CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz II** (19 Oct 2022)
Name				**20 marks** Page **1** of **2**
Roll No		Dept.		

Instructions:

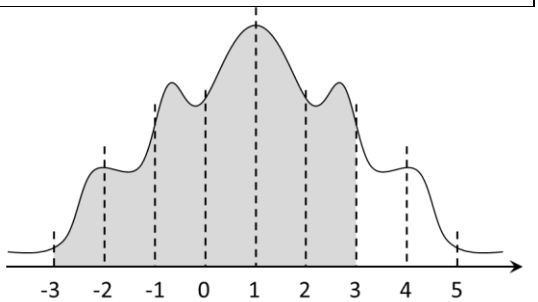
1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases will get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. Write **T** or **F** for True/False **in the box** and give justification below. **(4 x (1+2) = 12 marks)**

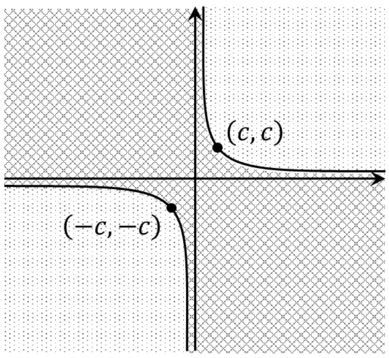
1	If $A, B \subset \mathbb{R}^2$ are convex sets, then the set $C = \frac{(A+B)}{2} \stackrel{\text{def}}{=} \left\{\frac{(a+b)}{2}, a \in A, b \in B\right\}$ is always convex. Give a brief proof if True else give counter example if False.	
2	The solution to $\min_{x \in \mathbb{R}} \frac{\lambda}{2} \cdot x^2 + \sum_{i=1}^{n} \left(x - a^i\right)^2$ approaches the mean of the real numbers $a^1, a^2, \ldots, a^n \in \mathbb{R}$ as $\lambda \to \infty$. Justify by deriving the solution below.	
3	For a doubly differentiable fn $f: \mathbb{R} \to \mathbb{R}$, if $f''(x_0) = 0$ at $x_0 \in \mathbb{R}$, then it must be that $f'(x_0) = 0$. Give brief justification if True else give counter example if False.	

4	Let X, Y be two random variables (may or may not be independent) such that $\mathrm{Var}[X] > 0$ and $\mathrm{Var}[Y] > 0$. Then it may happen that $\mathrm{Var}[X+Y] = 0$. Give an example if your answer is True else give a proof that $\mathrm{Var}[X+Y] > 0$ always.	

Q2. (Deviant behaviour) Melbo created a new class of $\mathcal{M}$ distributions with mean μ, std σ such that if $X \sim \mathcal{M}(\mu, \sigma^2)$, then for any $c > 0$ we have $\mathbb{P}[\mu - c\sigma \leq X \leq \mu + c\sigma] = \eta_c$ It is known that $\eta_1 = 0.75, \eta_2 = 0.9, \eta_3 = 0.95, \eta_4 = 0.99$. For $\mu = 1, \sigma = 1$, find out $\mathbb{P}[-3 \leq X \leq 3]$. Give brief calculations below and the final answer. **(2+2=4 marks)**



Q3. (XOR classifier) For a 2D rectangular hyperbola with equation $xy = c^2$ for $c \in \mathbb{R}$, give a feature map $\phi: \mathbb{R}^2 \to \mathbb{R}^D$ for some $D > 0$ and a corresponding linear classifier $\mathbf{W} \in \mathbb{R}^D$ so that for any $\mathbf{x} \in \mathbb{R}^2$, $\mathrm{sign}\left(\mathbf{W}^\top \phi(\mathbf{x})\right)$ takes value $z = -1$ in the light dotted region (see figure on the right) and $z = +1$ in the dark cross-hatched region. We don't care what happens on the boundary. Your map ϕ must **not depend on** c but $\mathbf{W}$ may depend on c. No need to show calculations. Just give the final answers. **(2 + 2 = 4 marks)**



$\phi(x, y) =$

$\mathbf{W} =$