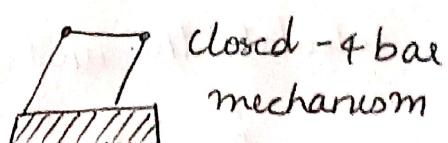


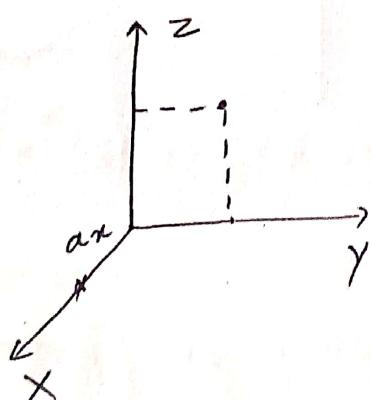
## Kinematics

### Kinematic chain

- It is a series of links connected by joints.
- When each and every link in a kinematic chain is coupled to almost 2 other links then the chain is referred to as simple kinematic chain.
- Simple kinematic chain can be closed or open.



How you represent a point in a space.



$$\vec{P} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Representation of a vector in space.

$$\vec{P} =$$

represent

$$\vec{P} = 3\hat{i} + 5\hat{j} + 2\hat{k}$$

represent vector in matrix form

$$P = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

represent the vector in matrix form with scaling factor of 2

$$P = \begin{bmatrix} 6 \\ 10 \\ 4 \end{bmatrix} \rightarrow \text{scaling factor}$$

calculate the unit vector. Represent the vector in unit matrix form.

$$\hat{P} = \frac{P}{|P|} = \begin{bmatrix} 3/\sqrt{38} \\ 5/\sqrt{38} \\ 2/\sqrt{38} \end{bmatrix}$$

$$\sqrt{36+100+16} \\ = \sqrt{152}.$$

$$= \sqrt{3^2+5^2+2^2} \\ = \sqrt{9+25+4} \\ = \sqrt{38}.$$

- A vector  $v$  is 5 units long and is in the direction of a unit vector  $q$ .  $q = \begin{bmatrix} 0.371 \\ 0.557 \\ q_z \\ 0 \end{bmatrix}$ . Express the vector in matrix form.

$$\sqrt{x^2+y^2+z^2} = 1$$

$$\sqrt{(0.371)^2 + (0.557)^2 + q_z^2} = 1$$

$$(0.391)^2 + (0.557)^2 + v_z^2 = 1.$$

$$v_z = 1 - (0.391)^2 - (0.557)^2 \\ = 0.55211$$

$$v_z = \underline{\underline{0.743}}$$

8) Write the vector in matrix form that describes direction of cross product of  $p = 5\hat{i} + 3\hat{k}$  &  $q = 3\hat{i} + 4\hat{j} + 5\hat{k}$

A vector  $p$  is 8 units long and is  $\perp$  to vector  $q$  and  $r$  that are described below.

Express the vector in matrix form

$$\vec{q} = \begin{bmatrix} 0.391 \\ 0.4 \\ 0 \end{bmatrix} \quad \vec{r} = \begin{bmatrix} r_x \\ 0.5 \\ 0.4 \\ 0 \end{bmatrix}$$

$$P \times q = \begin{vmatrix} i & j & k \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ 5 & 0 & 3 \\ 3 & 4 & 5 \end{vmatrix} = i(-12) - j(25 - 9) + k(20) \\ = -12i - 16j + 20k.$$

$$p \times q = \frac{-12i - 16j + 20k}{\sqrt{(-12)^2 + (-16)^2 + 20^2}} = \frac{-3i}{5\sqrt{2}} - \frac{4j}{5\sqrt{2}} + \frac{5k}{4\sqrt{2}}$$

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\vec{q} = \begin{bmatrix} 0.3 \\ q_y \\ q_z \\ 0 \end{bmatrix} \quad \vec{r} = \begin{bmatrix} r_x \\ 0.5 \\ 0.4 \\ 0 \end{bmatrix}$$

$$\vec{q} \times \vec{r} = \begin{vmatrix} i & j & k \\ 0.3 & q_y & 0.4 \\ r_x & 0.5 & 0.4 \end{vmatrix} = i(0.4q_y - 0.2) - j(0.12 - 0.4r_x) + k(0.15 - r_x q_y)$$

$$= \begin{bmatrix} 0.4q_y - 0.2 \\ 0.4r_x - 0.12 \\ 0.15 - r_x q_y \end{bmatrix}$$

$$= (0.4q_y - 0.2)^2 + (0.4r_x - 0.12)^2 + (0.15 - r_x q_y)^2$$

Representation of a frame at the origin of a fixed reference frame

A frame is generally represented by 3 mutually orthogonal axis  $x, y, z$ . These axes  $x, y$  &  $z$  is used to represent the fixed universal reference frame  $F(x, y, z)$ .

Suppose there is a set of axes  $n, o$  and  $a$  which represents another frame (moving)  $F(n, o, a)$  relative to the reference frame.

Each suppose each direction of each axis of the frame

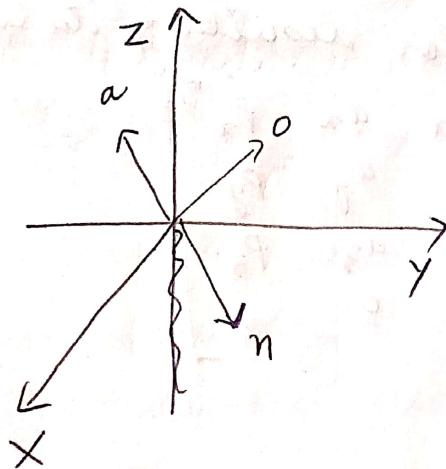
$F(n, o, a)$  is located at the origin of the reference frame

$F(x, y, z)$  is represented by 3 directional values

$$F(n, o, a) = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$

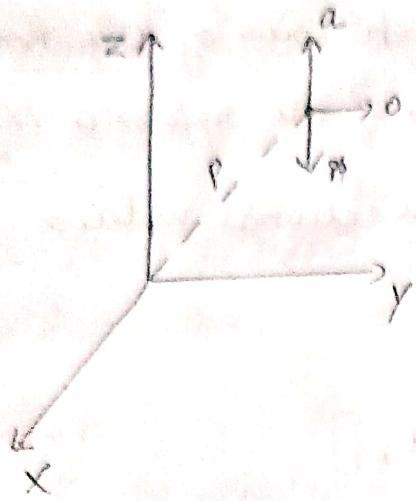
$n$  is a unit vector

$a$  is the unit vector for



### Representation of a frame relative to fixed reference frame

In order to describe a frame relative to another frame both the location of its origin & the direction of its axis must be specified. If the frame is not at the origin of the reference frame, its location relative to the reference frame is described by the vector b/w the origin of the frame & the origin of the reference frame.

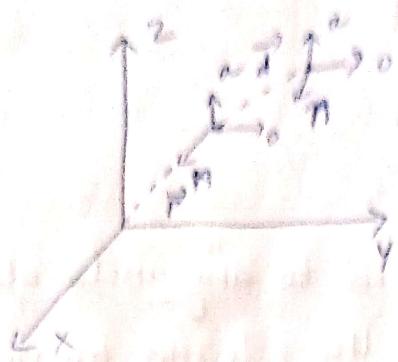


The vector is expressed by its components relative to the reference frame. Thus this frame can be expressed by 3 vectors describing its directional unit vector & the fourth vector describing its location.

as  $\begin{pmatrix} r \\ p \\ a \end{pmatrix} = \begin{bmatrix} r_x & a_x & a_x & P_x \\ r_y & a_y & a_y & P_y \\ r_z & a_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Representation of a rigid body

## Translation



only position of the frame changes.

If a frame moves in space without any change in its orientation then the transformation is pure translation. The directional unit vector remain in the same direction and do not change. The only thing that changes is the location of the origin of the frame relative to the reference frame. The new location of the frame relative to the reference frame can be found by adding the vector representing the translation to the vector representing the original location of the origin of the frame.

matrix representation of the frame

$$\begin{bmatrix} r_x & \alpha_x & a_x & p_x \\ r_y & \alpha_y & a_y & p_y \\ r_z & \alpha_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now the transformation  $T = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The new frame representation can be found by pre multiplying final position of the frame with the matrix with the representing the transformation

$$F_{\text{new}} = \begin{bmatrix} n_x & a_x & a_z & t_x \\ n_y & a_y & a_z & t_y \\ n_z & a_y & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- a) A frame F has been moved to  $10$  units along Y-axis &  $5$  units along Z-axis of the reference frame. Find the new location of the frame.

$$F = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.436 & 3 \\ -0.766 & 0 & 0.643 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$dx = 0$$

$$dy = 10$$

$$dz = 5 \quad F_{\text{new}} = \text{Trans}(dx, dy, dz) \times F \text{old}$$

$$F_{\text{new}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.436 & 3 \\ -0.766 & 0 & 0.643 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

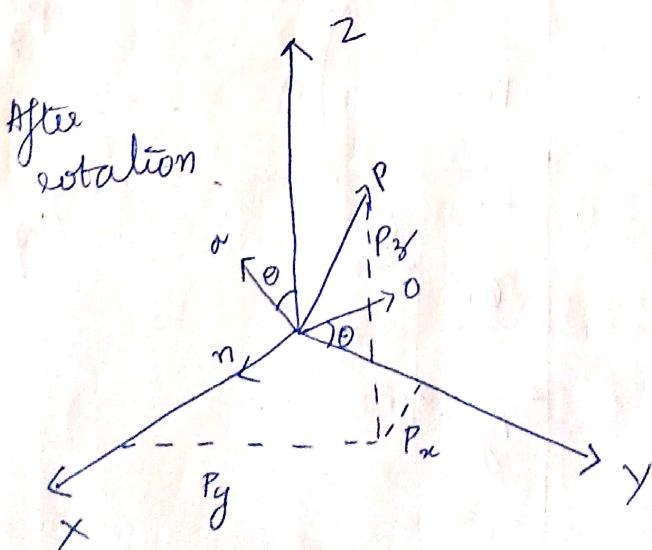
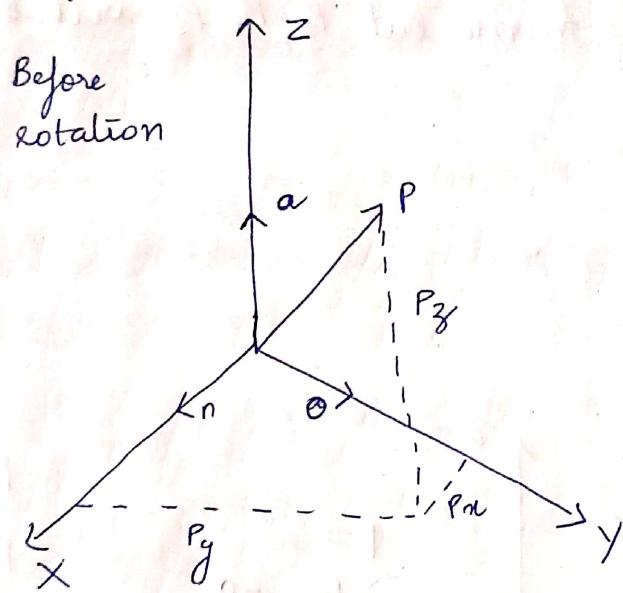
$$= \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.436 & 13 \\ -0.766 & 0 & 0.643 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- b) The translation matrix  $\begin{pmatrix} 2 & 3 & 4 & 1 \end{pmatrix}$  find the new location of the frame

$$F = \begin{bmatrix} 0.707 & 0.284 & 0 & 2 \\ 0.341 & 0.523 & 1 & 4 \\ 0.586 & -0.707 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
 t_{\text{new}} &= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.709 & 0.284 & 0 & 4 \\ 0.341 & 0.583 & 1 & 7 \\ 0.586 & 0.707 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &\approx \begin{bmatrix} 0.709 & 0.284 & 0 & 4 \\ 0.341 & 0.583 & 1 & 7 \\ 0.586 & 0.707 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

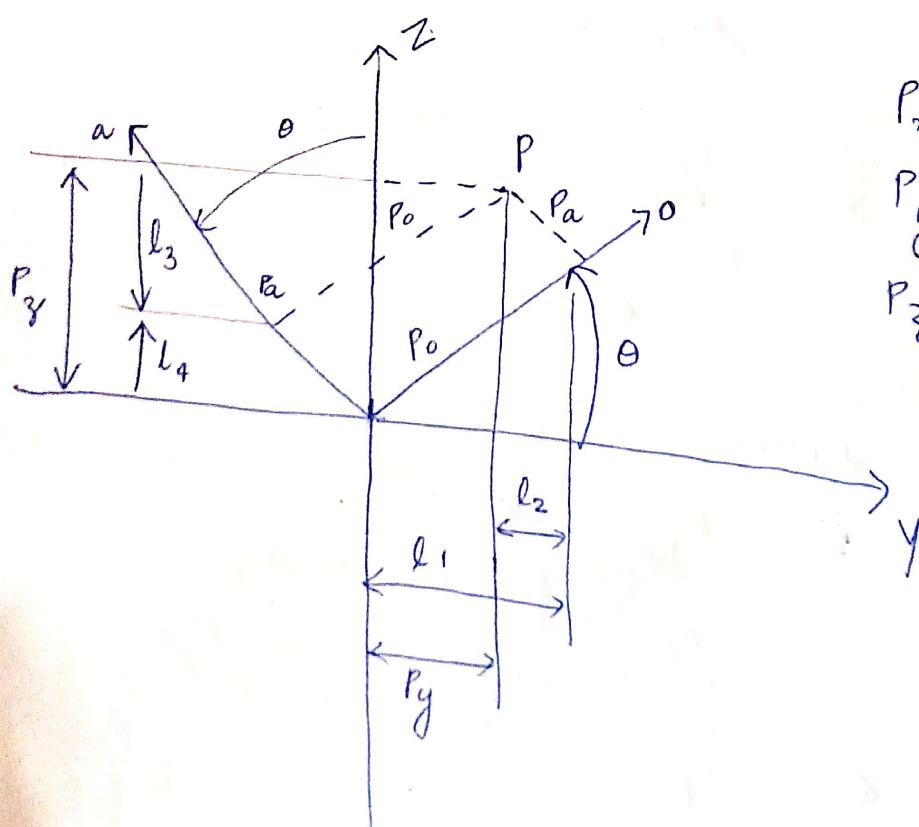
Representation of a pure rotation about an axis



Assume that the frame  $F_{n,o,a}$  is located at the origin of the reference frame  $F_{x,y,z}$ . It rotates at an angle  $\theta$  about  $x$ -axis of the reference frame. Assume that there is a point  $P$  is attached to the rotating frame  $F_{n,o,a}$  with coordinates  $P_x, P_y, P_z$  relating to the reference frame &  $P_n, P_o, P_a$  relative to the moving frame.

As the frame rotates about the  $x$ -axis the point  $P$  attached to the frame will also rotate with it. Before rotation the coordinates of the point in both frames are same.

After rotation  $P_n, P_o, P_a$  coordinates of the points remain the same in the rotating frame but  $P_x, P_y, P_z$  will be different in  $F_{x,y,z}$  frame.



$$P_x = P_n$$

$$P_y = l_1 - l_2$$

$$P_z = l_3 + l_4$$

The value of  $P_x$  do not change as the frame rotates about x-axis but the values of  $P_y$  &  $P_z$  do change.

$$P_x = P_{x0}$$

$$P_y = l_1 - l_2$$

$$P_z = l_3 + l_4$$



$$\cos\theta = \frac{P_y + l_2}{P_0}$$

$$\cos\theta = \frac{l_1}{P_0}$$

$$P_y = P_0 \cos\theta - P_a \sin\theta$$

$$P_z = l_3 + l_4$$

$$= P_0 \sin\theta + P_a \cos\theta$$

$$\begin{bmatrix} P_{x0} \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} P_A \\ P_0 \\ P_a \end{bmatrix}$$

$$(l_1 - \cos\theta) \cdot$$

$$P_{xyz} = \text{Rot}(z, \theta) \times P_{noa}$$

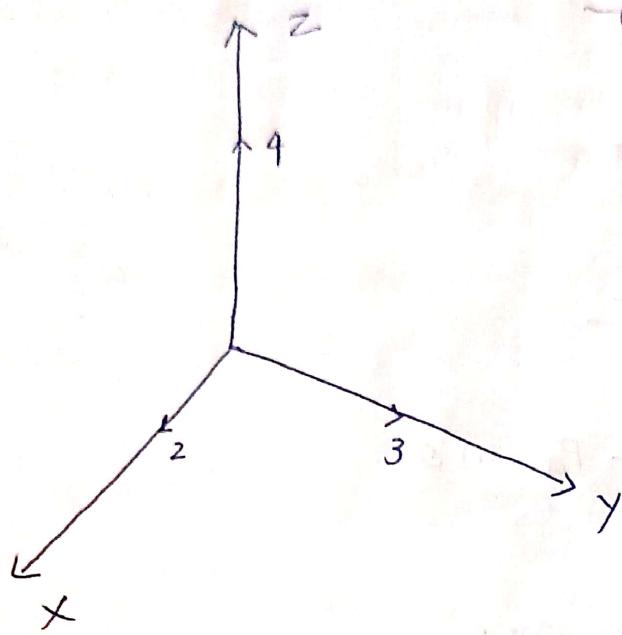
- Q) A point P(2, 3, 4) is attached to a rotating frame, the frame rotates  $90^\circ$  about the x-axis of the reference frame. Find the coordinates of the ~~frame~~<sup>point</sup> relative to the ref frame after rotation & verify the result graphically.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} P_n \\ P_0 \\ P_a \end{bmatrix}$$

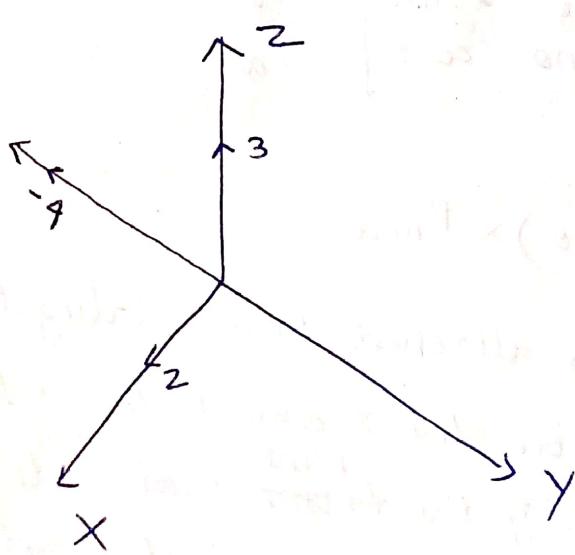
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3\cos\theta - 4\sin\theta \\ 3\sin\theta + 4\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$$

before rotation.



After rotation



Rotational matrix of rotation about y-axis:

$$P_{xyz} = \text{Rot}(y, \theta) \cdot P_{\text{noa}}$$

$$\text{Rot}(y, \theta) = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

About z axis:

$$(x, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

8) Find the coordinates of Point P(3, 5, 7) relative to the ref frame after a rotation of  $30^\circ$  about z axis.

$$P_{xyz} = \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3\cos 30 - 5\sin 30 \\ 3\sin 30 + 5\cos 30 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} + 3 \cdot \frac{\sqrt{3}}{2} - 5 \cdot \frac{1}{2} \\ \frac{3}{2} + 5 \cdot \frac{\sqrt{3}}{2} \\ 7 \end{bmatrix} \begin{matrix} \sqrt{3} \\ 2 \\ \frac{1}{2} \end{matrix}$$

$$= \begin{bmatrix} 0.098 \\ 5.83 \\ 7 \end{bmatrix}$$

9) A point P(7, 3, 1) is attached to a frame F<sub>noa</sub> and subjected to the following transformation. Find the coordinates of the point relative to the ref. frame

i) Rotation of  $90^\circ$  about z-axis

ii) followed by rotation of  $90^\circ$  about Y-axis

iii) followed by translation of (4, -3, 7)

$$P(x, y, z) = \text{Trans}(4, -3, 7) \times \text{Rot}(y, 90^\circ) \times \text{Rot}(z, 90^\circ)$$

$$\times P(7, 3, 1)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 90 & 0 & \sin 90 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90 & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ \sin 90 & \cos 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 4 \\ 10 \\ 1 \end{bmatrix}$$

Ref frame - reverse order  
 $n, o, g \rightarrow$  occurring order

- a) A frame B was rotated about the X axis by  $90^\circ$ . It was then translated about the current axis 3 inches in Z axis before being rotated about the Z axis by  $90^\circ$ . Finally it was translated about the current O axis by 5 inches. Write the eqns describing the motion. Find the location of a point P(1, 5, 4) attached to the frame relative to the reference frame

$$P(x, y, z) = \text{Trans}(0, 0, 3) \times \text{Rot}(x, 90^\circ) \times \text{Rot}(x, 90^\circ) \times$$

$$= \text{Rot}(Z, 90^\circ) \times \text{Rot}(X, 90^\circ) \times \text{Trans}(0, 0, 3) \times \text{Trans}(0, 5, 0) \times P(1, 5, 4)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ \sin 90 & \cos 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ \sin 90 & \cos 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 10 \\ 1 \end{bmatrix}$$

### Robot Kinematics

- It uses geometry to study the movement of robot structure
- It studies the relationship b/w the position, velocity & acceleration b/w joints & links
- Used to plan & control the movement of the structure & to compute the actuator forces & torque.

### Forward kinematics

In forward kinematics the joint & link parameters are given & by these values we calculate the position & orientation of the end effector.

Inverse of a transformation matrix: for finding the inverse of a homogeneous  $4 \times 4$  transformation matrix follow the steps:

Step 1: Rotation portion of the transformation matrix is transposed.

Step 2: The position portion of the transformation matrix is the -ve of the dot product of P with  $(n, o, a)$  vectors.

$E_B$

$$T = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transpose ( $T$ ) =

$$T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -(P \cdot n) \\ o_x & o_y & o_z & -(P \cdot o) \\ a_x & a_y & a_z & -(P \cdot a) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) calculate the inverse of the transformation

$$\begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0.866 & 0 & -0.5 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0.5 & 0.866 & 0 & -3.23^2 \\ 0 & 0 & 1 & -4.048 \\ 0.866 & -0.5 & 0 & -2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) calculate the inverse of given transformation matrix representing  $\text{Rot}(x, 40^\circ)$  inverse.

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \text{Rot}(x, 40^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.766 & -0.643 & 0 \\ 0 & 0.643 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(x, 40)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.766 & 0.643 & 0 \\ 0 & -0.643 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8) calculate the inverse of the following

$$T = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ 0.966 & 0 & 0.643 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0.527 & 0.369 & 0.966 & 6.01 \\ -0.574 & 0.819 & 0 & -5.497 \\ 0.628 & 0.439 & 0.643 & -5.38 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8) A frame F was rotated about the Y axis by  $90^\circ$  followed by a rotation about the Z axis by  $30^\circ$ . followed by a translation of 5 units along Z axis & finally a translation of 4 units along the X axis. Find the total transformation matrix.

$$P(x, y, z) = \text{Trans}(x) \times \text{Rot}(y) \times \text{Rot}(z) \times \text{Trans}(n)$$

$$= (4, 0, 0) \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \sqrt{3}/2 & 0 & y_2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times (5, 0, 0)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \sqrt{3}/2 & 0 & y_2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -0.5 & 0 & 0.866 & 1.5 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & -0.5 & -4.33 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we have to find the inverse of this matrix  
 To do this we will use the formula  
 $A^{-1} = \frac{1}{\det A} \text{adj } A$   
 where  $\det A$  is the determinant of the matrix and  
 $\text{adj } A$  is the adjoint of the matrix.

First we need to find the determinant of the matrix.

The determinant of a 4x4 matrix is given by:

$\det A = a_{11}(a_{22}a_{33}a_{44} - a_{23}a_{32}a_{41}) - a_{12}(a_{21}a_{33}a_{44} - a_{24}a_{31}a_{43}) + a_{13}(a_{21}a_{32}a_{44} - a_{24}a_{31}a_{42}) - a_{14}(a_{21}a_{33}a_{42} - a_{22}a_{31}a_{43})$

Substituting the values from the matrix, we get:

$\det A = (-0.5)(1)(-0.5) - (0)(0)(1.5) + (0)(0)(-4.33) - (0)(0)(1)$

$= -0.25 + 0 + 0 - 0$

$= -0.25$

Now we need to find the adjoint of the matrix.

The adjoint of a 4x4 matrix is given by:



Localization & MappingRepresenting Robot position

There are <sup>common</sup> 4 ways to represent the robot's position:

## 1) Cartesian coordinate:

This representation uses a 2 or 3 Dimensional coordinate system to represent position of the robot.

## 2) Polar coordinates:

In this method the position of the robot is described using radial method & angular direction

## 3) Homogeneous Transformation matrix

This rep uses a  $4 \times 4$  matrix to represent orientation & position of the robot in a 3D space.

## 4) Landmark based representation:

This representation describes the position of robot relative to a known landmark in the environment.

Robot localisation

The process of determining ~~where~~ where a mobile robot is located w.r.t to its environment.

Importance

The robot can plan possible paths with its location & map information & it can identify n - num of paths.

## Challenges of localisation

- Communication
- Translational errors
- Localisation may take time
- Noise

## Reactive Navigation

- It is a study of techniques for planning, executing & controlling the movement of a robot towards a desired location in a collision free path

## Map representation

Visualisation of the geographical information of the environment. It involves the translation of real world data to a digital representation.

Challenges:

1) Scaling & generalisation

2) Distortion

3) Data accuracy

4) Data integration & synchronisation

## General control schemes for mobile robot system

- Perception
- Localisation
- cognition
- motion control

localization requires perception. The sensors in the robot & environment collect the data & the data will be processed & given to the localization module. Then the robots create the maps or use the existing maps to do the localization.

### Methods for mobile robot localization

- 1) Odometry
- 2) Map based localization
- 3) Automation <sup>no sensor</sup> for map building

⇒ The localization method is called <sup>odometry</sup> if the robot uses its sensors to get the position & orientation.

The robot needs to know

The encoders attached to the robot will provide the when the velocity information. Then the velocity info can be used to get the position. The sensors will give the orientation of the

If this method is used for longer duration then the errors will be accumulated

#### Operation

The movement of the robot is sensed with wheel encoder or heading sensors or both. The internal sensors measure the state of the robot (wheel position or speed, battery charge etc) while the external sensors measure the state of environment (mapping, temp etc). These info are integrated to compute the position. The sensor measurement errors are integrated over time as a result the position error accumulates

Thus the position has to be updated from time to time by other localization mechanisms otherwise the robot is able to maintain a meaningful position estimate in long run.

→ There are many sources of odometry from environment factors to usolution

- 1) misalignment of wheels.
- 2) uncertainty in wheel diameter & in particular unequal wheel diameter.
- 3) variation of the contact point of the wheel.
- 4) unequal floor contact

→ Some errors may be deterministic thus they can be eliminated by proper calculation of the system. But there are still some non deterministic error that remain leading to uncertainties in position estimation over time.

→ From the geometric point of view, errors can be classified into 3 types:

- 1) Range error.
- 2) Turn error
- 3) Drift error

Turn & drift errors makes the position error non linear. So we need to establish an error model for odometric accuracy & see how the error propagates

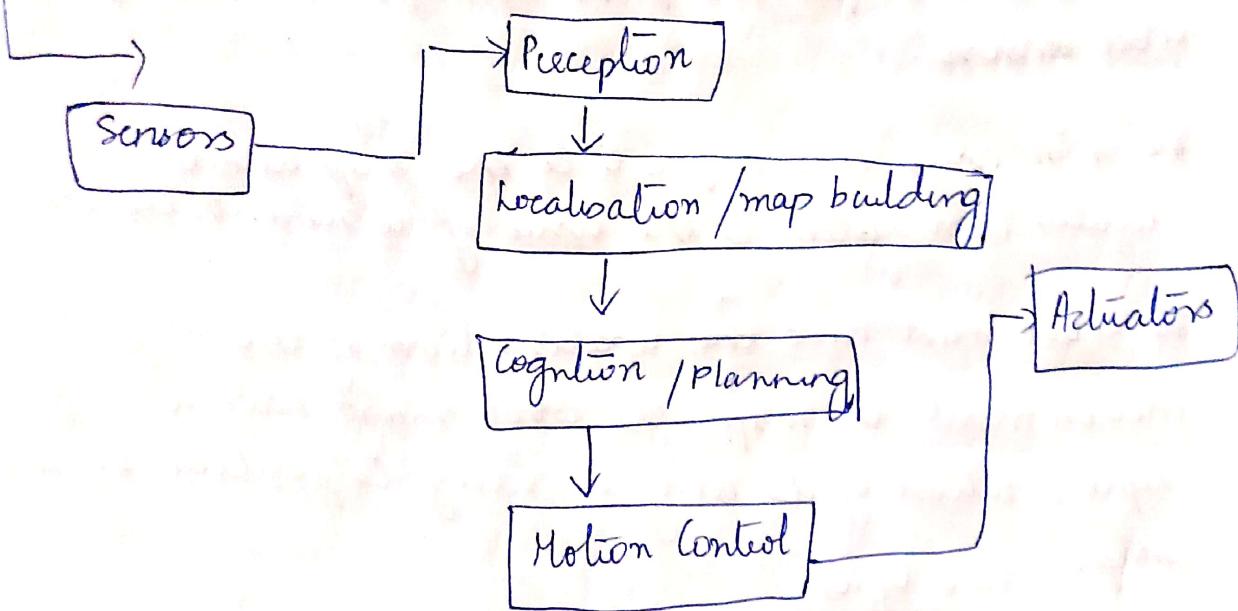
over time.

In map based localization the localization is done by using the odometry info & map info we have 2 methods for that we use probabilistic localization methods

1) Markov localization

2) Kalman filter localization

In probabilistic based localization odometric measures are used but we try to minimise the odometric errors by some method



In a known location the robot will be able to move based on odometry. After certain movement the robot will get uncertain about its position then we can apply probabilistic map based localization where the position is updated using the observation of its environment and then we take info along the map info to update robot position.

→ At every state the error must be reduced  
→ The problem of representing the environment or the  
problem of representing the robot possible position.

The decision made regarding the environmental  
representation can have impact on the choices available  
for robot position representation.

The precision of the map must appropriately match with  
the precision with which the robot needs to achieve goal

The precision of the map & the type of features represented  
must match the precision & the datatype returned by the  
robot sensors.

The main issue that differentiates the map based  
localisation system is the issue of representations.

The robot must have the representation of the  
environment as map. The robot must have a  
representation of its belief regarding its position on the  
map. Two types:

1) Continuous Representation:

It is a method for exact decomposition  
of the environment. Generally it uses a 2D representation.  
This generally assumes that all the environment  
objects are specified <sup>in</sup> the map. Thus the total  
storage needed in the map is proportional to the  
density of objects in the environment.

## Advantages

- high accuracy & expressiveness

## Disadvantage

- computational cost

## Decomposition strategies

A general decomposition is done if the environmental features are anticipated.

1) First decomposition strategy - Exact cell decomposition:

It is a std lossless form of decomposition.  
It does the decomposition by selecting the boundaries  
of the discrete cells based on geometric criticality.

Exact decomposition is not always appropriate.  
The exact decomposition is a function of the particular  
environment obstacles & free space.

2) ~~exact~~ fixed cell decomposition:

Robotic vision

The camera (external sensor) are used for interaction with the environment. The camera robot uses a camera for capturing the scene. These images are processed using image processing techniques for further analysis. This is known as computer vision.

### Image

The images obtained from normal cameras are 2D images. They lack depth information. Each point in the image will be represented using a pixel. A digital image is a rectangular array of dots or picture elements called pixels. Arranged in the form of a matrix with  $m$  rows &  $n$  columns. The expression  $m \times n$  is called the resolution of the image. Each pixel value stores the intensity of the light coming from the object. So image is a collection of data representing the light intensities of large no. of pixels.

### Stages of robotic vision

1. Sensing : The process of capturing an image. The images can be captured using analog or digital cameras. For computer vision images should be digitized. These digitized images are stored in computer memory in the binary form.
2. Preprocessing : Preprocessing is done to convert the image into a pure form that can be easily processed by the processor. It includes noise

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removal, inventions & enhancement.

3. Segmentation: The process of partitioning an image into objects of interest.
4. Description / feature extraction: Required features in an image is extracted to differentiate different objects in an image.
5. Recognition: Identifying the different objects in the images.
6. Interpretation: Assign meanings to the recognized objects.

## Image Processing techniques

- i. Histogram analysis:  
<sup>Hys</sup> Histogram is a graphical representation of total no. of pixels of an image at each gray level. It gives a rough sense of density of the pixel value distributed in the image. Horizontal axis of the graph represent the pixel value variation by vertical axis represents the no. of pixels in that particular value.

### Uses

1. For threshold
2. Adjusting the contrast

### 2. Thresholding :

Process of dividing the image into diff regions based on the pixel value. A particular <sup>pixel</sup> ~~fixed~~ value is set as a threshold & all the other pixels are compared with the threshold. Based on the comparison the pixels are categorized. <sup>There</sup> can be more than 1 threshold depending upon the application.

#### Uses

- converting grey scale image to binary
- object detection
- image segmentation

### 3. Edge detection

Edges are the regions where a sudden change in the intensity of the pixel occur. Usually edges occur at the boundary of the 2 objects. Edge detection techniques detect the edges of the objects in the image. The result of the edge detection will be a line giving the change in the intensities.

#### Methods :

1. Gradient based
2. Canny edge detection
3. Robert "
4. Zero crossing

2) Fixed cell decomposition  
In fixed cell decomposition

The continuous real environment is fixed  
is transformed into discrete approximation

Advantages

Disadvantages

- Inexact nature

- narrow passages are lost  
in the transformation.

### 3) Adaptive cell decomposition

It is the most popular map representation used in mobile robotics. Any cell Every cell is either filled or empty

white cells are outside the obstacles.

black cells are inside /considered as obstacles

grey cells are part of both the regions.

### Current challenges in map representation

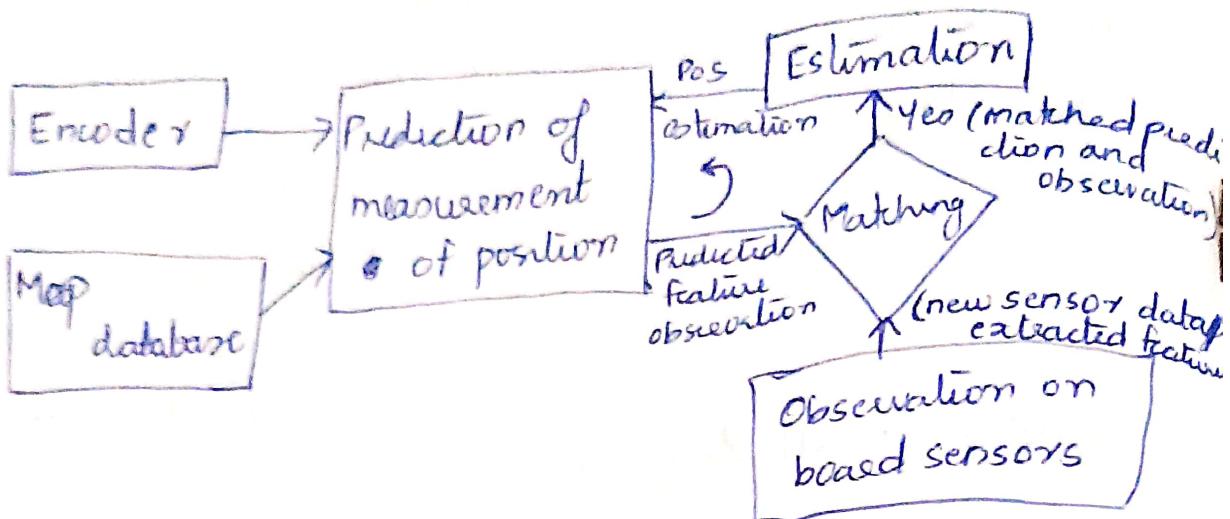
1. Real world is dynamic.

2. Mobile Robot organisation involves the traversal of open spaces

existing localisation techniques generally depend on local measures such as range thereby demanding the environments that are densely filled with objects that the sensors can detect & measure.

Thus wide open spaces such as parking lots, grass fields etc. cause difficulty.

## Probabilistic map based localisation



The basic <sup>man</sup> Kalman filter method allows multiple measurements to be incorporated optimally into a single estimate of the state. The first assumption made is that the state does not change b/w the acquisition of the first & second measurement. After presenting the static case, dynamic prediction is introduced.

The perception update using a Kalman filter is a multistep process. Given a set of possible features the Kalman filter is used to fuse the distance estimate from each feature to a matching object in the map. The first step is the action update / position prediction then the robot collects the actual sensor data & extract the appropriate features in the observation step. At the same time based on its predicted position in the map the robot

generates a measurement prediction which identifies the features that the robot expects to find & the position of those features. In matching the robot identifies the best pairings b/w the features actually extracted during observation & expected features due to measurement prediction. Then the Kalman filter fuses the information provided by all feature matches to update the robot belief state in estimation.

### SLAM (Simultaneous localisation and mapping)

estimate the pose of the robot & the map of the environment at the same time. SLAM problem is hard because in order to build a map we need the position. To determine the position we need a map. SLAM has multiple parts and each part can be executed in many diff ways.

- Parts : 1) Landmark detection
- 2) Data association
- 3) State estimation
- 4) State update
- 5) Landmark update

Landmarks are the features that can be easily be observed & distinguished from the environment. These are used to localise the robot.

## Applications of SLAM

SLAM is critical to a range of indoor, outdoor, marine & underwater application for both manned & autonomous vehicle.

e.g. drone, UAV,

## Types of SLAM

1. Visual SLAM in single camera (monocular SLAM):

It uses a single camera to estimate the position & orientation of a device while simultaneously mapping the environment.

### → Techniques

-SFM : (Structure from Motion)

reconstruct a 3D structure from multiple 2D images

-Visual Odometry : (VO)

-estimates the camera's motion by analysing the changes in consecutive images providing a local map of the environment.

-Feature based tracking:

Tracks the points of interest through successive camera frames to triangulate the 3D position.

## - Sensor fusion

Mitigating the information from other sensors to increase the accuracy & robustness.

## Applications

- monocular SLAM can be used for mobile robot & autonomous vehicles to navigate in map unknown environments.
- Augmented reality - AR application can use monocular slam to track the user movements.
- 3D reconstruction - can be used to reconstruct 3D model of environment or objects.

### a. Particle filter SLAM

- Particle filter SLAM uses Particle filter algorithm.
- It is a probabilistic method that uses a set of particles or samples to represent the robot's possible locations & environment.
- each particle carries information about the robot's pose and potentially a map of the environment.
- The particle filter uses sensor data to update the particle & refines the robot's location & the map.

#### Steps :

1. Prediction - Particles are moved according to the robot's motion model (odometry)

2. Observation - then the sensor data is used to weigh the particle based on how well they match the observed environment

3. Resampling: The particles are replicated with more copies given to the particle which helps to correct & refine the search

### Advantages

1. Handles non-linearity

2. Robustness

3. Adaptability

4. R

### Application

• mapping <sup>maps</sup> of

• creating indoor & outdoor spaces

• vehicle localization

• determining the position of vehicle in real time

• UAV navigation -

→ error model for odometric position estimation