B-Spline Interpolation

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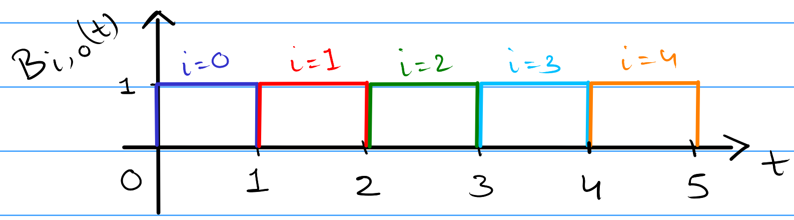
Q1) $f(0) = 2,\ f(1) = 3,\ f(2) = 4,\ f(3) = 2,\ f(4) = 5.$

find $f(2.5)$ using :

Ans) (a) 0-degree B-spline function:



$$B_{i,0}(t) = \begin{cases} 1, & \text{if } i \le t \le i+1 \\ 0, & \text{else} \end{cases} \quad — (1)$$

$$c(t) = \sum_{i=0}^{n=4} f(i) * B_{i,0}(t)$$

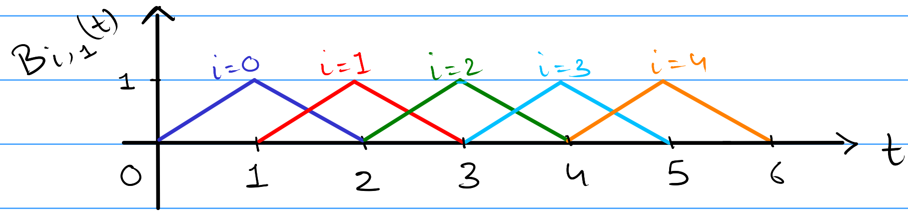
$$c(2.5) = f(0) * B_{0,0}(2.5) + f(1) * B_{1,0}(2.5) + f(2) * B_{2,0}(2.5) + f(3) * B_{3,0}(2.5) + f(4) * B_{4,0}(2.5) \quad — (2)$$

Using (1) to calculate $B_{i,0}(t)$ and subbing in 2

$$c(2.5) = 2*0 + 3*0 + 4*1 + 2*0 + 5*0$$

$$c(2.5) = 4$$

(b) 1-degree B-spline function:



$$B_{i,1}(t) = \begin{cases} \dfrac{t - t_i}{t_{i+1} - t_i}, & \text{if } t \in [t_i, t_{i+1}] \\ \dfrac{t_{i+2} - t}{t_{i+2} - t_{i+1}}, & \text{if } t \in [t_{i+1}, t_{i+2}] \\ 0, & \text{else} \end{cases} \quad — (3)$$

$$c(t) = \sum_{i=0}^{n=4} f(i) * B_{i,1}(t)$$

$$c(2.5) = f(0) * B_{0,1}(2.5) + f(1) * B_{1,1}(2.5) + f(2) * B_{2,1}(2.5) + f(3) * B_{3,1}(2.5) + f(4) * B_{4,1}(2.5) \quad — (4)$$

→ $B_{0,1}(t=2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}] = [0, 1, 2]$

So $t = 2.5 > t_{i+2} = 2$

$\therefore B_{0,1}(t=2.5) = 0$

→ $B_{1,1}(t=2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}] = [1, 2, 3]$

So $(t=2.5) \in [t_{i+1}, t_{i+2}] = [2, 3]$

$$B_{1,1}(t=2.5) = \frac{t_{i+2} - t}{t_{i+2} - t_{i+1}} = \frac{3 - 2.5}{3 - 2} = \frac{0.5}{1} = 1/2$$

$\rightarrow B_{2,1}(t=2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}] = [2, 3, 4]$

So $t=2.5) \in [t_i, t_{i+1}] = [2,3]$

$$B_{2,1}(t=2.5) = \frac{t-t_i}{t_{i+1}-t_i} = \frac{2.5-2}{3-2} = \frac{0.5}{1} = 1/2$$

$\rightarrow B_{3,1}(t=2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}] = [3, 4, 5]$

So $t=2.5 < t_i$

$\therefore B_{3,1}(t=2.5) = 0$

$\rightarrow B_{4,1}(t=2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}] = [4, 5, 6]$

So $t=2.5 < t_i$

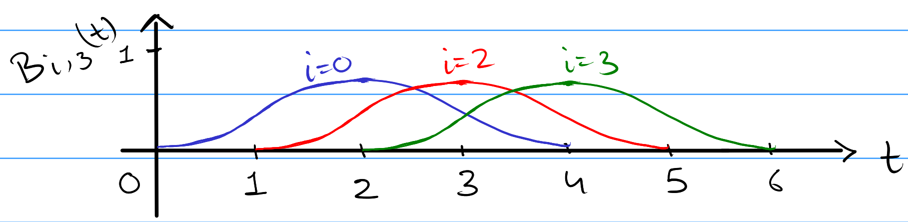
$\therefore B_{4,1}(t=2.5) = 0$

Using all the $B_{i,1}(2.5)$ values calculated and subbing in (4)

$$c(2.5) = 2*0 + 3*\frac{1}{2} + 4*\frac{1}{2} + 2*0 + 5*0$$

$$c(2.5) = \frac{3}{2} + \frac{4}{2} = \underline{\underline{\frac{7}{2}}}$$

(c) 3-degree B-spline function:



$$B_{i,3}(t) = \begin{cases} \text{formula same as the one in} \\ \text{the PPT sent in G-class.} \end{cases}$$

$$c(t) = \sum_{i=0}^{n=4} f(i) * B_{i,3}(t)$$

$$c(2.5) = f(0) * B_{0,3}(2.5) + f(1) * B_{1,3}(2.5) + f(2) * B_{2,3}(2.5) + \underbrace{f(3) * B_{3,3}(2.5)}_{0} + \underbrace{f(4) * B_{4,3}(2.5)}_{0}$$

$$B_{3,3}(2.5) = B_{4,3}(2.5) = 0 \quad \text{as} \quad 2.5 \notin [3, 8]$$

$$\therefore c(2.5) = f(0) * B_{0,3}(2.5) + f(1) * B_{1,3}(2.5) + f(2) * B_{2,3}(2.5) \quad -(5)$$

$\rightarrow$ $B_{0,3}(2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}, t_{i+3}, t_{i+4}] = [0, 1, 2, 3, 4]$

So $t = 2.5 \in [t_{i+2}, t_{i+3}] = [2, 3]$

$$B_{0,3}(2.5) = \left(\frac{2.5-0}{3-0} \times \frac{3-2.5}{3-1} \times \frac{3-2.5}{3-2}\right) + \frac{4-2.5}{4-1} \times \left(\frac{2.5-1}{3-1} \times \frac{3-2.5}{3-2} + \frac{4-2.5}{4-2} \times \frac{2.5-2}{3-2}\right)$$

$$= 23/48$$

$\rightarrow$ $B_{1,3}(2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}, t_{i+3}, t_{i+4}] = [1, 2, 3, 4, 5]$

So $t = 2.5 \in [t_{i+1}, t_{i+2}] = [2, 3]$

$$B_{1,3}(2.5) = \frac{2.5-1}{4-1}\left(\frac{2.5-1}{3-1} \times \frac{3-2.5}{3-2} + \frac{4-2.5}{4-2} \times \frac{2.5-2}{3-2}\right) + \frac{5-2.5}{5-2} \times \frac{2.5-2}{4-2} \times \frac{2.5-2}{3-2}$$

$$= 23/48$$

$\rightarrow B_{2,3}(2.5) \Rightarrow [t_i, t_{i+1}, t_{i+2}, t_{i+3}, t_{i+4}] = [2, 3, 4, 5, 6]$

So $t = 2.5 \in [t_i, t_{i+1}] = [2, 3]$

$$B_{2,3}(2.5) = \frac{2.5-2}{5-2} \times \frac{2.5-2}{4-2} \times \frac{2.5-2}{3-2} = \frac{0.5}{3} \times \frac{0.5}{2} \times \frac{0.5}{1} = 1/48$$

Using all the $B_{i,3}(2.5)$ values calculated and subbing in (5)

$$c(2.5) = 2 * 23/48 + 3 * 23/48 + 4 * 1/48 = \frac{46}{48} + \frac{69}{48} + \frac{4}{48}$$

$$c(2.5) = \frac{119}{48}$$