Introduction to Numerical Methods (CMPUT 340)

1. (Heath 2018) Consider the nonlinear equation

$$f(x) = x^2 - 2 = 0$$

   a) With $x_0 = 1$ as a starting point, what is the value of x_1 if you use Newton's method for solving this problem?
   b) With $x_0 = 1$ and $x_1 = 2$ as a starting points, what is the value of x_w if you use the secant method for solving this problem?

2. (Heath 2018) Write out Newton's iteration for solving each of the following nonlinear equations:
   a) $x^3 - 2x - 5 = 0$
   b) $e^{-x} = x$
   c) $x \sin(x) = 1$

3. (Heath 2018) On a computer with no functional unit for floating-point division, one might instead use multiplication by the reciprocal of the divisor. Apply Newton's method to produce an iterative scheme for approximating the reciprocal of a number $y > 0$. Considering the intended application, your formula should contain no divisions.

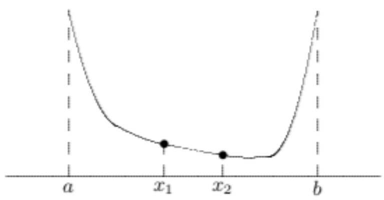
4. Consider a function $f : \mathbb{R}^n \to \mathbb{R}$ given by

$$f(x) = \frac{1}{2}x^T Ax - x^T b + c\,,$$

   where x and b are vectors, A is a matrix, and c is a constant. Explain in English why Newton's method is able to find the minimum of the equation above in a single step for any starting point x_0.

5. Prove that any local minimum of a convex function f on a convex set $S \subseteq \mathbb{R}^n$ is a global minimum of f on S.

6. Consider a procedure for finding the minimum of the function depicted in the image below in the interval $[a, b]$. We will assume that the function is convex in the interval.



a) Given that $f(x_1) > f(x_2)$, what is the smallest interval involving values a, x_1, x_2, b that will contain the minimum of the function?

b) The bisection algorithm reduces the interval $[a, b]$ using the procedure described in (a) to reduce the bracket containing the minimum of the function. The algorithm's efficiency clearly depends on the choice of x_1 and x_2. Describe a method for choosing x_1 and x_2 and explain how this is more efficient than using arbitrary values of x_1 and x_2.

7. (Heath 2018) Given the three data points $(-1, 1), (0, 0), (1, 1)$, determine the interpolating degree two:

   a) Using the monomial basis
   b) Using the Lagrange basis
   c) Using the Newton basis

   Can you see that all three approaches give the same polynomial?

8. (Heath 2018) Express the following polynomial in the correct form for evaluation by Horner's method (the nested form we studied in class)

$$p(t) = 5t^3 - 3t^2 + 7t - 2$$

   Why is the nested form more efficient than the exponential form written above?

9. Using the composite midpoint quadrature rule, compute the approximate value for the integral $\int_0^1 x^3 dx$, using a mesh size (subinterval length) of $h = 1$ and also using $h = 0.5$. Compare the results.

10. Using the computational graph below compute the partial derivatives of the following function with respect to x_0, x_1, w_0, w_1, b: $g(w_0, w_1, b, x_0, x_1) = \frac{1}{1+\exp(-(w_0 \cdot x_0 + w_1 \cdot x_1 + b))}$.