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Assignment - Epipolar Geometry

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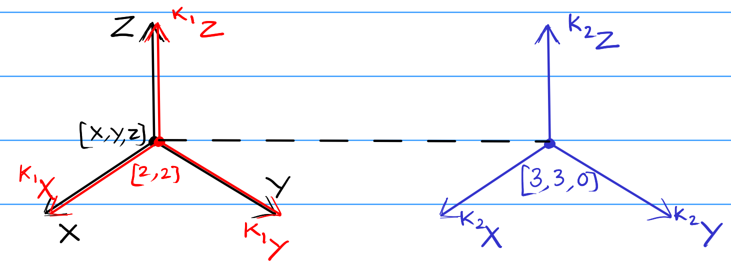
Q1)

FIGURE :



GIVEN :

Camera 1	Camera 2
Image Coord = [2, 2]	Image Coord = [2, 2]
c = -2	c = -2
axis is alligned.	axis is shifted

→ First let's consider Camera 1 :

$$\underbrace{\begin{bmatrix}2\\2\end{bmatrix} \xrightarrow{H} \begin{bmatrix} 2 * {}^{C}w_{\bar p} \\ 2 * {}^{C}w_{\bar p} \\ {}^{C}w_{\bar p}\end{bmatrix}}_{\text{Image Coord}} = \underbrace{\begin{bmatrix}-2&0&0&0\\0&-2&0&0\\0&0&1&0\end{bmatrix}}_{\text{Calibration Matrix.}} \underbrace{\begin{bmatrix}{}^{K_1}x_p\\{}^{K_1}y_p\\{}^{K_1}z_p\\1\end{bmatrix}}_{\text{Camera Coord.}} \xrightarrow{E} \begin{bmatrix}-2\,{}^{K_1}x_p\\-2\,{}^{K_1}y_p\\{}^{K_1}z_p\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 * {}^{C}w_{\bar p} \\ 2 * {}^{C}w_{\bar p} \\ {}^{C}w_{\bar p}\end{bmatrix} = \begin{bmatrix}-2\,{}^{K_1}x_p\\-2\,{}^{K_1}y_p\\{}^{K_1}z_p\end{bmatrix}$$

As $\boxed{{}^{C}w_{\bar p} = {}^{K_1}z_p}$ we get :

$$\begin{matrix} 2\,{}^{C}w_{\bar p} = \\ 2\,{}^{C}w_{\bar p} = \end{matrix} \begin{bmatrix} 2\,{}^{K_1}z_p = -2\,{}^{K_1}x_p \\ 2\,{}^{K_1}z_p = -2\,{}^{K_1}y_p \end{bmatrix} \Rightarrow \begin{matrix} {}^{K_1}x_p = -{}^{K_1}z_p \\ {}^{K_2}y_p = -{}^{K_1}z_p \end{matrix}$$

$$\text{Camera Coordinate} = \left[-{}^{K_1}Z_p ,\ -{}^{K_1}Z_p ,\ {}^{K_1}Z_p \right]$$

$$\text{World Coordinate} = \left[-{}^{K_1}Z_p ,\ -{}^{K_1}Z_p ,\ {}^{K_1}Z_p \right] \quad \text{(Axis is aligned)}$$

→ <u>Now let's consider Camera 2:</u>

$$\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\mathcal{H}} \begin{bmatrix} 1 * C_{w_{\bar{p}}} \\ 1 * C_{w_{\bar{p}}} \\ C_{w_p} \end{bmatrix}}_{\text{Image Coord}} = \underbrace{\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Calibration Matrix.}} \underbrace{\begin{bmatrix} {}^{K_2}x_p \\ {}^{K_2}y_p \\ {}^{K_2}z_p \\ 1 \end{bmatrix}}_{\text{Camera Coord.}} \xrightarrow{E} \begin{bmatrix} -2\ {}^{K_2}x_p \\ -2\ {}^{K_2}y_p \\ {}^{K_2}z_p \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 * C_{w_{\bar{p}}} \\ 1 * C_{w_{\bar{p}}} \\ C_{w_{\bar{p}}} \end{bmatrix} = \begin{bmatrix} -2\ {}^{K_2}x_p \\ -2\ {}^{K_2}y_p \\ {}^{K_2}z_p \end{bmatrix}$$

As $\boxed{C_{w_{\bar{p}}} = {}^{K_2}z_p}$ we get:

$$\begin{bmatrix} C_{w_{\bar{p}}} = {}^{K_2}z_p = -2\ {}^{K_2}x_p \\ C_{w_{\bar{p}}} = {}^{K_2}z_p = -2\ {}^{K_2}y_p \end{bmatrix}$$

$$\Rightarrow \begin{aligned} {}^{K_1}x_p &= (-1/2)\ {}^{K_2}z_p \\ {}^{K_2}y_p &= (-1/2)\ {}^{K_2}z_p \end{aligned}$$

$$\text{Camera Coordinate} = \left[(-1/2)\ {}^{K_2}Z_p ,\ (-1/2)\ {}^{K_2}Z_p ,\ {}^{K_2}Z_p \right]$$

As the axis is shifted to $[3, 3, 0]$,

$$\underbrace{\begin{bmatrix} (-\tfrac{1}{2}) \cdot {}^{K_2}Z_p \\ (-\tfrac{1}{2}) \cdot {}^{K_2}Z_p \\ {}^{K_2}Z_p \\ 1 \end{bmatrix}}_{\text{Camera coord}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Translation Matrix}} \underbrace{\begin{bmatrix} X_p \\ Y_p \\ Z_p \\ 1 \end{bmatrix}}_{\to \text{World coord}} = \begin{bmatrix} X_p - 3 \\ Y_p - 3 \\ Z_p \\ 1 \end{bmatrix}$$

As $\boxed{{}^{K_2}Z_p = Z_p}$ we get :

$$\left.\begin{aligned} (-\tfrac{1}{2})\,{}^{K_2}Z_p &= (-\tfrac{1}{2}) Z_p = X_p - 3 \\ (-\tfrac{1}{2})\,{}^{K_2}Z_p &= (-\tfrac{1}{2}) Z_p = Y_p - 3 \end{aligned}\right\}$$

$$\therefore \; X_p = (-\tfrac{1}{2}) Z_p + 3$$

$$Y_p = (-\tfrac{1}{2}) Z_p + 3$$

World Coordinates $= [(-\tfrac{1}{2}) Z_p + 3, (-\tfrac{1}{2}) Z_p + 3, Z_p]$

→ <u>Comparing World Coordinates from both Cameras :</u>

Camera 1 → World Coordinate $= [-{}^{K_1}Z_p, -{}^{K_1}Z_p, {}^{K_1}Z_p]$

Camera 2 → World Coordinates $= [(-\tfrac{1}{2}) Z_p + 3, (-\tfrac{1}{2}) Z_p + 3, Z_p]$

$$-{}^{K_1}Z_p = (-\tfrac{1}{2}) Z_p + 3 \quad - (\text{i})$$

$$-{}^{K_1}Z_p = (-\tfrac{1}{2}) Z_p + 3 \quad - (\text{ii})$$

$${}^{K_1}Z_p = Z_p \quad - (\text{iii})$$

Subbing (iii) in (i) we get :

$$-Z_p = \left(\tfrac{-1}{2}\right) Z_p + 3$$

$$\left(\tfrac{-1}{2}\right) Z_p - 3 = 0$$

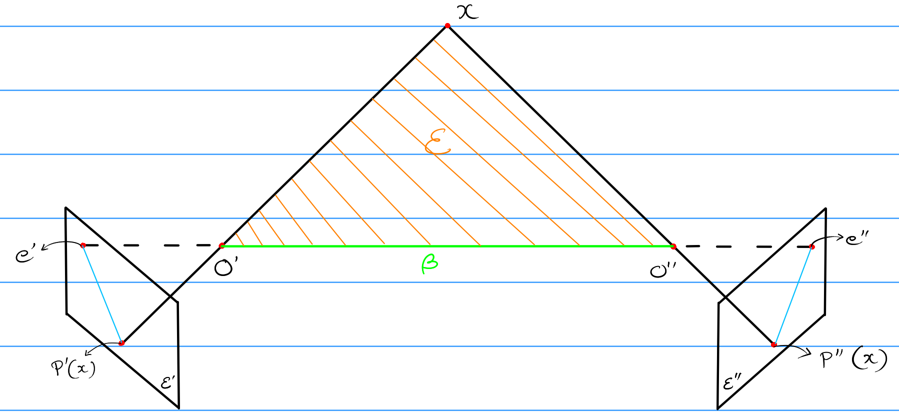
$$Z_p = \frac{3}{\left(-\tfrac{1}{2}\right)} = -6 \rightarrow \underbrace{\text{Z-coord of World CS.}}$$

$$X_p = \left(\tfrac{-1}{2}\right) Z_p + 3 = \tfrac{6}{2} + 3 = 3 + 3 = \underline{\underline{6}}$$

$$Y_p = \left(\tfrac{1}{2}\right) Z_p + 3 = 3 + 3 = \underline{\underline{6}}$$

World Coord $= \underline{\underline{[6, 6, -6]}}$

Q2) Figure:



- **Epipolar Axis:** The line connecting the two projection centers, O' & O'' denoted in the figure as 'β'.

- **Epipolar Line:** The line joining the epipole on an Image plane to the projection of the real world point 'x', on the Image plane. Denoted by the 'blue lines'.

⊛ **Epipole :** The projection of the opposite projection center on the initial image plane.

from the figure:

e' is the projection of O'' on $\mathcal{E}'$.

& e'' is the projection of O' on $\mathcal{E}''$.

Geometrically:

$e' = (O'O'') \cap \mathcal{E}'$

$e'' = (O'O'') \cap \mathcal{E}''$

⊛ **Epipolar Plane :** This is the plane formed by connecting the projection centers and the point in the real world 'x'. Denoted by '$\mathcal{E}$' in the figure. ($\mathcal{E} = O'O''x$)

⊛ **Fundamental Matrix :** It is used to encapsulate the intrinsic geometry that epipolar geometry follows. The fundamental matrix 'F' is a 3x3 matrix of rank 2.

From the above figure,

$$P''(x) \cdot F \cdot P'(x) = 0$$

⊛ **Projection Matrix :** When converting from 3D to 2D or vice-versa we make use of a transformation matrix, we call this matrix the Projection Matrix.

Its also used in finding Epipoles.

$\boxed{e' = P' X_{O''}}$ $\boxed{e'' = P'' X_{O'}}$

* **Calibrated Camera:** A camera is said to be calibrated if the parameters of the camera are estimated and set.

* **Uncalibrated Camera:** In contrary, if the parameters are not yet estimated, then its uncalibrated.

* **Intrinsic Parameters:** In simple terms, all the parameters that have to do with the Image to Camera planes are called Intrinsic.

  They include focal length, optical center etc.

  During the Intrinsic parameter calibration there is loss of information, due to 3D to 2D conversion.

* **Extrinsic Parameters:** In simple terms, those parameters that are affected in the world to camera planes are called Extrinsic.

  Rotation and translation are good examples of these parameters. There is generally no loss of data due to them being transformations from 3D to 3D.