UNIVERSITY OF BIRMINGHAM

Visualisation

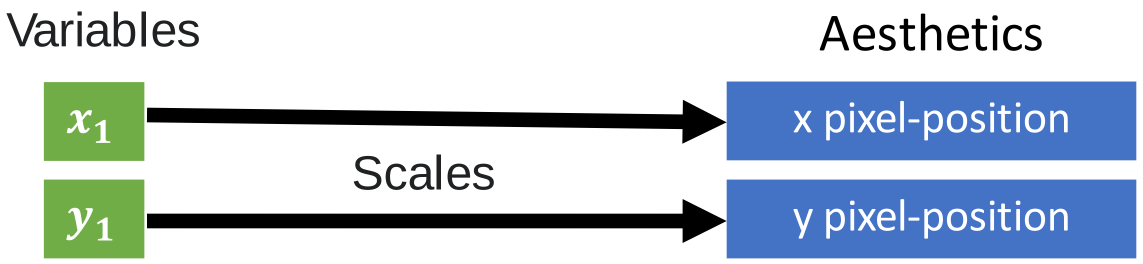
Week 2

Tuesday Session

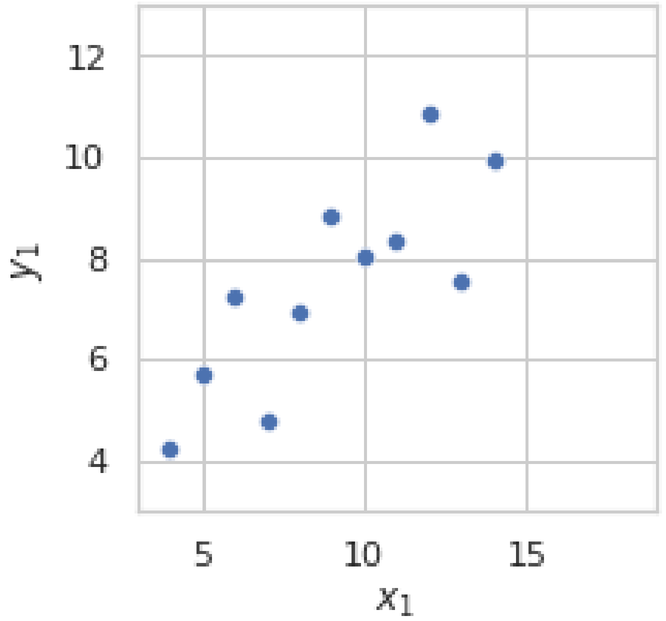
Mapping Variables to Aesthetics

Data

x_1	y_1
10	8.04
8	6.95
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68



Scatter Plot



Mapping Variables to Aesthetics

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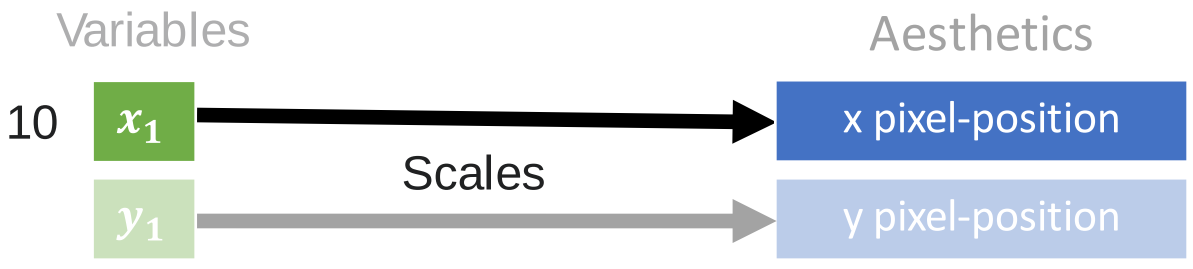

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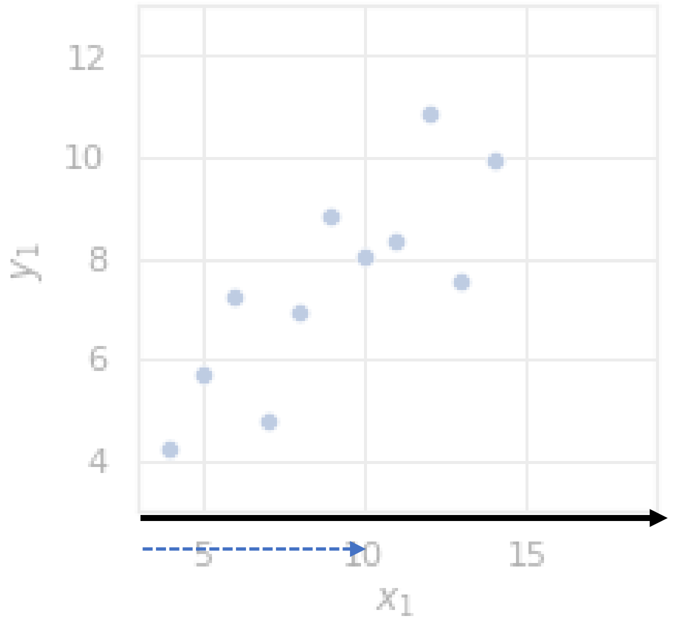

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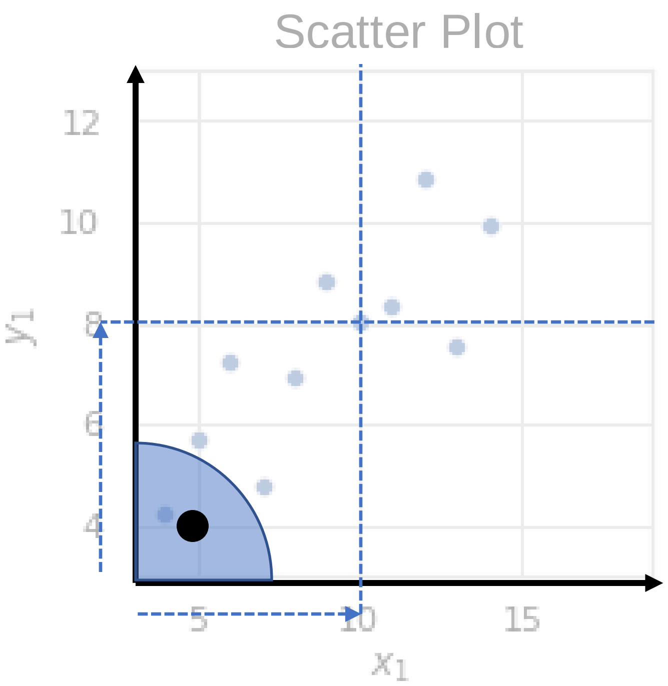
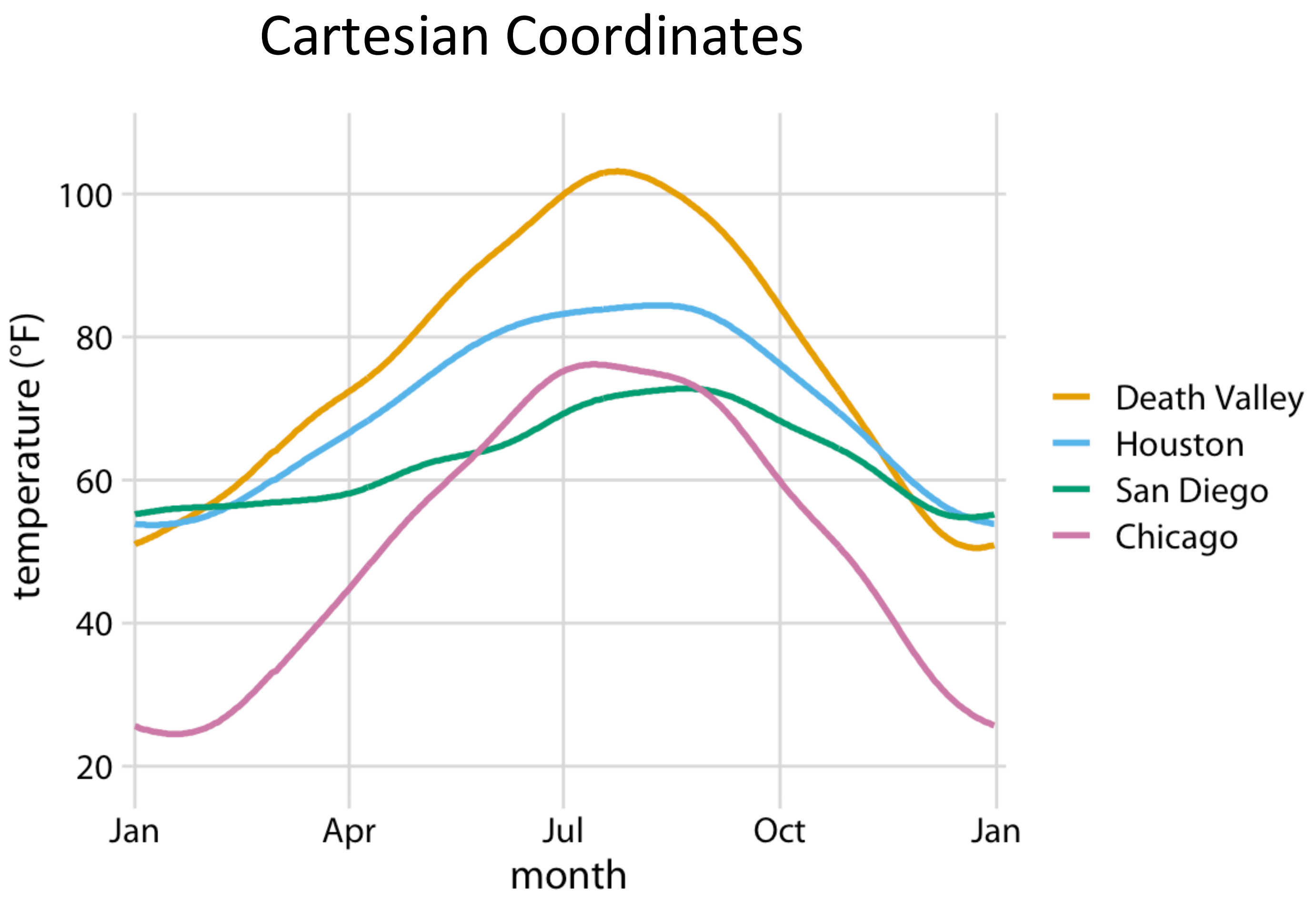
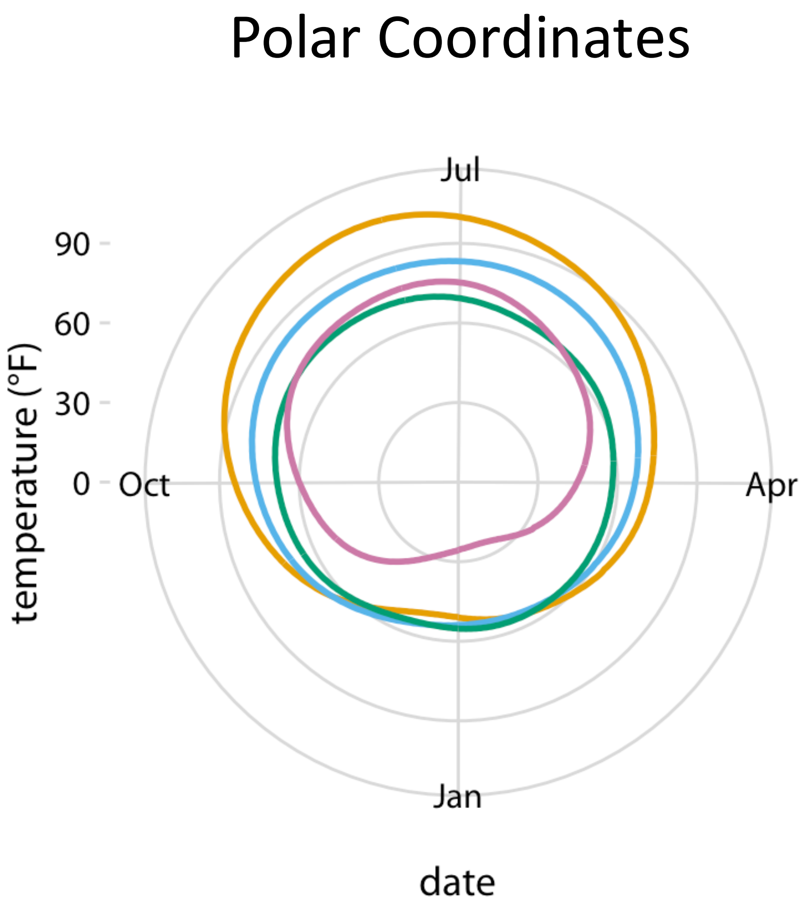


Cartesian Coordinates

Coordinate Systems





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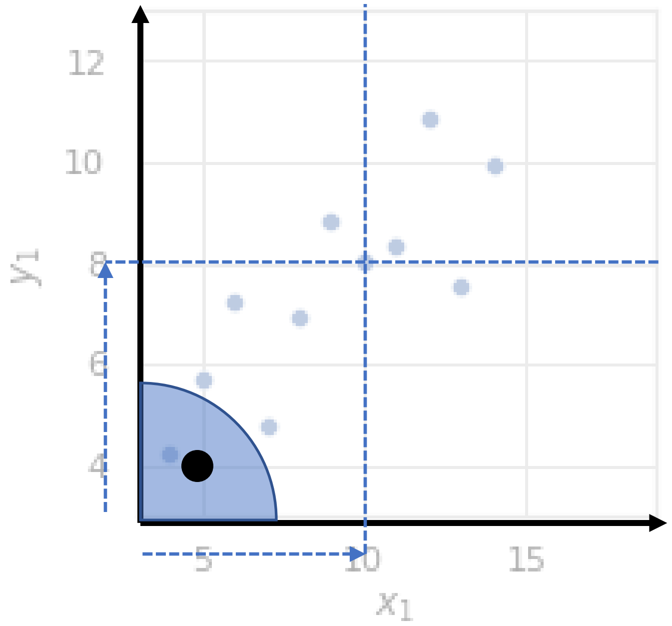
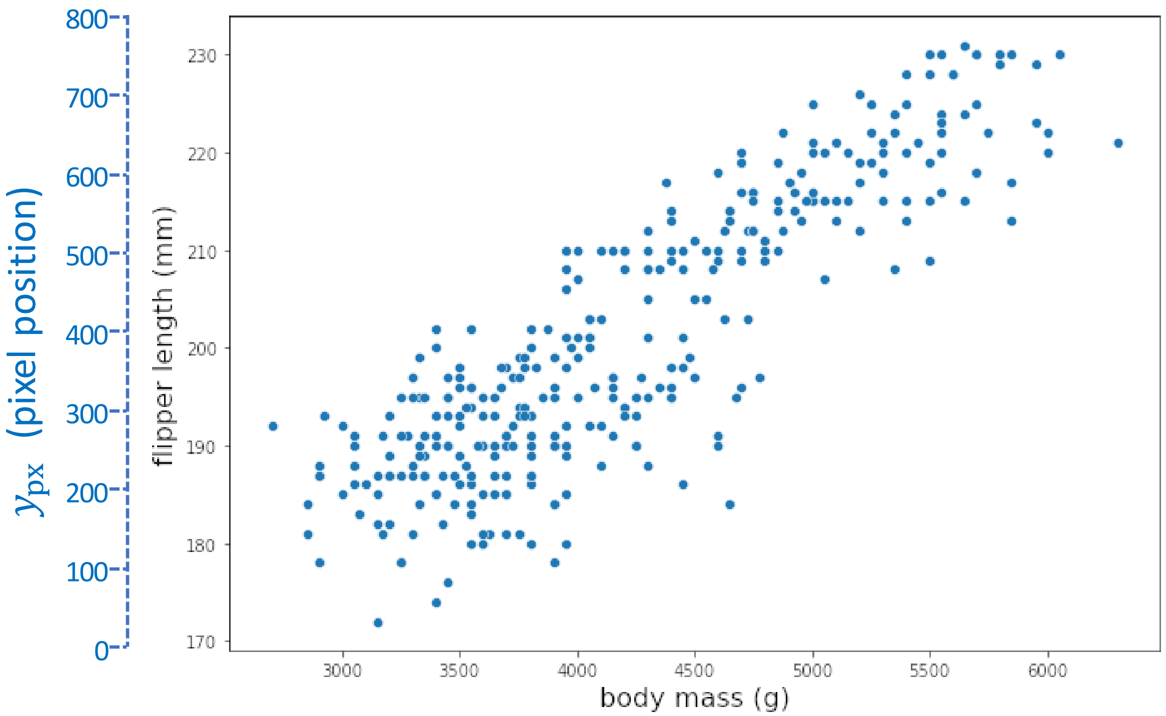


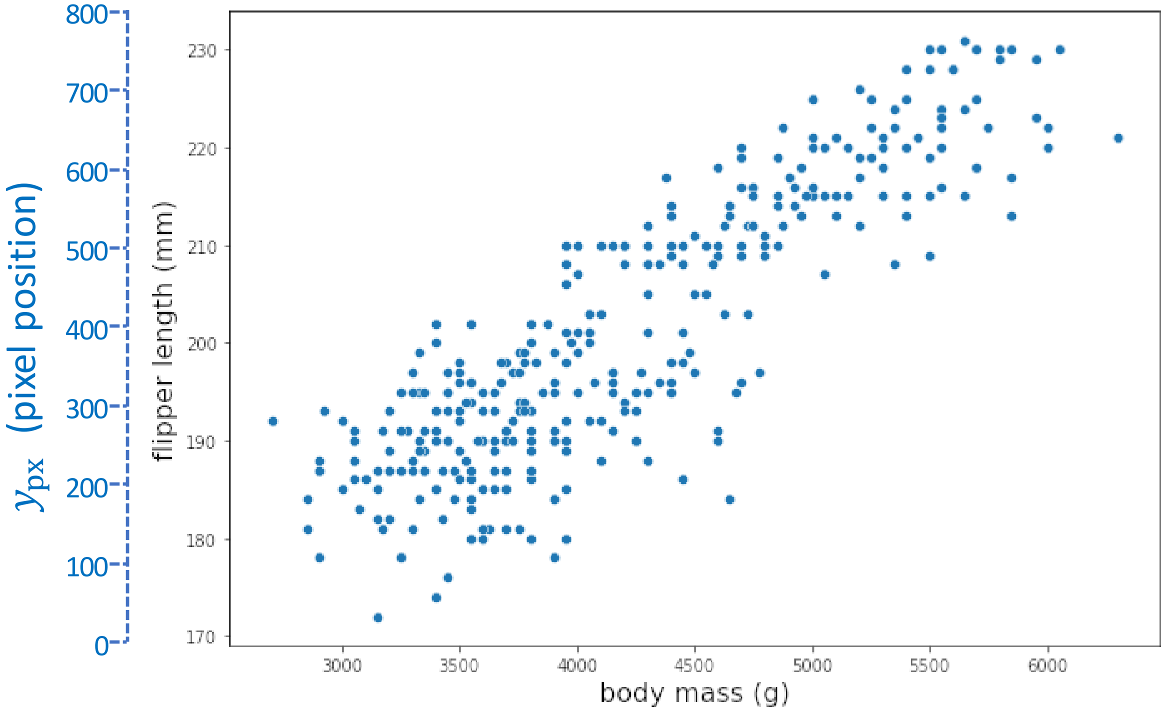
Cartesian Coordinates

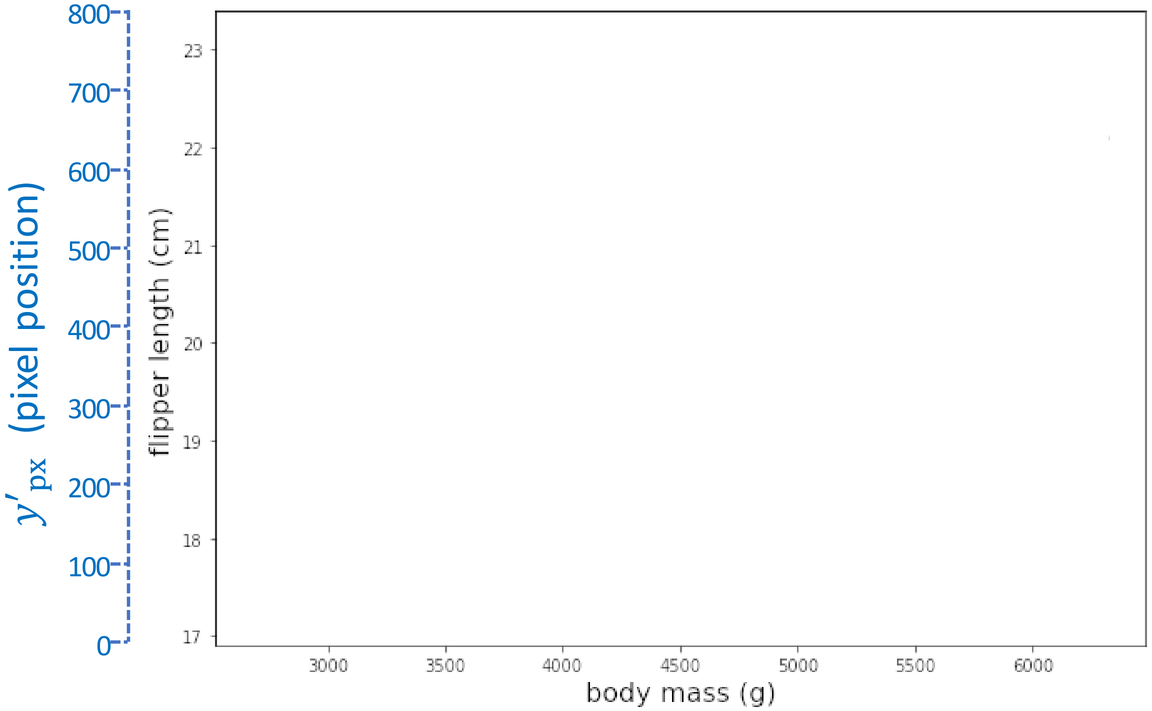
Linear Scales



$$y_{\mathrm{px}} = a_{\mathrm{y}}\, y_{\mathrm{mm}} + b_{\mathrm{y}}$$

Linear Scales



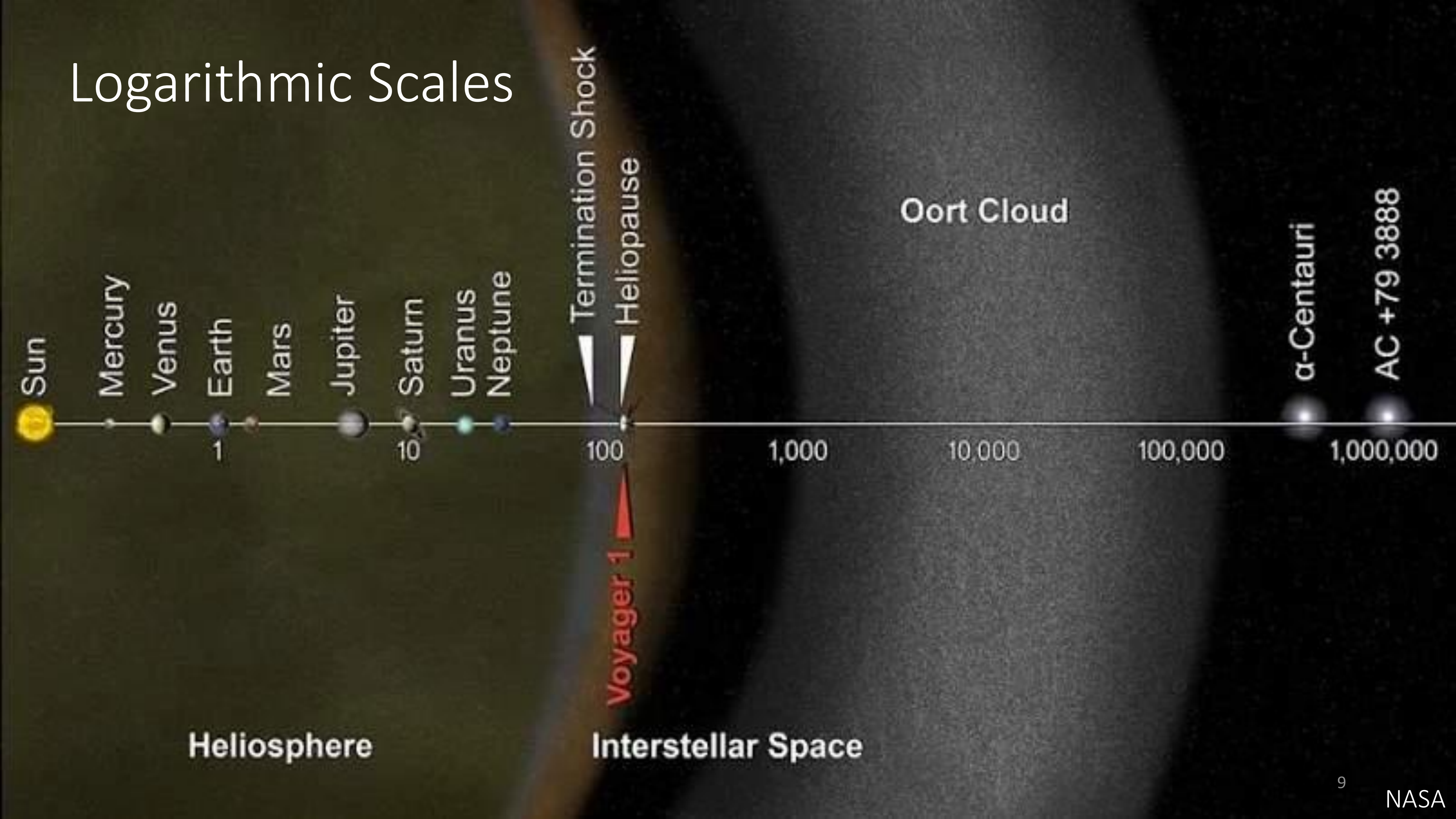


$y_{\text{px}} = a_{\text{y}}\, y_{\text{mm}} + b_{\text{y}}$

$y'_{\text{px}} = a'_{\text{y}}\, y_{\text{cm}} + b_{\text{y}} \quad | \; a'_{\text{y}} = 10 a_{\text{y}}$

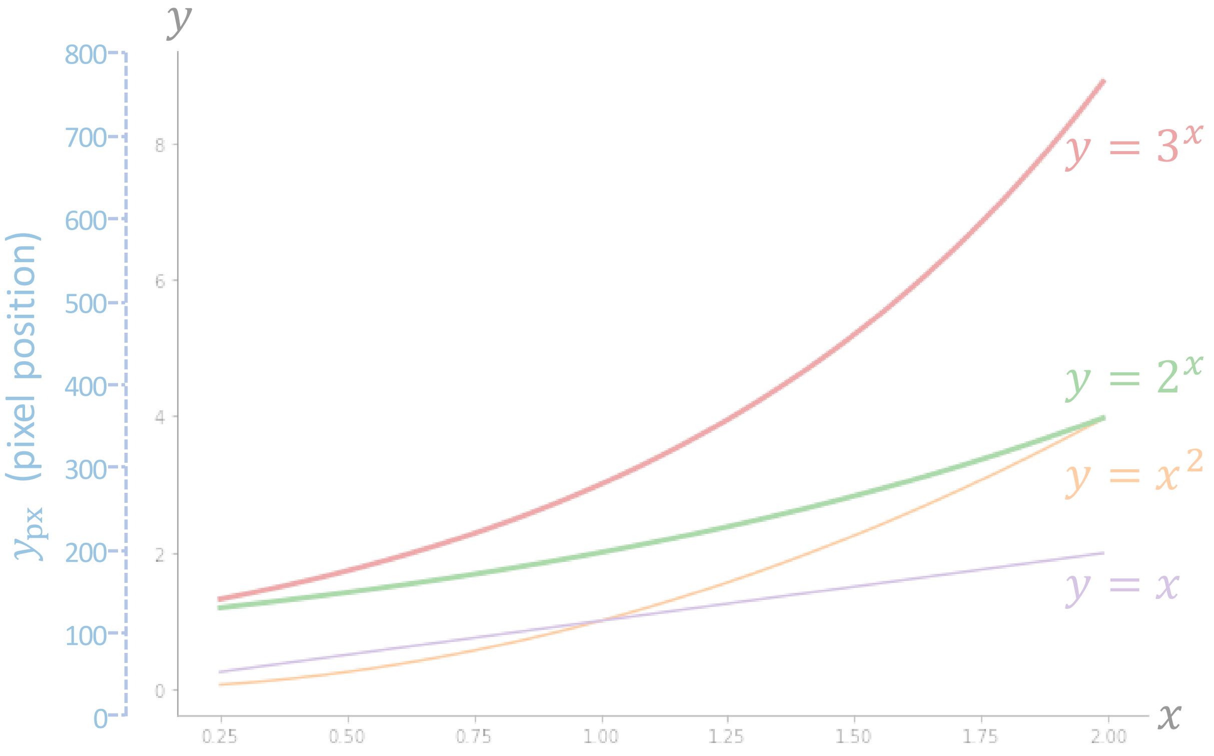
- “Invariant” to:
  - scaling and shifting
  - unit change

Logarithmic Scales

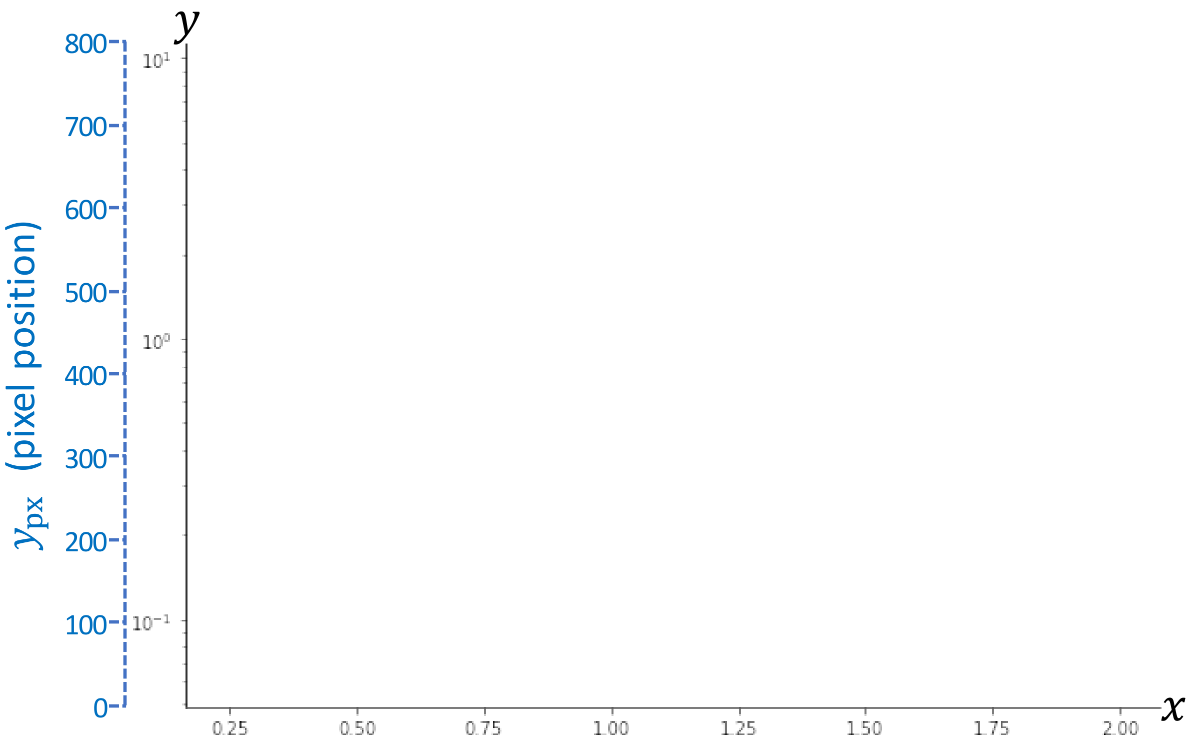


NASA

Logarithmic Scales – Log-Linear Plot



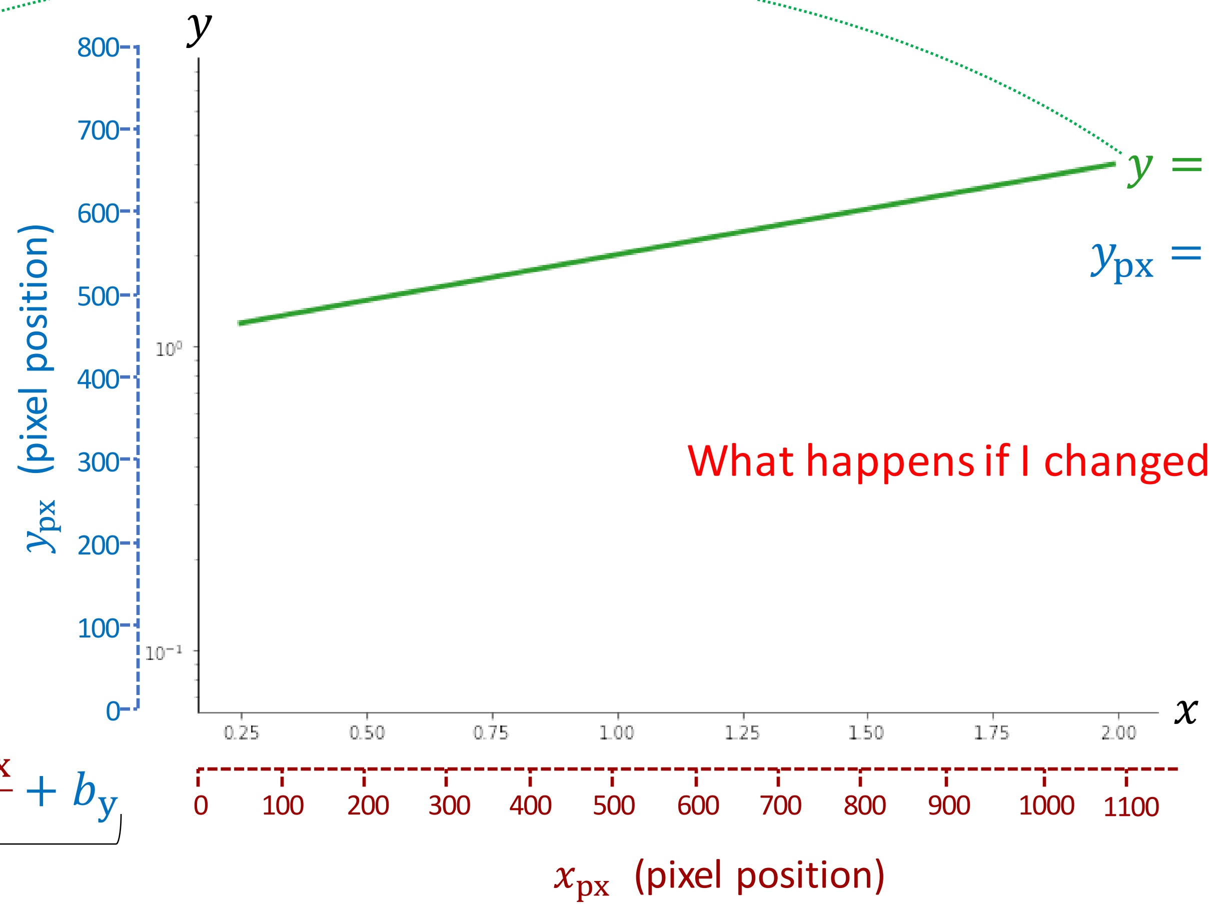
$$y_{\mathrm{px}} = a_{\mathrm{y}}\, y_{\mathrm{y}} + b_{\mathrm{y}}$$



$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y) + b_{\mathrm{y}}$$

Logarithmic Scales – Log-Linear Plot

$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(\alpha^{x}) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(\alpha) x + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(\alpha) \left(\frac{x_{\mathrm{px}}}{a_x} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}} \right) + b_{\mathrm{y}}$$
$$= \underbrace{\frac{a_{\mathrm{y}}}{a_{\mathrm{x}}} \log(\alpha)}_{a} \, x_{px} \underbrace{- \frac{a_{\mathrm{y}} \log(\alpha)\, b_{\mathrm{x}}}{a_{\mathrm{x}}} + b_{\mathrm{y}}}_{b}$$
$$y_{\mathrm{px}} = a \quad x_{px} \quad + \quad b$$



$y = \alpha^{x}$

$y_{\mathrm{px}} = \mathrm{f}(x_{\mathrm{px}})$

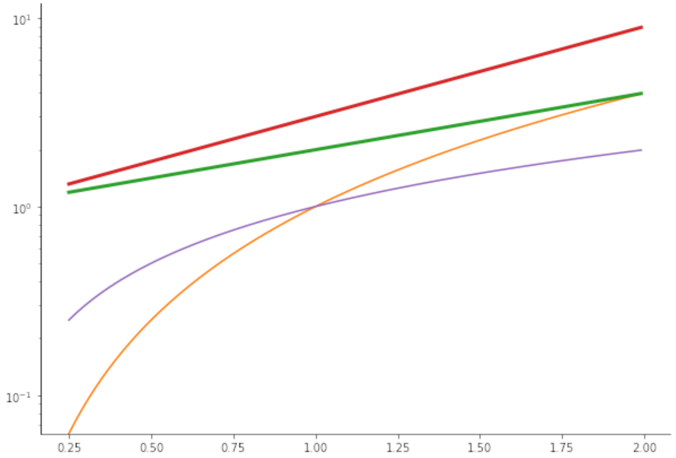
What happens if I changed the unit from meters to cm?

$$x_{\mathrm{px}} = a_{\mathrm{x}}\, x + b_{\mathrm{x}}$$
$$x = \frac{x_{\mathrm{px}}}{a_{\mathrm{x}}} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}}$$

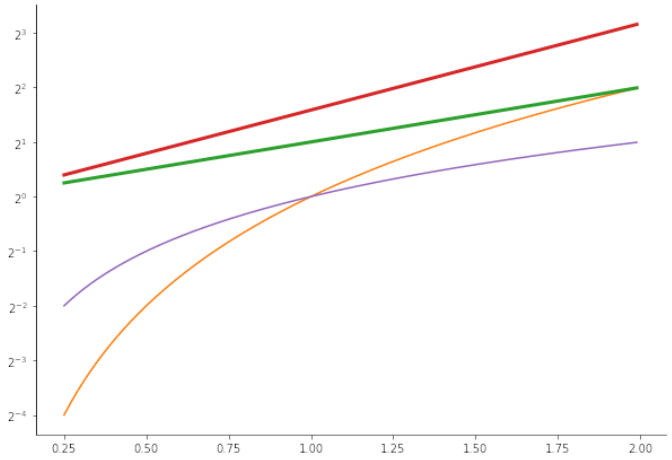
Logarithmic Scales – Log-Linear Plot

- All functions of the form: $y = \lambda \alpha^x$
  - Result in straight line.
  - Slope is proportional to: $\log(\alpha)$

- “Invariant” to:
  - ~~scaling and shifting~~
  - unit change
  - changing base



Base 10



Base 2

Invariance to Scaling but NOT Shifting

Scaling:

$$\begin{aligned} y_{\mathrm{px}} &= a_{\mathrm{y}} \log(yc) + b_{\mathrm{y}} \\ &= a_{\mathrm{y}} \log(y) + \log(c) + b_{\mathrm{y}} \end{aligned}$$

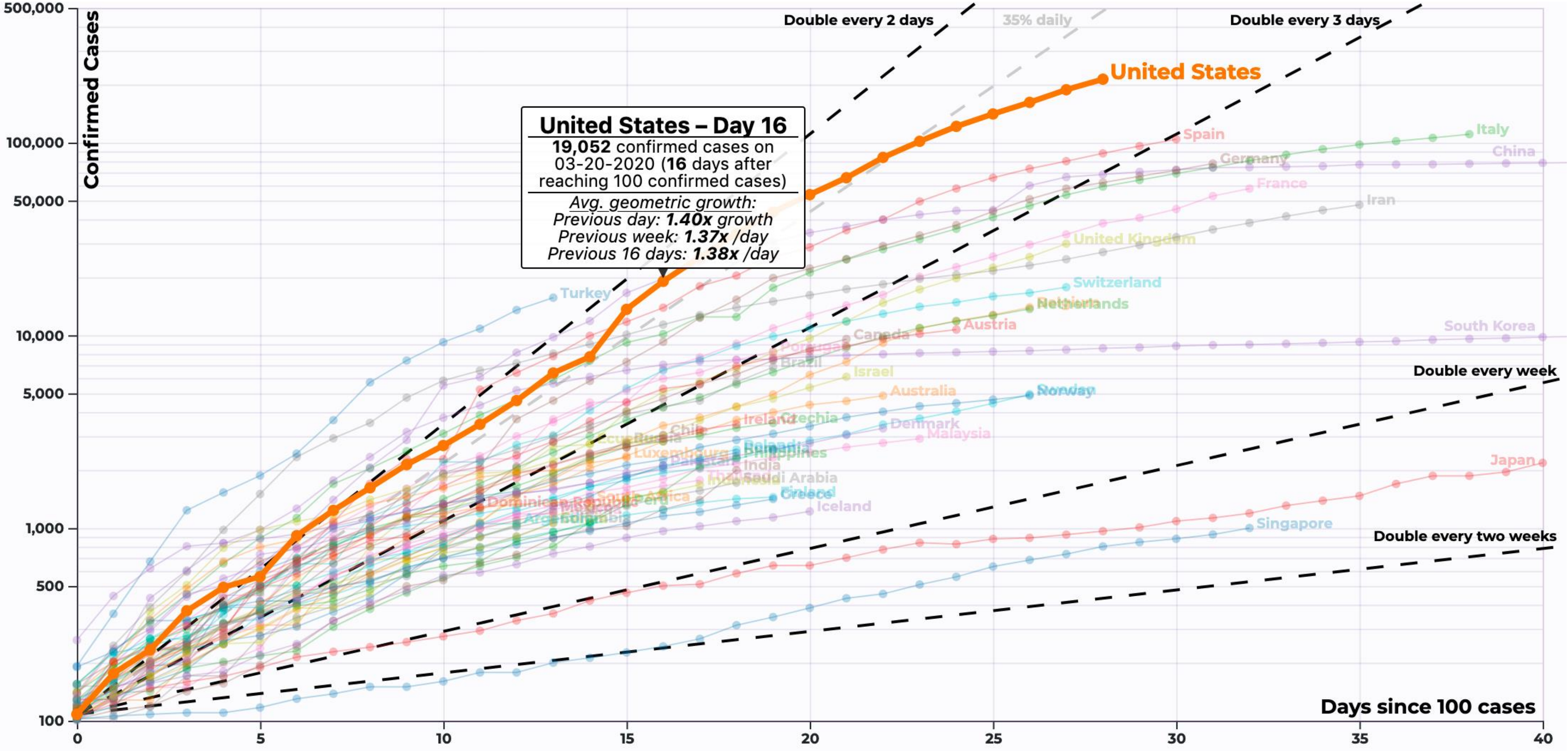
- We can adjust b_{y}

Shifting:

$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y + c) + b_{\mathrm{y}}$$

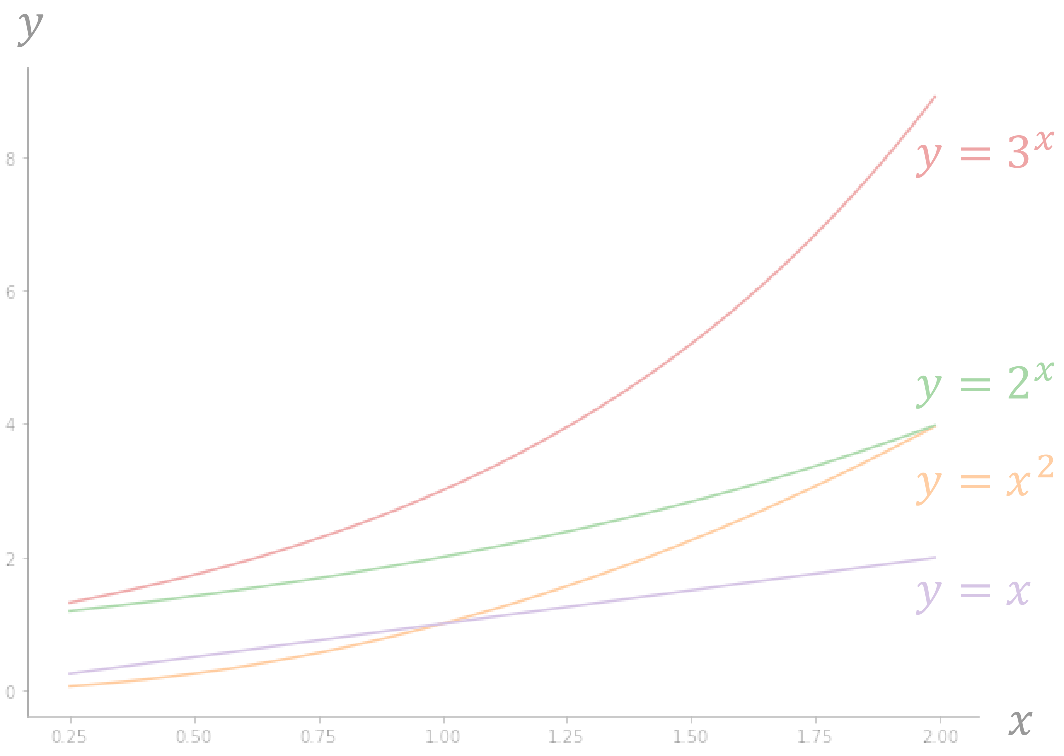
- Nothing we can do!

Logarithmic Scales – Log-Linear Plot

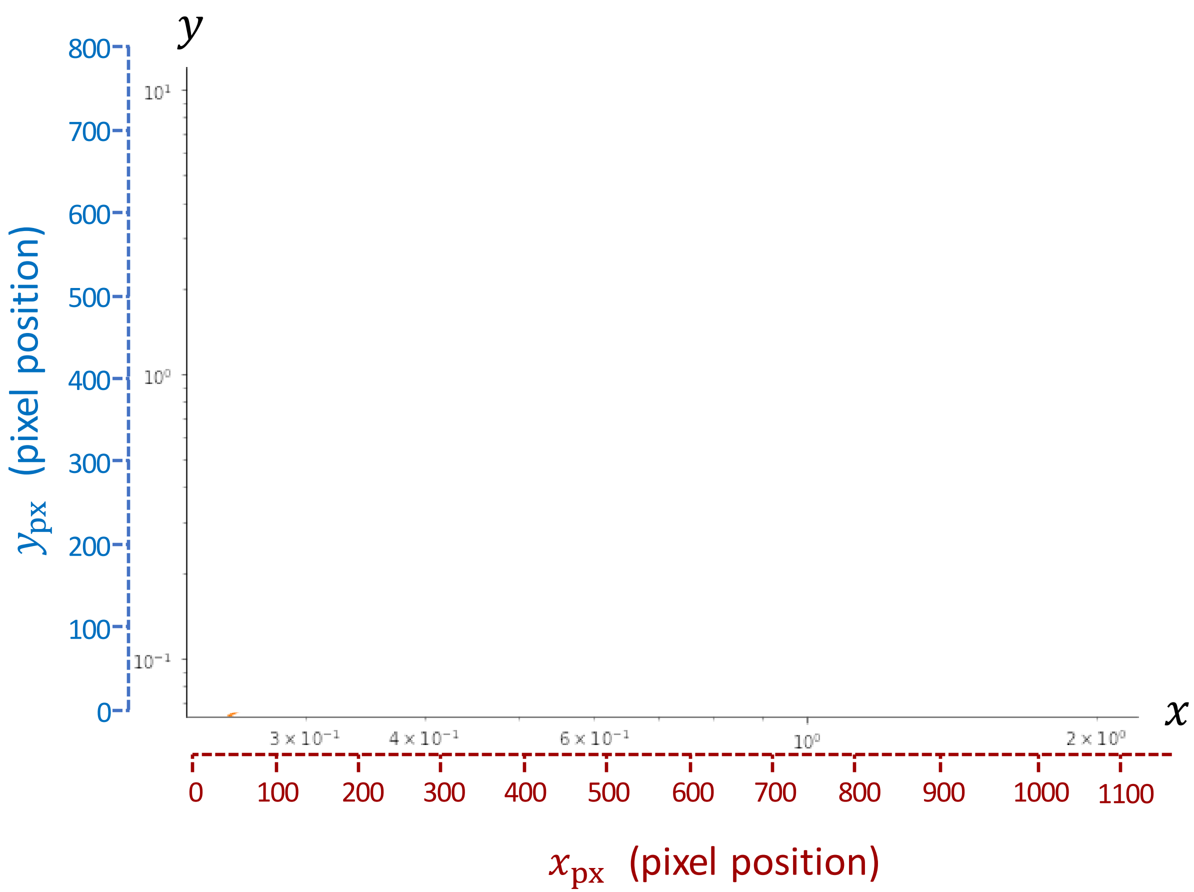


The Verge, 91-divoc.com

Logarithmic Scales – Log-Log Plot



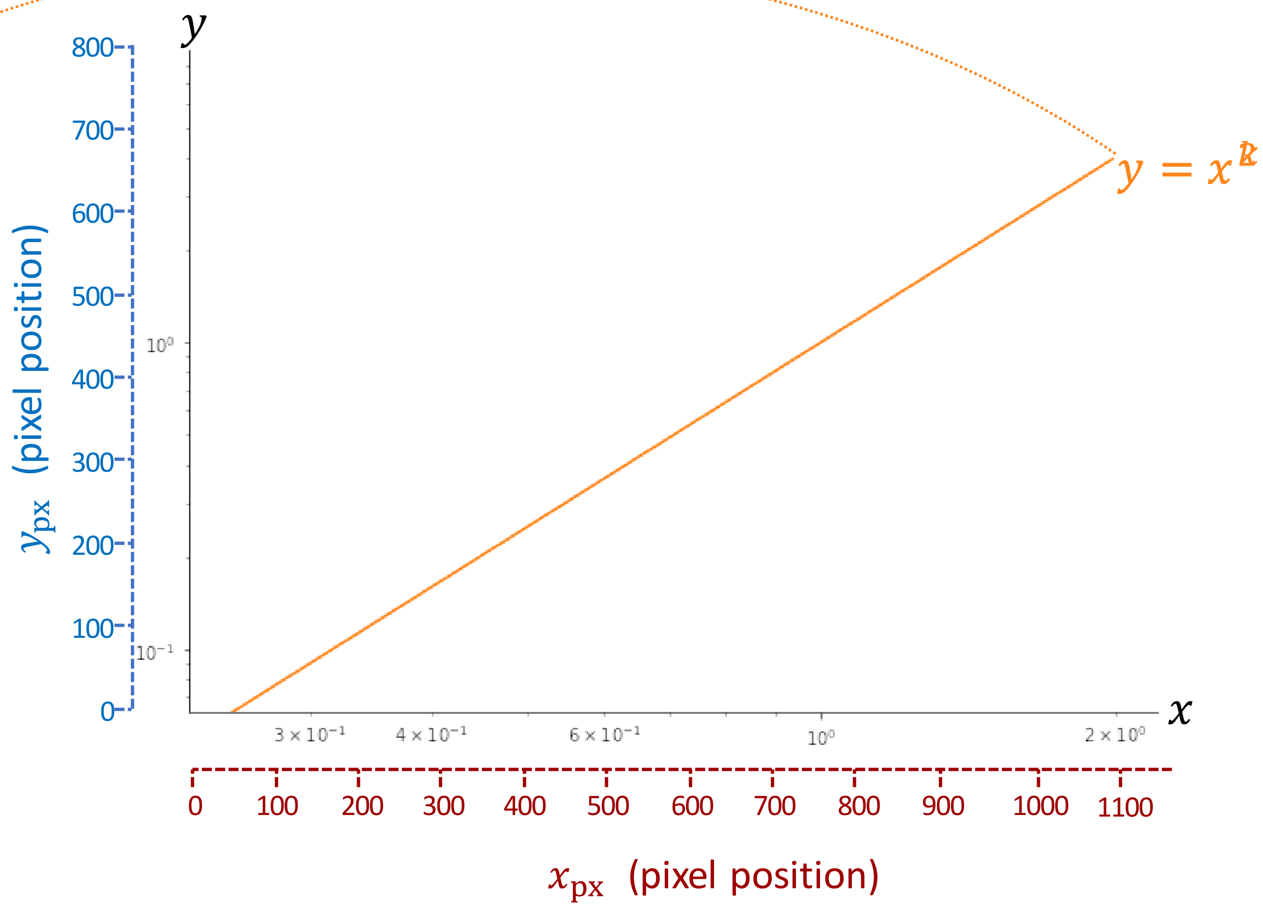
$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y) + b_{\mathrm{y}}$$



$$x_{\mathrm{px}} = a_{\mathrm{x}} \log(x) + b_{\mathrm{x}}$$

Logarithmic Scales – Log-Log Plot

$$
\begin{aligned}
y_{\mathrm{px}} &= a_{\mathrm{y}} \log(y) + b_{\mathrm{y}} \\
&= a_{\mathrm{y}} \log(x^{k}) + b_{\mathrm{y}} \\
&= a_{\mathrm{y}} \log(x) k + b_{\mathrm{y}} \\
&= a_{\mathrm{y}} \left(\frac{x_{\mathrm{px}}}{a_{\mathrm{x}}} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}}\right) k + b_{\mathrm{y}} \\
&= \underbrace{\frac{a_{\mathrm{y}} k}{a_{\mathrm{x}}}}_{a} x_{\mathrm{px}} \underbrace{- \frac{a_{\mathrm{y}} k b_{\mathrm{x}}}{a_{\mathrm{x}}} + b_{\mathrm{y}}}_{b} \\
y_{\mathrm{px}} &= a \;\; x_{px} + b
\end{aligned}
$$



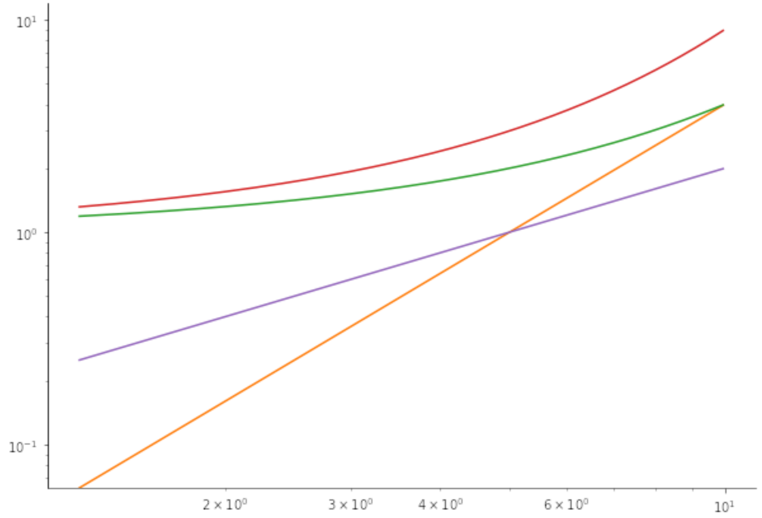
$$x_{\mathrm{px}} = a_{\mathrm{x}} \log(x) + b_{\mathrm{x}}$$

$$x = \exp\left(\frac{x_{\mathrm{px}}}{a_{\mathrm{x}}} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}}\right)$$

Logarithmic Scales – Log-Log Plot

- All functions of the form: $y = \alpha x^k$
  - Result in straight line.
  - Slope proportional to: k

- "Invariant" to:
  - ~~scaling and shifting~~
  - unit change
  - changing base



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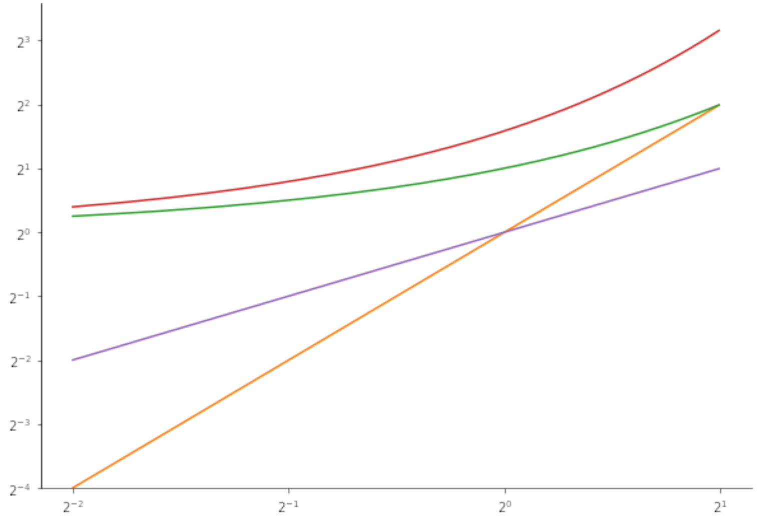


Base 2

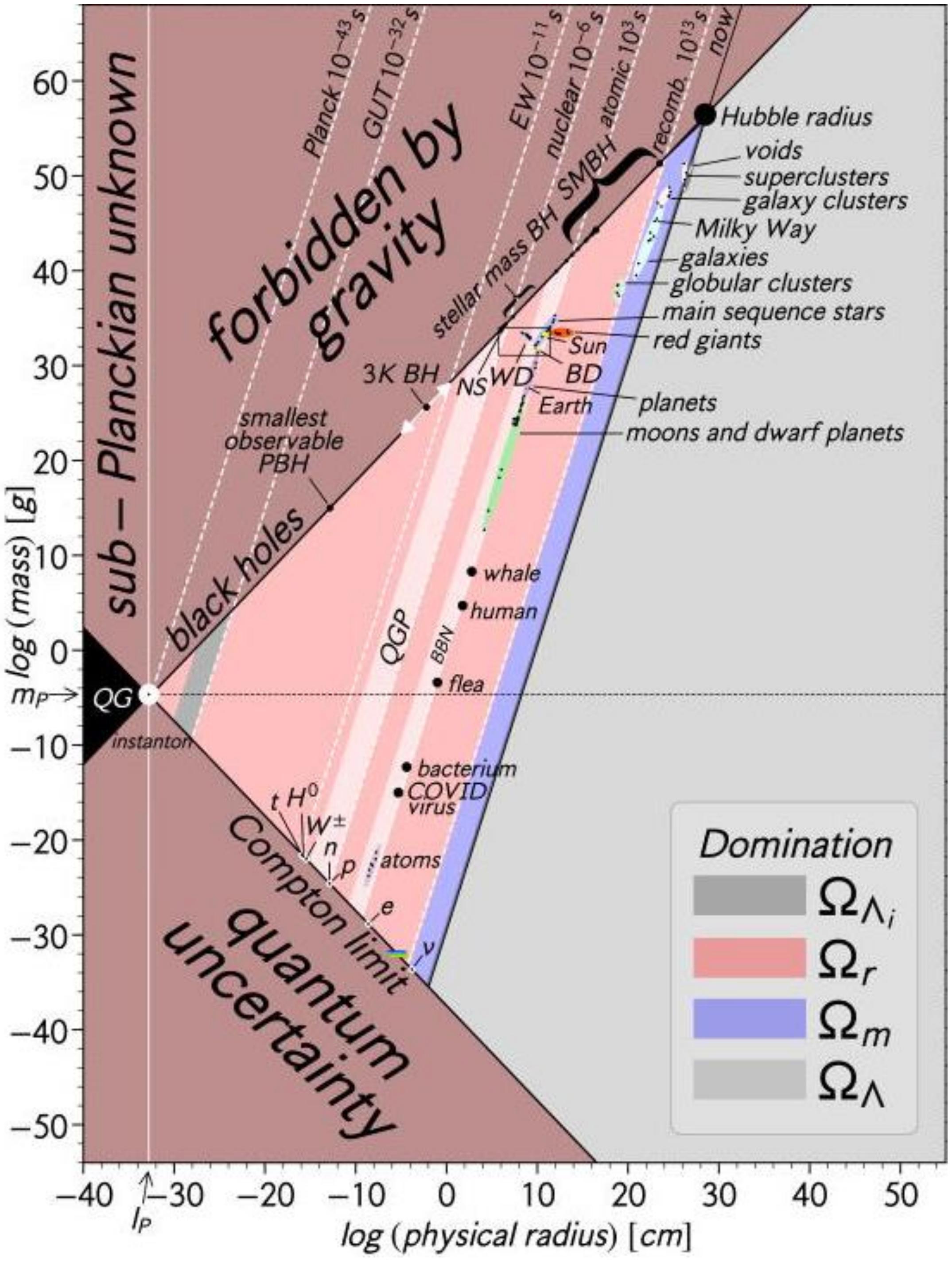
Chart of Everything

Why are most objects on sloped line?
Why is the slope approximately a=3?

$$y = x^k$$

$$y_{px} = \frac{a_y k}{a_x} x_{px} - \frac{a_y k b_x}{a_x} + b_y$$

$$y_{px} = a \; x_{px} + b$$



Charles H. Lineweaver; Vihan M. Patel: All objects and some questions