Assignment - Camera Calibration

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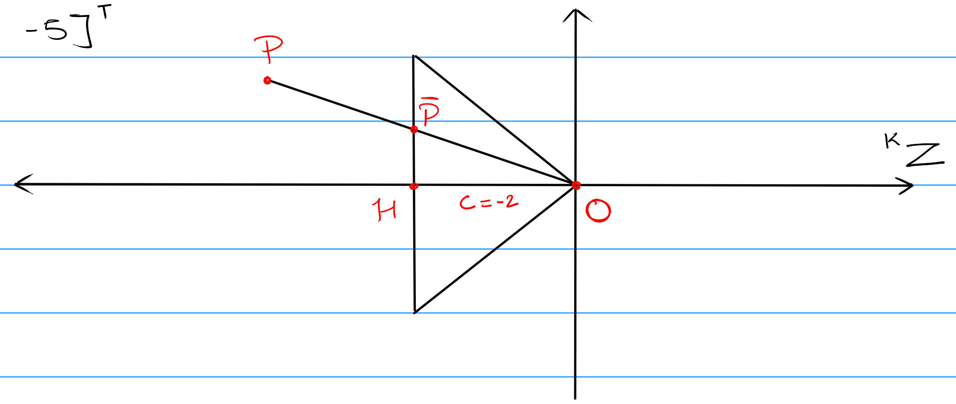
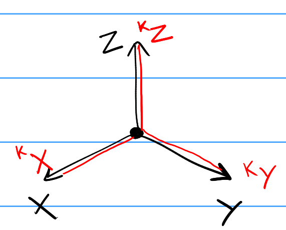
Q1) $P = [6 \quad 6 \quad -5]^T$

$c = -2$



- First we need to convert from world coordinates to camera coordinate

$$\underbrace{\begin{bmatrix} {}^K X_P \\ 1 \end{bmatrix}}_{\text{Camera Coord. in Homogeneous coord.}} = \underbrace{\begin{bmatrix} R & 0 \\ 0^T & 1 \end{bmatrix}}_{\text{Rotation}} \cdot \underbrace{\begin{bmatrix} I_3 & -X_0 \\ 0^T & 1 \end{bmatrix}}_{\text{Translation}} \cdot \underbrace{\begin{bmatrix} X_P \\ 1 \end{bmatrix}}_{\text{World Coord. in Homogenous Coord.}} \quad — (1)$$



As mentioned camera coord. and world coord. are aligned, ie same place.

$\therefore$ No Rotation & No translation

In (1); $R = I_3$ due to no rotation and $-X_0 = 0$ due to no translation

$$\begin{bmatrix} {}^K X_P \\ 1 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0^T & 1 \end{bmatrix} \cdot \begin{bmatrix} I_3 & 0 \\ 0^T & 1 \end{bmatrix} \cdot \begin{bmatrix} X_P \\ 1 \end{bmatrix}$$

$$= [I_4] \cdot [I_4] \cdot \begin{bmatrix} X_P \\ 1 \end{bmatrix} = \begin{bmatrix} X_P \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} {}^K X_p \\ 1 \end{bmatrix} = \begin{bmatrix} {}^K X_p \\ {}^K Y_p \\ {}^K Z_p \\ 1 \end{bmatrix} = \begin{bmatrix} X_p \\ Y_p \\ Z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -5 \\ 1 \end{bmatrix}$$

Camera Coord. $= [6 \;\; 6 \;\; -5]^T$

- Now we convert from Camera coord. to Image Coord.

$$\underbrace{{}^c x_{\bar{p}} = \begin{bmatrix} {}^c u_{\bar{p}} \\ {}^c v_{\bar{p}} \\ {}^c w_{\bar{p}} \end{bmatrix}}_{\text{Image Coord.}} = \underbrace{\begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{translation of } c} \cdot \underbrace{\begin{bmatrix} {}^K X_p \\ {}^K Y_p \\ {}^K Z_p \\ 1 \end{bmatrix}}_{\text{Camera Coord.}} \quad — \quad (2)$$

Subbing $c = -2$ in (2) we get

$$ {}^c x_{\bar{p}} = \begin{bmatrix} {}^c u_{\bar{p}} \\ {}^c v_{\bar{p}} \\ {}^c w_{\bar{p}} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 6 \\ -5 \\ 1 \end{bmatrix}$$

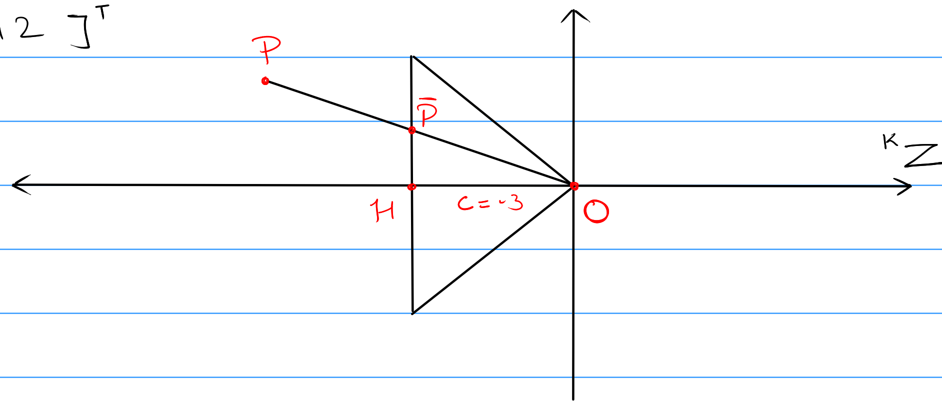
$${}^c x_{\bar{p}} = \begin{bmatrix} -12 \\ -12 \\ -5 \end{bmatrix} \xrightarrow[\text{to Euclidian}]{\text{from Homognous}} \begin{bmatrix} -12/5 \\ -12/5 \end{bmatrix}$$

Ans) Camera Coord. $= [6 \;\; 6 \;\; -5]^T$

Image Coord. $= [-12/5 \;\; -12/5]^T$ & ${}^K Z_{\bar{p}} = -2$

Q2) $\bar{P} = [12 \quad 12]^T$

$c = -3$



- First we need to convert from Image coordinates to camera coordinate.

$$\begin{bmatrix} 12 \\ 12 \end{bmatrix} \xrightarrow[\text{transformation}]{\text{Homogeneous}} \begin{bmatrix} 12 * {}^c w_{\bar{p}} \\ 12 * {}^c w_{\bar{p}} \\ {}^c w_{\bar{p}} \end{bmatrix} = \begin{bmatrix} -3 \cdot {}^K x_p \\ -3 \cdot {}^K y_p \\ {}^K z_p \end{bmatrix} \leftarrow \text{using } (2)$$

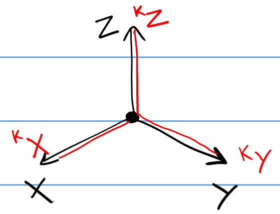
$\therefore \; {}^c w_{\bar{p}} = {}^K z_p$

$\therefore \; {}^K x_p = -4 \cdot {}^K z_p$

${}^K y_p = -4 \cdot {}^K z_p$

${}^K z_p$ = any real number (because this information is lost)

Camera Coord. = $[-4 \cdot {}^K z_p \,,\, -4 \, {}^K z_p \,,\, {}^K z_p]$



As mentioned camera coord. and world coord. are aligned, ie same place.

$\therefore$ No Rotation & No translation

$\therefore$ World Coord. = Camera Coord. $\leftarrow$ from (1)

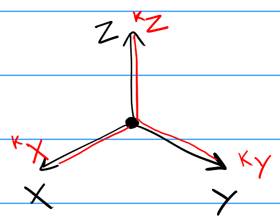
Ans) Camera Coord. = $[-4 \cdot {}^K z_p \,,\, -4 \cdot {}^K z_p \,,\, {}^K z_p]$

World Coord. = $[-4 \cdot {}^K z_p \,,\, -4 \cdot {}^K z_p \,,\, {}^K z_p]$

Q3) world cs 1 = $[12 \quad 12 \quad -4]^T$ image cs 1 = $[6, 6]^T$

world cs 2 = $[6 \quad 6 \quad -5]^T$ image cs 2 = ?

- first we need to find the value of c, using world cs.1 and image cs.2



As mentioned camera coord. and world coord. are aligned, ie same place.

$\therefore$ No Rotation & No translation

$\therefore$ world cs.1 = camera cs.1 = $[12 \quad 12 \quad -4]$

$$ {}^{c}x_{\bar{p}} = \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 12 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \cdot c \\ 12 \cdot c \\ -4 \end{bmatrix} \xrightarrow{H} \begin{bmatrix} 12c/-4 \\ 12c/-4 \end{bmatrix} $$

$$ \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} -3c \\ -3c \end{bmatrix} \qquad \therefore \boxed{c = -2} $$

- now using $c = -2$ and world cs. 2 = $[6 \quad 6 \quad -5]^T$ we calculate Image cs.2 .

world cs. 2 = camera cs. 2 = $[6 \quad 6 \quad -5]$ (using above axis dia)

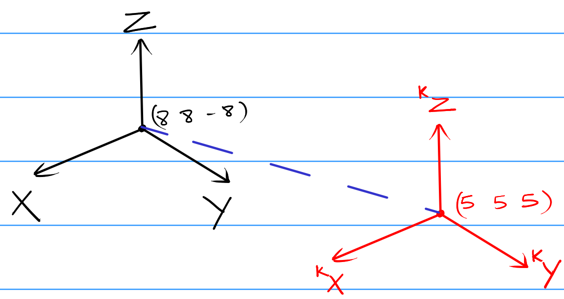
$$ {}^{c}x_{\bar{p}} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 6 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -12 \\ -12 \\ -5 \end{bmatrix} \xrightarrow{H} \begin{bmatrix} -12/-5 \\ -12/-5 \end{bmatrix} $$

Ans) Image cs.2 = $[12/5 \quad 12/5]^T$

Q4)

world cs. $= [8 \quad 8 \quad -8]^T$, $c = -2$

camera cs. = ?, image cs = ?



As mentioned camera coord. and world coord. are shifted but axis is $\parallel^{l}$.

$\therefore$ No Rotation

- first we find the camera cs using world cs & $c = -2$

$$\begin{bmatrix} {}^{K}X_p \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} R & 0 \\ 0^T & 1 \end{bmatrix}}_{\substack{R = I_3 \\ \text{No Rotation}}} \cdot \underbrace{\begin{bmatrix} I_3 & -X_0 \\ 0^T & 1 \end{bmatrix}}_{X_0 = [5\ 5\ 5]^T} \cdot \begin{bmatrix} X_p \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} {}^{K}x_p \\ {}^{K}y_p \\ {}^{K}z_p \\ 1 \end{bmatrix} = [I_4] \cdot \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 8 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -13 \\ 1 \end{bmatrix}$$

- Now we convert from Camera coord. to Image Coord.

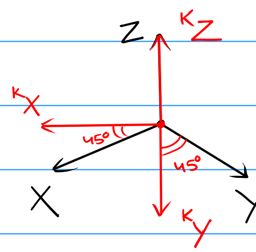
$${}^{c}x_{\bar{p}} = \begin{bmatrix} {}^{c}u_{\bar{p}} \\ {}^{c}v_{\bar{p}} \\ {}^{c}w_{\bar{p}} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 3 \\ -13 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -6 \\ -13 \end{bmatrix} \xrightarrow{H} \begin{bmatrix} -6/_{-13} \\ -6/_{-13} \end{bmatrix}$$

Ans)

Camera Coord. $= [3 \quad 3 \quad -13]^T$

Image Coord $= [6/_{13} \quad 6/_{13}]^T$

Q5) world cs $= [8 \;\; 8 \;\; -8]^T$, $c = -2$



As mentioned camera coord. and world coord. coincide but rotated about z axis.

$\therefore$ No Translation.

- first we find the camera cs using world cs & $c = -2$

$$\begin{bmatrix} {}^K X_P \\ 1 \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0^T & 1 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} I_3 & -X_0 \\ 0^T & 1 \end{bmatrix}}_{\text{No translation}} \cdot \begin{bmatrix} X_P \\ 1 \end{bmatrix}$$

$\therefore X_0 = \vec{0}$

→ As we are camera 45° clockwise about z axis (ie $\theta = -45$)

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} {}^K X_P \\ {}^K Y_P \\ {}^K Z_P \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 8 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 4\sqrt{2} + 4\sqrt{2} \\ 4\sqrt{2} - 4\sqrt{2} \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 8\sqrt{2} \\ 0 \\ -8 \\ 1 \end{bmatrix}$$

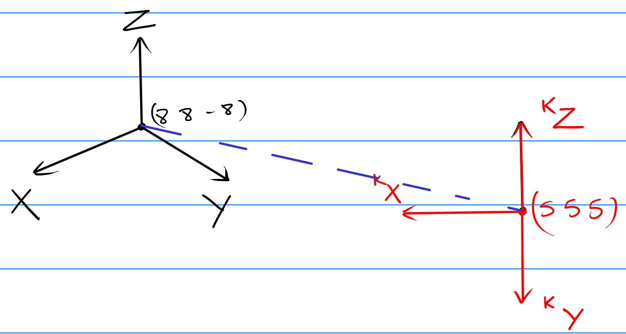
- Now we convert from Camera coord. to Image Coord.

$${}^c x_{\bar{P}} = \begin{bmatrix} {}^c u_{\bar{P}} \\ {}^c v_{\bar{P}} \\ {}^c w_{\bar{P}} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 8\sqrt{2} \\ 0 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} -16\sqrt{2} \\ 0 \\ -8 \end{bmatrix} \xrightarrow{H} \begin{bmatrix} -16\sqrt{2}/-8 \\ 0 \end{bmatrix}$$

Ans) Camera Coord. = $[8\sqrt{2} \quad 0 \quad -8]^T$

Image coord = $[2\sqrt{2} \quad 0]^T$

Q6) world coord. = $[8 \quad 8 \quad -8]$, $c = -2$



As mentioned camera coord. and world coord. are shifted and rotated.

- first we find the camera cs using world cs & $c = -2$

$$\begin{bmatrix} {}^{K}X_P \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} R & 0 \\ 0^T & 1 \end{bmatrix}}_{R \text{ is same as Q5}} \cdot \underbrace{\begin{bmatrix} I_3 & -X_0 \\ 0^T & 1 \end{bmatrix}}_{X_0 = [5\ 5\ 5]^T} \cdot \begin{bmatrix} X_P \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} {}^{K}X_P \\ {}^{K}Y_P \\ {}^{K}Z_P \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 8 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -13 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 3 \\ -13 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/\sqrt{2} + 3/\sqrt{2} \\ 3/\sqrt{2} - 3/\sqrt{2} \\ -13 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\sqrt{2} \\ 0 \\ -13 \\ 1 \end{bmatrix}$$

- Now we convert from Camera coord. to Image Coord.

$$ {}^{c}x_{\bar{P}} = \begin{bmatrix} {}^{c}u_{\bar{P}} \\ {}^{c}v_{\bar{P}} \\ {}^{c}w_{\bar{P}} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3\sqrt{2} \\ 0 \\ -13 \\ 1 \end{bmatrix} = \begin{bmatrix} -6\sqrt{2} \\ 0 \\ -13 \end{bmatrix} \xrightarrow{\mathcal{H}} \begin{bmatrix} -6\sqrt{2}/-13 \\ 0 \end{bmatrix} $$

Ans) Camera Coord. = $[3\sqrt{2} \quad 0 \quad -13]^T$

Image Coord. = $[6\sqrt{2}/13 \quad 0]^T$

Q7)

World Coord.	Image Coord.
$[9 \quad 9 \quad 9]^T$	$[4 \quad 4]^T$
$[9 \quad 7 \quad 9]^T$	$[4 \quad 2]^T$
$[7 \quad 9 \quad 9]^T$	$[2 \quad 4]^T$
$[7 \quad 7 \quad 9]^T$	$[2 \quad 2]^T$
$[9 \quad 9 \quad 11]^T$	$[2 \quad 2]^T$
$[9 \quad 7 \quad 11]^T$	$[2 \quad 1]^T$

$$ \underbrace{x_i}_{\text{Image Coord.}} = \underbrace{P}_{\text{Transformation Matrix.}} \underbrace{X_i}_{\text{World Coord.}} $$

- Let's try to find the dimension of P

$$ P = \underset{3\times3}{{}^{c}K} \; \underset{3\times3}{R} \; \underset{3\times4}{[I_3 \mid -X_0]} \qquad \therefore \; \mathrm{Dim}(P) = 3\times4 $$

Now, $x_i = P X_i$

$$ \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} (P_{11} & P_{12} & P_{13} & P_{14}) \\ (P_{21} & P_{22} & P_{23} & P_{24}) \\ (P_{31} & P_{32} & P_{33} & P_{34}) \end{bmatrix} \cdot \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix} = \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} \cdot X_i = \begin{bmatrix} A^T X_i \\ B^T X_i \\ C^T X_i \end{bmatrix} $$

where $A^T = [P_{11} \quad P_{12} \quad P_{13} \quad P_{14}]$, $B^T = [P_{21} \quad P_{22} \quad P_{23} \quad P_{24}]$, $C^T = [P_{31} \quad P_{32} \quad P_{33} \quad P_{34}]$

$$\therefore \quad x_i = \frac{A^T X_i}{C^T X_i} \qquad y_i = \frac{B^T X_i}{C^T X_i}$$

$$\Rightarrow \quad x_i C^T X_i - A^T X_i = 0 \quad \& \quad y_i \cdot C^T X_i - B^T X_i = 0$$

$$\Rightarrow \quad a_{x_i}^T \cdot p = 0 \quad \& \quad a_{y_i}^T \cdot p = 0$$

where

$$\boxed{\begin{aligned} p &= \mathrm{vec}(P^T) = [P_{11}\ P_{12} \cdots P_{21}\ P_{22} \cdots P_{31} \cdots P_{34}]^T_{(12\times 1)} \\ a_{x_i}^T &= [-X_i, -Y_i, -Z_i, -1, 0, 0, 0, 0, x_i X_i, x_i Y_i, x_i Z_i, x_i]_{(1\times 12)} \\ a_{y_i}^T &= [0, 0, 0, 0, -X_i, -Y_i, -Z_i, -1, y_i X_i, y_i Y_i, y_i Z_i, y_i]_{(1\times 12)} \end{aligned}}$$

- To get the value of P we will stack $a_{x_i}^T$ $a_{y_i}^T$ for all points.

$$\begin{bmatrix} a_{x_1}^T \\ a_{y_1}^T \\ \cdots \\ a_{x_i}^T \\ a_{y_i}^T \\ \cdots \\ a_{x_I}^T \\ a_{y_I}^T \end{bmatrix} \cdot p = \underset{2I\times 12}{M} \cdot \underset{12\times 1}{p} \quad \text{should be } 0$$

- We need to take the SVD of 'M', to get the value of p.

$$\underset{12\times 12}{M} = \underset{12\times 12}{U} \cdot \underset{12\times 12}{\Sigma} \cdot \underset{12\times 12}{V^T} \qquad [\text{since } I = 6]$$

- As we need to make $M \cdot p = 0$ we choose the lowest contributing value of V^T so it becomes 0.

* I used np.linalg.svd(M) to calculate U, Σ & V^T

then,

$V^T[11] = [-0.124, 0, 0, 0.622, 0, -0.124, 0, 0.622, 0, 0, -0.062, 0.435]$

Ans) So $P = \begin{bmatrix} -0.124 & 0 & 0 & 0.622 \\ 0 & -0.124 & 0 & 0.622 \\ 0 & 0 & -0.062 & 0.432 \end{bmatrix}$