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Canny Edge Detection

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Q) Perform each step of Canny Edge Detection on the image.

Original Image =

50	50	50	50	50
50	50	50	100	100
50	100	100	100	100
100	100	100	100	100
100	100	100	100	100

Ans) (i) Noise Reduction : (Gaussian Blur)

$$G_{2D}(x,y) = \frac{1}{2\pi\sigma^2} \times e^{\frac{-(x^2+y^2)}{2\sigma^2}} \quad \text{(Gaussian Filter)}$$

→ Taking filter size = 3, $\sigma = 1$ and using [-1, 0, 1] indexing.

$$G_{2D}(x,y) = \frac{1}{2\pi} \times e^{\frac{-(x^2+y^2)}{2}} = \frac{1}{2\pi} \times e^{\begin{bmatrix} -1 & -1/2 & -1 \\ -1/2 & 0 & -1/2 \\ -1 & -1/2 & -1 \end{bmatrix}}$$

$$= \frac{1}{2\pi}\begin{bmatrix} e^{-1} & e^{-1/2} & e^{-1} \\ e^{-1/2} & e^{0} & e^{-1/2} \\ e^{-1} & e^{-1/2} & e^{-1} \end{bmatrix}$$

Convolving $G_{2D}(x,y)$ with the image we get the following

Conv($G_{2D}(x,y)$, Original Image) = Blured Image

$$\frac{1}{2\pi}\begin{bmatrix} e^{-1} & e^{-1/2} & e^{-1} \\ e^{-1/2} & e^{0} & e^{-1/2} \\ e^{-1} & e^{-1/2} & e^{-1} \end{bmatrix} \xrightarrow{\text{Rotate } 180^\circ} \frac{1}{2\pi}\begin{bmatrix} e^{-1} & e^{-1/2} & e^{-1} \\ e^{-1/2} & e^{0} & e^{-1/2} \\ e^{-1} & e^{-1/2} & e^{-1} \end{bmatrix}$$

Using Reflective Padding $\Rightarrow$

50	50	50	50	50	50	50
50	50	50	50	50	50	50
50	50	50	50	100	100	100
50	50	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100
100	100	100	100	100	100	100

$$\text{Blured Image }(0,0) = \frac{1}{2\pi}\begin{bmatrix} e^{-1} & e^{-1/2} & e^{-1} \\ e^{-1/2} & e^{0} & e^{-1/2} \\ e^{-1} & e^{-1/2} & e^{-1} \end{bmatrix} \times \begin{bmatrix} 50 & 50 & 50 \\ 50 & 50 & 50 \\ 50 & 50 & 50 \end{bmatrix}$$

$$\begin{array}{c|ccc} 50 & 50 & 50 & 50 \\ \hline 50 & 50 & 50 & 50 \\ 50 & 50 & 50 & 50 \\ 50 & 50 & 100 & 100 \end{array}$$

$$= \frac{1}{2\pi}\Big(e^{-1} * 50 + e^{-1/2} * 50 + e^{-1} * 50 + e^{-1/2} * 50 + e^{0} * 50 + e^{-1/2} * 50 + e^{-1} * 50 + e^{-1/2} * 50 + e^{-1} * 50\Big)$$

$$= \frac{50}{2\pi} * \left(4/e + 4/e^{1/2} + 1\right) = \frac{25}{\pi}\left(1.47 + 2.426 + 1\right)$$

$$= \frac{25 \times 7}{22}\left(4.896\right) = \lfloor 38.94 \rfloor = \underline{\underline{38}}$$

$\rightarrow$ Similarly for all the indices,

Blured Image =

38	38	41	46	49
41	46	54	62	67
54	62	70	75	77
70	75	77	77	77
77	77	77	77	77

we dont need to blur the image because it is very small and we need it to be exact for furthur calculations to work properly.

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 50 & 50 & 50 \\ 50 & 50 & 50 \\ 50 & 50 & 100 \end{bmatrix}$$

50

(ii) Gradient Calculation : (Apply Sobel)

$$|G| = \sqrt{I_x^2 + I_y^2}$$

$$\theta(x,y) = \arctan\left(\frac{I_y}{I_x}\right)$$

$$K_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad K_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

I_x =

0	0	206	206	0
206	206	206	206	0
206	206	206	206	0
206	206	0	0	0
0	0	0	0	0

Conv (original, K_x)

I_y =

0	0	50	150	150
50	100	150	150	150
150	150	100	50	0
100	50	0	0	0
0	0	0	0	0

Conv(Original, K_y)

$|G|$ =

0	0	212	229.1	150
212	229.1	255	255	150
255	255	229.1	212	0
229.1	212	0	0	0
0	0	0	0	0

$\theta(x,y)$ =

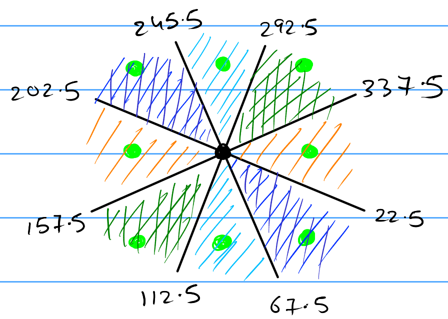
0	0	0.238	0.452	1.57
0.238	0.452	0.629	0.629	1.57
0.629	0.629	0.452	0.238	0
0.452	0.238	0	0	0
0	0	0	0	0

(iii) Non Max Suppression

Converting $\theta(x,y)$ from Radians to Degrees:

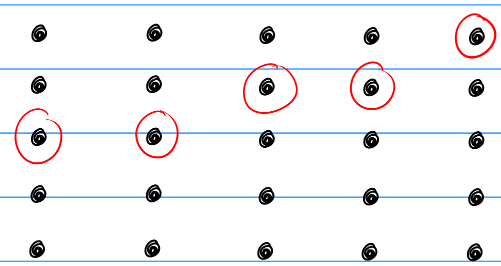
$\theta(x,y)_{deg}$ =

0	0	13.65	25.89	90
13.65	25.89	36.1	36.1	90
36.1	36.1	25.89	13.65	0
25.89	13.65	0	0	0
0	0	0	0	0



We find q and r pixels for every pixel in the image. q and r are the pixel values that are in the direction of θ and $180 - \theta$.

Then we check $|G|_p \geq (|G|_q \,\&\, |G|_r)$ if this holds the the pixel value remains same else it becomes '0'.



○ → denotes the pixels following the inequality mentioned above.

Non Max Suppressed Image =

0	0	0	0	255
0	0	255	255	0
255	255	0	0	0
0	0	0	0	0
0	0	0	0	0

(iv) Double Threshold :

This step is not necessary for our example as the image pixel values are already either 0 or 255.

(v) Edge Tracking by Hysteresis :

As all the pixels are already 255, taking any range X, Y such that

$$\begin{cases} 0 & img[x,y] \leq X \\ \text{weak} & X < img[x,y] \leq Y \\ \text{strong} & img[x,y] > Y \end{cases}$$

will always result in the same image

∴ Final Image =

0	0	0	0	255
0	0	255	255	0
255	255	0	0	0
0	0	0	0	0
0	0	0	0	0