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Indirect Method to Estimate Distance Measurement Based on Single Visual Cameras

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Abstract— **In this paper, we proposed an indirect method to measure the distance of an object accurately by single visual cameras using triangulation. The object can be seen as the third point of a triangle with two known sides and one known angle. Distance to object can be determined indirectly on the base of known sides and angle, rather than being measured directly. It would be very useful in case there is no line of sight to object (inaccessible) or an obstacle interrupts it. Furthermore, the results show that the measured distance using the indirect method has a less measurement error than the one using the direct method. This method establishes a basis for the implementation of the position algorithm into the navigation subsystem of swarm robots and will be very helpful especially in robot cooperation.**

Keywords: distance measurement; single visual camera; indirect method; uncertainty; measurement error

I. Introduction

Robot vision includes using a combination of camera hardware and computer algorithms to allow robots to process visual data. Finding the location and direction of the objects in the surrounding space, which is relative to the reference frame, is one of the main tasks in robotic vision. Determining the distance between the camera and the objects accurately is essential for localizing, navigating, and performing some high-level task planning.

Nowadays, on robotic systems, we have many algorithms and techniques to measure the distance to objects or targets: image-based distance measurement techniques [1], [2], [10], [11]; photogrammetry, stereo vision, structured light, time of flight, laser triangulation, single camera [4] etc.

In this paper, we apply the image-based techniques using a single visual camera to estimate the distance from a robot to an object directly/indirectly and their uncertainties.

II. Measurement Method

A. Distance measurement using a single visual camera

To determine a distance "directly", we use a single visual camera to capture a photo of the object. This camera is usually mounted on a mobile robot and applied to mapping, localization and obstacle detection. Based on the size of the object in the image, focal length and real size of the object via some geometry transformation, we can estimate accurately the distance from the camera to object. Here we use the term "directly" to distinct from the "indirect" method, which will be discussed later, although this method is actually considered an indirect measurement.

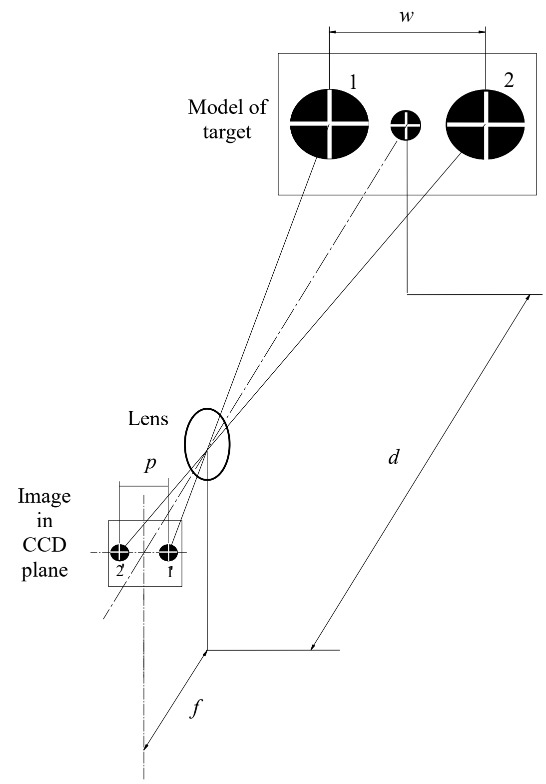

Figure 1. The model of the direct measurement method.

There is a lot of work to do in the preparation phase for the experiment's results to achieve the highest possible accuracy: selection of the area for the experiments with accuracy reference points, a model of the object, a camera with high enough resolution and software to analyze the images etc. The robots are supposed to be on a flat ground, the target is in the field of view and perpendicular to the optical axis of the camera.

The preparation must proceed very carefully and accurately to limit errors which may appear due to establishing the measurement system.

In order to determine the distance from a camera to a known object, we are going to utilize triangle similarity. The triangle similarity goes something like this: As shown in Fig. 1, an object with a known width w [mm] is placed some distance d [mm] from our position. We take a picture of the object by using a camera and then measure the apparent width p [mm] of the object in the image. When we know the focal length f [mm] of our camera, it allows us to derive the distance d [mm] [2]:

$$d = \frac{wf}{p}, \tag{1}$$

Notice that all components in the above equation need to be in the same unit of length, for example, in millimeters or meters.

To get the value of the image's size in SI unit of length, we have to make a transformation. Based on the dimension and the resolution of the camera's CCD sensor, the size of one pixel can easily be found.

For example, our camera used in the experiment has these parameters (as shown in Fig. 2):

Resolution: 3008x2000 [pixels] ~ $X.Y$ [pixels]

CCD sensor's size: 23.7x15.6 [mm] ~ $A.B$ [mm]

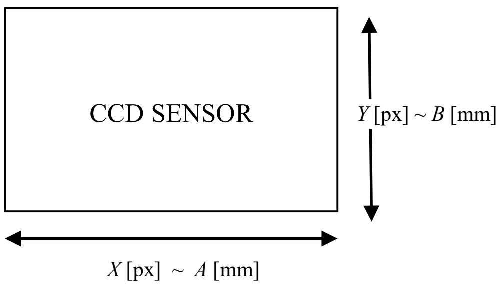

Figure 2. CCD sensor's size.

Then, an image taken by this camera in the best resolution 3008x2000 [pixel] has its size 23.7x15.6 [mm]. It means that one pixel in the horizontal plane has a size of 23.7/3008 [mm] and in the vertical plane has a size of 15.6/2000 [mm].

After obtaining the object's image, it can be analyzed by using a software to calculate the number of pixels which characterize the object's size. The principle of measuring the object's size in pixels is that, assuming that we have the coordinates of two points on the image plane (in pixel): Point 1 (X_1, Y_1) and point 2 (X_2, Y_2), thus the distance from point 1 to point 2 will be calculated by this equation:

$$p_{[pixel]} = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}\ , \tag{2}$$

$$p_{[mm]} = p_{[pixel]} \frac{A}{X}\ , \text{(horizontal)}$$

$$p_{[mm]} = p_{[pixel]} \frac{B}{Y}\ , \text{(vertical)}$$

With a specific model of the object, the camera and its settings, we have already these information: the object's width w [mm]; the focal length f [mm] and as presented above, by using the software, we can determine the object's size $p_{[px]}$ on the image plane. Therefore, we can calculate the distance d [mm] from the camera to the object:

$$d = \frac{wf}{p_{[px]} \dfrac{A_{mm}}{X_{px}}}\ , \tag{3}$$

B. Mathematical model of the indirect method

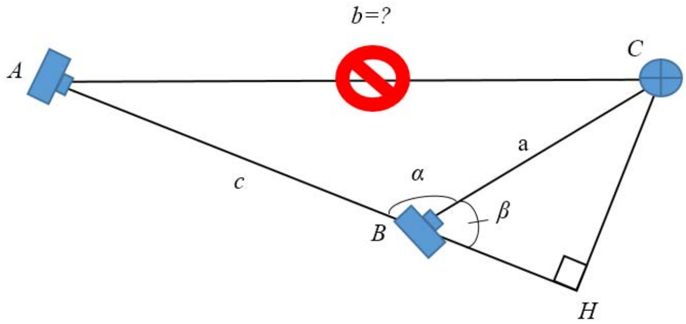

Figure 3. The model of the indirect measurement method.

In Fig. 3, we have the illustration of the model of the method. Assuming that we need to determine distance b from the robot located at the point A to an object located at the point C. Because of the existence of an obstacle between point A and point C, there is no line of sight from the robot to the object, so that obviously we cannot use above approach to determine the robot-object distance. In this case, we can determine this distance from the information provided by another robot located at the point B.

The distance measurement is based on the trigonometric proposition that if two sides and one angle of a triangle are known, the remaining side can be computed. In this case, we have known the distance c from the robot located at the point A to the second robot located at the point B, the distance a from the second robot located at the point B to the object located at the point C and the angle $\widehat{ABC} = \alpha$ between them. Thus to calculate the distance b from the robot located at the point A to the object located at the point C, the following calculations must be executed:

Consider right triangle ΔBHC:

$$BH \ = \ BC.\cos\beta = -a\ \cos\alpha, \tag{4}$$

where β is the angle $\widehat{CBH}$, the supplementary angle to the angle α.

Applying Pythagorean theorem to determine the last side's length of ΔBHC:

$$CH^2 \ = \ BC^2 - BH^2 \ = \ a^2 - \left(a\ \cos\alpha\right)^2 = a^2 \sin^2\alpha, \tag{5}$$

The length of AH is a sum of two lengths of AB and BH, then we have:

$$AH = AB + BH = c - a \cos\alpha,$$
$$AH^2 = c^2 + a^2 \cos^2\alpha - 2ac \cos\alpha, \quad (6)$$

Consider right triangle ΔAHC, the distance b can be expressed as follows:

$$AC^2 = AH^2 + CH^2,$$
$$b^2 = c^2 + a^2 - 2ac\cos\alpha,$$
$$b = \sqrt{a^2 + c^2 - 2ac\cos\alpha}, \quad (7)$$

Therefore, by using this method, the distance b from the robot located at the point A to the object located at the point C is determined indirectly by the help of the second robot located at the point B.

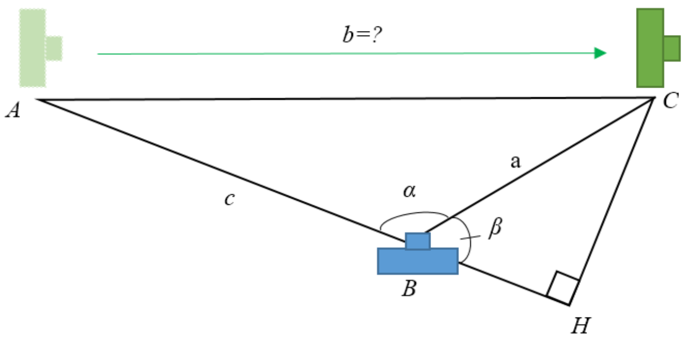

Figure 4. The model of the indirect measurement method.

This indirect method can also be applied to the case, in which the distance that a robot transfers from point A to point C can be determined by using only one camera placed at the point B (Fig. 4). Firstly, the camera at the point B measures the distance from the point B to the robot location at the point A at the time. After the interval of time, the robot arrives to point C. Then the camera measures the distance from point B to point C. Using the obtained data and knowing the angle created by the two sides BA and BC, we can determine the distance AC which the robot traveled.

III. MEASUREMENT ERROR AND UNCERTAINTY

Error analysis is the study and evaluation of uncertainty in measurement. An error in a scientific measurement is not the usual connotations of the terms “mistake” or “blunder” but rather the inevitable occurrence of uncertainty that attends all measurements. Because of not being a mistake, it cannot be eliminated by measuring very carefully [10].

It has shown that all measurements, which however are carried out carefully and scientifically, are inaccurate operations and cannot be completely free of uncertainties. The word “uncertainty” means doubt about the validity of the result of a measurement [5]. It means that the value obtained as the measurement’s result is only an approximation or estimate of the physical object quantity’s value. This result differs from its true value and it is only complete when accompanied by a statement of the uncertainty of that estimate. Measurement accuracy is defined as the closeness of agreement between a measured quantity value and a true quantity value [9].

The correct way to express the result of a measurement is to give the best estimate of the quantity value and the interval which the true value of the quantity lies within. Due to formal reasons, the numerical values of uncertainties are always positive quantities, which we afterward have to provide with a sign “±”. The interval “estimator ± measurement uncertainty” defines the result of a measurement. This range is required to localize the true value of the measuring (or quantity to be measured) [6].

In general, the result of a measurement of a quantity x is usually stated as follows [10]:

$$measured\ value\ of\ x = x_{best\ estimate} \pm \Delta x \quad (8)$$

where Δx is called the uncertainty in the measurement of x.

The true value of x is somewhere between $x_{best\ estimate}$ - Δx and $x_{best\ estimate}$ $+\Delta x$.

On the whole, measurement uncertainties are composed of two parts, one is due to random errors and the other is due to unknown systematic errors. Random errors make results of the repeated measures being scattered over a range. The best estimate of the measured quantity is the mean of the distributed data; the error is associated with the distribution of values around this mean. Systematic errors cause the measured quantity to be shifted away from the accepted, or predicted value. Measurements where this shift is small (relative to the error) are described as accurate [6], [7].

Standard uncertainty or combined standard uncertainty components can be evaluated by two methods [5], [9]:

- By the statistical analysis of a series of observations.

- Based on assumptions about the possible variation of given uncertainty components (a type of distribution, the variation range), allowing to estimate standard deviation.

Uncertainty components are divided into two categories, depending on their calculation method:

(A) Uncertainties calculated by statistical methods.

(B) Uncertainties estimated by other methods.

The standard uncertainty type A, expressed as a standard deviation, is calculated from the probability density function obtained from the observed frequency distribution. The standard uncertainty type B is calculated on the basis of the assumed probability density function, based on the confidence degree of probability of the given event appearance.

The formula commonly used for uncertainty evaluation of measurement results is based on the following equation [3], [5], [8]:

$$u^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i), \quad (9)$$

This equation is called as “the law of uncertainty propagation” and determines relations between variations of quantities described in probabilistic categories. Quantities x_i, i = 1, 2, …, N, are measured directly and then, at the base of its estimates, the value of the quantity y is calculated by assuming that the relation between these quantities is known as the function which generally can be written as $y = f(x_1, x_2, ..., x_N)$.

Since the measured quantities have errors, it is inevitable that the quantities computed from them will have errors. Variances $u^2(x_i)$, $i = 1, 2, \ldots, N$, in (9) are defined as the squares of suitable so-called "standard uncertainties" which are in fact standard deviations (or their estimates) of quantities measured directly. Variation $u^2(y)$_ is treated as a square of standard uncertainty $u(y)$ of the quantity measured indirectly. In above relationship, it is assumed that quantities x_i, $i = 1, 2, \ldots, N$ are independent. An independent variable having a certain value does not depend on the values of any other variable's parameter.

For the direct measurement method, we have the difference between real and measured distances as follows:

$$\Delta d = d_{real} - d_{measured}, \quad (10)$$

And the error percentage is calculated by:

$$e = \left| \frac{d_{real} - d_{measured}}{d_{real}} \right| .100\%, \quad (11)$$

For the indirect method, we calculate the distance and its measurement error based on other known information.

Consider triangle ABC, we have its sides, angles, and their measurement errors as following:

$$\begin{aligned} BC &= a \pm \Delta a, \qquad AC = b \pm \Delta b, \\ AB &= c \pm \Delta c, \qquad \widehat{ABC} = \alpha \pm \Delta \alpha, \end{aligned} \quad (12)$$

Based on a mathematical model of the indirect method presented above, the distance b is a function of the components a, c and α:

$$b = f(a,c,\alpha) = \sqrt{a^2 + c^2 - 2ac\cos\alpha}, \quad (13)$$

For multi-variable functions, the total error obtained by adding the components from each variable in quadrature (provided the variables are independent). The uncertainties in a, c and α are uncorrelated; or in other words, a, c and α are independent variables. From (9) we have:

$$\Delta b^2 = \left(\frac{\partial f}{\partial a}\right)^2 \Delta a^2 + \left(\frac{\partial f}{\partial c}\right)^2 \Delta c^2 + \left(\frac{\partial f}{\partial \alpha}\right)^2 \Delta \alpha^2, \quad (14)$$

Therefore, we can calculate the uncertainty of the measurement of distance b as the following equation:

$$\Delta b = \sqrt{\frac{(a - c\cos\alpha)^2 \Delta a^2 + (c - a\cos\alpha)^2 \Delta c^2 + (ac\sin\alpha)^2 \Delta \alpha^2}{a^2 + c^2 - 2ac\cos\alpha}}, \quad (15)$$

IV. EXAMPLES

Based on the results of the practical experiments, which were implemented by the research team of the Department of Air Defense Systems measuring the distance to an object in the range from 1 to 6 meters using a single camera [1], we have the Table I of practical data:

TABLE I. RESULTS USING THE DIRECT METHOD

Real Distance	p	p(1)	Measured distance	Difference	Error
[mm]	[px]	[mm]	[mm]	[mm]	%
1002	2274.179	17.918	976.659	-25.341	-2.529
1250	1797.111	14.159	1235.926	-14.074	-1.126
1503	1492.586	11.760	1488.087	-14.913	-0.992
1750	1267.067	9.983	1752.944	2.944	0.168
2003	1108.029	8.730	2004.548	1.548	0.077
2250	979.074	7.714	2268.570	18.570	0.825
2505	882.111	6.950	2517.933	12.933	0.516
2750	796.141	6.273	2789.828	39.828	1.448
3004	732.549	5.772	3032.010	28.010	0.932
3250	672.036	5.295	3305.025	55.025	1.693
3503	626.020	4.932	3547.965	44.965	1.284
3750	581.022	4.578	3822.745	72.745	1.940
4002	547.033	4.310	4060.262	58.262	1.456
4250	512.024	4.034	4337.873	87.873	2.068
4503	486.002	3.829	4570.137	67.137	1.491
4750	458.017	3.609	4849.372	99.372	2.092
5003	436.014	3.435	5094.095	91.095	1.821
5250	414.011	3.262	5364.828	114.828	2.187
5504	396.020	3.120	5608.545	104.545	1.899
5750	377.021	2.971	5891.173	141.173	2.455
6004	363.012	2.860	6118.516	114.516	1.907

All the measurements were implemented with a fixed focal length of 35 [mm] and other settings of the camera.

As it is illustrated in Fig. 5, the object's size, which is obtained on the image plane and transformed from [pixel] into [mm] by using software, is inversely proportional to the distance. By using this direct method, the difference between real and measured distances is from 1.548 to 141.173 [mm] (Table I) and the measurement error [%] is from 0.077 to 2.529 % (Fig. 6).

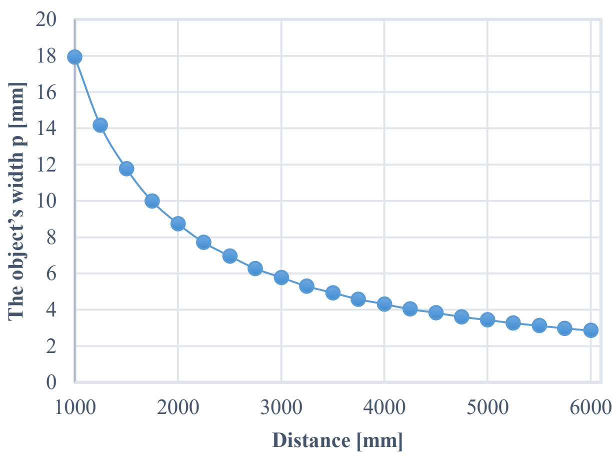

Figure 5. The object's width [mm] in the image.

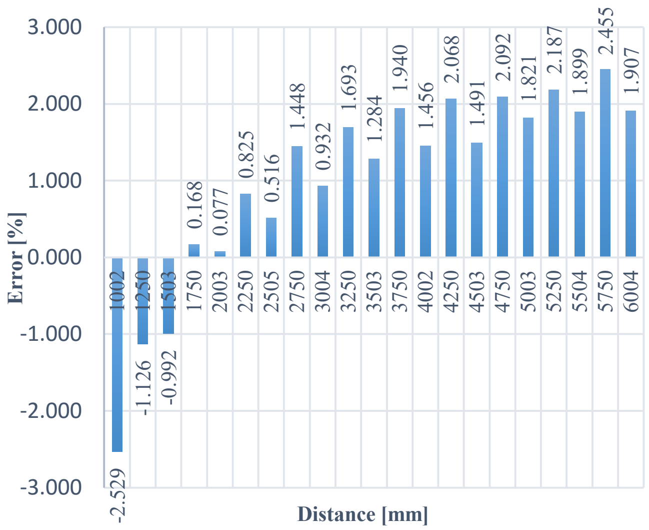

Figure 6. Measurement error [%] using direct method.

For the indirect method, consider a special case when $\alpha = 90^0$, with a little help from Pythagorean theory, then $\cos\alpha = 0$, so that we can eliminate component α in (7). Thus, we have:

$$b = \sqrt{a^2 + c^2}, \tag{16}$$

In addition, distance uncertainty can be calculated by following equations:

$$\Delta b = \sqrt{\frac{a^2 \Delta a^2 + c^2 \Delta c^2}{a^2 + c^2}}, \tag{17}$$

We use the practically obtained data above to continue the analysis and get the table of new data. High precision is not possible for short distances (smaller than 2000 mm) because of the distortion of images [1], so that to compare two methods, we eliminate all the cases in which the distance measurement was smaller than 2000 mm.

TABLE II. RESULTS USING THE INDIRECT METHOD

a [mm]	Δa [mm]	c [mm]	Δc [mm]	Indirectly measured b [mm]	Indirectly measured Δb [mm]
2003	1.548	2250	18.570	3012.393	13.908
2003	1.548	2505	12.933	3207.341	10.147
2250	18.570	2750	39.828	3553.168	32.992
2250	18.570	3004	28.010	3753.201	25.031
2003	1.548	3503	44.965	4035.222	39.042
2003	1.548	3750	72.745	4251.413	64.170
2505	12.933	3750	72.745	4509.715	60.915
3250	55.025	3503	44.965	4778.442	49.871
3004	28.010	4002	58.262	5004.000	49.537
2250	18.570	4750	99.372	5255.949	90.157
3503	44.965	4250	87.873	5507.586	73.593
3250	55.025	4750	99.372	5755.432	87.701
4250	87.873	4250	87.873	6010.408	87.873

In both methods, the measurement uncertainty tends to increase inversely proportional to the distance.

Comparing the data in Table I and Table II, we can see that the distance uncertainties using the indirect method are smaller than the one using the direct method. It ranges from 10.147 to 90.157 [mm] using the indirect method and from 28.010 to 141.173 [mm] using the direct method, as illustrated in Fig. 7.

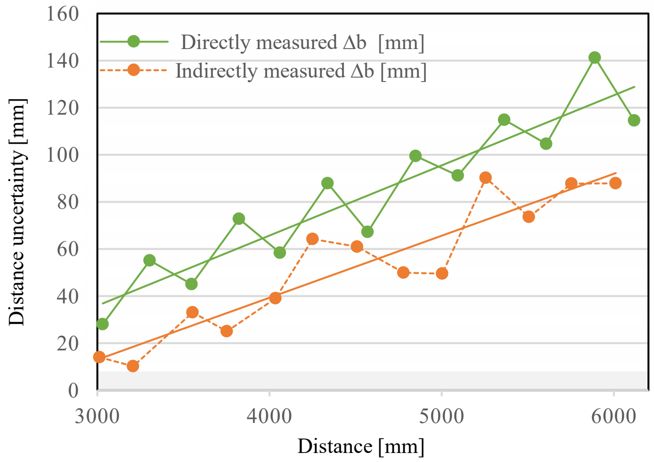

Figure 7. Comparison of distance uncertainties of two methods.

Now we consider when we use more robots to take part in the progress of distance determination. For example, we will calculate the distance from the point A_1 to the point A_N from information provided by other robots located at the points A_2, A_3,…, A_{N-1} (Fig. 8). We repeat the steps as above to calculate the distance from point A_1 to point A_2, and then the distance from point A_2 to point A_3 and so on. The distance A_1A_N and its measurement error can be calculated by means of the distances A_1A_2, A_2A_3,…, $A_{N-1}A_N$ and their measurement uncertainties.

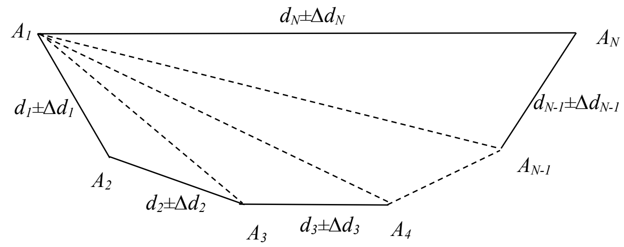

Figure 8. The model of multiple indirect measurements.

Here are two examples:

TABLE III. EXAMPLE 1 OF MULTIPLE INDIRECT MEASUREMENTS

a [mm]	Δa [mm]	c [mm]	Δc [mm]	Mul.indirectly measured b [mm]	Mul.indirectly measured Δb [mm]
2250.000	18.570	3004.000	28.010	3753.201	25.031
2003.000	1.548	3753.201	25.031	4254.236	22.095
3503.000	44.965	4254.236	22.095	5510.856	33.285
2505.000	12.933	5510.856	33.285	6053.475	30.770

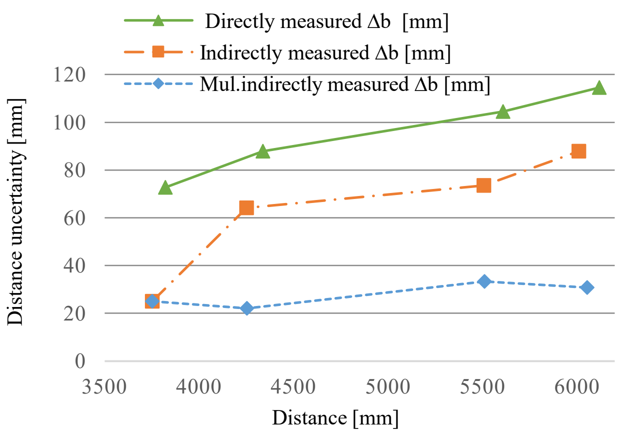

Figure 9. Comparison of distance uncertainties in example 1.

The results show us that accuracy is improved significantly. As it is illustrated in the Table III and Fig. 9, in example 1 at the distance about of 6000 [mm], the measurement uncertainty decreases from 114.516 [mm] using the direct method to 87.873 [mm] using the indirect method once. When we apply the indirect method continuously 4 times to calculate the distance, the measurement uncertainty decreases to only 30.770 [mm].

In the second example, as shown in Table IV and Fig. 10, at the distance about of 5250 [mm], the measurement uncertainty decreases from 114.828 [mm] to 90.157 [mm] and to 24.321 [mm] correspondingly.

TABLE IV. EXAMPLE 2 OF MULTIPLE INDIRECT MEASUREMENTS

a [mm]	Δa [mm]	c [mm]	Δc [mm]	Mul.indirectly measured b [mm]	Mul.indirectly measured Δb [mm]
2003.000	1.548	2250	18.570	3012.393	13.908
2250.000	18.570	3012.393	13.908	3759.921	15.737
2505.000	12.933	3759.921	15.737	4517.968	14.932
2750.000	39.828	4517.968	14.932	5289.096	24.321

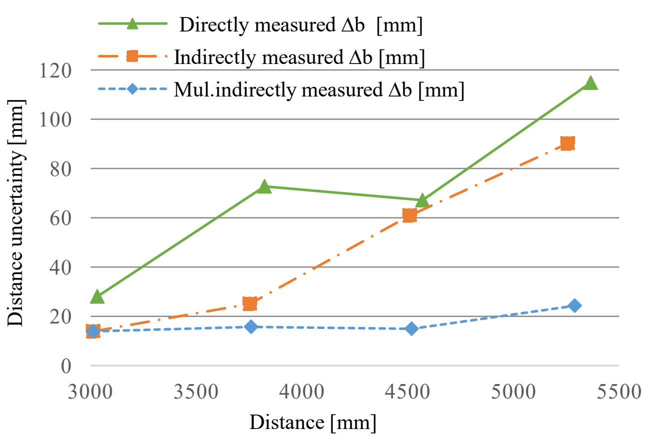

Figure 10. Comparison of distance uncertainties in example 2.

V. CONCLUSIONS

In this paper, we have proposed an indirect method to measure object distance using a single camera. The results obtained are relatively accurate in the range up to 6m. Although there are some problems which limit the measurement results, it can be applied in cooperative swarm robot to improve the measurement accuracy. However, it needs to be carried out with some changes of more various parameters and a number of problems need to be solved in order to evaluate more the validity of the method. Our future work is to evaluate the influence of different local lengths, camera's resolution and also the rotation of the object model for the distance measurement.

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REFERENCES

[1] A. de La Bourdonnaye, R. Doskocil, V. Krivanek and A. Stefek, "Practical Experience with Distance Measurement Based on Single Visual Camera," Advances in Military Technology Vol. 7, No. 2, December 2012.

[2] Y. Bergeon, "Calculation of The Distance Covered by a Robot Thanks to Image Analysis With a Two-robot Team," Proceedings of ICMT'11, International Conference on Military Technologies, Brno, University of Defence, 2011, p. 849-854.

[3] A. M. Chandra, Surveying Problem Solution With Theory And Objective Type Questions, New Age International, 2005.

[4] Alizadeh Peyman, "Object distance measurement using a single camera for robotic applications," Diss. Laurentian University of Sudbury, 2015.

[5] BIPM, IEC, et al., Evaluation of measurement data - guide to the expression of uncertainty in measurement, JCGM 100: 2008.

[6] Michael Grabe, Measurement uncertainties in science and technology, Springer, 2014.

[7] Ifan G.Hughes and Thomas Hase, Measurements and their uncertainties: A practical guide to modern error analysis, Oxford University Press, 2010.

[8] J. Jakubiec, "A new conception of measurement uncertainty calculation," Acta Physica Polonica A 124.3, 2013.

[9] Jerzy A. Sładek, Coordinate metrology - Accuracy of Systems and Measurements, Springer, 2015.

[10] John R. Taylor, An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, University Science Books, 1998.

[11] Neha Shukla and Anurag Trivedi, "Image Based Distance Measurement Technique for Robot Vision using New LBPA approach," International Journal of Novel Research in Electrical and Mechanical Engineering Vol. 2, Issue 1, 2015.