UNIVERSITY OF BIRMINGHAM

Visualisation

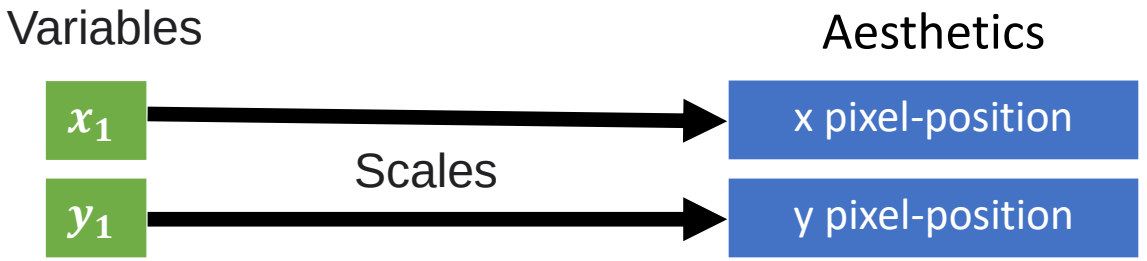
Week 1

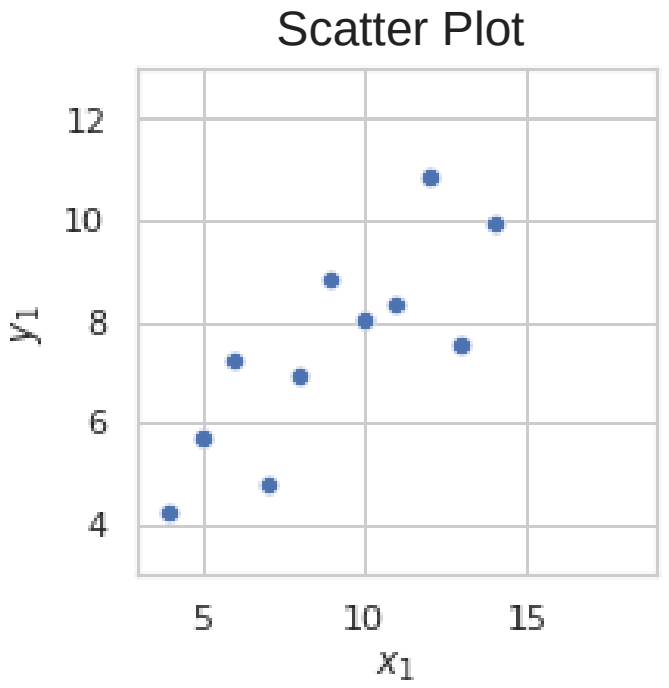
Position Scales

Mapping Variables to Aesthetics

Data

x_1	y_1
10	8.04
8	6.95
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68





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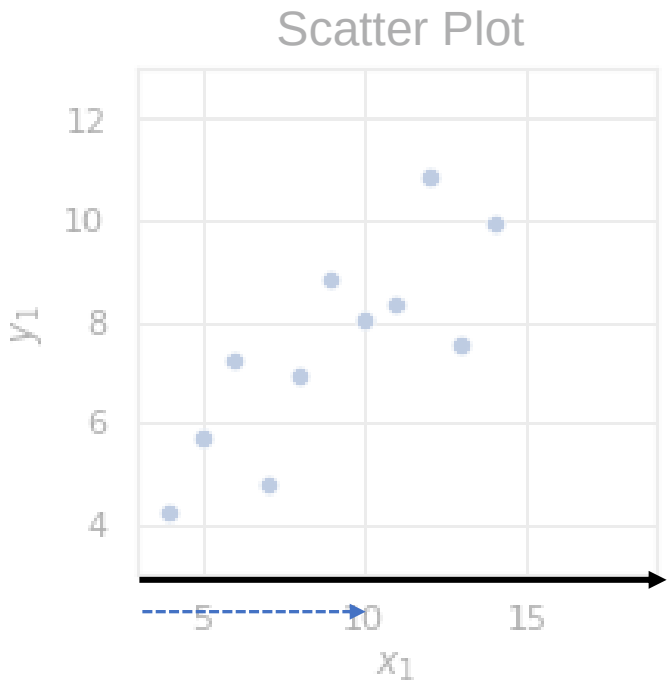

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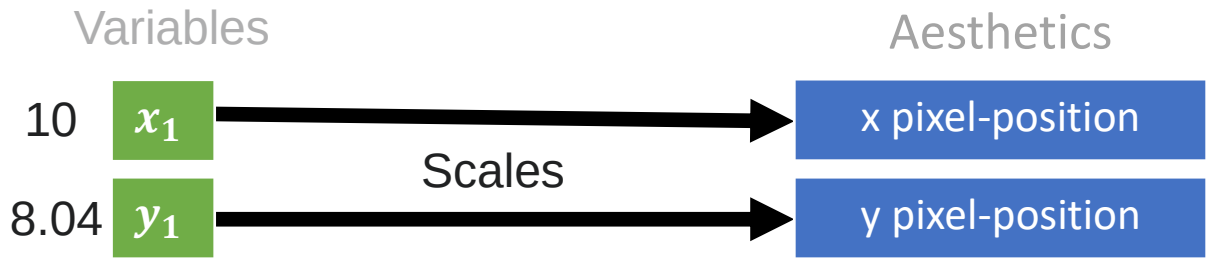
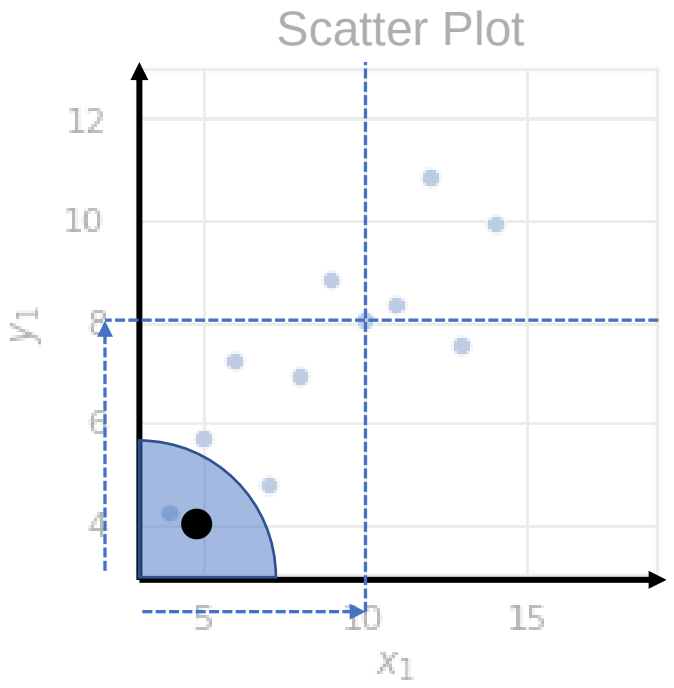
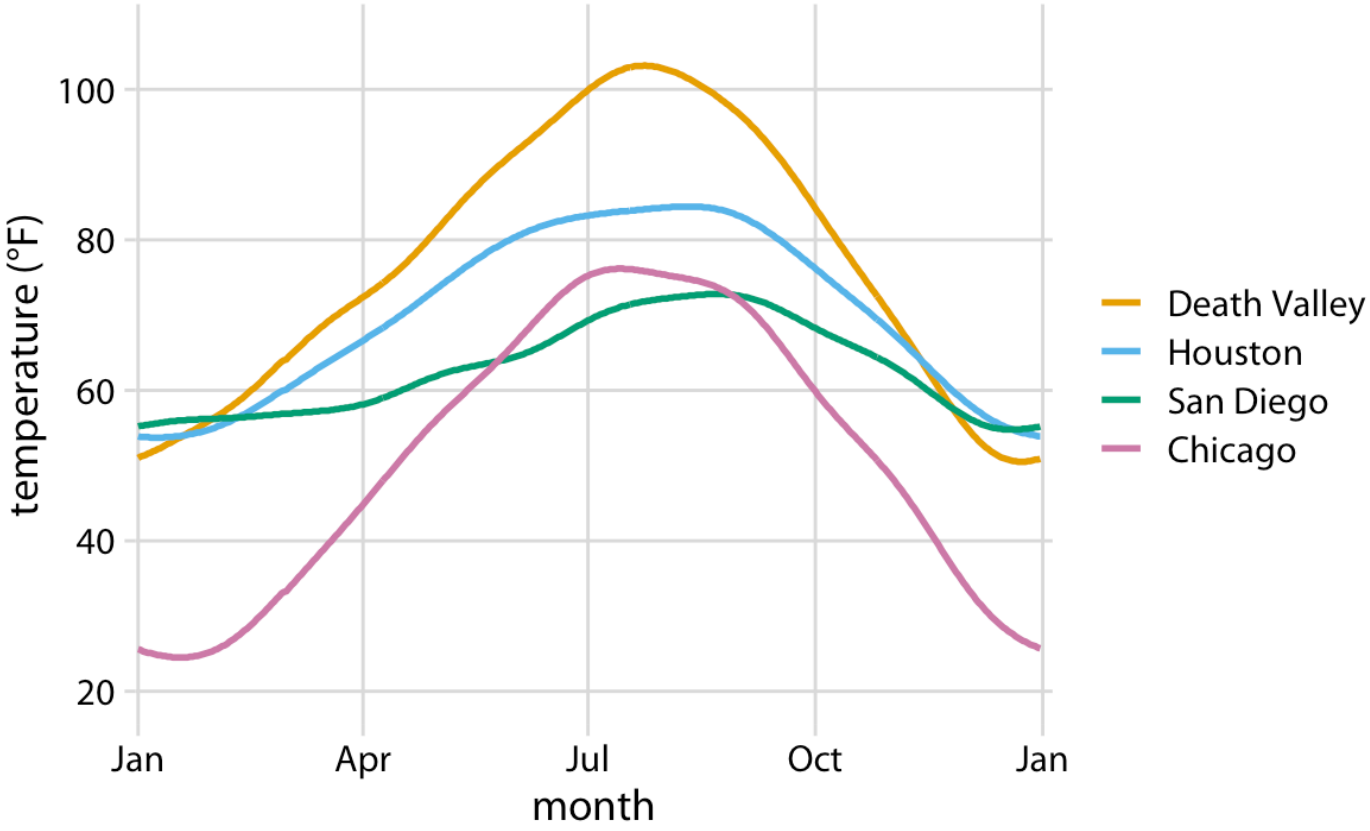




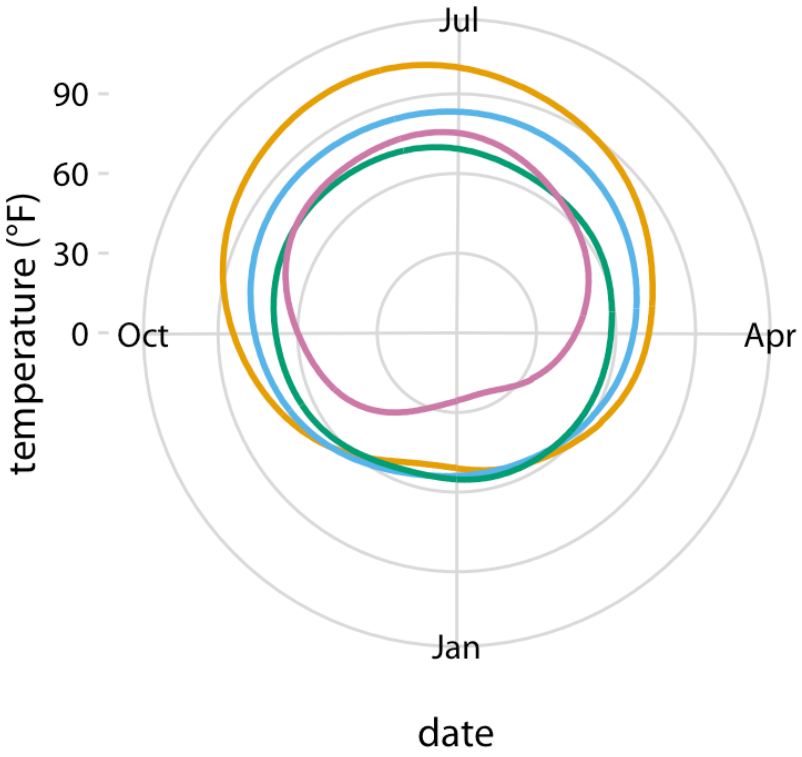
Cartesian Coordinates

Coordinate Systems

Cartesian Coordinates



Polar Coordinates



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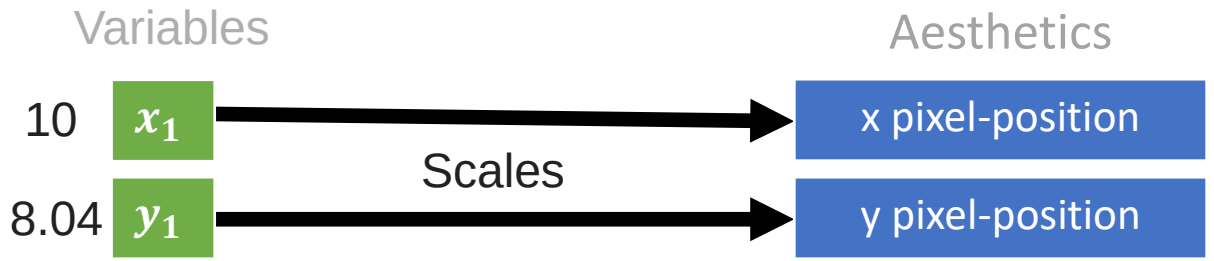
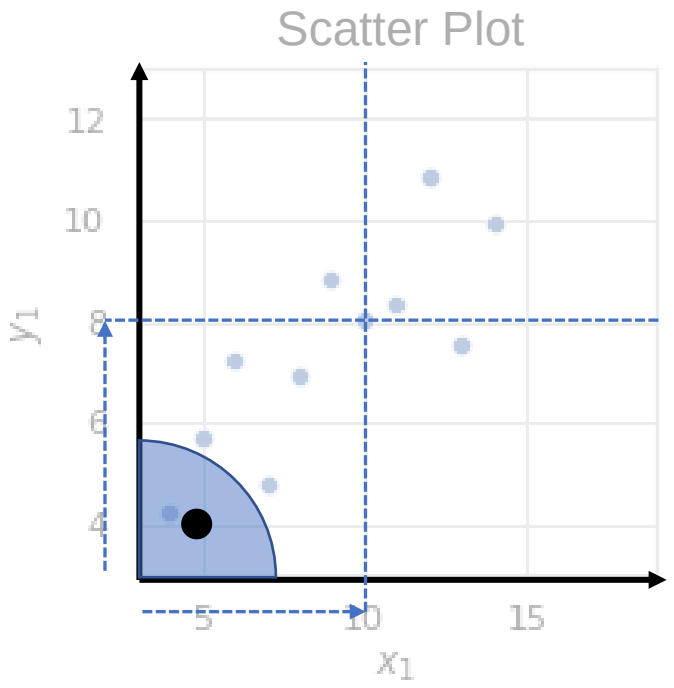
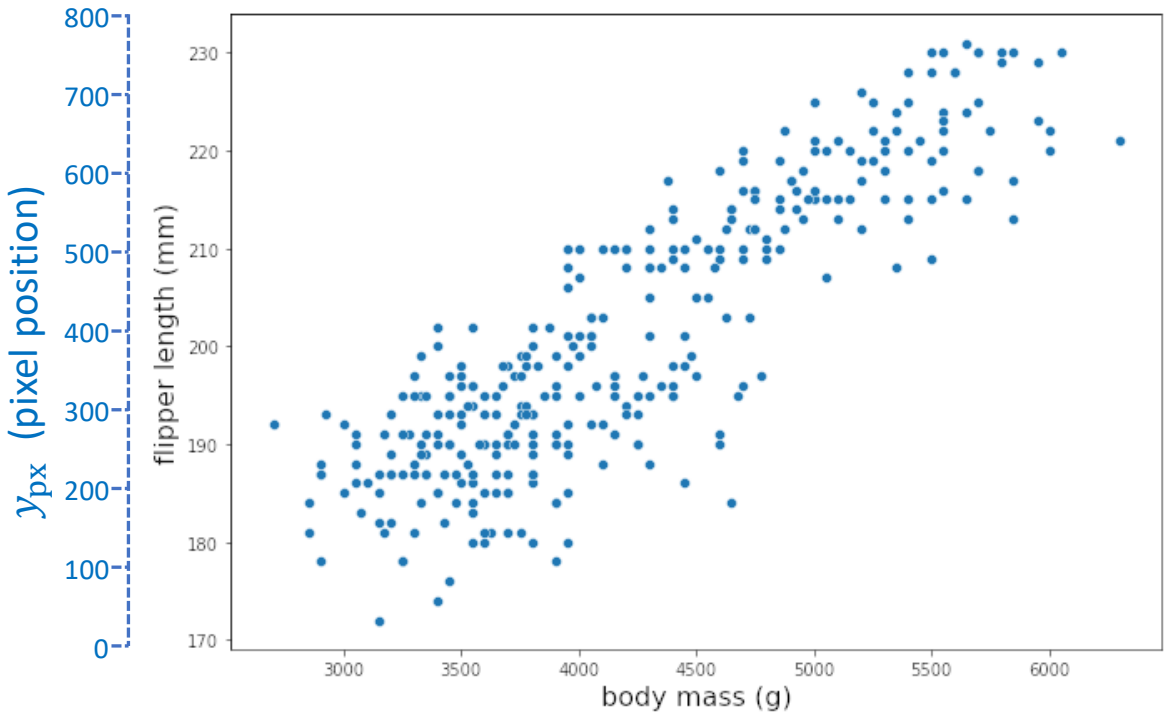




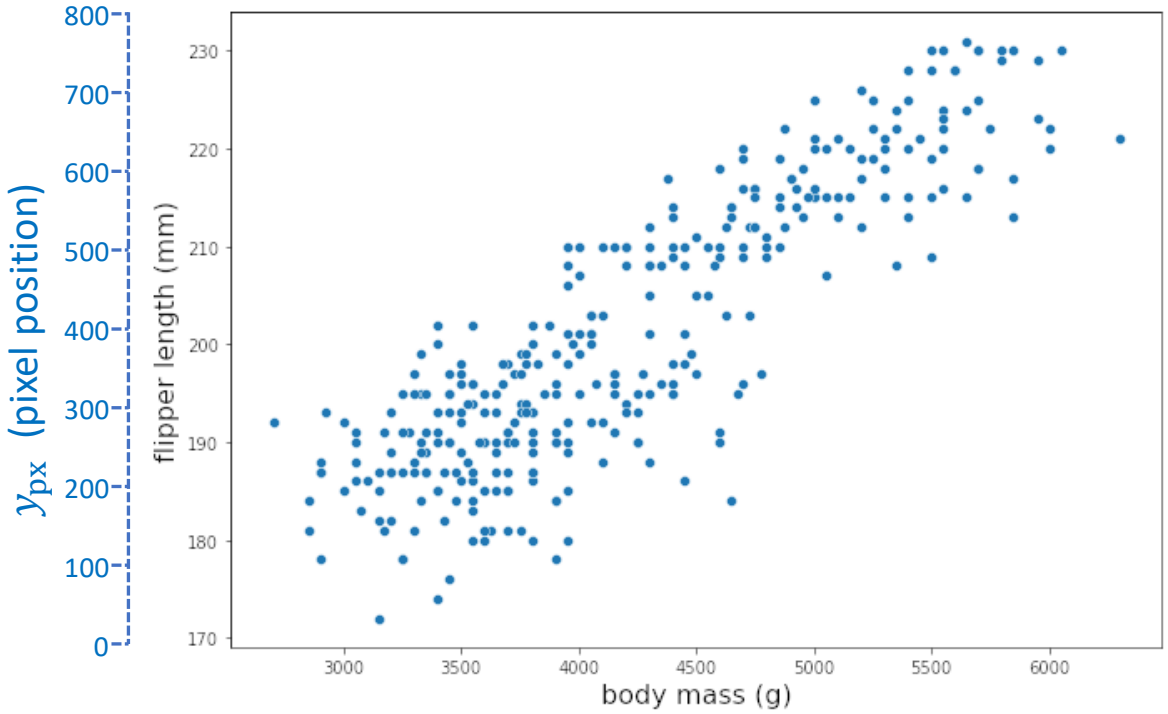
Cartesian Coordinates

Linear Scales

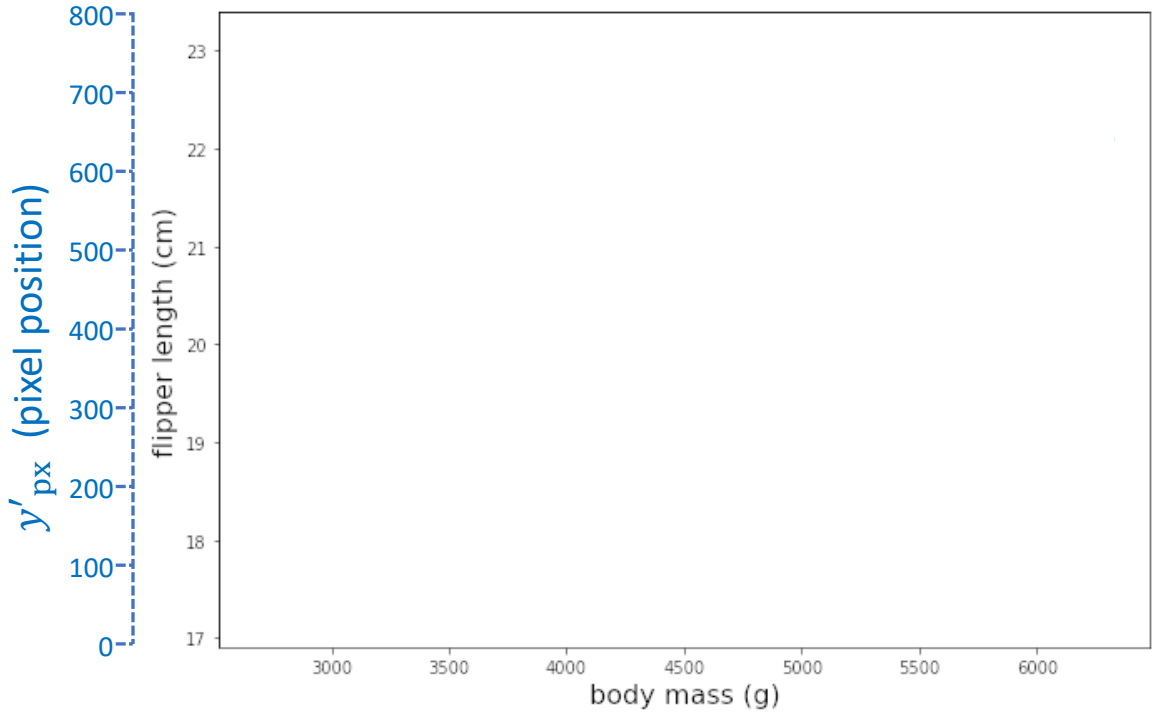


$$y_{\mathrm{px}} = a_{\mathrm{y}}\, y_{\mathrm{mm}} + b_{\mathrm{y}}$$

Linear Scales



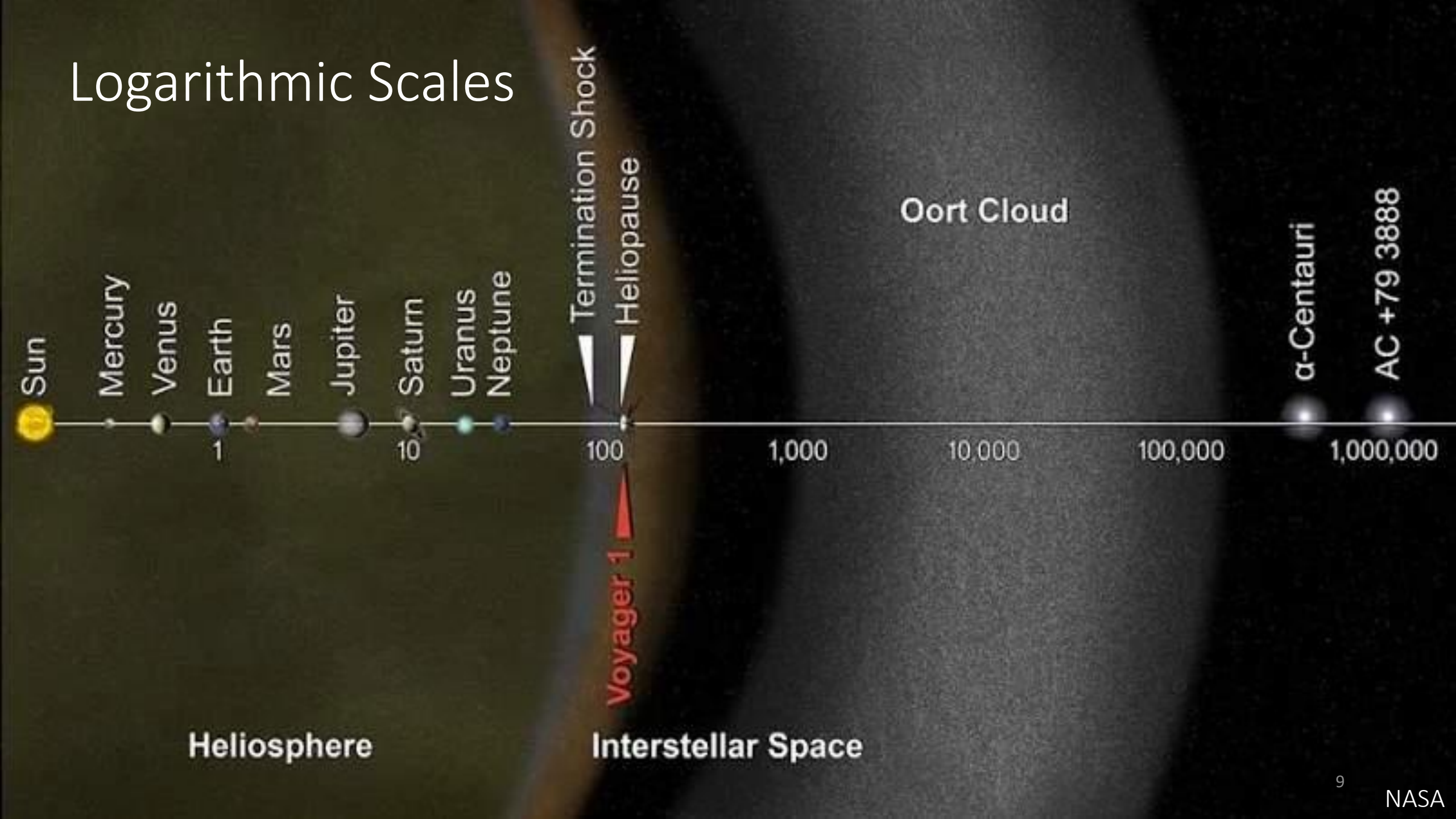
$$y_{\mathrm{px}} = a_{\mathrm{y}}\, y_{\mathrm{mm}} + b_{\mathrm{y}}$$



$$y'_{\mathrm{px}} = a'_{\mathrm{y}}\, y_{\mathrm{cm}} + b_{\mathrm{y}} \qquad | \; a'_{\mathrm{y}} = 10 a_{\mathrm{y}}$$

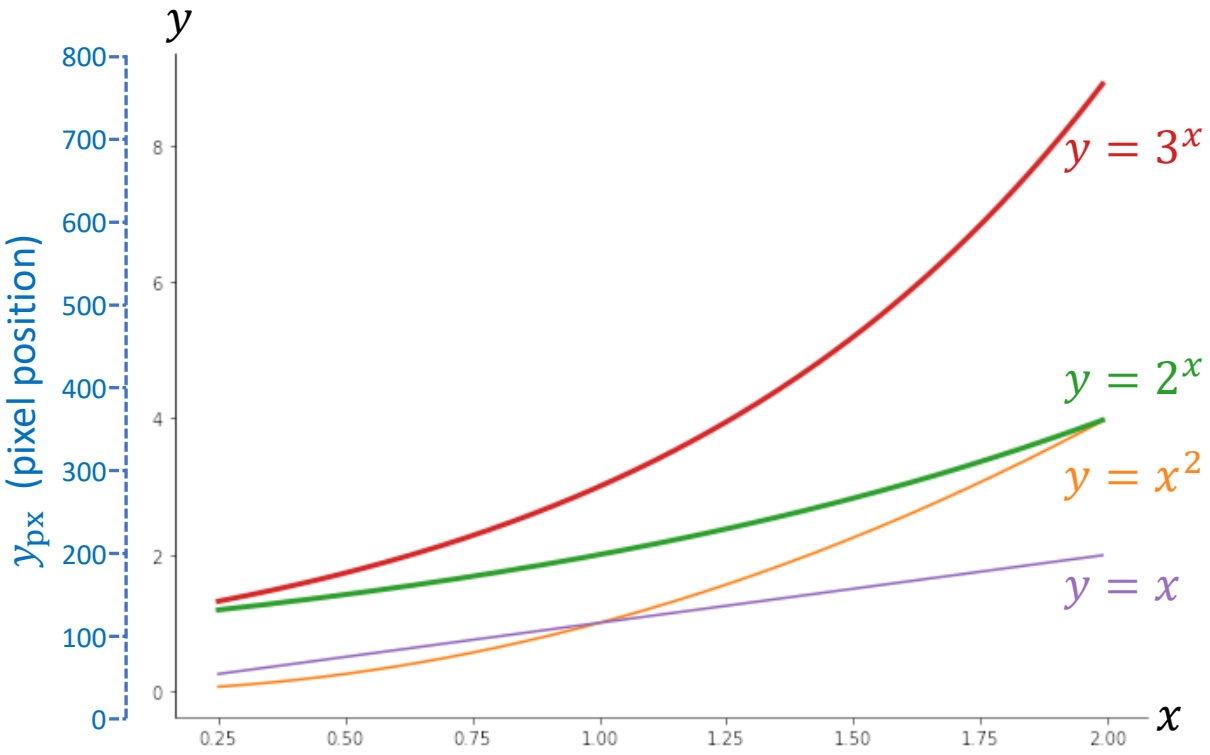
- "Invariant" to:
  - scaling and shifting
  - unit change

Logarithmic Scales

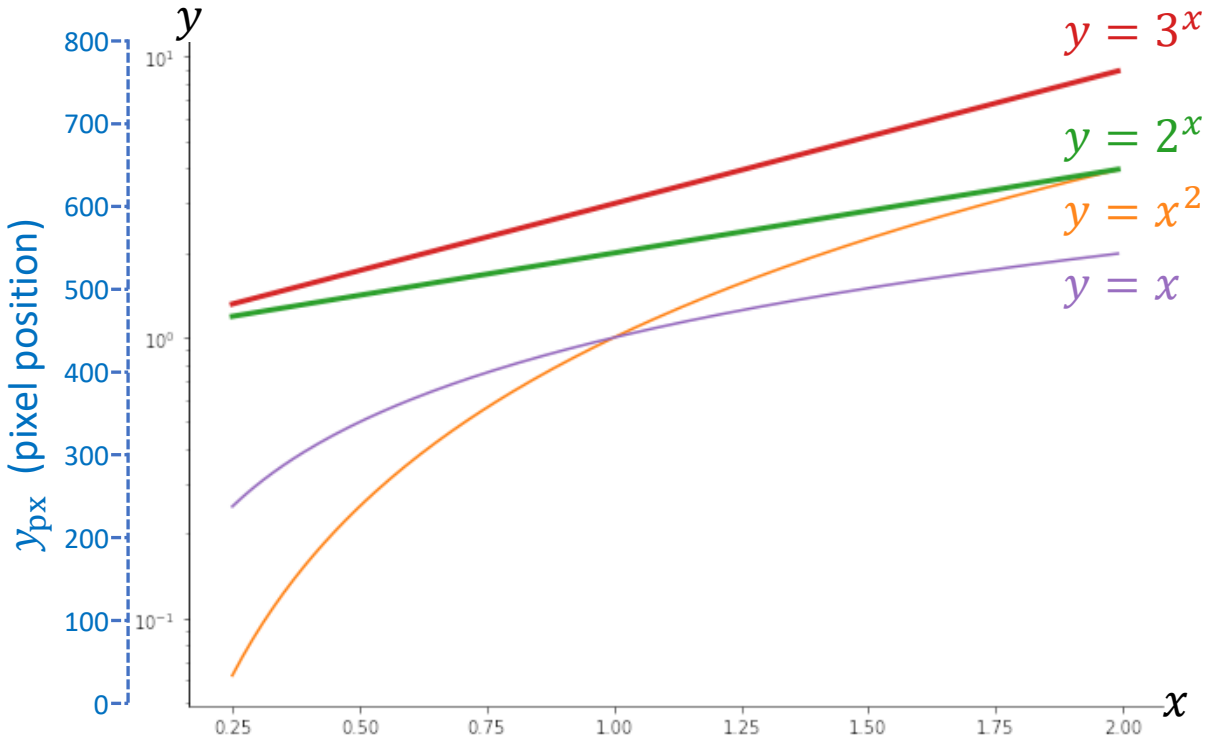


NASA

Logarithmic Scales – Log-Linear Plot

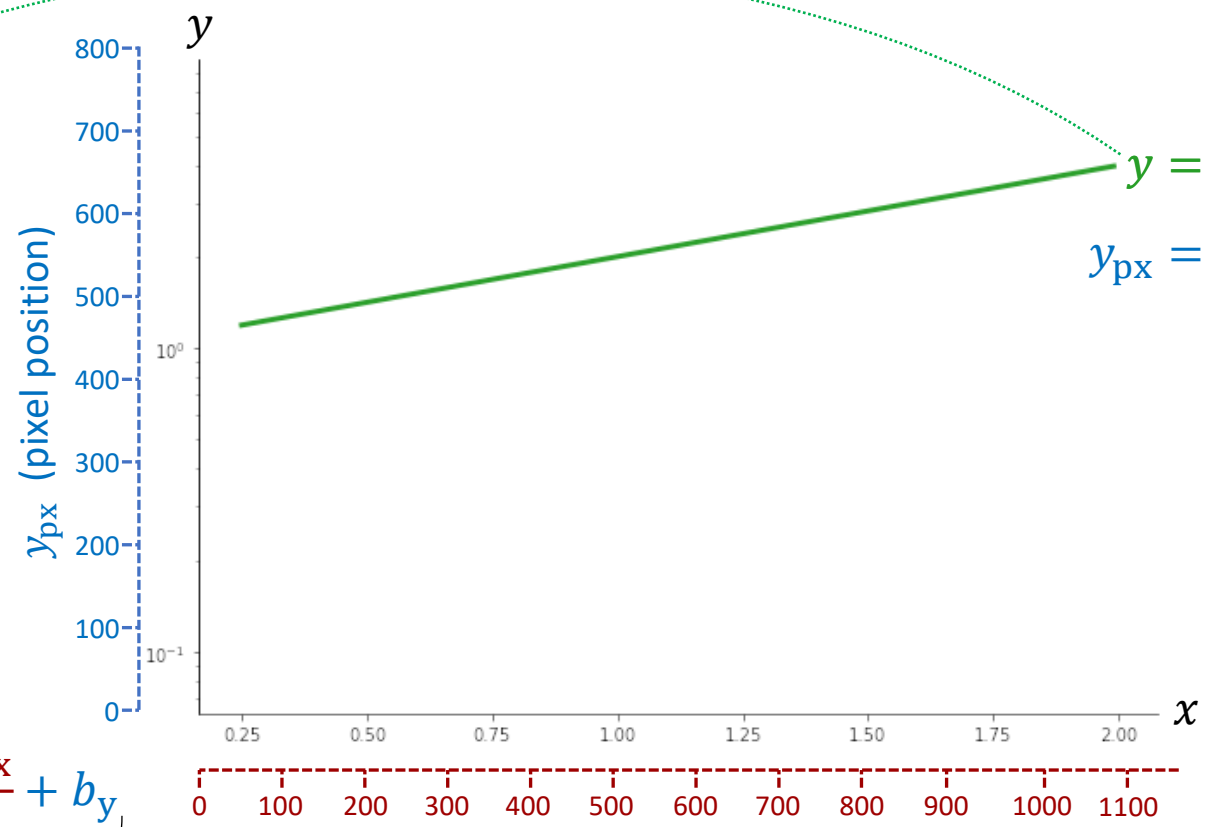


$$y_{px} = a_y\, y_y + b_y$$



$$y_{px} = a_y \log(y) + b_y$$

Logarithmic Scales – Log-Linear Plot



$y = \alpha^x$

$y_{\mathrm{px}} = \mathrm{f}(x_{\mathrm{px}})$

$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(\alpha^x) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(\alpha) x + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(\alpha) \left(\frac{x_{\mathrm{px}}}{a_x} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}} \right) + b_{\mathrm{y}}$$
$$= \underbrace{\frac{a_{\mathrm{y}}}{a_{\mathrm{x}}} \log(\alpha)}_{a} \; \underbrace{x_{px}}_{x_{px}} \; \underbrace{- \frac{a_{\mathrm{y}} \log(\alpha)\, b_{\mathrm{x}}}{a_{\mathrm{x}}} + b_{\mathrm{y}}}_{b}$$
$$y_{\mathrm{px}} = a \quad x_{px} \quad + \quad b$$

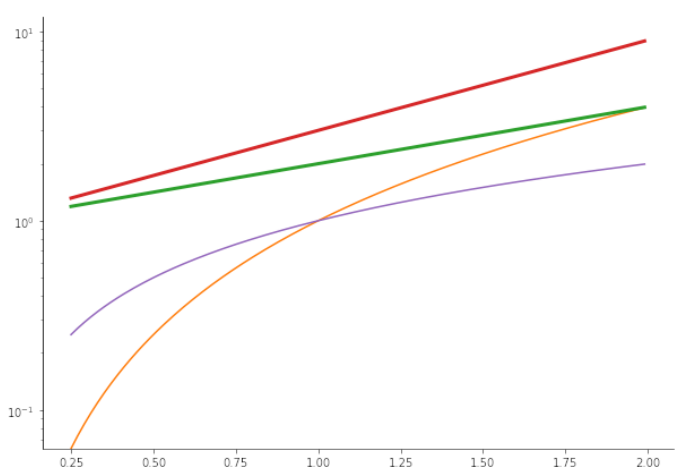
x_{px} (pixel position)

$$x_{\mathrm{px}} = a_{\mathrm{x}}\, x + b_{\mathrm{x}}$$
$$x = \frac{x_{\mathrm{px}}}{a_{\mathrm{x}}} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}}$$

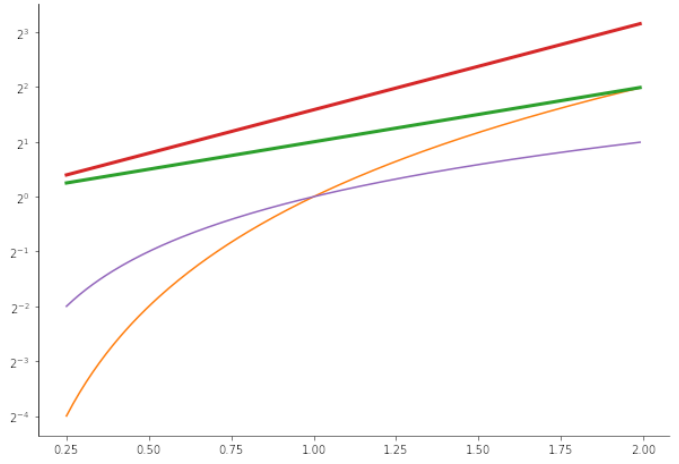
Logarithmic Scales – Log-Linear Plot

- All functions of the form: $y = \lambda\alpha^x$
  - Result in straight line.
  - Slope is proportional to: $\log(\alpha)$

- "Invariant" to:
  - scaling and shifting
  - unit change
  - changing base



Base 10



Base 2

Invariance to Scaling but NOT Shifting

Scaling:

$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(yc) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(y) + \log(c) + b_{\mathrm{y}}$$

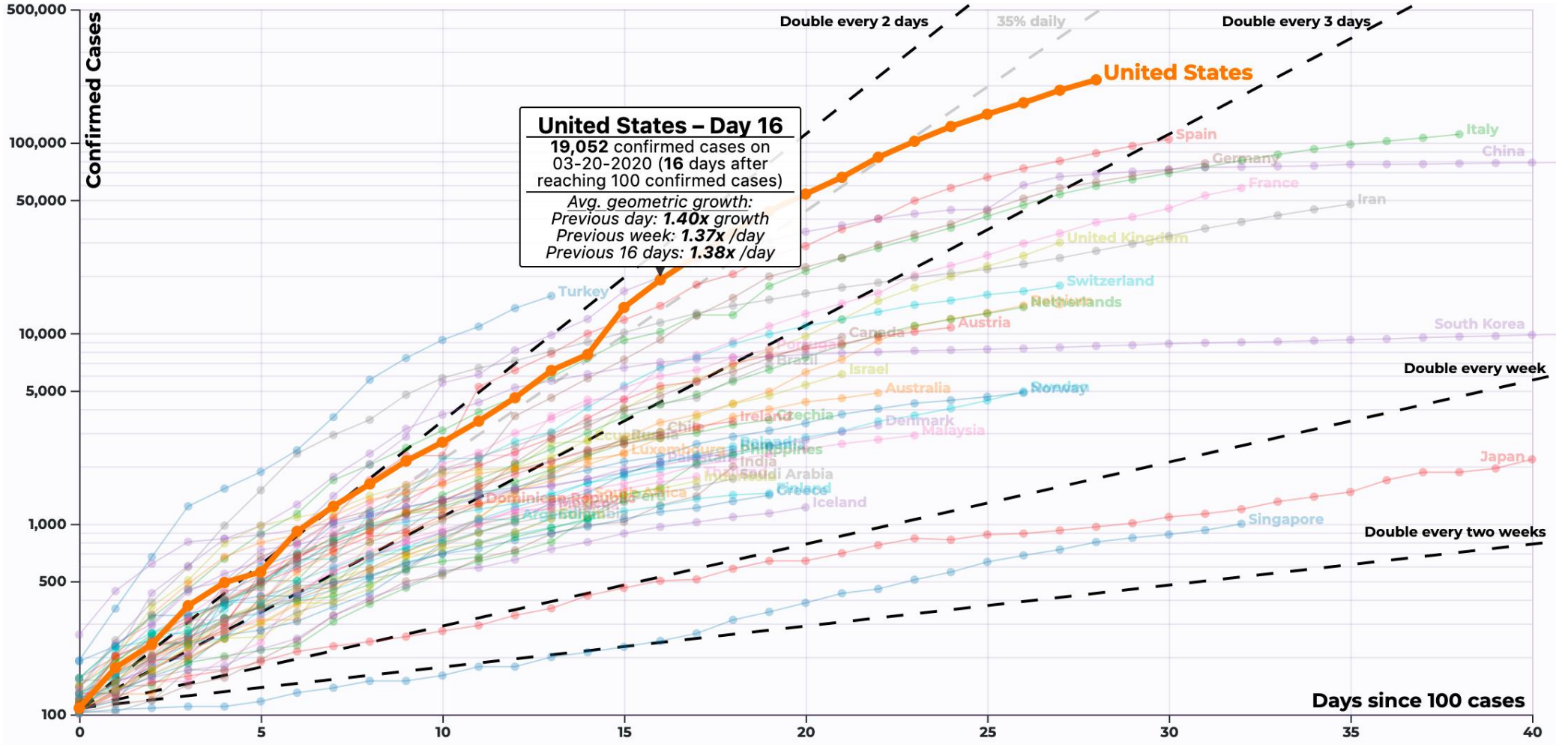
- We can adjust b_{y}

Shifting:

$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y + c) + b_{\mathrm{y}}$$

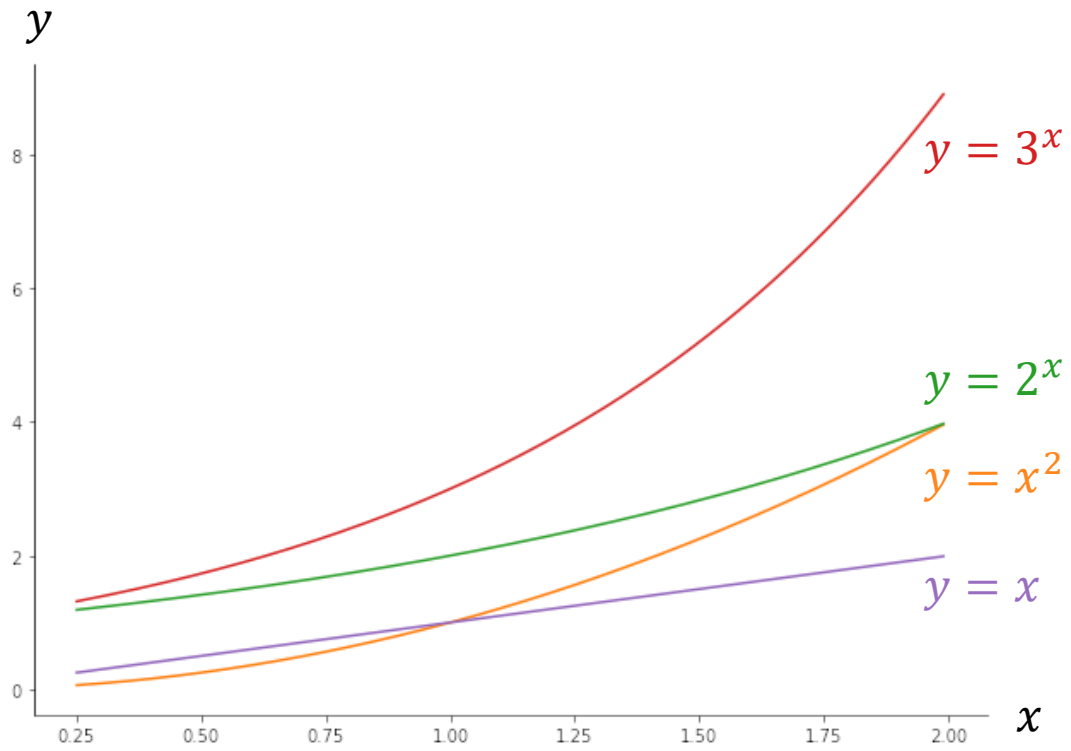
- Nothing we can do!

Logarithmic Scales – Log-Linear Plot

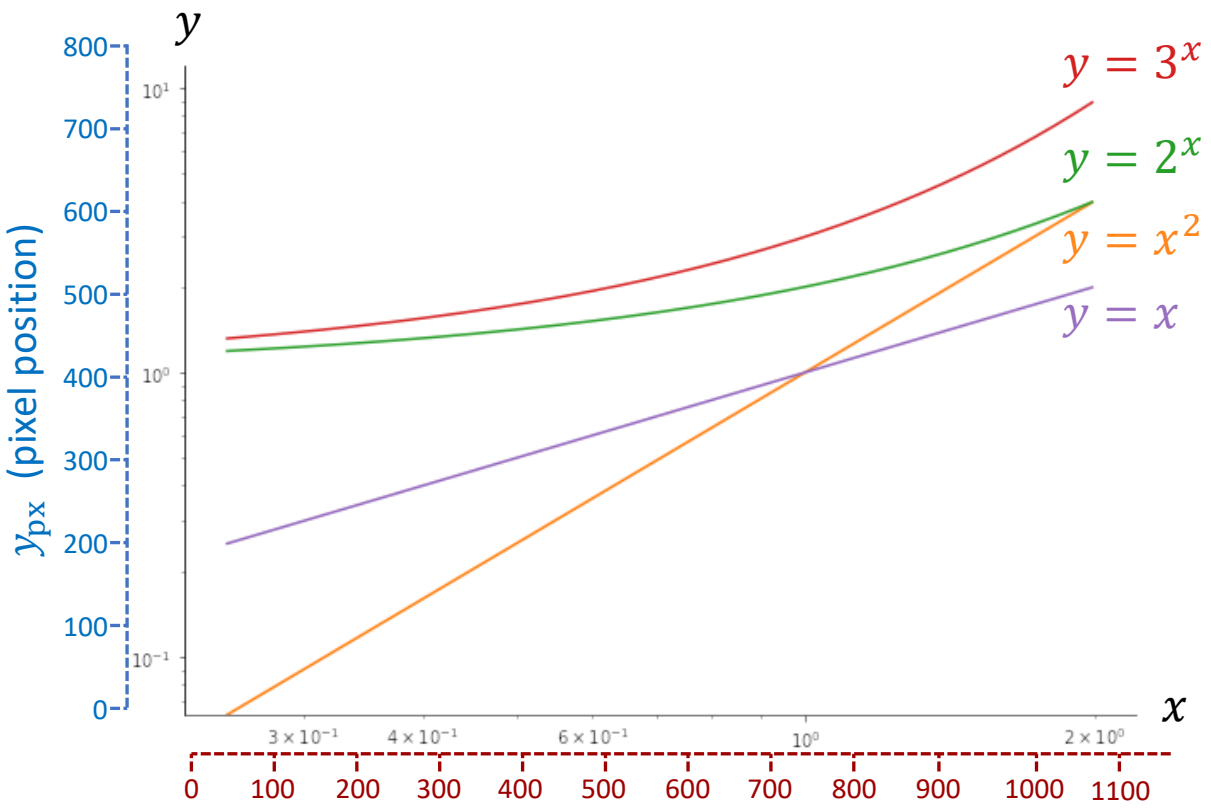


The Verge, 91-divoc.com

Logarithmic Scales – Log-Log Plot



$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y) + b_{\mathrm{y}}$$

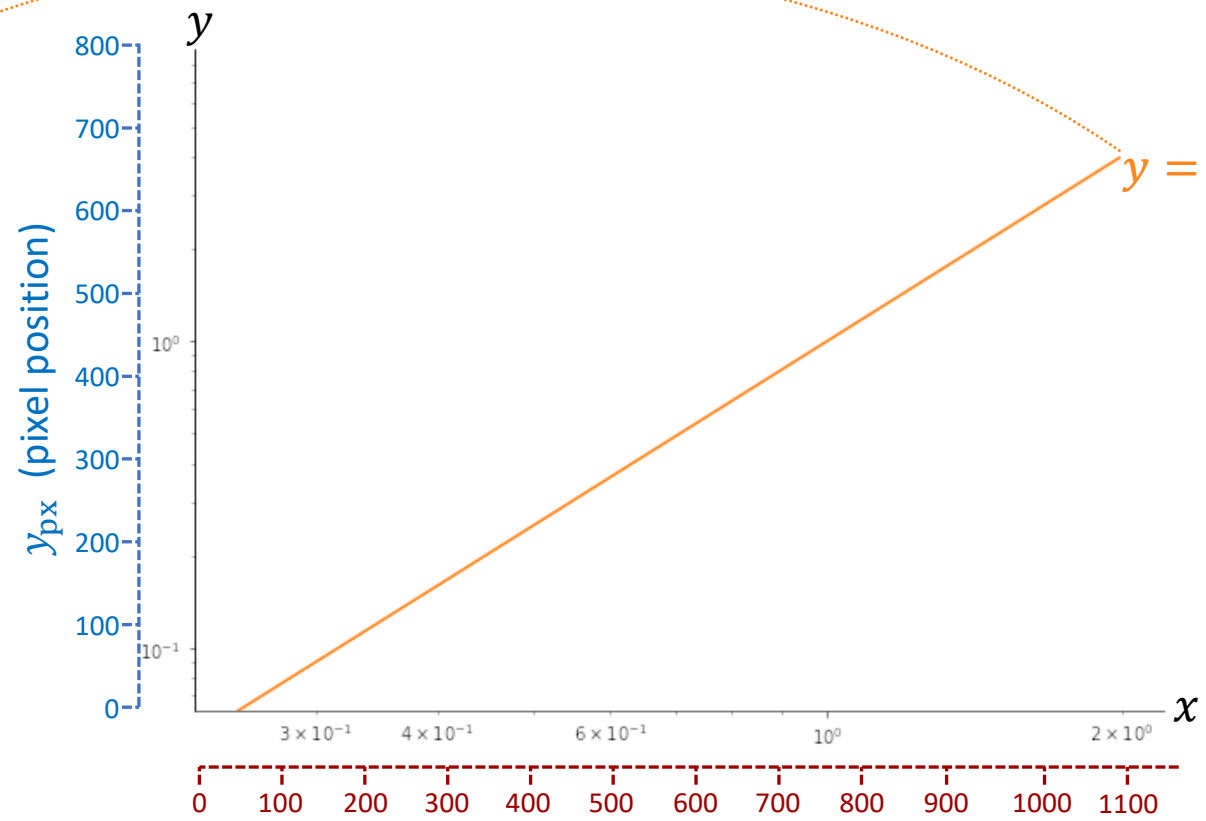


x_{px} (pixel position)

$$x_{\mathrm{px}} = a_{\mathrm{x}} \log(x) + b_{\mathrm{x}}$$

Logarithmic Scales – Log-Log Plot

$$y_{\mathrm{px}} = a_{\mathrm{y}} \log(y) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(x^k) + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \log(x) k + b_{\mathrm{y}}$$
$$= a_{\mathrm{y}} \left(\frac{x_{\mathrm{px}}}{a_{\mathrm{x}}} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}} \right) k + b_{\mathrm{y}}$$
$$= \underbrace{\frac{a_{\mathrm{y}} k}{a_{\mathrm{x}}}}_{a} x_{\mathrm{px}} \underbrace{- \frac{a_{\mathrm{y}} k b_{\mathrm{x}}}{a_{\mathrm{x}}} + b_{\mathrm{y}}}_{b}$$
$$y_{\mathrm{px}} = a \; x_{px} + b$$



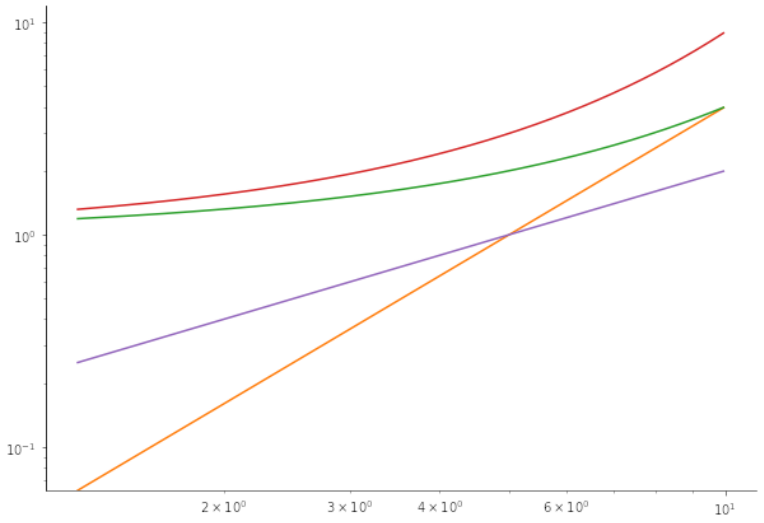
$$x_{\mathrm{px}} = a_{\mathrm{x}} \log(x) + b_{\mathrm{x}}$$

$$x = \exp\left(\frac{x_{\mathrm{px}}}{a_{\mathrm{x}}} - \frac{b_{\mathrm{x}}}{a_{\mathrm{x}}} \right)$$

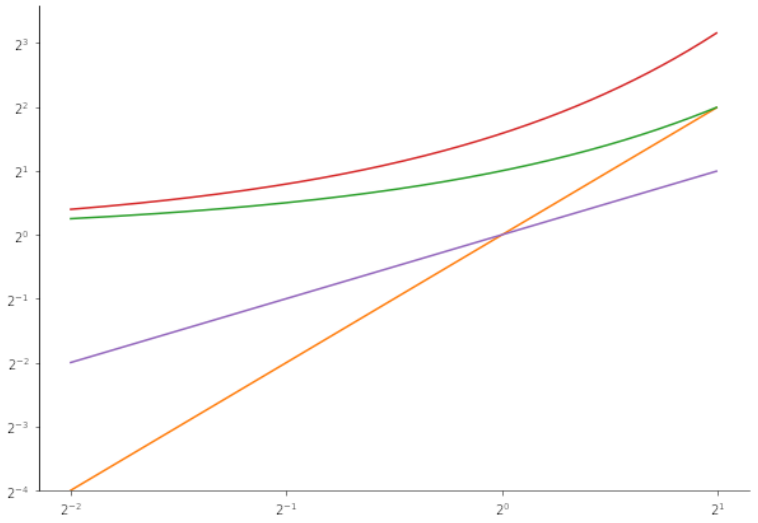
Logarithmic Scales – Log-Log Plot

- All functions of the form: $y = \alpha x^k$
  - Result in straight line.
  - Slope proportional to: k

- "Invariant" to:
  - scaling and shifting
  - unit change
  - changing base

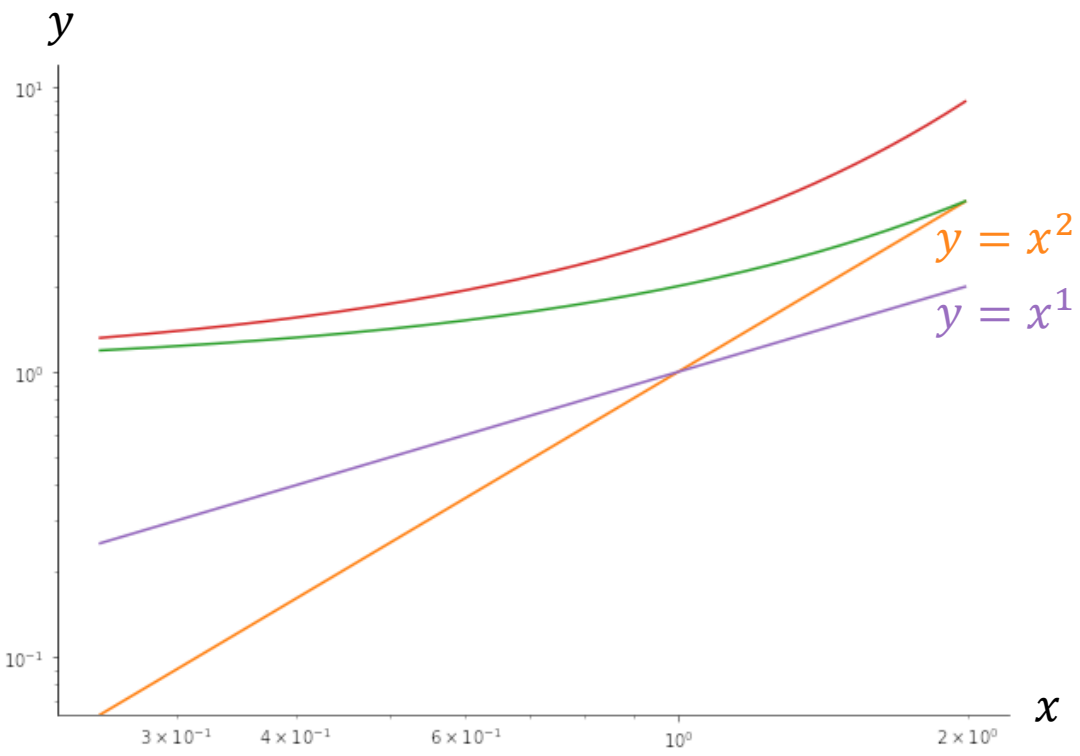


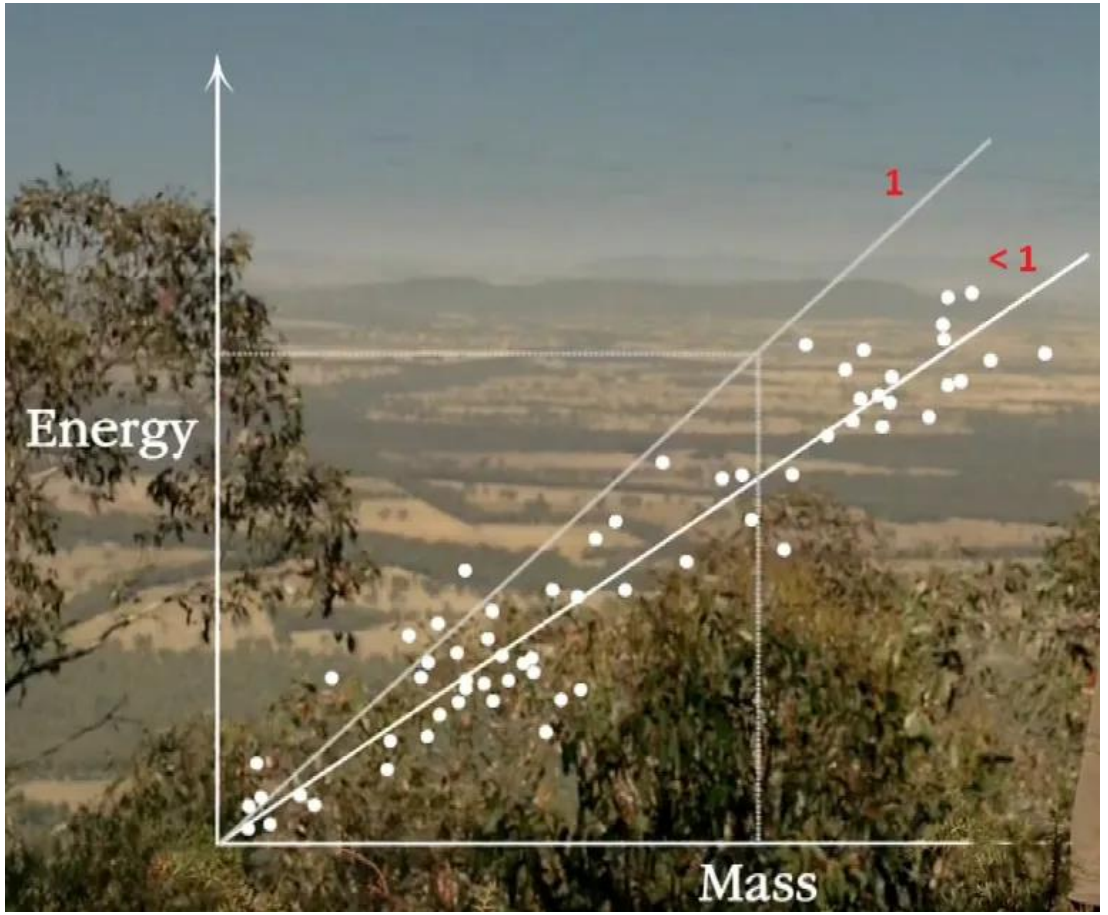
Base 10



Base 2

Log-Log Plots Application

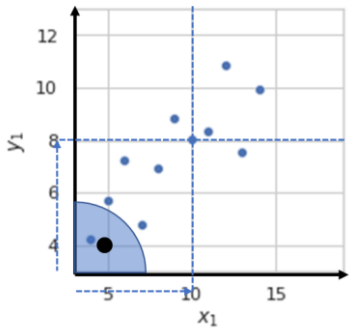


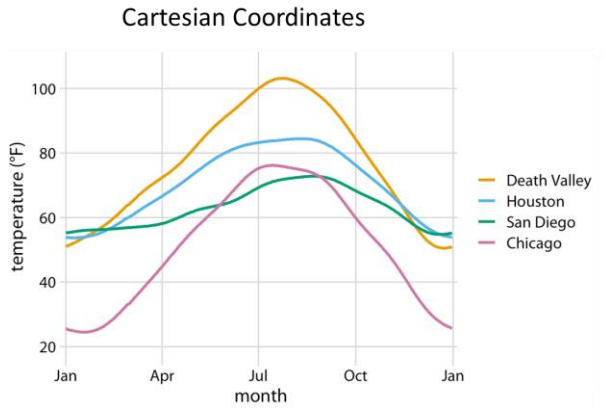


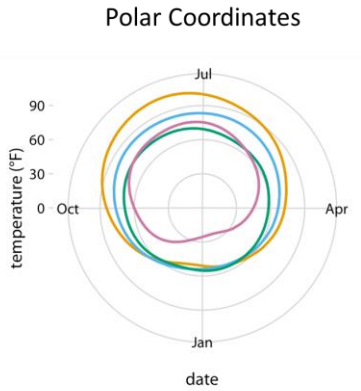
BBC Wonder of Life (Brian Cox) via statisticsbyjim.com

Summary

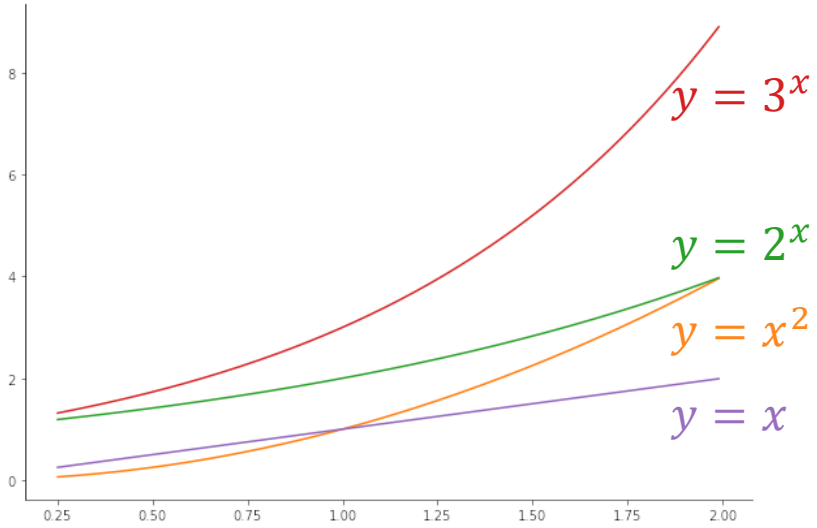
- Pos. scales map variables to positions along an axis.





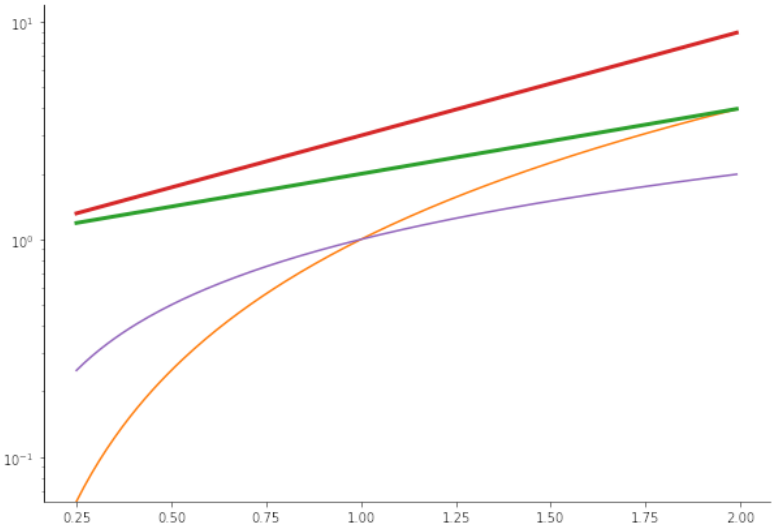


- Plots using different scales:



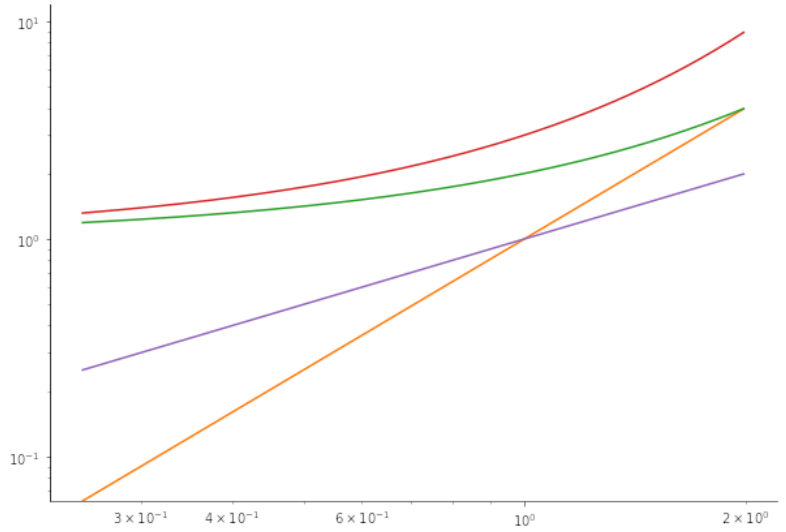
Linear

$y = \alpha x + b$ is a line



Log-Linear

$y = \lambda\alpha^{\gamma x}$ is a line



Log-Log

$y = \alpha x^k$ is a line