# Uninformed Search

CMPUT 366: Intelligent Systems

P&M §3.5

# Logistics

- TA office hours begin this week
  - See eClass page for times and meeting links
- Assignment #1 released next week

# Recap: Graph Search

- Many Al tasks can be represented as search problems
  - A single generic graph search algorithm can then solve them all!
- A search problem consists of states, actions, start states, a successor function, a goal function, optionally a cost function
- Solution quality can be represented by labelling arcs of the search graph with costs

# Recap: Generic Graph Search Algorithm

Input: a graph; a set of start nodes; a goal function

```
frontier := { <s> | s is a start node}

while frontier is not empty:

select a path <n<sub>1</sub>, n_2, ..., n_k> from frontier

remove <n<sub>1</sub>, n_2, ..., n_k> from frontier

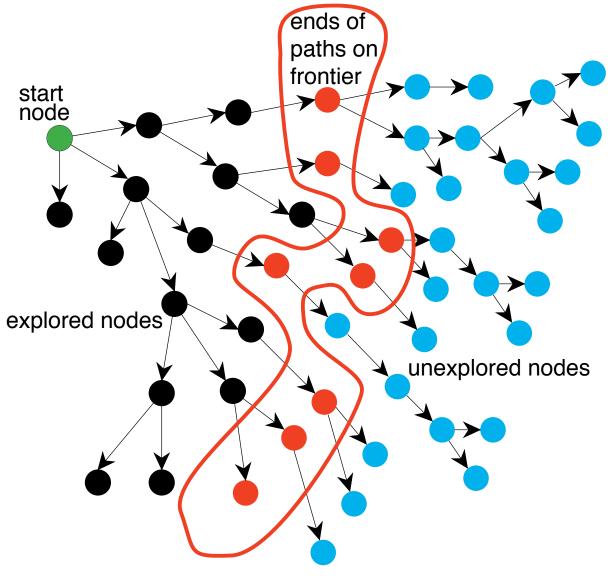
if goal(n_k):

return <n<sub>1</sub>, n_2, ..., n_k>

for each neighbour n of n_k: (i.e., expand node n_k)

add <n<sub>1</sub>, n_2, ..., n_k, n> to frontier

end while
```



https://artint.info/2e/html/ArtInt2e.Ch3.S4.html

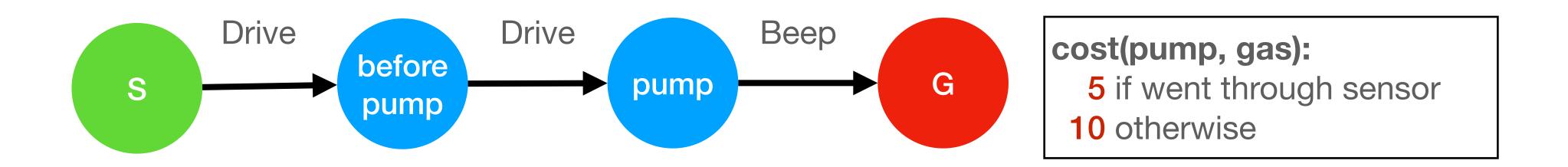
Which value is selected from the frontier defines the search strategy

# Lecture Outline

- 1. Logistics & Recap
- 2. Markov Assumption
- 3. Properties of Algorithms and Search Graphs
- 4. Depth First Search
- 5. Breadth First Search

# Markov Assumption: GasBot

The Markov assumption is crucial to the graph search algorithm



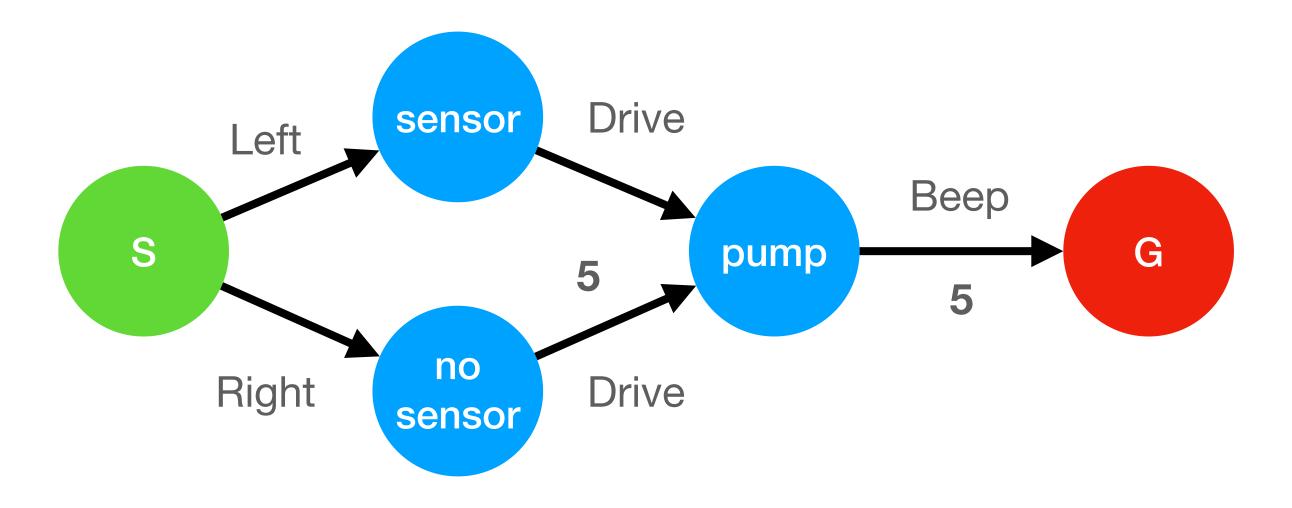
#### **Getting to the pump:**

from the **left** goes through sensor from the **right** does not

**Question:** Does this environment satisfy the Markov assumption? Why or why not?

# Markov Assumption: GasBot

The Markov assumption is crucial to the graph search algorithm



- 1. Does *this* environment satisfy the Markov assumption? Why or why not?
- 2. How else could we have fixed up the previous example?

# Algorithm Properties

What properties of algorithms do we want to analyze?

- A search algorithm is complete if it is guaranteed to find a solution within a finite amount of time whenever a solution exists.
- The time complexity of a search algorithm is a measure of how much time the algorithm will take to run, in the worst case.
  - In this section we measure by number of paths added to the frontier.
- The space complexity of a search algorithm is a measure of how much space the algorithm will use, in the worst case.
  - We measure by maximum number of paths in the frontier.

# Search Graph Properties

What properties of the search graph do algorithmic properties depend on?

- Forward branch factor: Maximum number of neighbours Notation: *b*
- Maximum path length. (Could be infinite!)
  Notation: *m*
- Presence of cycles
- Length of the shortest path to a goal node

# Depth First Search

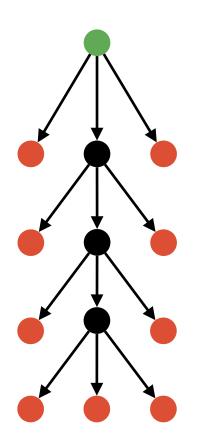
**Input:** a *graph*; a set of *start nodes*; a *goal* function

```
frontier := \{ \langle s \rangle \mid s \text{ is a start node} \}
while frontier is not empty:
   select the newest path \langle n_1, n_2, ..., n_k \rangle from frontier
   remove \langle n_1, n_2, ..., n_k \rangle from frontier
   if goal(n_k):
       return < n_1, n_2, ..., n_k >
   for each neighbour n of n_k:
       add \langle n_1, n_2, ..., n_k, n \rangle to frontier
end while
```

#### **Question:**

What **data structure** for the frontier implements this search strategy?

# Depth First Search



Depth-first search always removes one of the longest paths from the frontier.

#### **Example**:

Frontier:  $[p_1, p_2, p_3, p_4]$ 

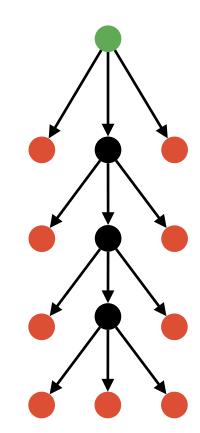
successors( $p_1$ ) = { $n_1, n_2, n_3$ }

#### What happens?

- 1. Remove  $p_1$ ; test  $p_1$  for goal
- 2. Add  $\{\langle p_1, n_1 \rangle, \langle p_1, n_2 \rangle, \langle p_1, n_3 \rangle\}$  to **front** of frontier
- 3. New frontier:  $[<p_1,n_1>, <p_1,n_2>, <p_1,n_3>, p_2,p_3,p_4]$
- 4. p2 is selected only after all paths starting with p1 have been explored

**Question:** When is  $\langle p_1, n_3 \rangle$  selected?

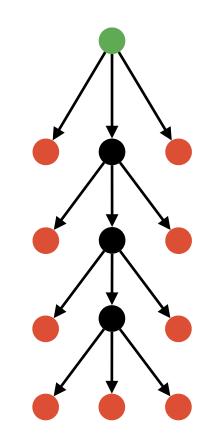
# Depth First Search Analysis



For a search graph with maximum branch factor *b* and maximum path length *m...* 

- 1. What is the worst-case time complexity?
  - [A: O(m)] [B: O(mb)] [C: O(bm)] [D: it depends]
- 2. When is depth-first search complete?
- 3. What is the worst-case space complexity?
  - [A: O(m)] [B: O(mb)] [C:  $O(b^m)$ ] [D: it depends]

# When to Use Depth First Search



- When is depth-first search appropriate?
  - Memory is restricted
  - All solutions at same approximate depth
  - Order in which neighbours are searched can be tuned to find solution quickly
- When is depth-first search inappropriate?
  - Infinite paths exist
  - When there are likely to be shallow solutions
    - Especially if some other solutions are very deep

### Breadth First Search

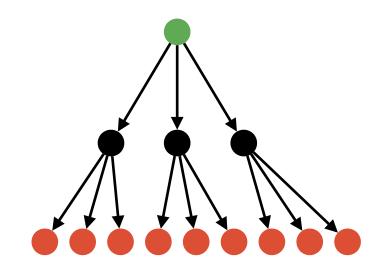
**Input:** a *graph*; a set of *start nodes*; a *goal* function

```
frontier := \{ \langle s \rangle \mid s \text{ is a start node} \}
while frontier is not empty:
   select the oldest path \langle n_1, n_2, ..., n_k \rangle from frontier
   remove \langle n_1, n_2, ..., n_k \rangle from frontier
   if goal(n_k):
       return < n_1, n_2, ..., n_k >
   for each neighbour n of n_k:
       add \langle n_1, n_2, ..., n_k, n \rangle to frontier
end while
```

#### **Question:**

What **data structure** for the frontier implements this search strategy?

## Breadth First Search



Breadth-first search always removes one of the **shortest** paths from the frontier.

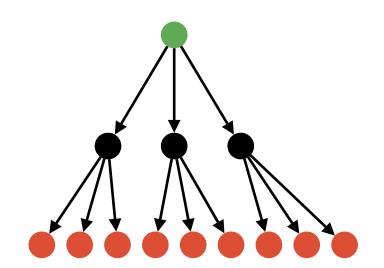
#### **Example**:

Frontier:  $[p_1, p_2, p_3, p_4]$ successors $(p_1) = \{n_1, n_2, n_3\}$ 

#### What happens?

- 1. Remove  $p_1$ ; test  $p_1$  for goal
- 2. Add  $\{\langle p_1, n_1 \rangle, \langle p_1, n_2 \rangle, \langle p_1, n_3 \rangle\}$  to **end** of frontier:
- 3. New frontier:  $[p_2, p_3, p_4, \langle p_1, n_1 \rangle, \langle p_1, n_2 \rangle, \langle p_1, n_3 \rangle,]$
- 4. p<sub>2</sub> is selected next

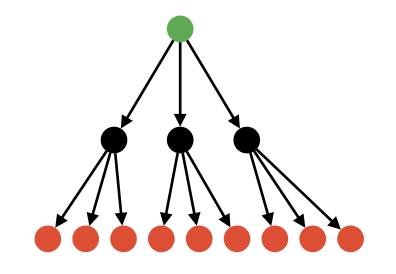
# Breadth First Search Analysis



For a search graph with maximum branch factor b and maximum path length m...

- 1. What is the worst-case time complexity?
  - [A: O(m)] [B: O(mb)] [C:  $O(b^m)$ ] [D: it depends]
- 2. When is breadth-first search complete?
- 3. What is the worst-case space complexity?
  - [A: O(m)] [B: O(mb)] [C:  $O(b^m)$ ] [D: it depends]

# When to Use Breadth First Search



- When is breadth-first search appropriate?
  - When there might be infinite paths
  - When there are likely to be shallow solutions, or
  - When we want to guarantee a solution with fewest arcs
- When is breadth-first search inappropriate?
  - Large branching factor
  - All solutions located deep in the tree
  - Memory is restricted

# Comparing DFS vs. BFS

|                  | Depth-first            | Breadth-first      |
|------------------|------------------------|--------------------|
| Complete?        | Only for finite graphs | Complete           |
| Space complexity | O(mb)                  | O(b <sup>m</sup> ) |
| Time complexity  | $O(b^m)$               | $O(b^m)$           |

- Can we get the space benefits of depth-first search without giving up completeness?
- Run depth-first search to a maximum depth
  - then try again with a larger maximum
  - until either goal found or graph completely searched

# Iterative Deepening Search

Input: a graph; a set of start nodes; a goal function

for max\_depth from 1 to ∞:

Perform **depth-first search** to a maximum depth *max\_depth* **end for**