Roll No	000001	Dept.	AWSM	Page **1** of **2**

Instructions:
1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases will get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. Write **T** or **F** for True/False (write **only in the box on the right-hand side**) **(5x1=5 marks)**

1	For a linear classifier with model parameters: vector $\mathbf{w} \in \mathbb{R}^d$ and bias $b = 0$, the origin point (i.e., the vector $\mathbf{0} \in \mathbb{R}^d$) must always lie on the decision boundary.	T
2	Let $f: \mathbb{R} \to \mathbb{R}$ be a doubly differentiable function (i.e., first and second derivatives exist). If $f''(x^0) = 0$ at $x^0 \in \mathbb{R}$, then it is always the case that $f'(x^0) = 0$ too.	F
3	For any dimension $d \in \mathbb{N}$, the dot product of two d-dimensional vectors is always another d-dimensional vector.	F
4	If a set $\mathcal{C} \subset \mathbb{R}^2$ is convex, then its translation $\mathcal{C}' = \mathcal{C} + \mathbf{a}$ must be convex too for any vector $\mathbf{a} \in \mathbb{R}^2$ where we define the translation as $\mathcal{C}' \stackrel{\text{def}}{=} \{\mathbf{v} + \mathbf{a} : \mathbf{v} \in \mathcal{C}\}$.	T
5	Feature vectors used to describe data points to an ML model are never allowed to have negative values in their coordinates.	F

Q2. (Melbo's claim) Melbo makes another claim that for some values of $m, b \in \mathbb{R}$, the function on the right is both continuous and differentiable for all $x \in \mathbb{R}$. Find these magical values of m, b. Show the major steps in your derivation. Answers in fractions and using terms like e okay. No need for decimal answers. **(5 marks)**

$$f(x) = \begin{cases} e^x & x \leq 2 \\ mx + b & x > 2 \end{cases}$$

For a function to be differentiable, it must have a unique derivative at each point. At $x = 2$, if we approach from the left, we get the slope to be e^2 since $\frac{de^x}{dx} = e^x$. However, if we approach from the right, the slope is $\frac{d(mx+b)}{dx} = m$. For the function to be differentiable we must have

$$m = e^2$$

The function value at $x = 2$ is e^2 if we approach from the left and $e^2 \cdot x + b$ if we approached from the right. For the function to be continuous, these two values must be the same which gives us $e^2 = 2e^2 + b$ which gives us

$$b = -e^2$$

Thus, we have

$$f(x) = \begin{cases} e^x & x \leq 2 \\ e^2 x - e^2 & x > 2 \end{cases}$$

Q3. (Vector line-up) Give examples of 4D vectors (fill-in the 4 boxes) with the following properties. Any example will get full marks so long as it satisfies the property mentioned in the question part. Your answers to the parts a, b, c, d, e may be same/different. **(5x1 = marks)**

a. A vector $\mathbf{v} \in \mathbb{R}^4$ with L_1 norm of two i.e., $\|\mathbf{v}\|_1 = 2$.

1	0	-1	0

b. A vector $\mathbf{v} \in \mathbb{R}^4$ with unit L_2 norm i.e., $\|\mathbf{v}\|_2 = 1$.

0	0	0	-1

c. A vector $\mathbf{v} \in \mathbb{R}^4$ equal to its own negative i.e., $\mathbf{v} = -\mathbf{v}$.

0	0	0	0

d. A vector $\mathbf{v} \in \mathbb{R}^4$ with same L_1 and L_2 norm i.e $\|\mathbf{v}\|_1 = \|\mathbf{v}\|_2$.

0	0	5	0

e. A vector $\mathbf{v} \in \mathbb{R}^4$ whose L_2 norm is half its L_1 norm i.e., $\|\mathbf{v}\|_2 = \frac{1}{2}\|\mathbf{v}\|_1$.

$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

Q4. (Melbo takes a break) When not being the star of ML YouTube videos, Melbo likes to play volleyball. Melbo finds that if thrown straight up from a height of 1 metre (assuming $g = 10\ m/s^2$), the height of the ball t seconds after being launched is $h = 1 + 5t - 5t^2$. Find out:

1. The maximum height attained by the ball
2. Time taken to reach the highest point
3. Time taken for the ball to hit the ground initially
4. The velocity with which Melbo threw the ball
5. The velocity of the ball at its highest point

Hint: In this case, velocity would be defined as $v = \frac{dh}{dt}$. The "up" direction is considered positive.

Answers in fractions are okay – no need for decimals. Show main calculations. **(5x1=5 marks)**

It is notable that this height equation is valid only till the ball first hits the ground since bouncing off the ground will cause a non-differentiable change in height.

Since maxima are stationary points, we set $\frac{dh}{dt} = 5 - 10t = 0$ to get $t = \frac{1}{2}$ at which point we have $h(t) = 1 + \frac{5}{2} - \frac{5}{4} = \frac{9}{4}$. Thus, the maximum height attained is $\frac{9}{4}$ meters and this takes $\frac{1}{2}$ seconds to reach this height.

Setting $1 + 5t - 5t^2 = 0$ gives us $t = \frac{1}{2} + \frac{3\sqrt{5}}{10}$. The other root give to a negative time which corresponds to a time before the ball was thrown and can be discarded since that solution assumes that the ball was launched from the ground or below the ground.

We have $v = \frac{dh}{dt} = 5 - 10t$. Thus, at $t = 0$, velocity was $5\ m/s^2$ (upward)

Since the highest point is a stationary point, the velocity there is 0.