A Unified IRIS Mathematical Foundation

This appendix presents the complete mathematical foundation of **IRIS (Intentional Reflexive Intelligence System)**. It specifies the formal operators, cybernetic indices, graph-theoretic constructs, resonance metrics, regularization mechanisms, classical and distributed recursion models, diagnostic stability functions, and the quantum-inspired density-matrix extension used throughout the paper.

Appendix A is formal and definitional. Interpretation, empirical behavior, and limitations are discussed in Sections 4–6 and Appendix B.

A.1 Core Notation

A.1.1 Agents and State Variables

Let N denote the number of agents, and let $i, j \in \{1, \dots, N\}$.

Each agent i possesses a reflexive cognitive state at discrete time t:

$$x_t^{(i)} \in \mathbb{R}.$$

(Vector-valued extensions $x_t^{(i)} \in \mathbb{R}^d$ are immediate.)

The global system state is

$$\mathbf{x}_t = \left(x_t^{(1)}, \dots, x_t^{(N)}\right)^\top \in \mathbb{R}^N.$$

Each agent maintains a two-step internal memory:

$$\mathbf{X}_t^{(i)} = \begin{pmatrix} x_t^{(i)} \\ x_{t-1}^{(i)} \end{pmatrix}.$$

A.2 Graph-Theoretic Structure

Distributed reflexivity in IRIS is modeled via a directed interaction graph.

Adjacency matrix

$$A_{ij} = \begin{cases} 1, & \text{if agent } j \text{ influences agent } i, \\ 0, & \text{otherwise.} \end{cases}$$

Degree matrix

$$D = \mathrm{diag}(\deg(i)).$$

Graph Laplacian

$$L = D - A.$$

Normalized Laplacian

$$L_{\text{norm}} = I - D^{-1/2} A D^{-1/2}.$$

Let the eigenvalues satisfy

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N.$$

Unless otherwise stated, spectral quantities are computed from the normalized operator consistent with the coupling term.

A.3 Cybernetic Indices

A.3.1 Reflexive Stability Index (RSI)

Raw reflexive deviation is defined as

$$d_t^{(i)} = \|x_t^{(i)} - x_{t-1}^{(i)}\|.$$

The normalized Reflexive Stability Index is

$$\text{RSI}_t^{(i)} = \exp\left(-\lambda d_t^{(i)}\right), \qquad \lambda > 0.$$

RSI is bounded in $(0, 1]$ and functions as a *diagnostic–regulatory signal*, not a formal stability guarantee.

A.3.2 Entropy Drift Bound (EDB)

Unregulated recursion is defined as

$$r_t^{(i)} = a x_t^{(i)} + b x_{t-1}^{(i)}.$$

Regularized recursion is given by

$$\tilde{r}_t^{(i)} = R_\varepsilon\left(r_t^{(i)}\right).$$

The Entropy Drift Bound proxy is

$$\text{EDB}_t^{(i)} = |\tilde{r}_t^{(i)} - r_t^{(i)}|.$$

EDB serves as a computationally tractable proxy for long-horizon uncertainty accumulation.

A.4 Ramanujan-Inspired Regularization

Recursive awareness loops may diverge under repeated self-reference. IRIS stabilizes such dynamics using a Ramanujan-style rational regularizer.

$$R_\varepsilon(s) = \frac{s}{1+\varepsilon|s|}, \qquad \varepsilon > 0.$$

This operator assigns finite values to otherwise divergent recursive terms while preserving local linear behavior.

A.5 Resonance Metrics

Resonance Metrics quantify how interaction topology supports stabilizing reflexive feedback.

A.5.1 Global Resonance Metric

Define the spectral gap

$$\Delta = \lambda_1 - \lambda_2.$$

Normalized expansion quality:

$$E_{\text{spec}} = \frac{\Delta}{\lambda_1 + \eta}, \qquad \eta > 0.$$

The global resonance metric is

$$\mathrm{RM}_{\text{global}} = w_1 E_{\text{spec}} + w_2\, \mathrm{norm}\big(C(A)\big),$$

where $w_1, w_2 \in [0,1]$ are design parameters.

A.5.2 Per-Agent Resonance Metric

$$\mathrm{RM}_i = \alpha \frac{v_i^{(1)}}{\max_j v_j^{(1)}} + (1-\alpha)\left[\beta\, \mathrm{norm}\big(\Delta_i^{(k)}\big) + (1-\beta)\, \mathrm{norm}(\kappa_i)\right].$$

A.6 Unified IRIS Update Equation

The IRIS recursion incorporating RSI, EDB, and RM is

$$\begin{aligned} x_{t+1}^{(i)} = R_\varepsilon\Big(& a x_t^{(i)} + b x_{t-1}^{(i)} \\ &+ (\lambda + \mu\,\mathrm{RM}_i)\mathrm{RSI}_t^{(i)} \\ &- (\sigma - \nu\,\mathrm{RM}_i)\mathrm{EDB}_t^{(i)} \\ &+ \gamma \mathrm{RM}_i \sum_j A_{ij}(x_t^{(j)} - x_t^{(i)})\Big). \end{aligned}$$

A.7 Diagnostic Stability Function

Define the diagnostic Lyapunov-like candidate

$$\begin{aligned} V_{\mathrm{RM}}(t) = \|\mathbf{x}_t - \mathbf{x}_{t-1}\|^2 &+ \beta\|\mathrm{EDB}_t\|_1 \\ &- \alpha\|\mathrm{RSI}_t\|_1 - \delta\,\mathrm{RM}^\top \mathrm{RSI}_t. \end{aligned}$$

A decreasing $V_{\mathrm{RM}}(t)$ indicates empirical stabilization. This function is diagnostic, not a formal Lyapunov proof.

A.8 Quantum-Inspired IRIS Extension

Each agent's classical state is mapped to a density matrix

$$\rho_t^{(i)} = \frac{1}{2}(I + x_t^{(i)}\sigma_x + y_t^{(i)}\sigma_y + z_t^{(i)}\sigma_z).$$

A.8.1 RM-Modulated Quantum Evolution

$$\begin{aligned} \rho_{t+1}^{(i)} = (1-\varepsilon)U(\mathrm{RM}_i)\rho_t^{(i)}U^\dagger(\mathrm{RM}_i) + \varepsilon\frac{I}{2} \\ + (\lambda + \mu\,\mathrm{RM}_i)\mathrm{RSI}_t^{(i)}C_1(\rho_t) - (\sigma - \nu\,\mathrm{RM}_i)\mathrm{EDB}_t^{(i)}C_2(\rho_t). \end{aligned}$$

A.9 Classical–Quantum Correspondence

$$\mathrm{RSI}_t^{(i)} \uparrow \iff P_t^{(i)} \uparrow, \qquad \mathrm{EDB}_t^{(i)} \uparrow \iff S_t^{(i)} \uparrow.$$

Resonance metrics strengthen both trends by stabilizing coupling structure.

B Scope, Diagnostic Interpretation, and Empirical Limits

This appendix documents the experimental protocol used to evaluate **IRIS (Intentional Reflexive Intelligence System)**. Its purpose is methodological transparency and reproducibility, not task optimization or benchmark comparison.

B.1 Experimental Objectives and Scope

Experiments are designed to assess the following intrinsic properties:

- Lyapunov-bounded stability,
- Reflexive coherence (RSI),
- Long-horizon uncertainty accumulation (EDB),
- Structural coordination and alignment (RM),
- Failure modes under controlled ablation, and
- Efficiency under bounded quantum-inspired perturbations.

No experiment evaluates:

- Task performance,
- Semantic correctness, or
- External benchmarks.

B.2 Simulation Environment

All experiments are conducted in a discrete-time simulation environment implementing the equations defined in Appendix A.

B.2.1 Core Properties

The simulation environment supports:

- Two-step recursive memory,
- Modular activation and deactivation of IRIS components,
- Configurable interaction topologies,
- Deterministic execution under fixed random seeds, and
- Metric logging at every iteration.

B.3 Experimental Configurations

Fourteen experimental configurations are evaluated, grouped into:

- Baseline recursive dynamics,
- Spectral and topological structure variations,

- Component ablation studies,
- Scaling experiments, and
- Quantum-inspired extensions.

Each configuration differs along a single explicit axis only, ensuring mechanism-level isolation.

B.4 Parameter Control and Isolation

Across all experiments:

- Only one control variable is varied at a time,
- Recursive coefficients are held fixed,
- Initial conditions are identical across configurations, and
- Coupling strength is fixed unless topology is explicitly varied.

This design enables causal attribution of observed effects to specific IRIS mechanisms.

B.5 Initialization and Time Horizon

Initial states are sampled from bounded, zero-mean distributions. All simulations run for a fixed horizon T sufficient to observe both transient dynamics and steady-state behavior.

B.6 Randomization and Replication

For each bounded configuration:

- 50 independent runs are executed,
- Each run uses a distinct random seed, and
- Two-sided 95% confidence intervals are computed using Student-t statistics.

Divergent regimes do not admit steady-state statistics and are excluded from numerical aggregation by design.

B.7 Metric Logging

The following internal diagnostics are logged at every iteration:

- Diagnostic energy $V(t)$,
- Reflexive Stability Index (RSI),
- Entropy Drift Bound (EDB),
- Resonance Metrics (RM), and
- Quantum purity and entropy (where applicable).

These metrics provide interpretable, mechanism-localized signals for stability, coherence, and sustainability.

B.8 Treatment of Divergent Configurations

The `Unstable-Fail` configuration:

- Diverges within a small number of iterations,
- Exhibits unbounded diagnostic energy, and
- Is reported qualitatively only.

The absence of steady-state statistics in this regime is itself a diagnostic outcome.

B.9 Quantum-Inspired Experiments

Quantum-inspired perturbations are:

- Bounded in magnitude,
- Exploratory rather than stabilizing, and
- Strictly subordinate to classical IRIS control mechanisms.

No claims are made regarding quantum hardware implementation or computational advantage.

B.10 Reproducibility Guarantees

Reproducibility is ensured through:

- Explicit configuration definitions embedded directly in the executable notebook,
- Deterministic execution under fixed random seeds,
- Complete logging of all internal diagnostics and observables, and
- Regeneration of all figures and tables from first principles at runtime.

No external datasets, preprocessing pipelines, or network access are required.

B.11 Limitations and Non-Claims

The IRIS framework does not claim:

- Universal optimality,
- Task transferability, or
- Formal worst-case guarantees.

B.12 Summary

Appendix B establishes that the empirical evaluation of IRIS is:

- Controlled,
- Mechanism-isolating,
- Statistically grounded,
- Fully reproducible, and
- Scope-appropriate.