## Graph Search

CMPUT 366: Intelligent Systems

P&M §3.1-3.4

# "Recap": Perfect Rationality vs. Bounded Rationality

Is it feasible for the agent to achieve the ideal optimum, or must it trade off solution quality against computational cost?

- Perfect rationality: The agent can derive the best course of action without accounting for computational limitations.
- Bounded rationality: Agent decides on best action that it can find within its computational limitations
  - Anytime algorithm: Solution quality improves with time

#### Recap: Dimensions

- Static vs. sequential action
- Goals vs. complex preferences
- Episodic vs. continuing
- State representation scheme
- Perfect vs. bounded rationality

- 1. Uncertainty
- 2. Interaction
- 3. Number of agents

Different dimensions interact; you can't just set them arbitrarily

#### Lecture Outline

- 1. Recap
- 2. Search Problems
- 3. Graph Search
- 4. Markov Assumption

#### Search

- It is often easier to recognize a solution than to compute it
- For fully-observable, deterministic, offline, single-agent problems, search exploits this property!
- Agent searches internal representation to find solution
  - All computation is purely internal to the agent.
  - Environment is fully deterministic, so no need for observations, just remember actions
- Formally represent as searching a directed graph for a path to a goal state
- Question: Why might this be a good idea?
  - Because it is very general. Many Al problems can be represented in this form, and the same algorithms can solve them all.

#### State Space

- A state describes all the relevant information about a possible configuration of the environment
- Markov assumption: How the environment got to a given configuration doesn't matter, just the current configuration.
- A state is an assignment of values to one or more variables
  - A single variable called "state"
  - x and y coordinates, temperature, battery charge, etc.
- Actions change the environment from one state to another

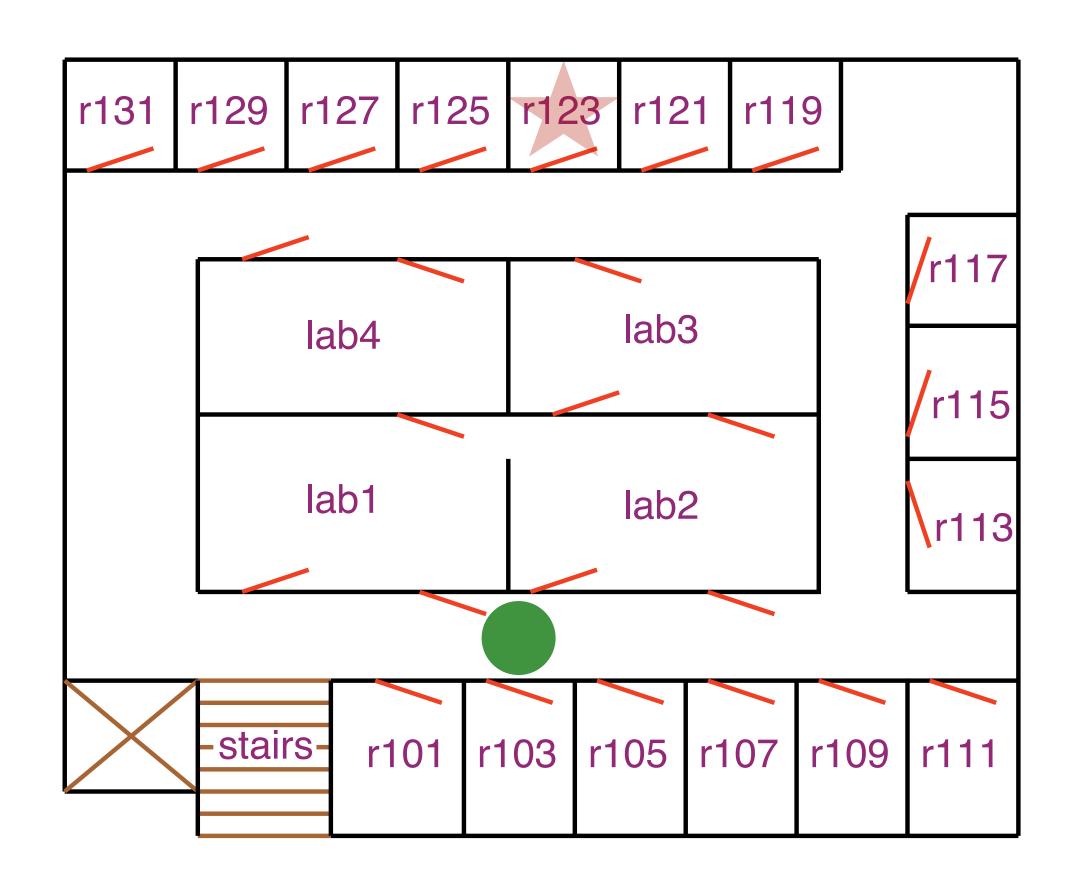
#### Search Problem

Definition: Search problem (textbook: state-space problem)

- A set of states
- A start state (or set of start states)
- A set of actions available at each state
- A successor function that maps from a state to a set of next states
  - The textbook calls this an action function
- A goal function that returns true when a state satisfies the goal

## Example: DeliveryBot

DeliveryBot wants to get from outside room 103 to inside room 123



# DeliveryBot as a Search Problem

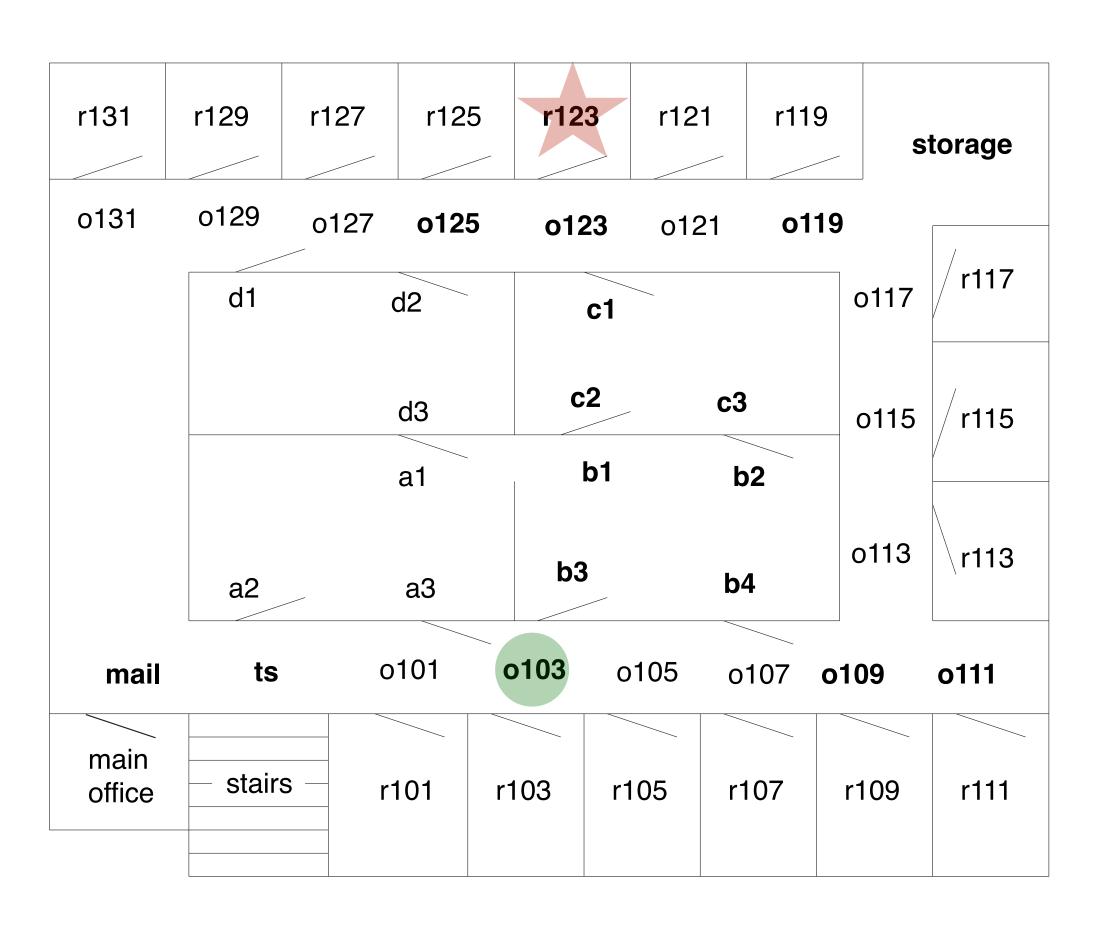
**States** {r131, o131, r129, o129, ...}

Actions {go-north, go-south, go-east, go-west}

Start state o103

**Successor function** succ(r101) = {r101, o101}, succ(o101) = {o101, lab1, r101,o105, ts}, ...

**Goal function** goal(state): (state == r123)



### Example: VacuumBot

- Two rooms, one cleaning robot
- Each room can be clean or dirty
- Robot has two actions:
  - clean: makes the room the robot is in clean
  - move: moves to the other room

#### **Questions:**

- 1. How many **states** are there?
- 2. How many goal states?

# Solving Search Problems, informally

- 1. Consider each start state
- 2. Consider every state that can be **reached** from some state that has been previously considered
- 3. Stop when you encounter a goal state

#### Directed Graphs

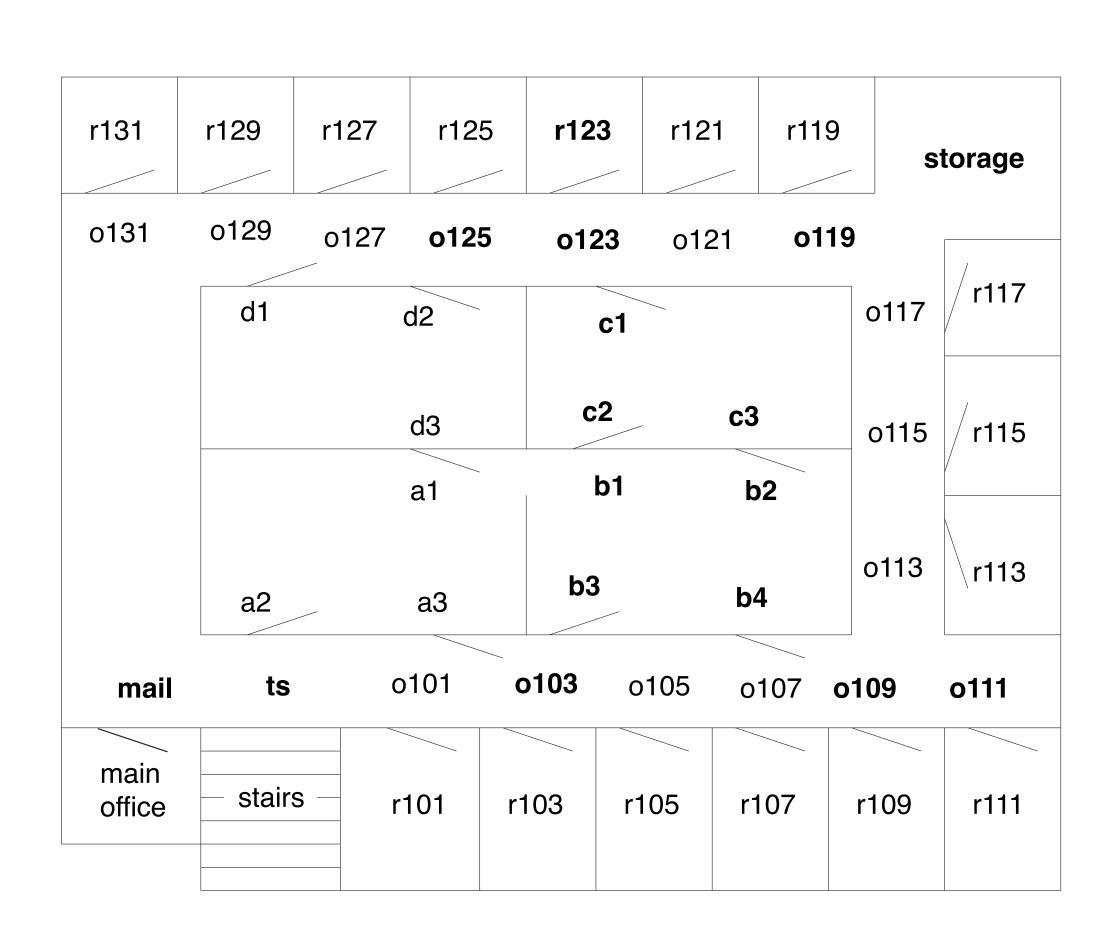
- A directed graph is a pair G = (N, A)
  - N is a set of nodes
  - A is a set of ordered pairs called arcs
- Node  $n_2$  is a **neighbour** of  $n_1$  if there is an arc from  $n_1$  to  $n_2$ 
  - i.e.,  $\langle n_1, n_2 \rangle \in A$
- A path is a sequence of nodes  $\langle n_1, n_2, ..., n_k \rangle$  with  $\langle n_{i-1}, n_i \rangle \in A$
- A solution is a path  $\langle n_1, n_2, ..., n_k \rangle$  from a start node to a goal node

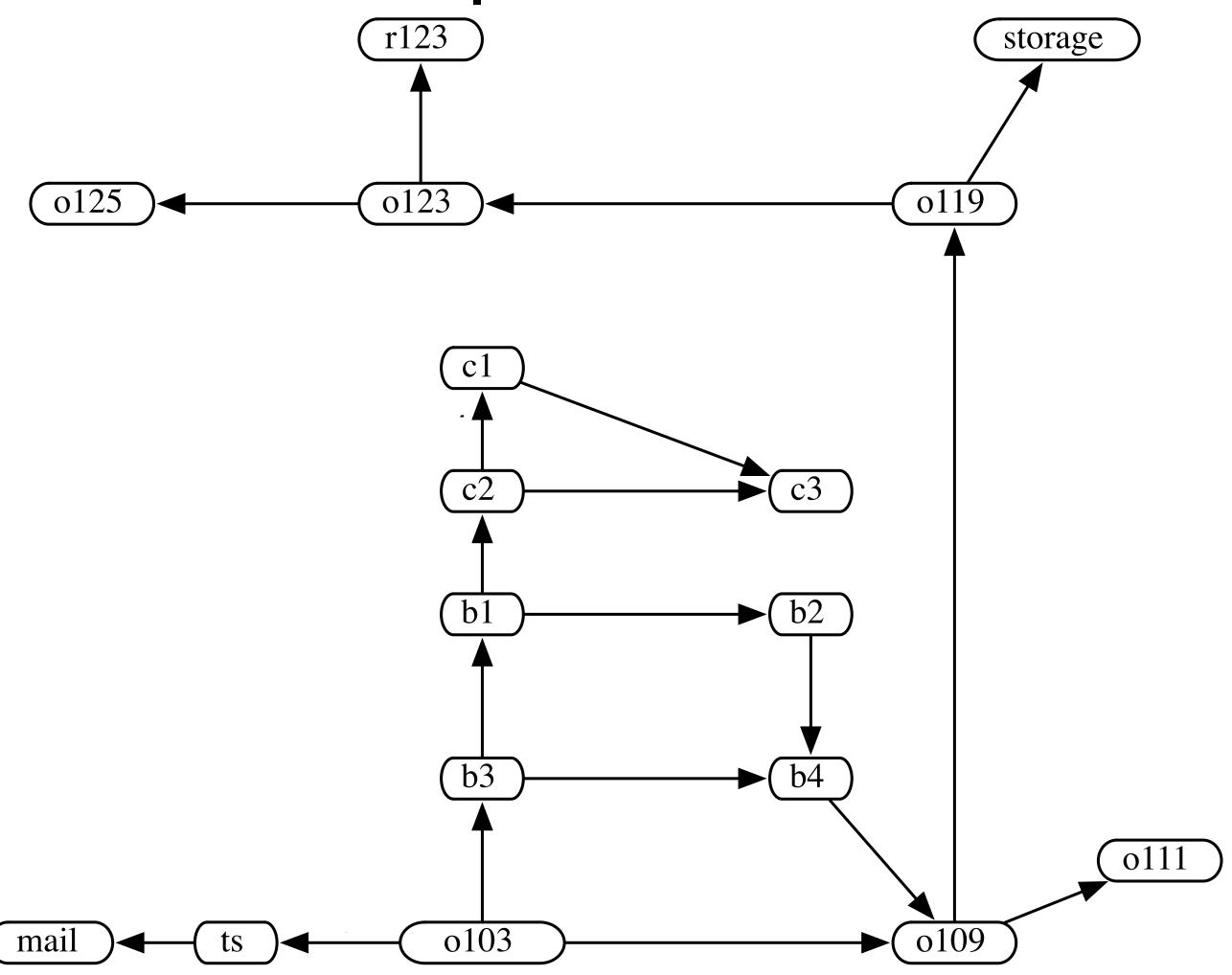
#### Search Graph

We can represent any state space problem as a search graph:

- 1. Nodes are the **states**
- 2. Neighbours are the successors of a state
  - i.e., add one arc from state s to each of s's successors
- 3. Optional: Label each arc with the action that leads to the successor state

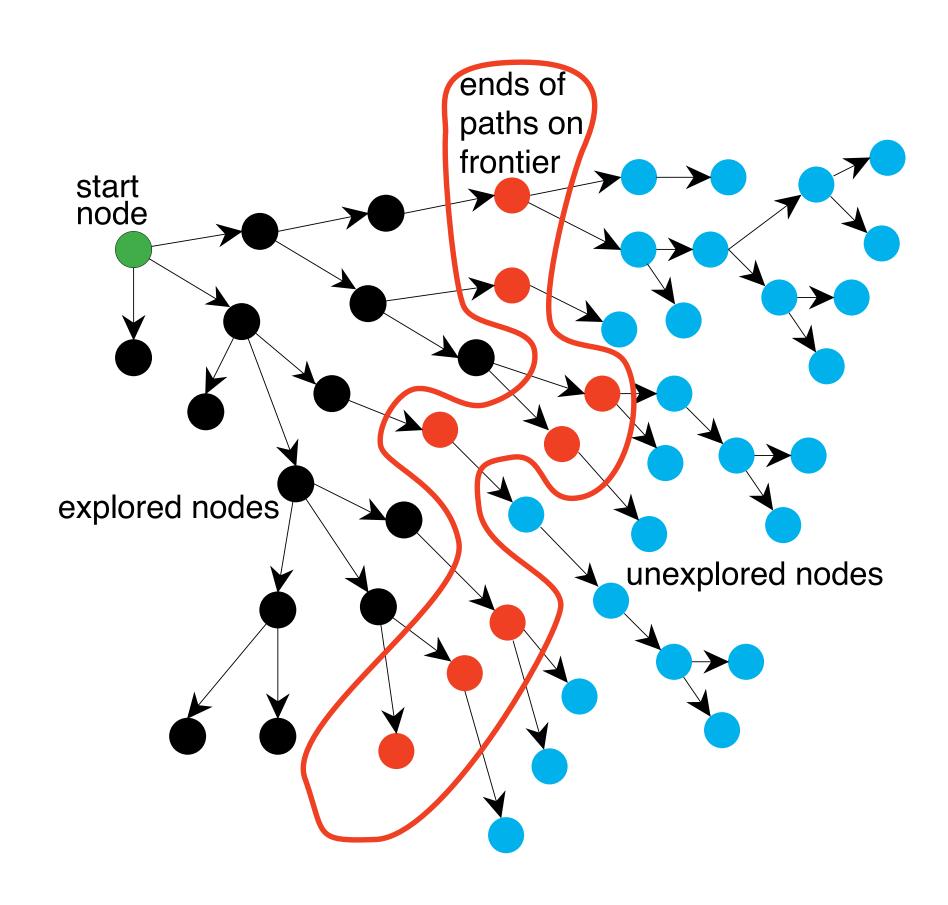
# DeliveryBot: State Space Graph





### Generic Graph Search Algorithm

- Given a graph, start nodes, and goal, incrementally explore paths from the start nodes
- Maintain a frontier of paths that have been explored
- As search proceeds, the frontier **expands** into the unexplored nodes until a goal is encountered.
- The way the frontier is expanded defines the search strategy



https://artint.info/2e/html/ArtInt2e.Ch3.S4.html

## Generic Graph Search Algorithm

**Input:** a *graph*; a set of *start nodes*; a *goal* function

```
frontier := { <s> | s is a start node}

while frontier is not empty:

select and remove a path < n_1, n_2, ..., n_k > from frontier

if goal(n_k):

return < n_1, n_2, ..., n_k >

for each neighbour n of n_k:

add < n_1, n_2, ..., n_k, n > to frontier

end while
```

- Can continue the procedure after algorithm returns
- Which value is selected from the frontier defines the search strategy

#### Search Problem with Costs

What if solutions have differing qualities?

- Add costs to each arc: cost(<n<sub>i-1</sub>,n<sub>i</sub>>)
- Cost of a solution is the sum of the arc costs:

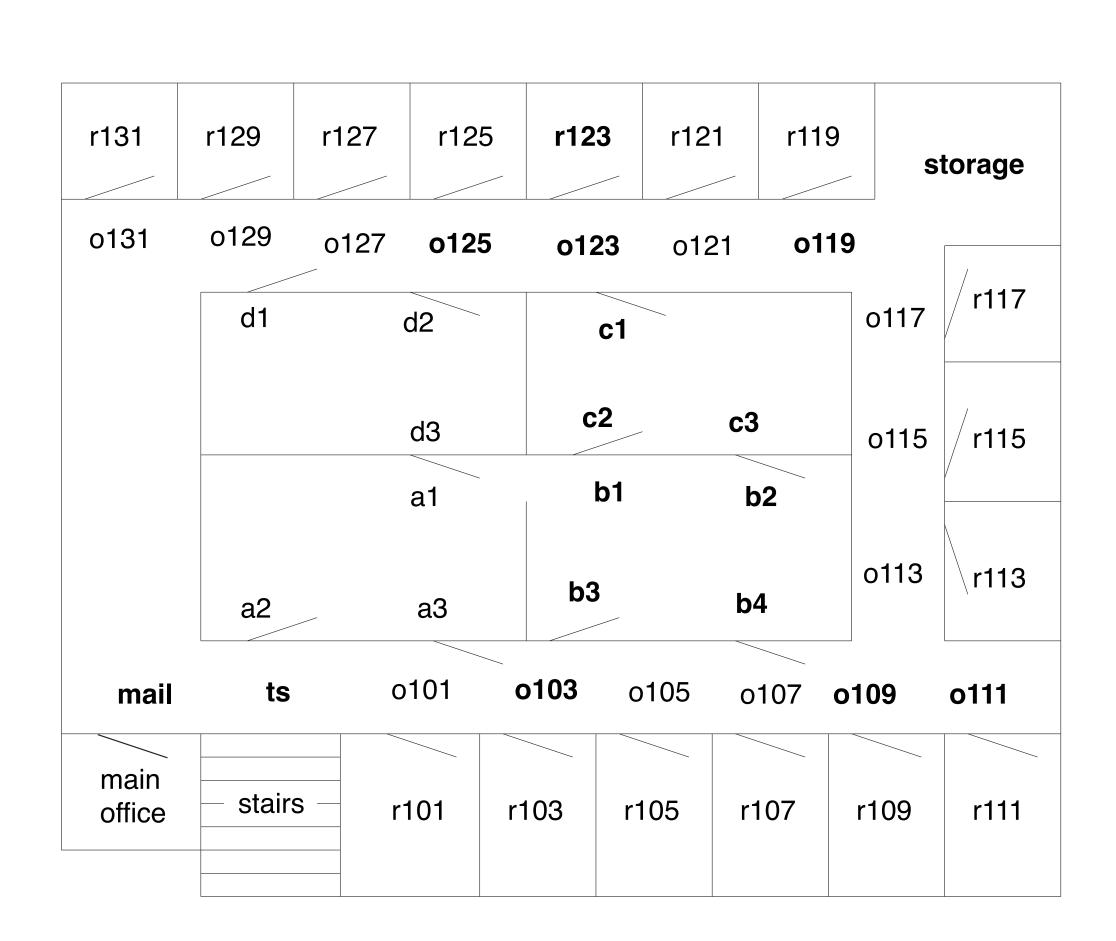
$$\operatorname{cost}\left(\langle n_0, n_1, \dots, n_k \rangle\right) = \sum_{i=1}^k \operatorname{cost}\left(\langle n_{i-1}, n_i \rangle\right)$$

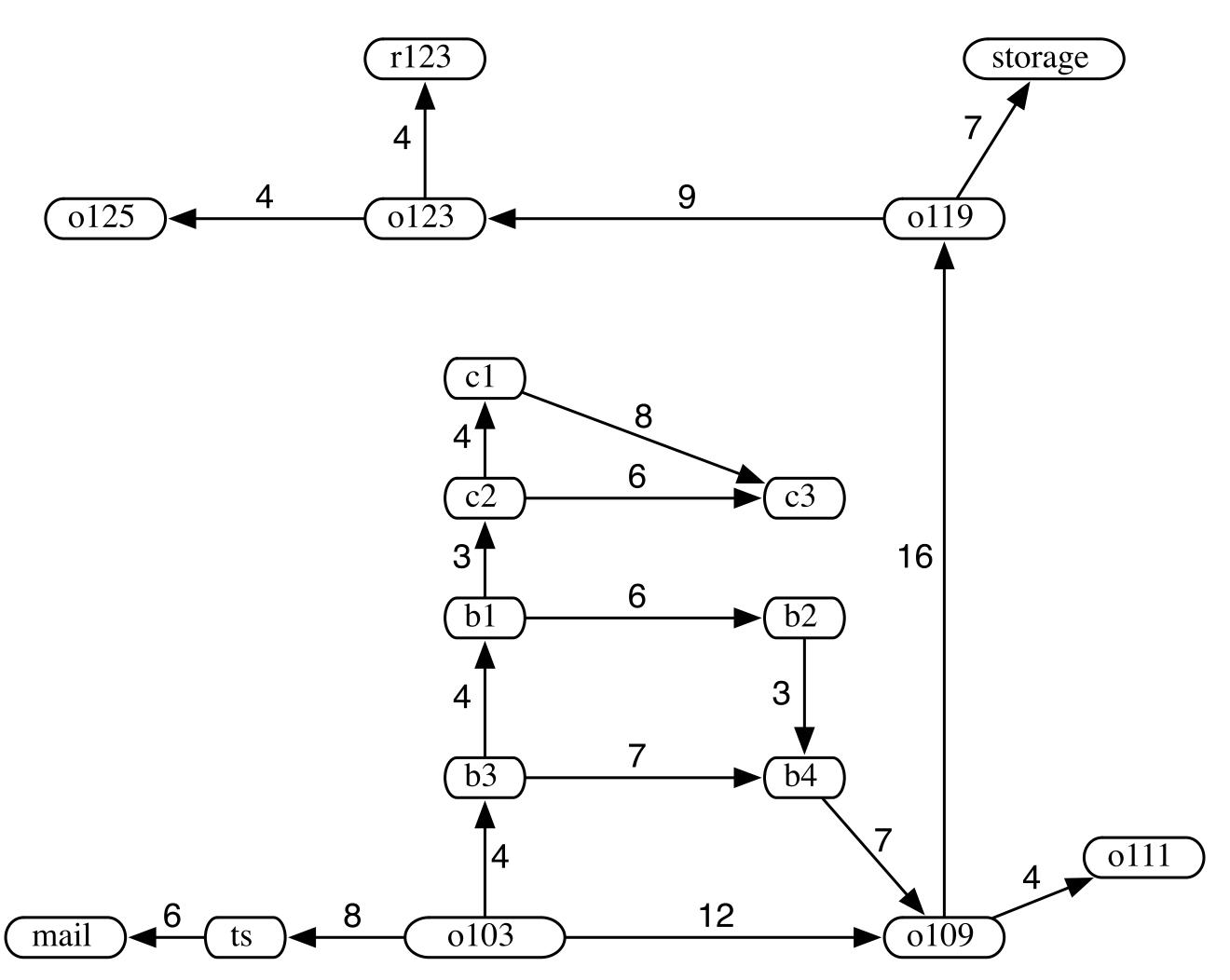
An optimal solution is one with the lowest cost

#### **Questions:**

- 1. Is this scheme sufficiently general?
- 2. What if we only care about the number of actions that the agent takes?
- 3. What if we only care about the quality of the solution we find?

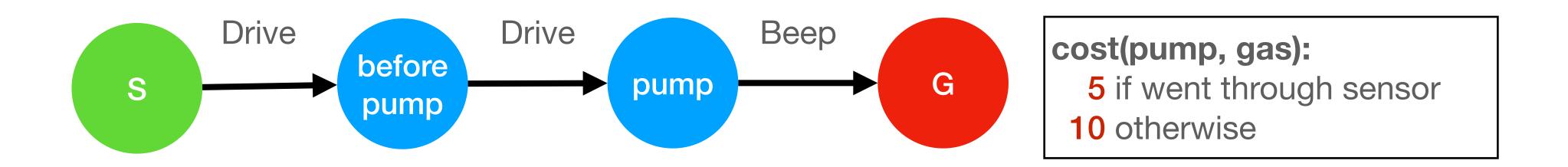
## DeliveryBot with Costs





## Markov Assumption: GasBot

The Markov assumption is crucial to the graph search algorithm



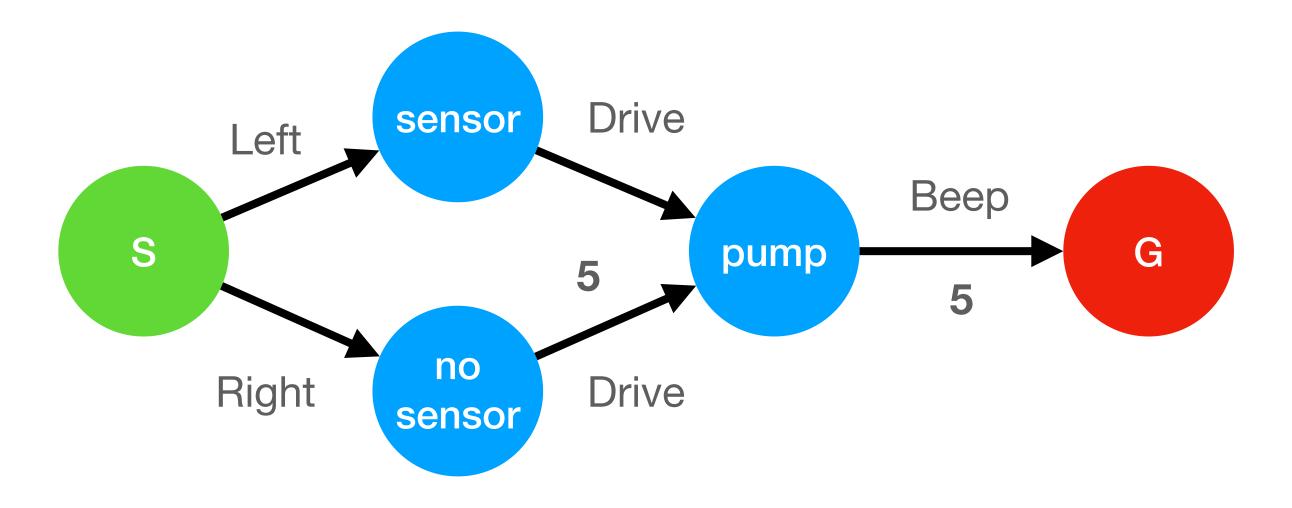
#### **Getting to the pump:**

from the **left** goes through sensor from the **right** does not

**Question:** Does this environment satisfy the Markov assumption? Why or why not?

## Markov Assumption: GasBot

The Markov assumption is crucial to the graph search algorithm



- 1. Does *this* environment satisfy the Markov assumption? Why or why not?
- 2. How else could we have fixed up the previous example?

#### Summary

- Many Al tasks can be represented as search problems
  - A single generic graph search algorithm can then solve them all!
- A search problem consists of states, actions, start states, a successor function, a goal function, optionally a cost function
- Solution quality can be represented by labelling arcs of the search graph with costs