UNIVERSITY OF BIRMINGHAM

Visualisation

Week 3

Fourier Transform

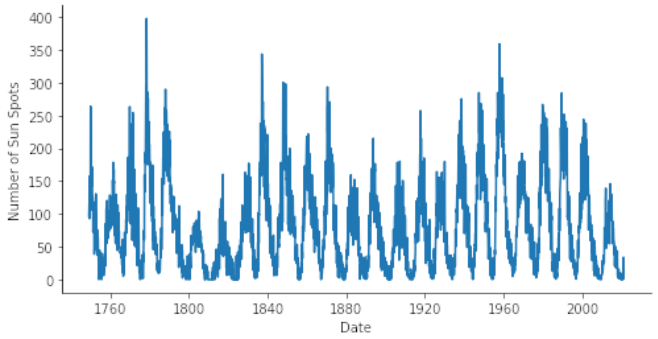
What is the Fourier Transform?

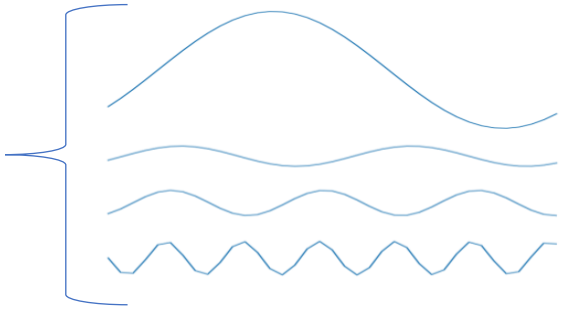
Time Domain:

- What happens at each time point?

Frequency Domain:

- What happens at each frequency?





Jean Baptiste Joseph Fourier (1768 - 1830)

- Will help us visualising hidden aspects.
- Will help us with smoothing and filtering time series.

Terminology

Fourier Transform:

- Mathematical mapping from time domain to frequency domain.

Discrete Fourier Transform (DFT):

- Discretized version, mapping evenly spaced time series to discretized frequency domain.

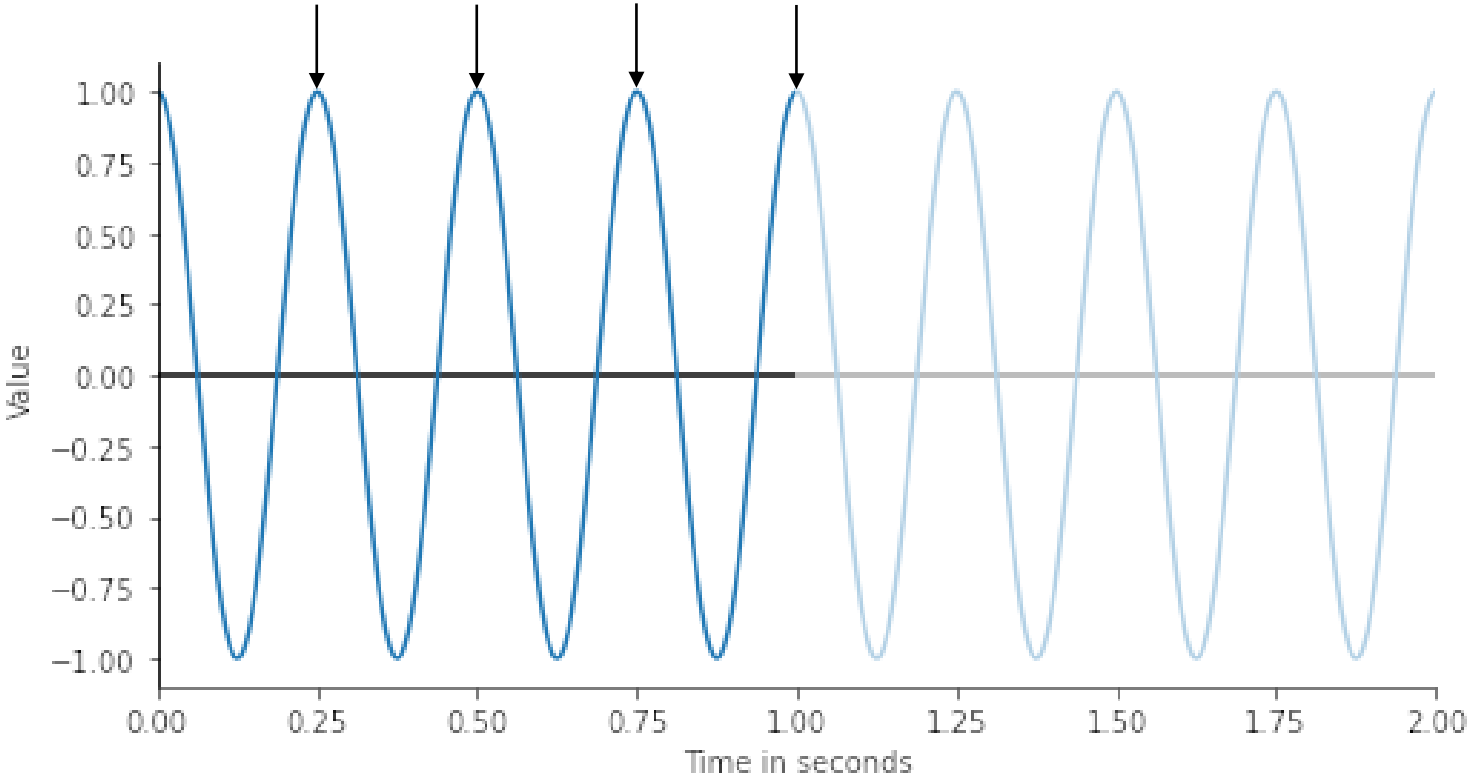
Fast Fourier Transform (FFT):

- Highly efficient algorithm implementing DFT.

$$O(n \log(n))$$

Jean Baptiste Joseph Fourier (1768 - 1830)

Period and Frequency



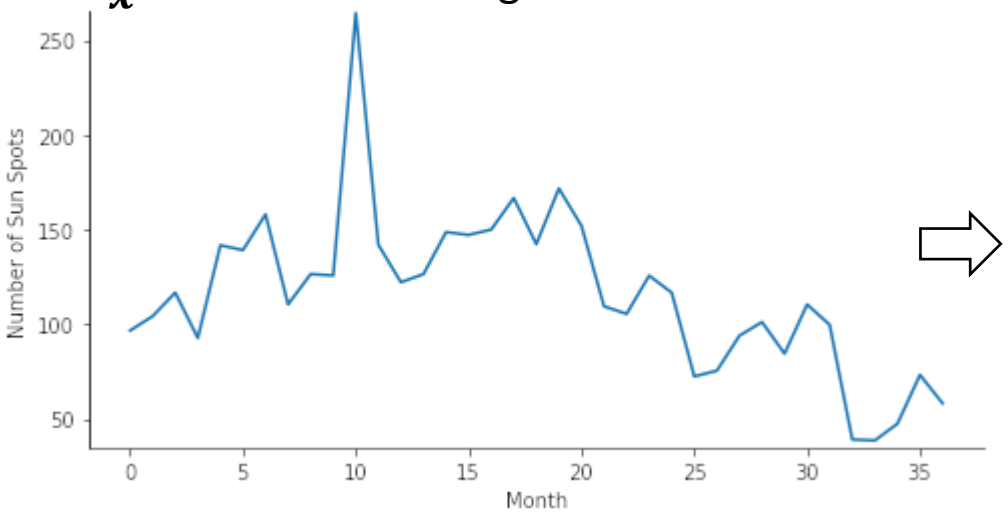
Period:
How long does the pattern take to repeat itself?
$\boldsymbol{P = 0.25s.}$

Frequency:
How often does the pattern repeat per unit time?
$\boldsymbol{f = 4s^{-1} = 4}$ **Hz.**

$$\boldsymbol{f = \frac{1}{P}}$$

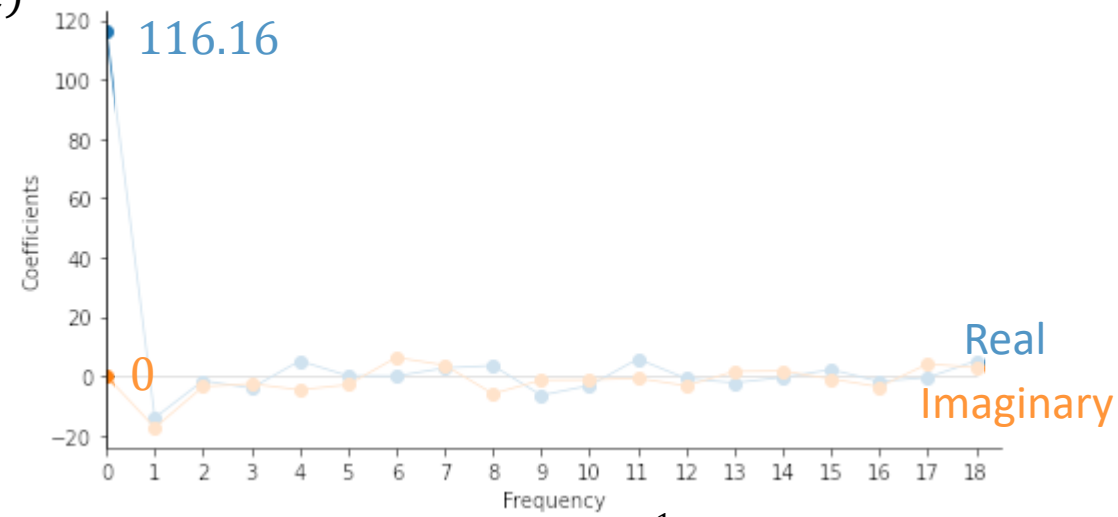
x

Original



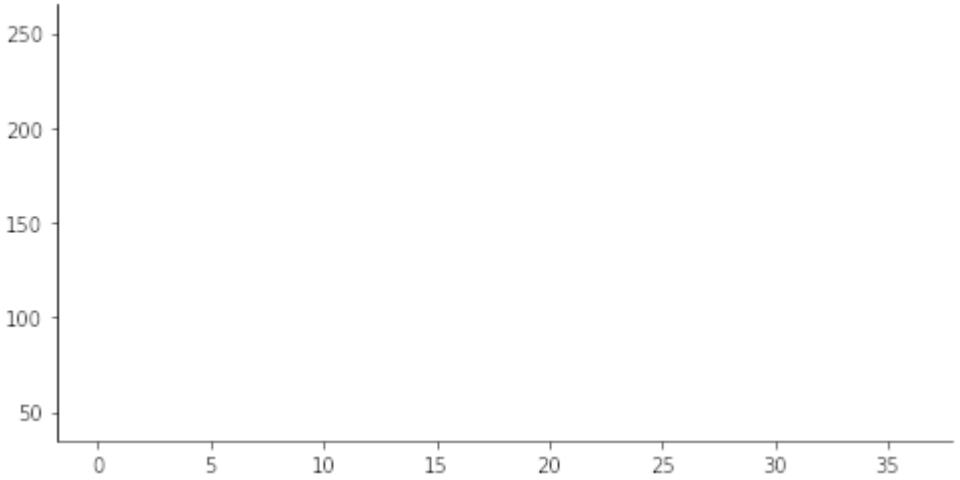
Discrete Fourier Transform

$F(x)$



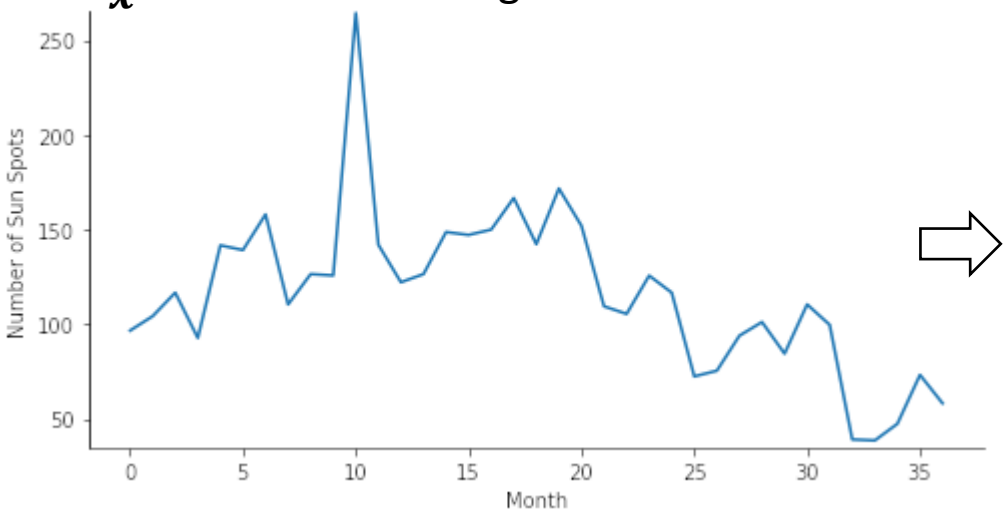
(series length^{-1})

Reconstruction



Original

x



Discrete Fourier Transform

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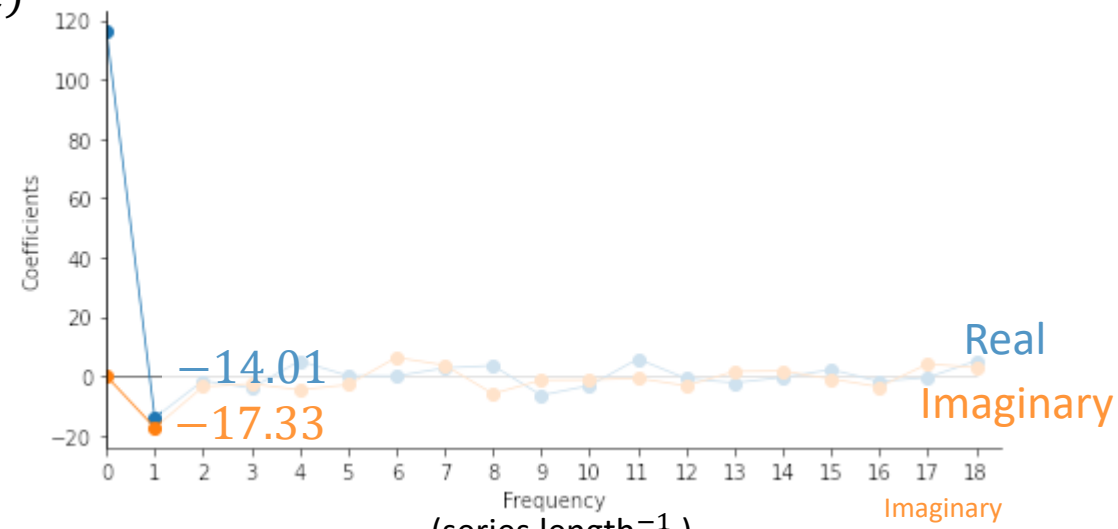

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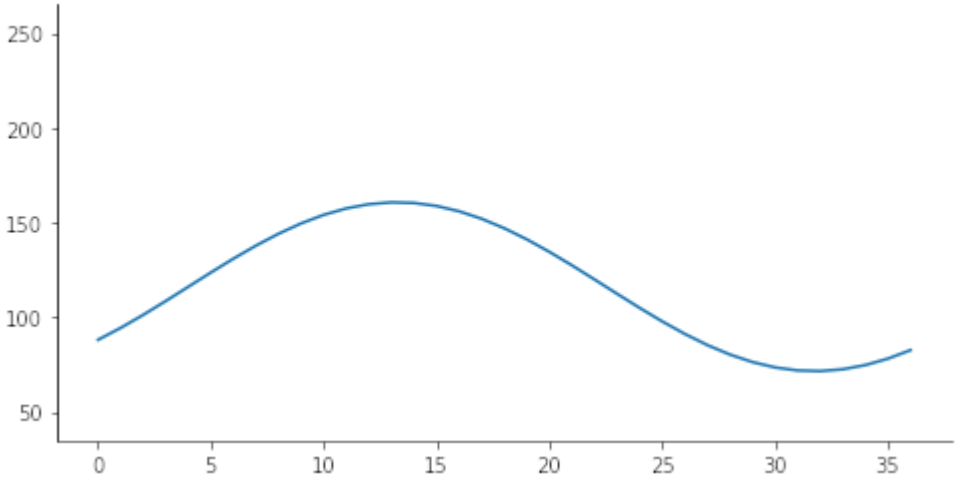
Reconstruction



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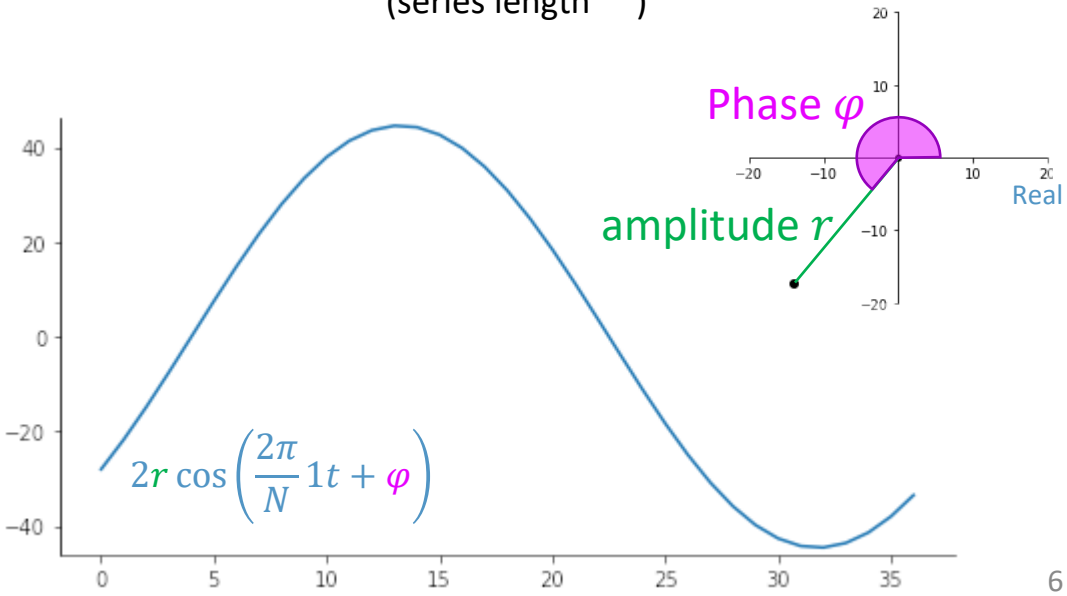
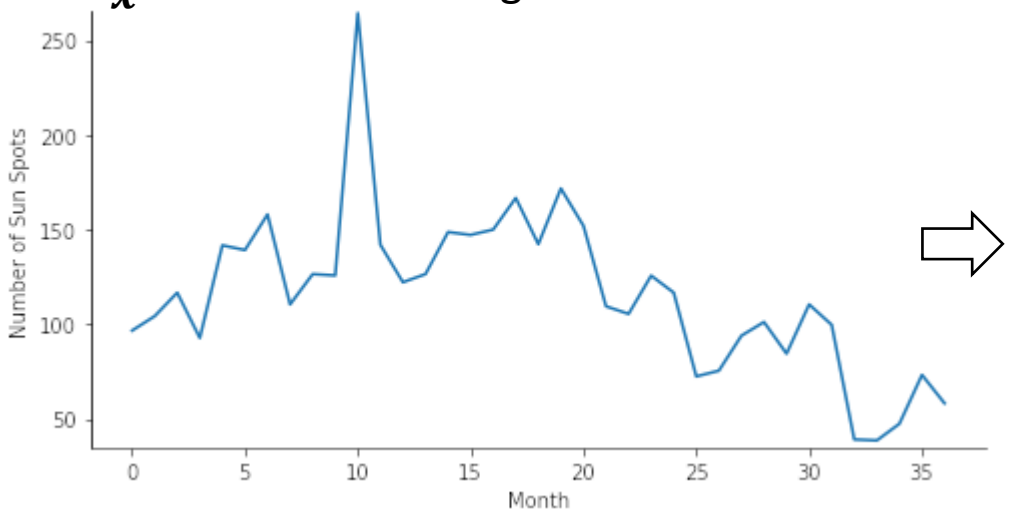


x

Original



Discrete Fourier Transform

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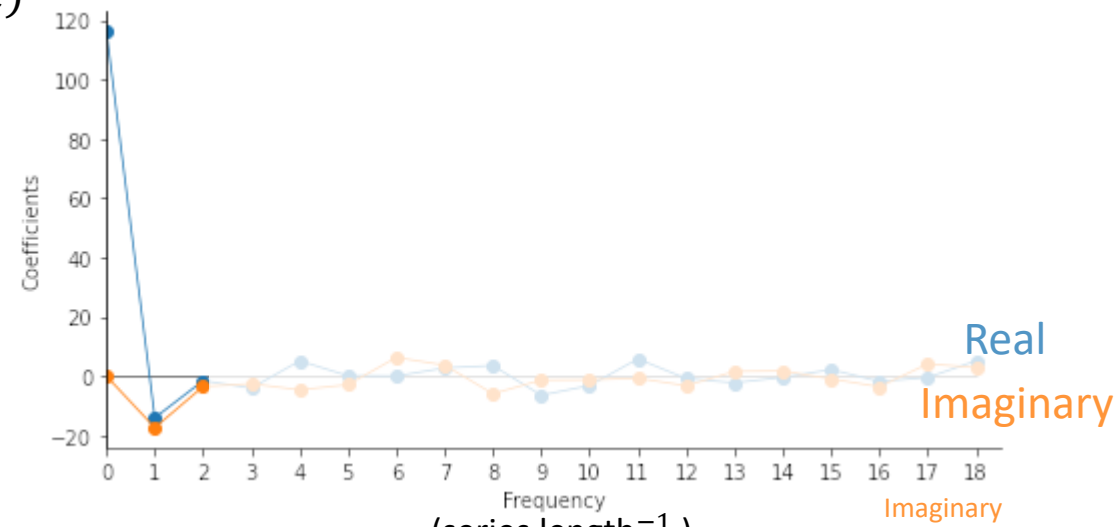

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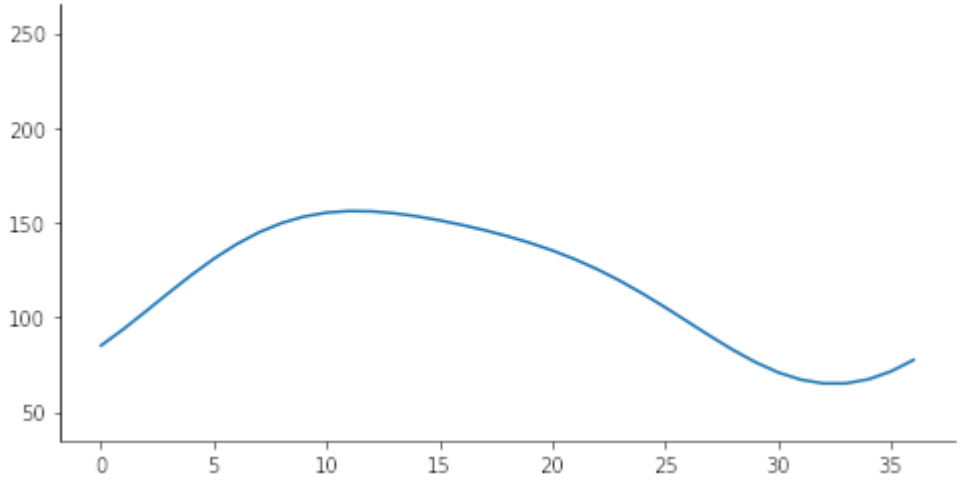
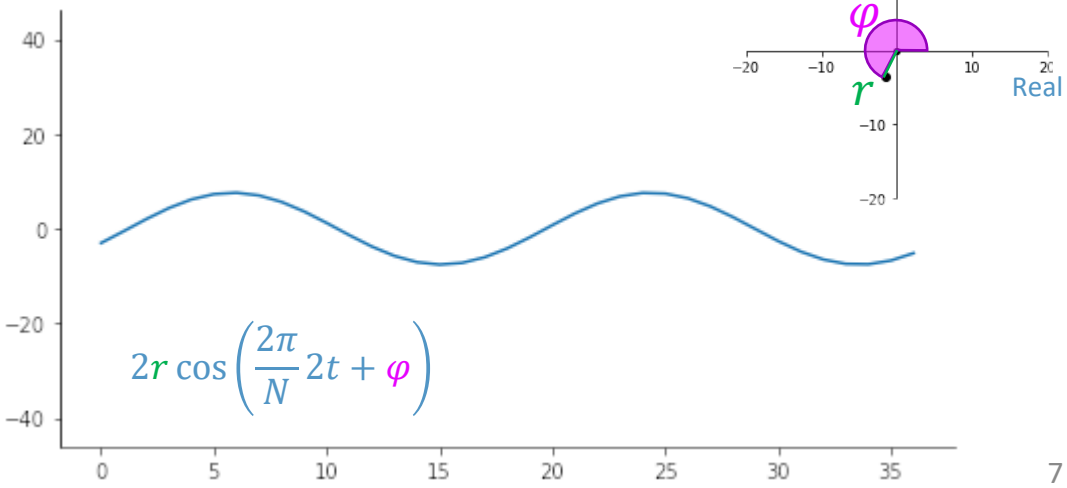
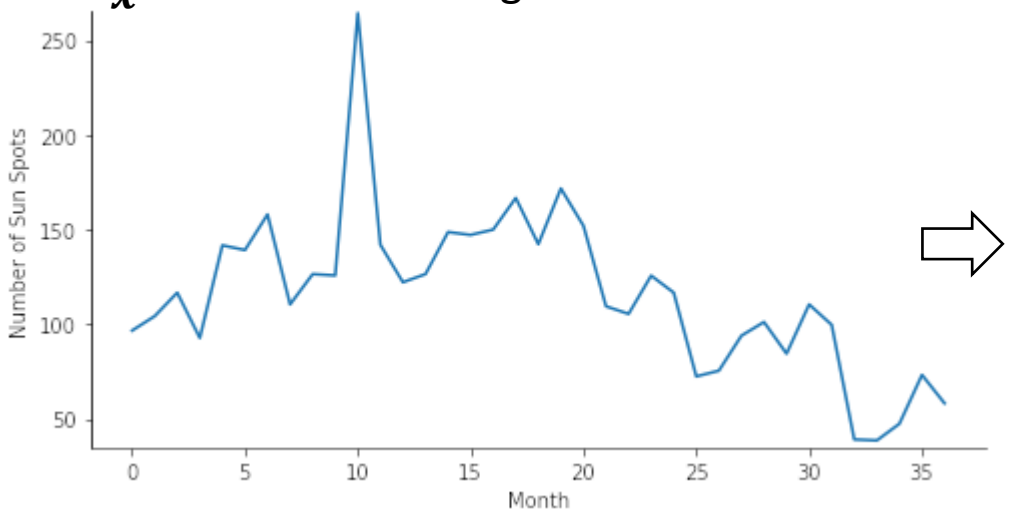




x

Original



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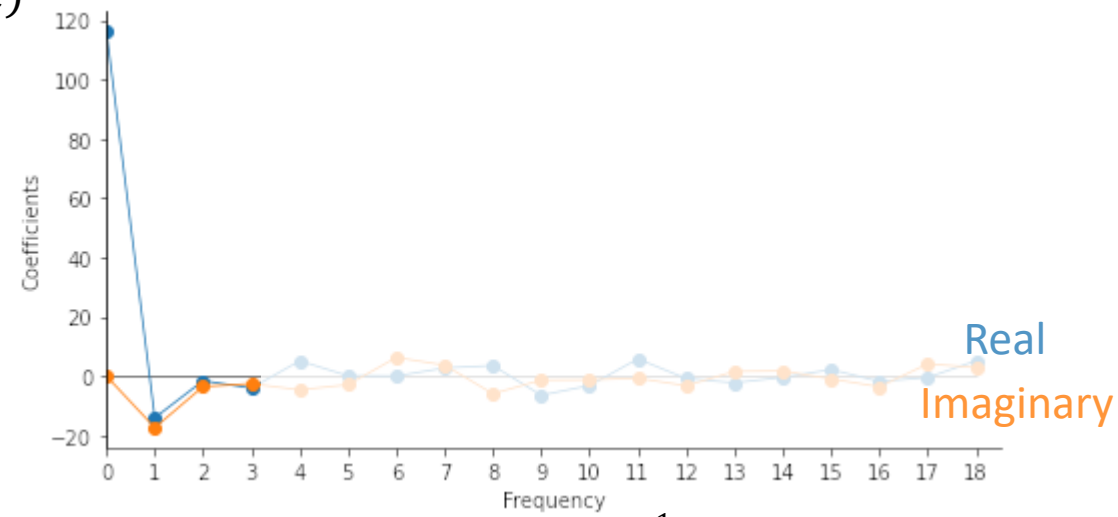
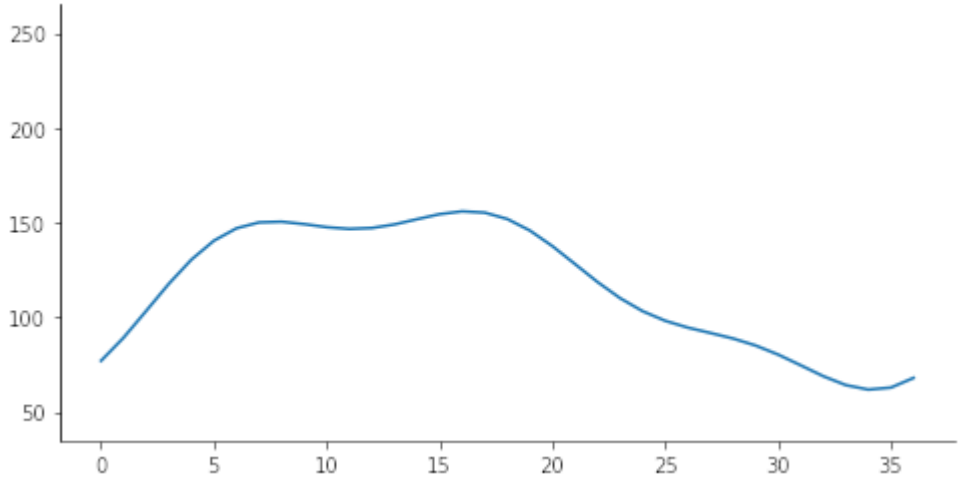


(series length^{-1})

Reconstruction



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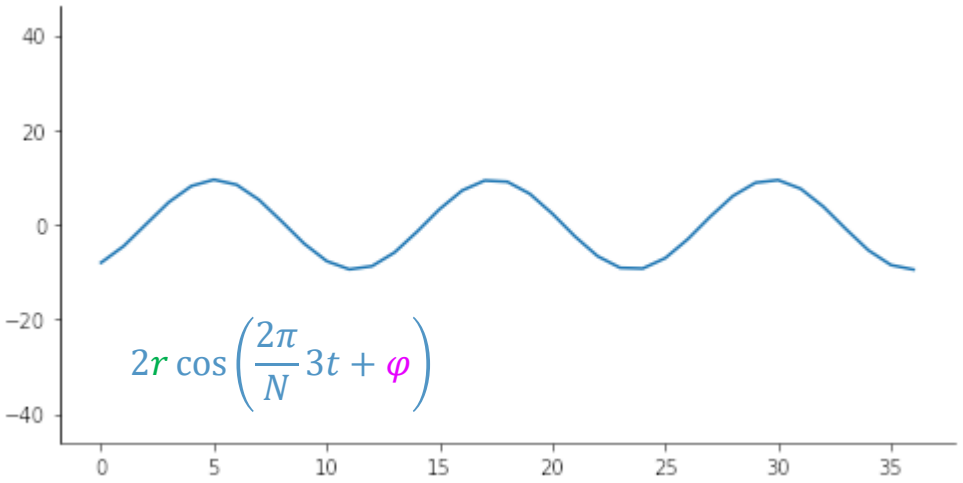
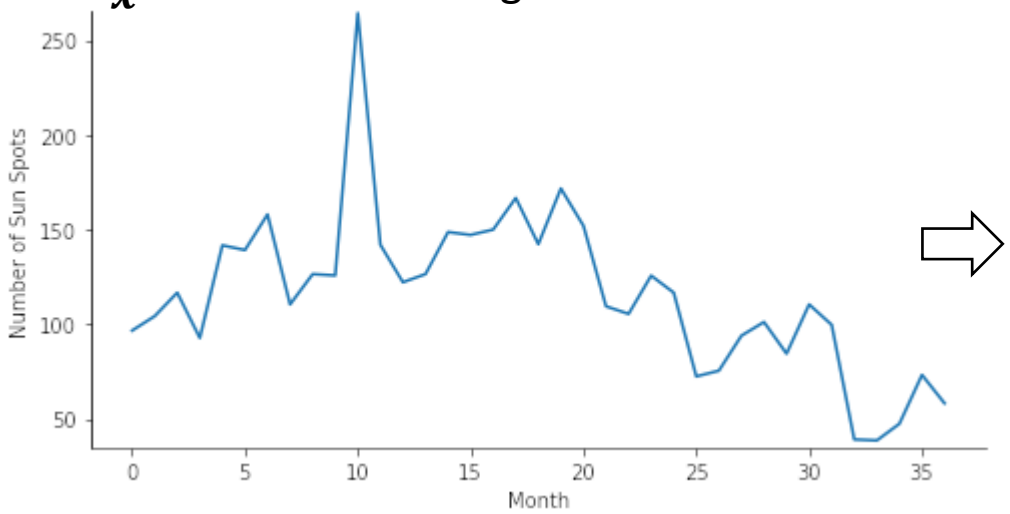


x

Original



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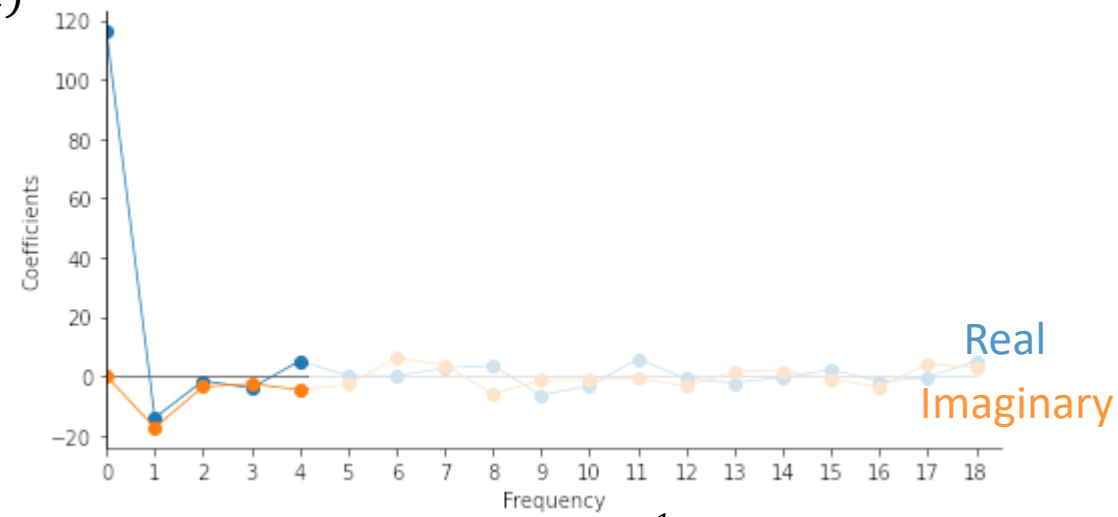
$F(x)$



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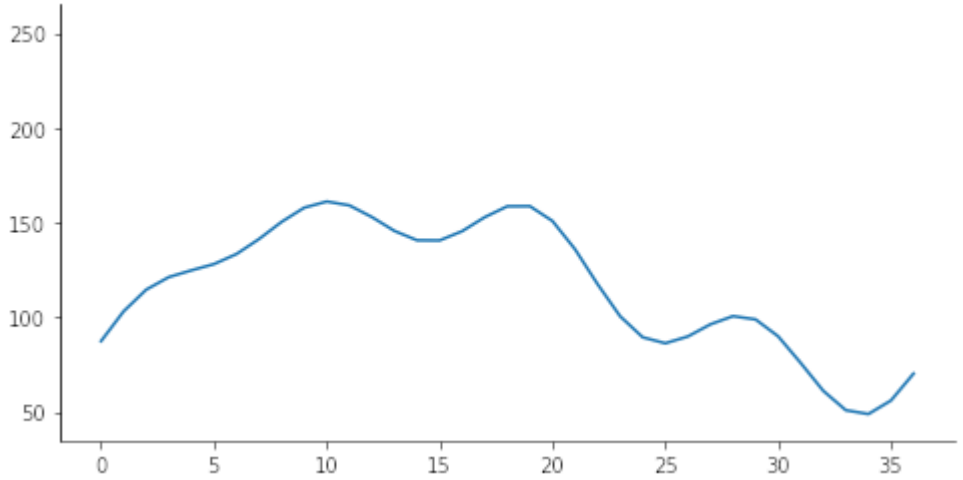
Reconstruction



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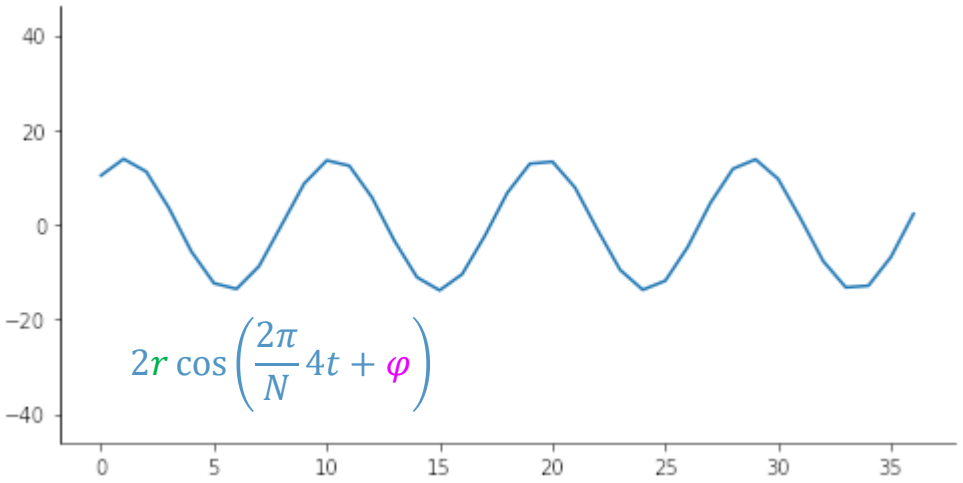
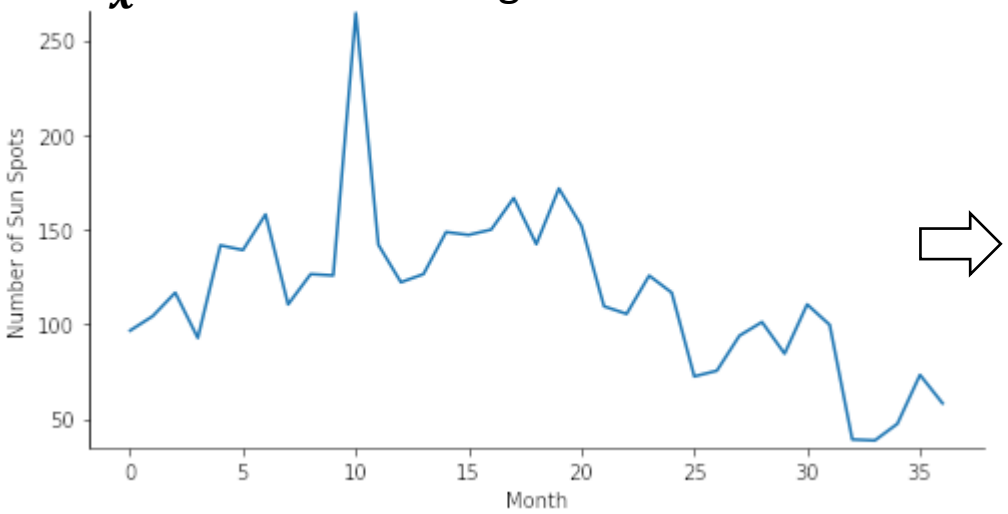


x

Original



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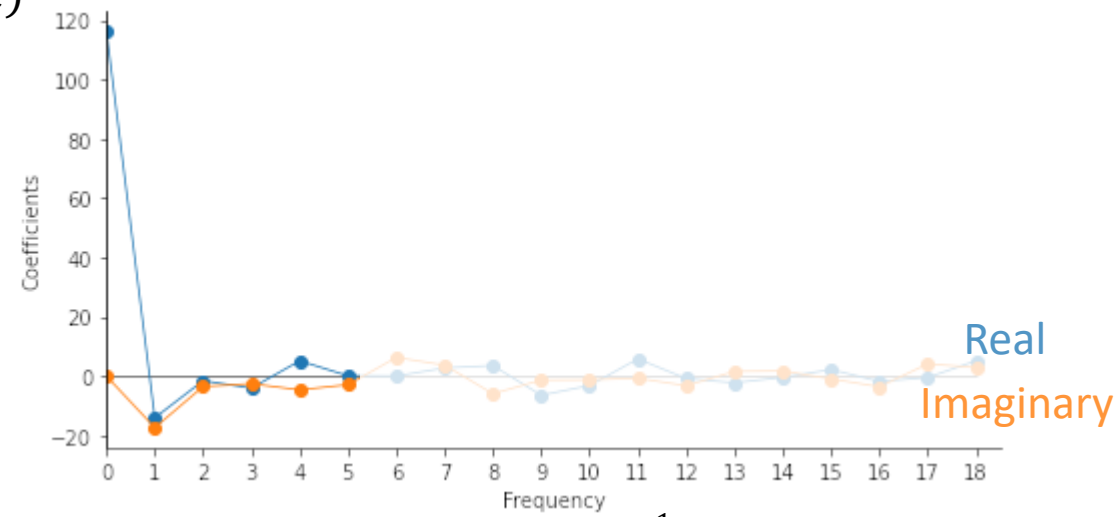
$F(x)$



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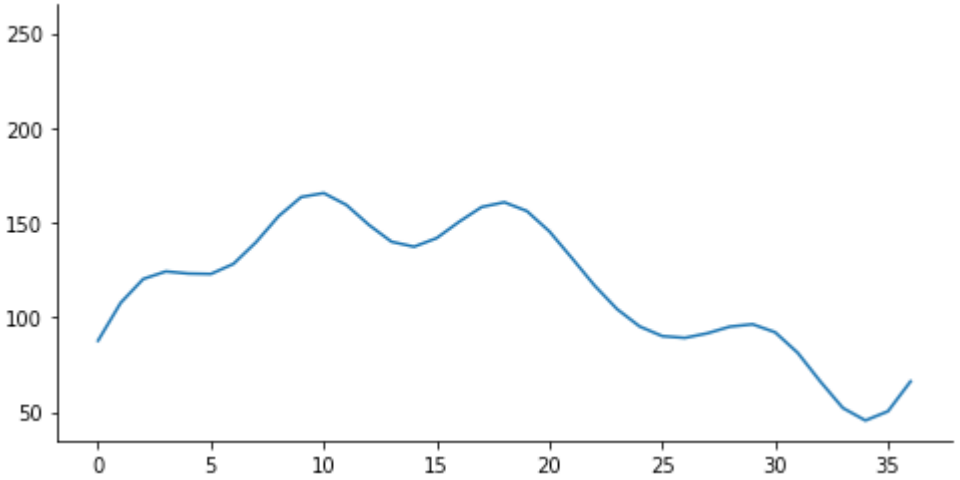
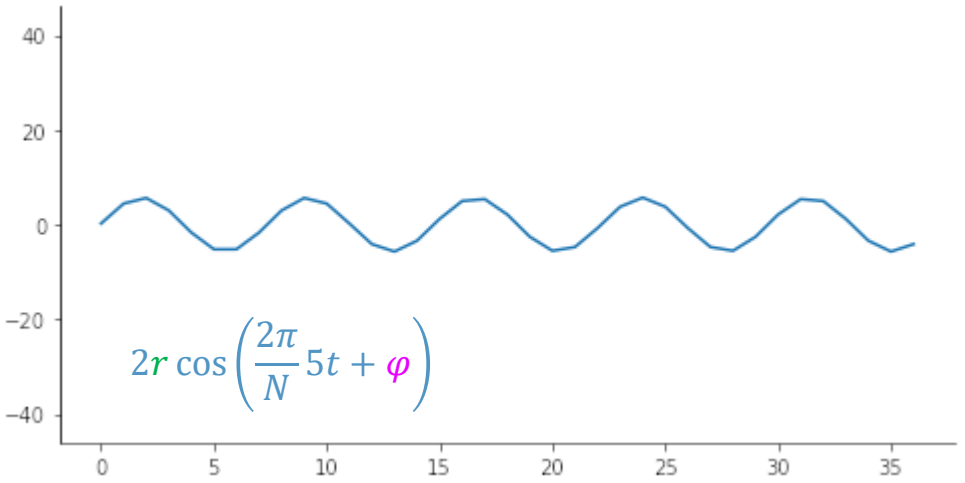
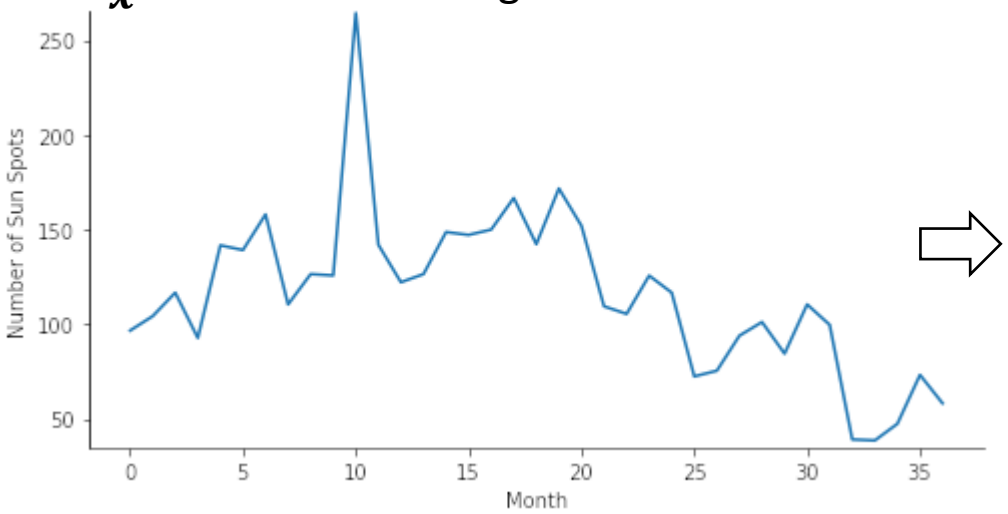
Reconstruction





Original

x



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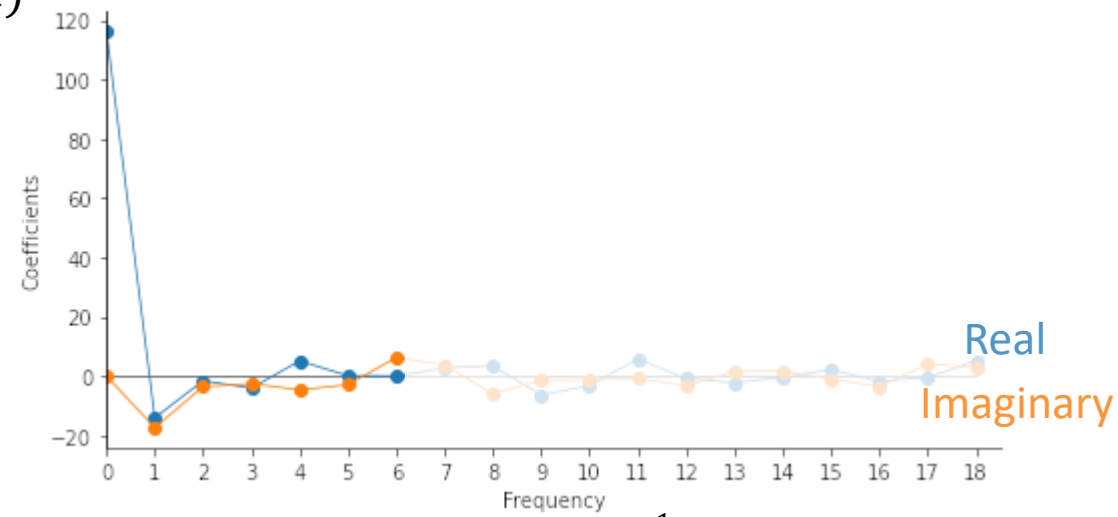

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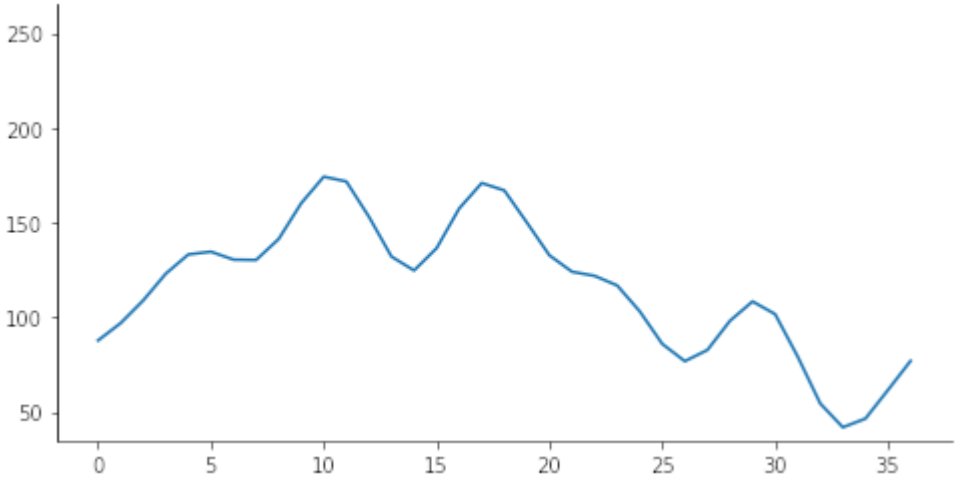

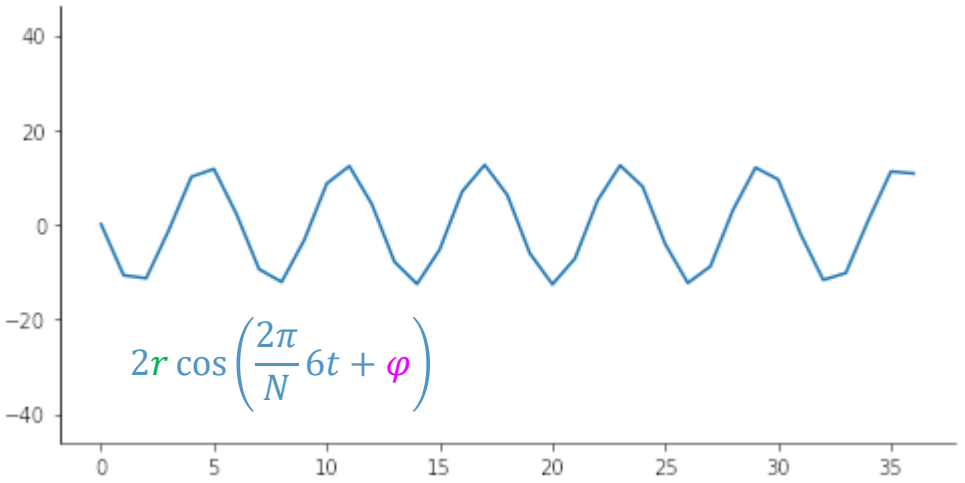

x

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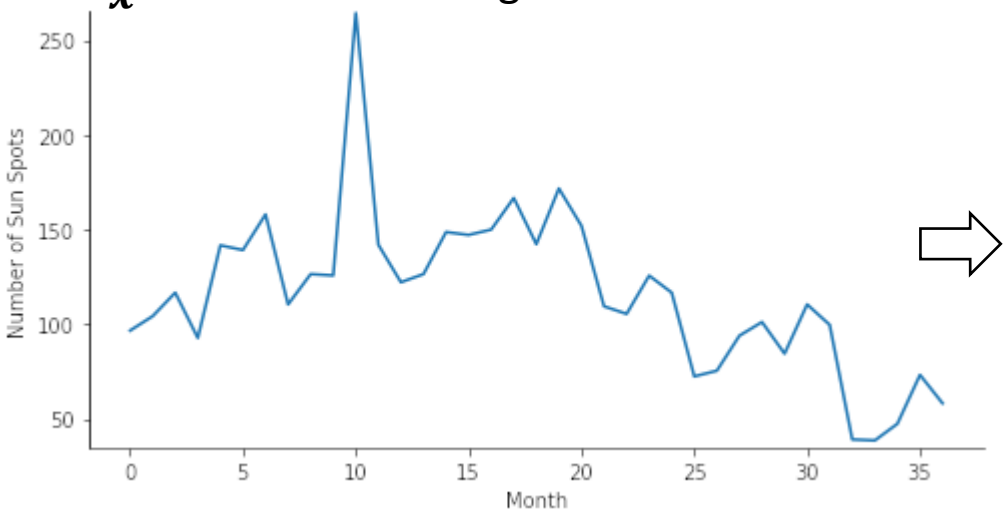

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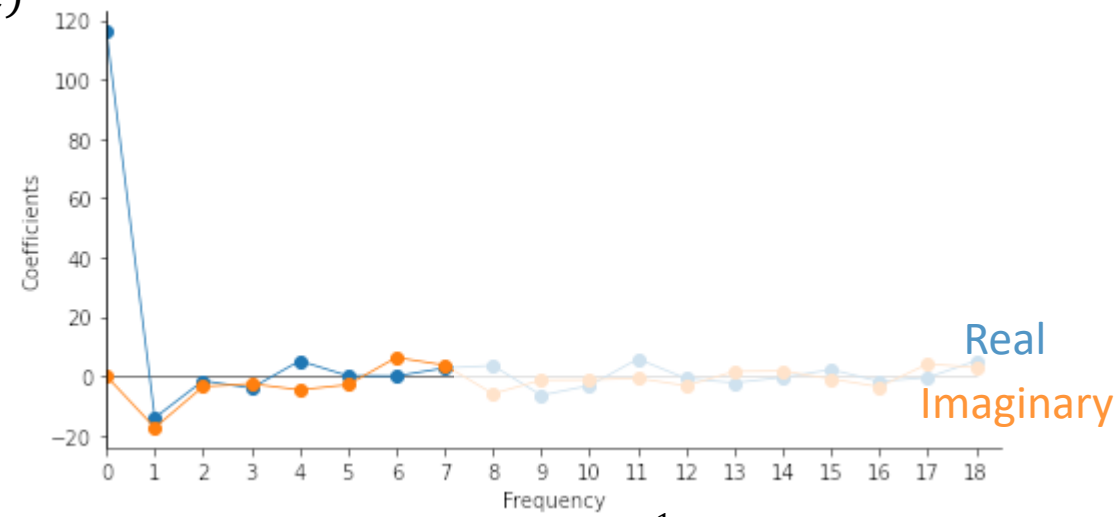

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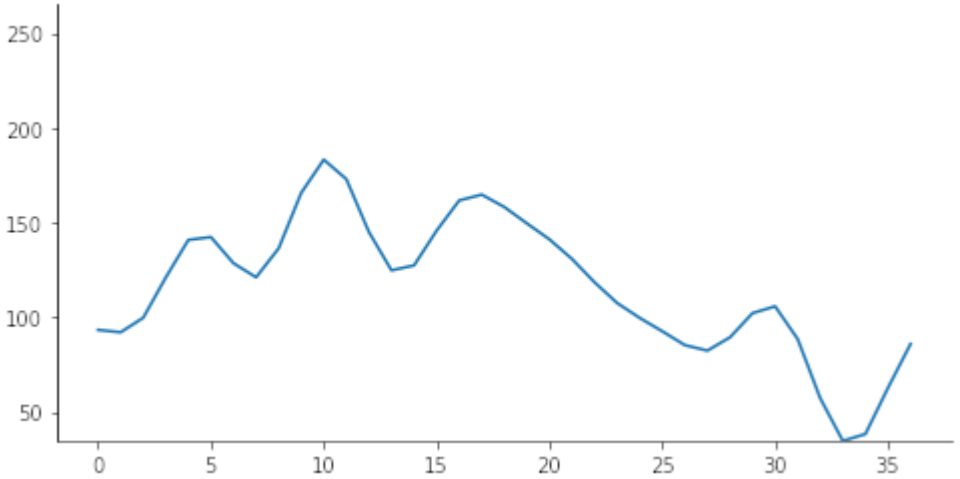
Reconstruction



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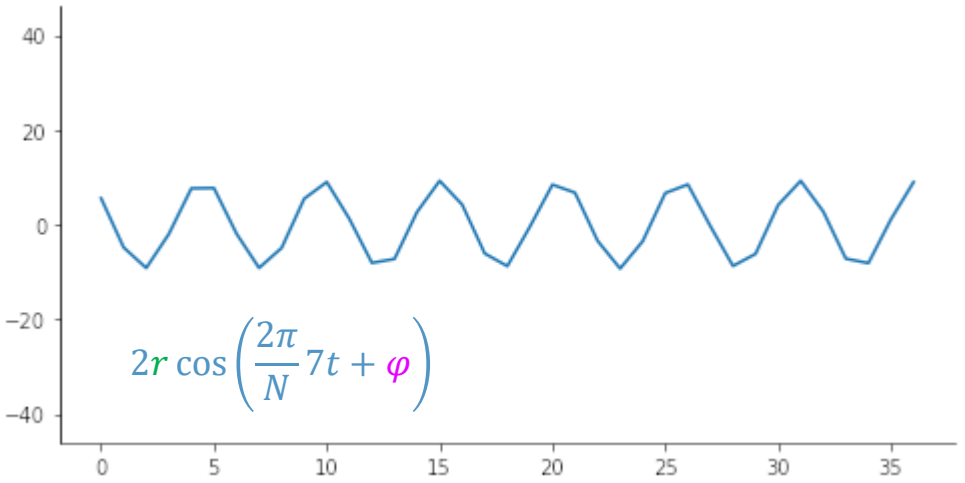
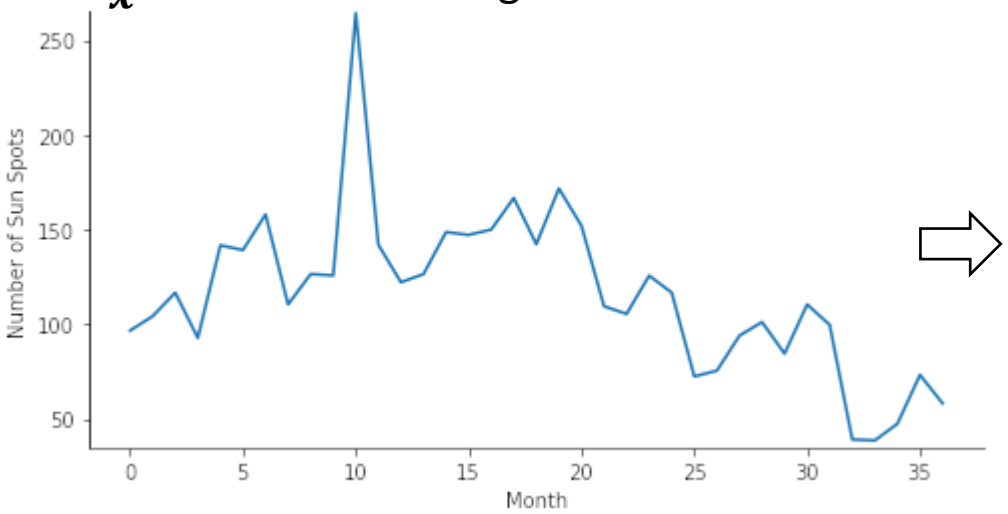


x

Original



Discrete Fourier Transform

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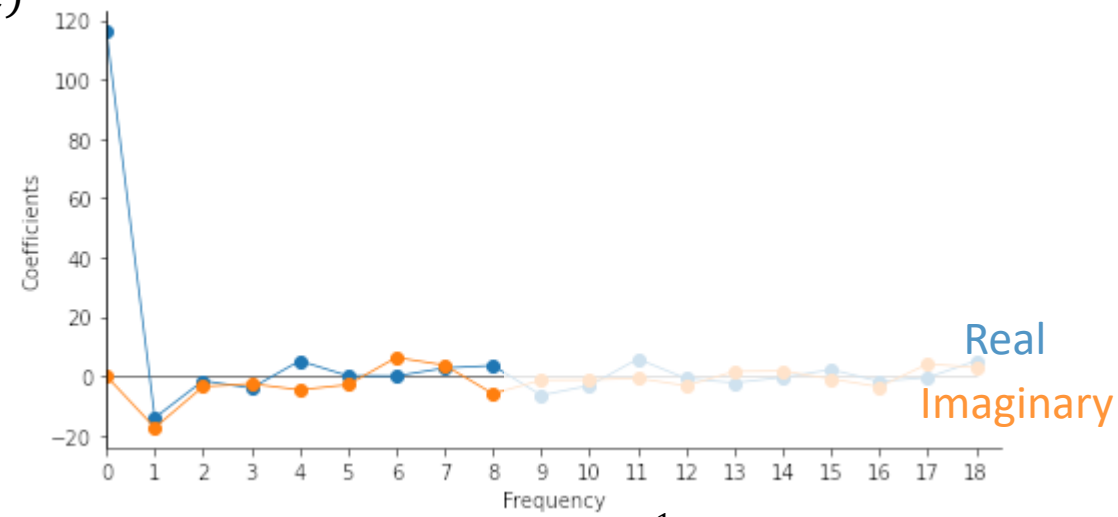

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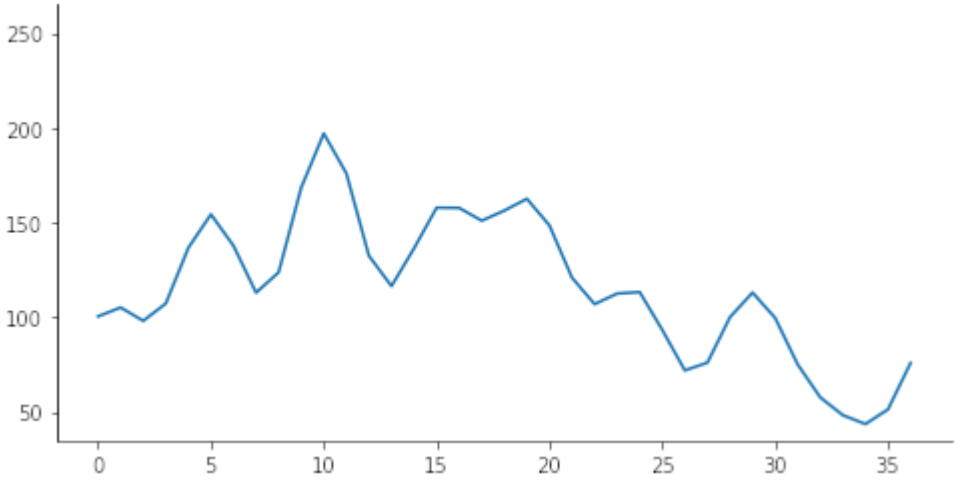
Reconstruction



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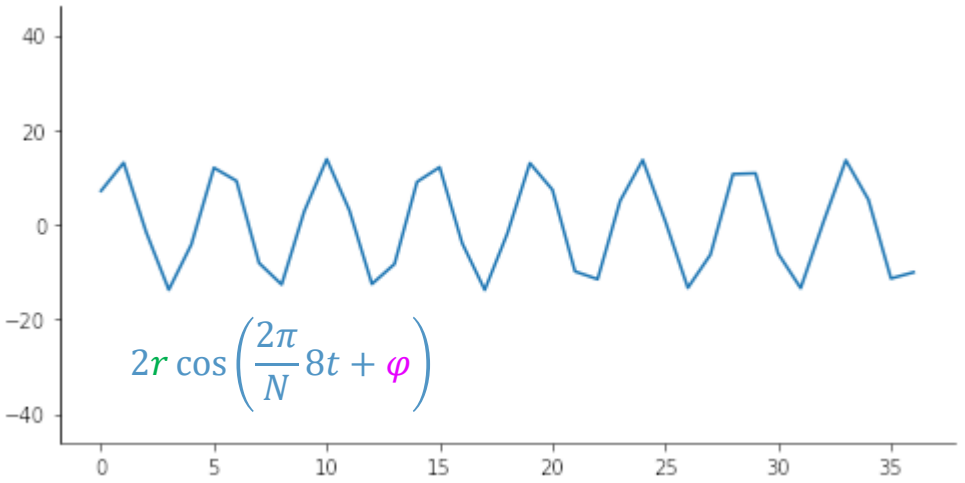
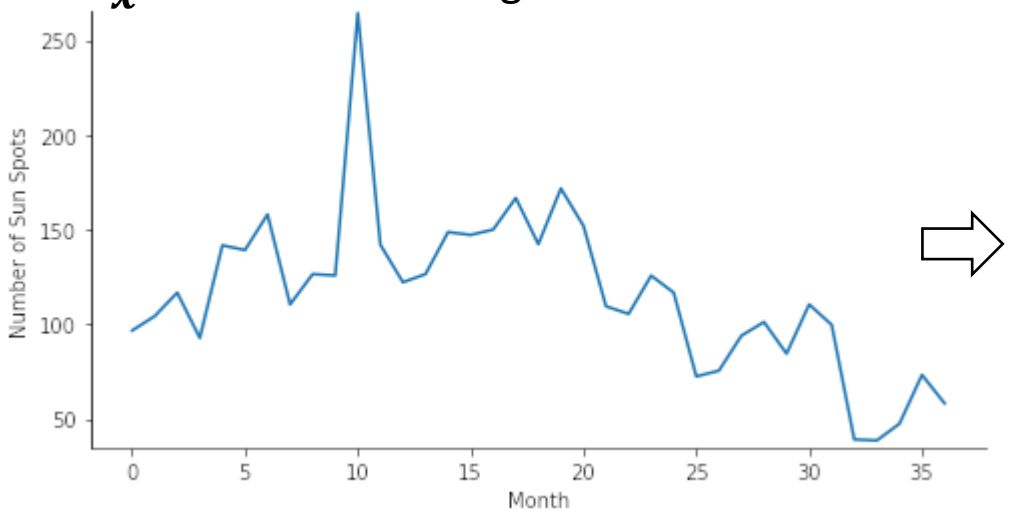


x

Original



Discrete Fourier Transform

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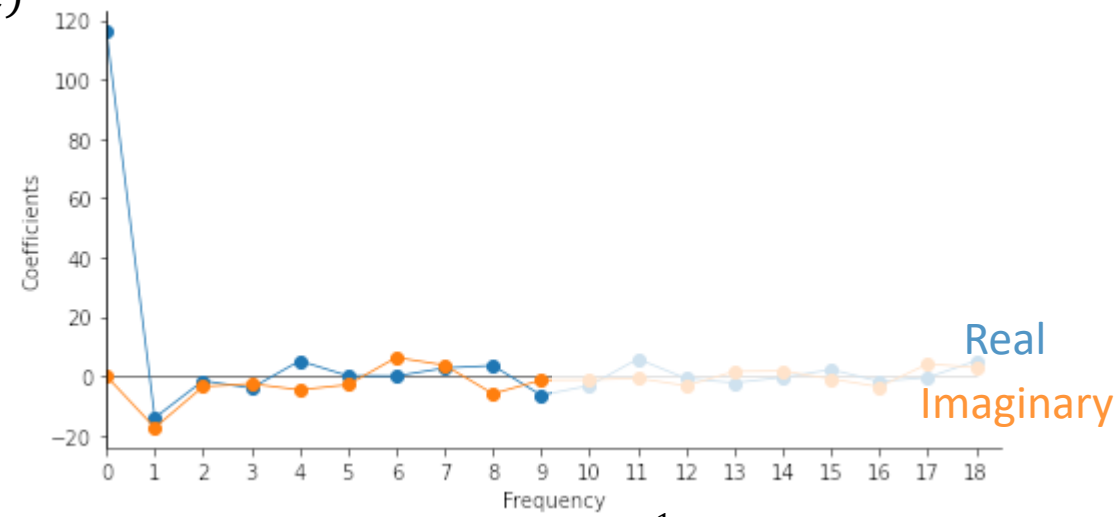

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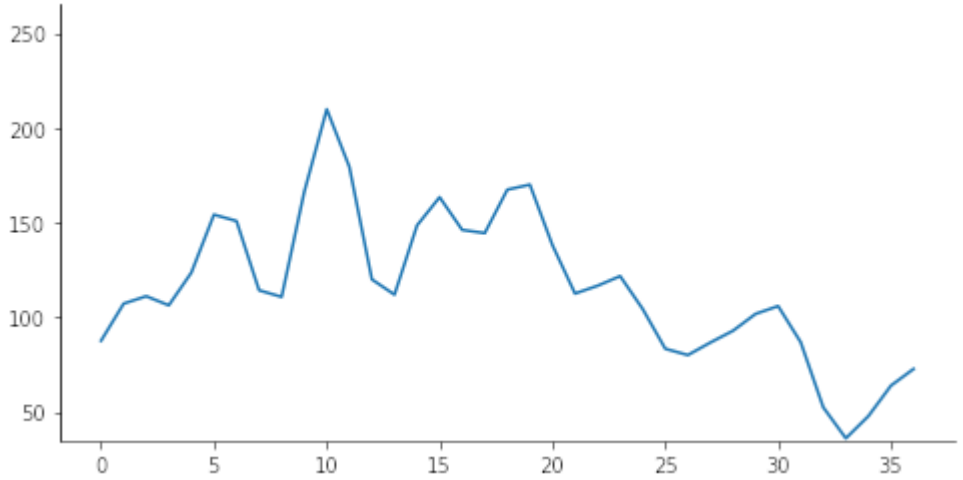

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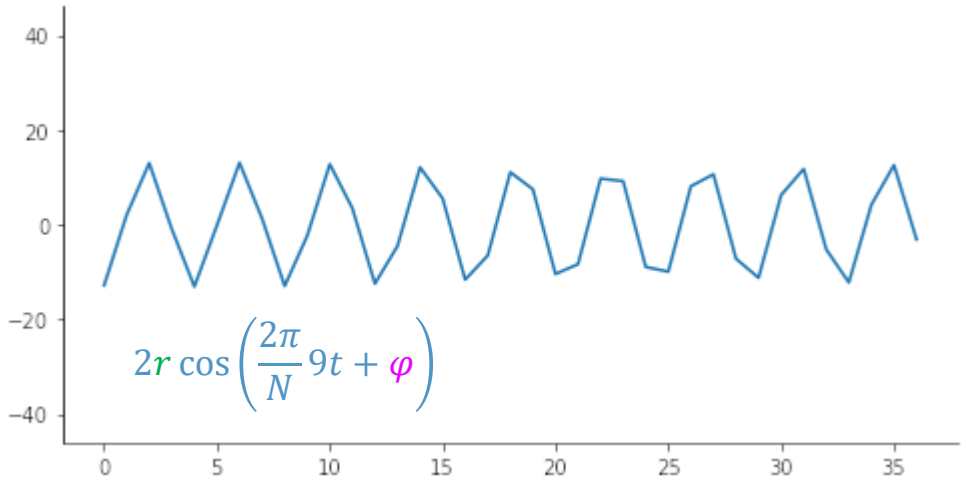
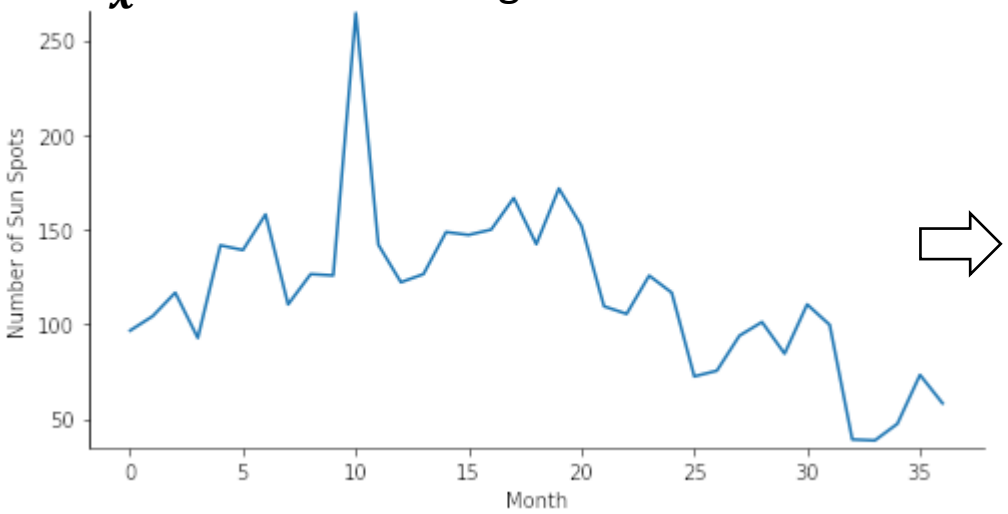


x

Original



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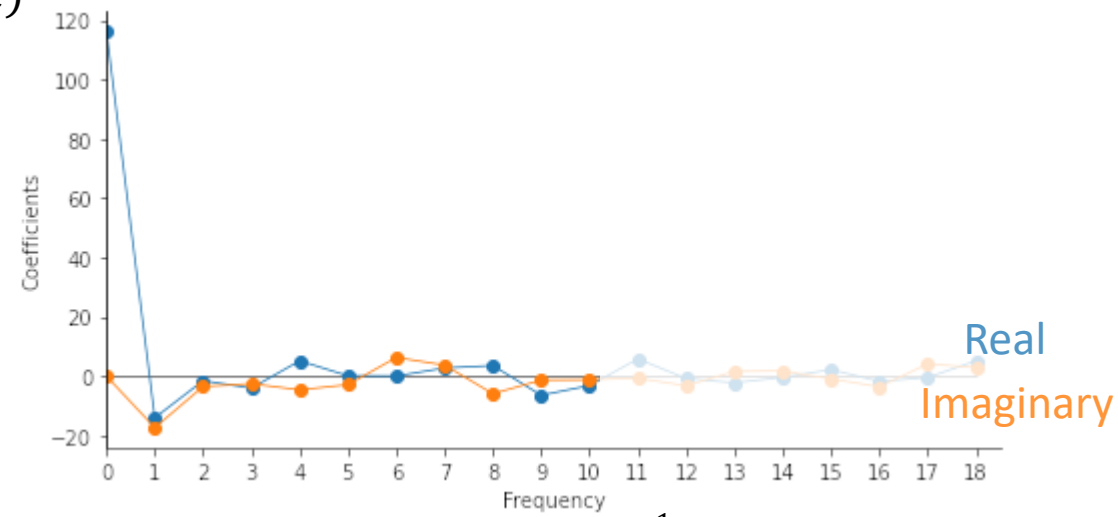

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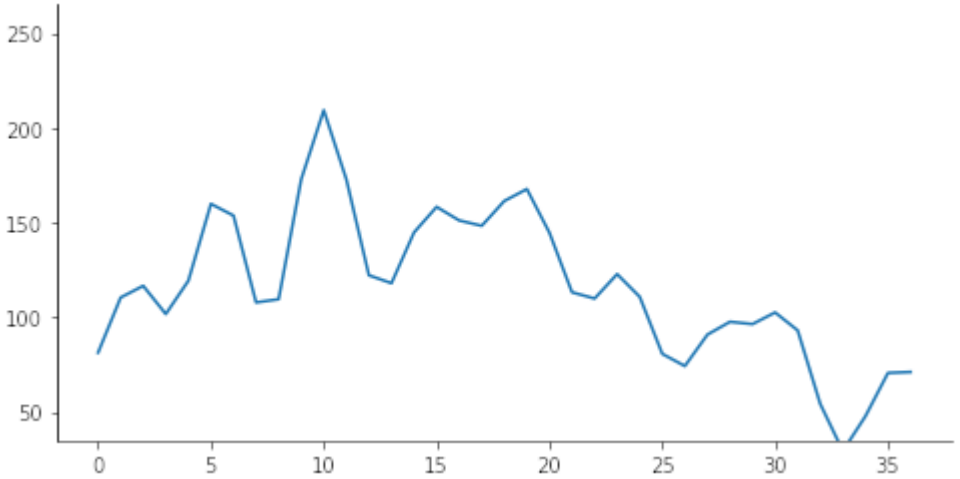
Reconstruction



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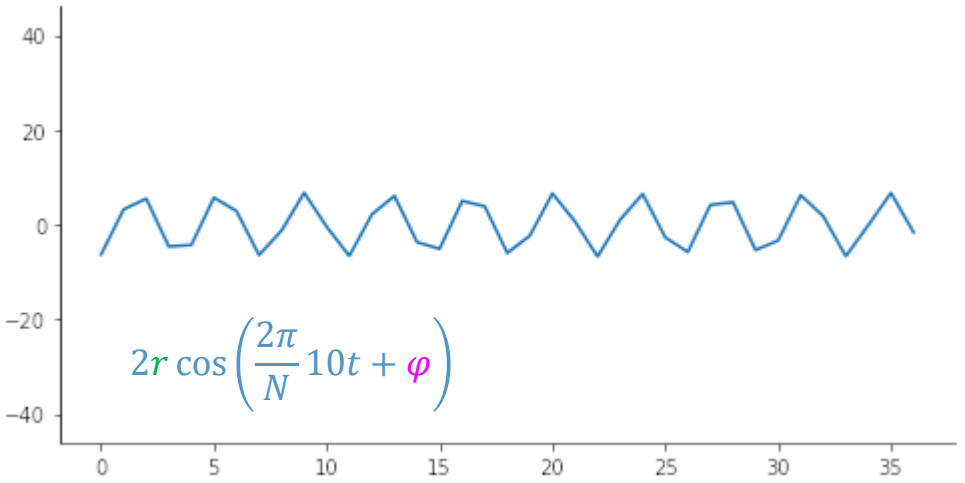
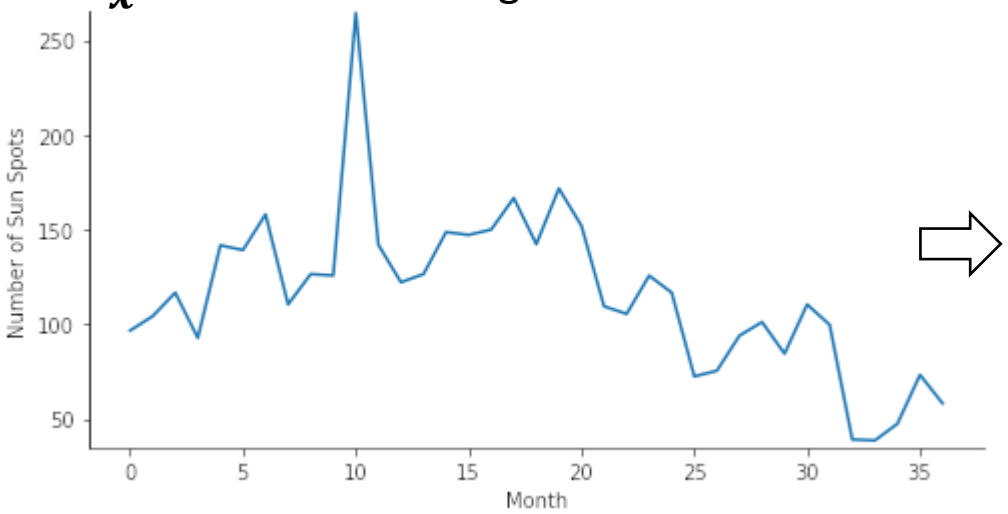


x

Original



Discrete Fourier Transform

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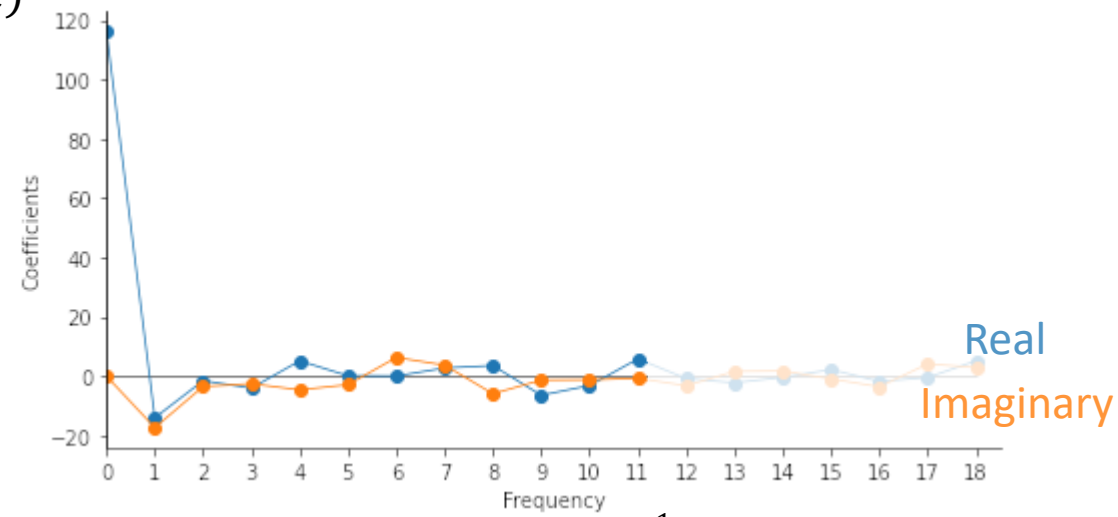

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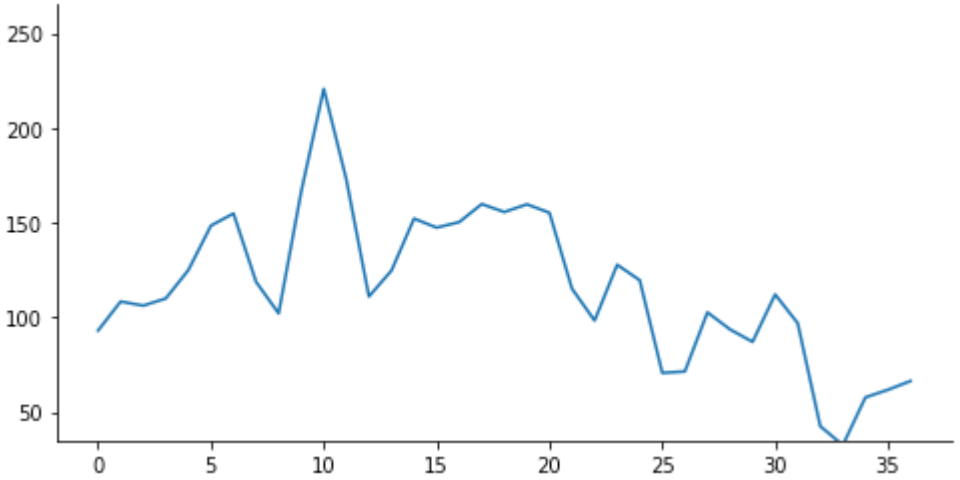
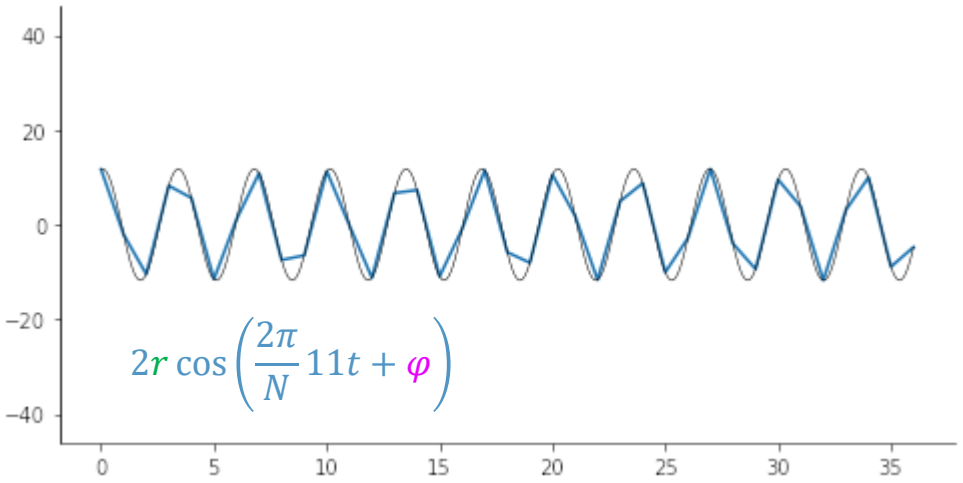
Reconstruction

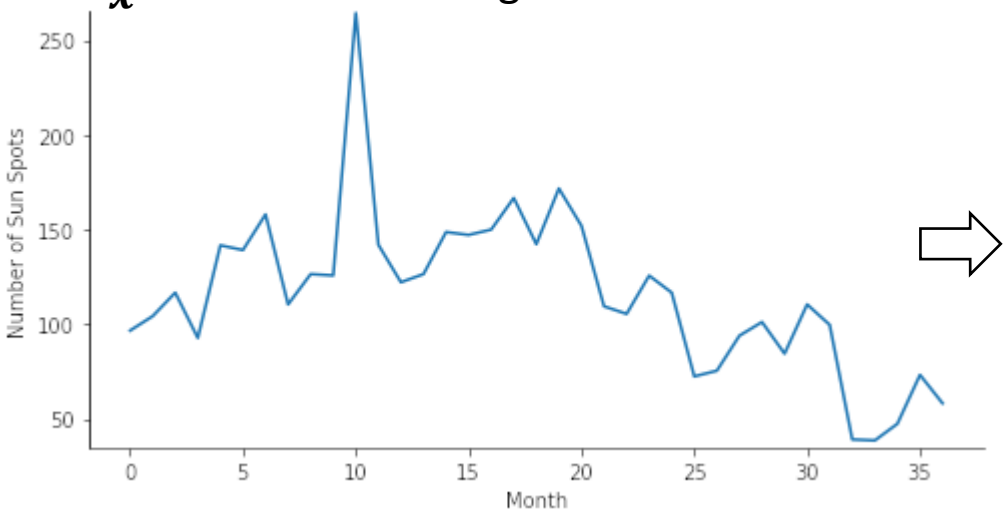




$$2r\cos\left(\frac{2\pi}{N}11t+\varphi\right)$$

x

Original



Discrete Fourier Transform

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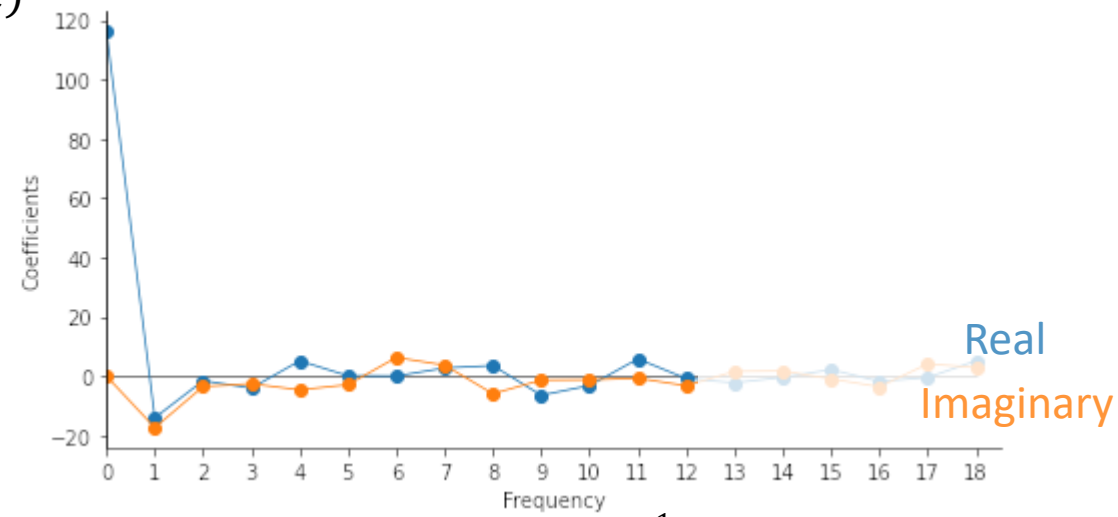

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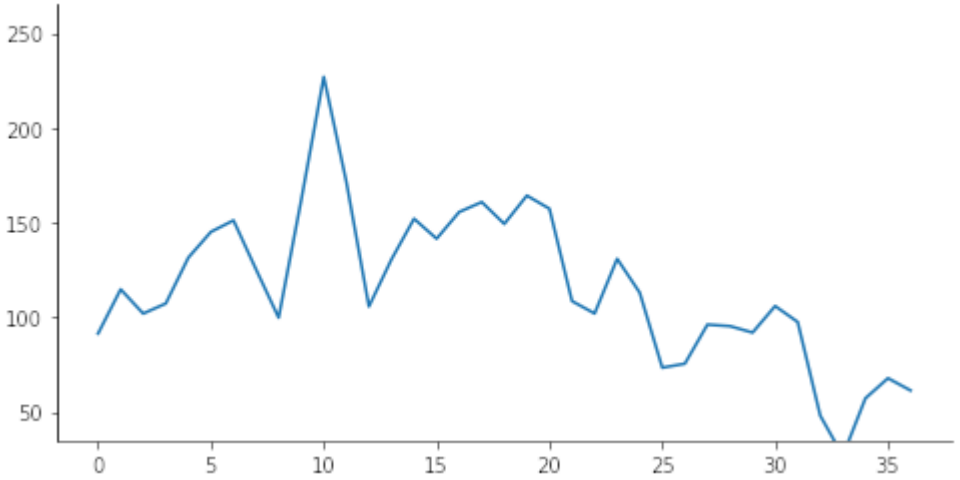
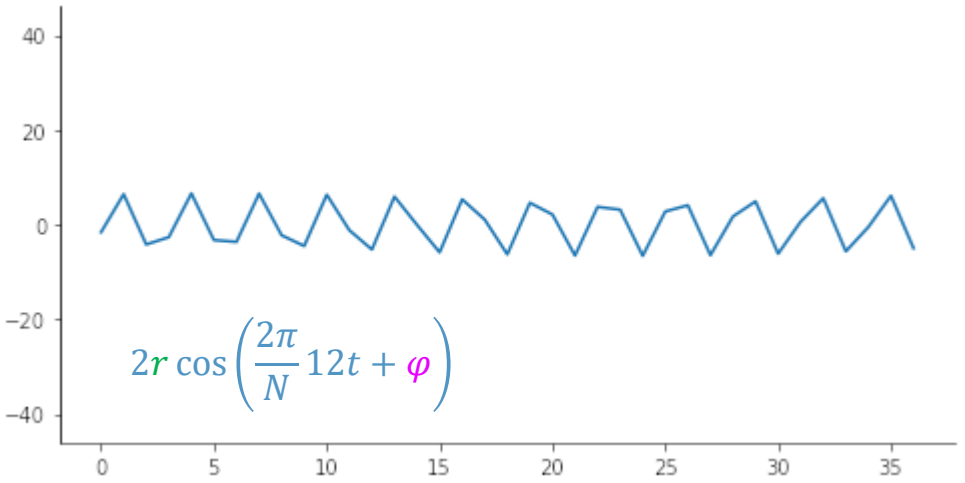
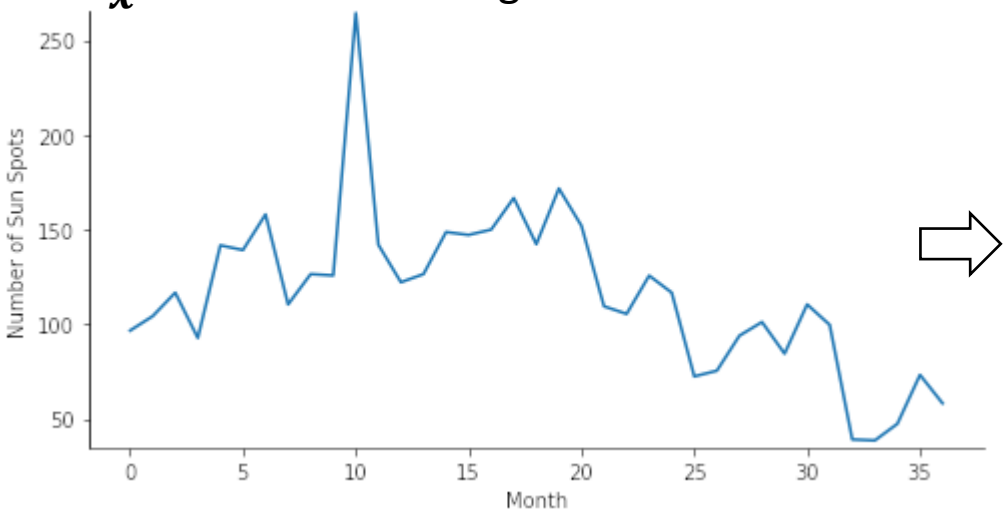




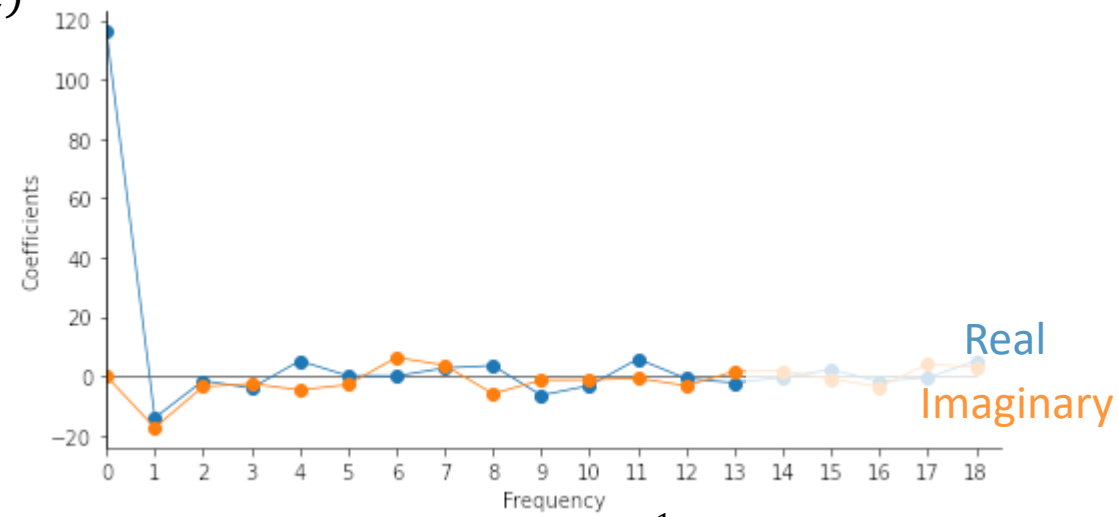
x

Original



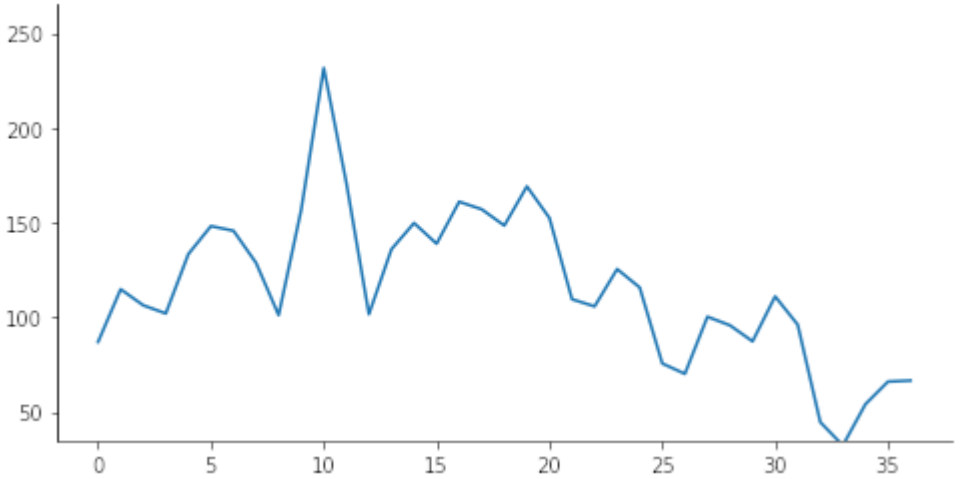
Discrete Fourier Transform

$F(x)$

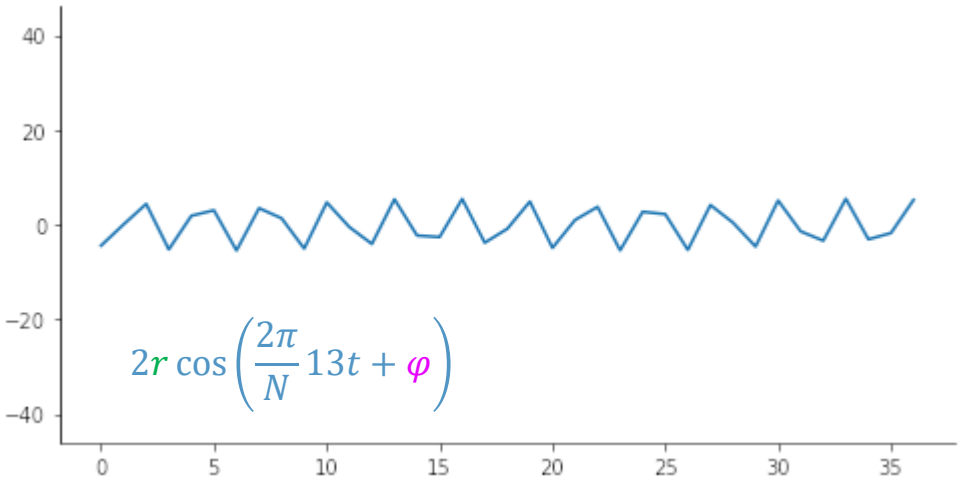


(series length^{-1})

Reconstruction

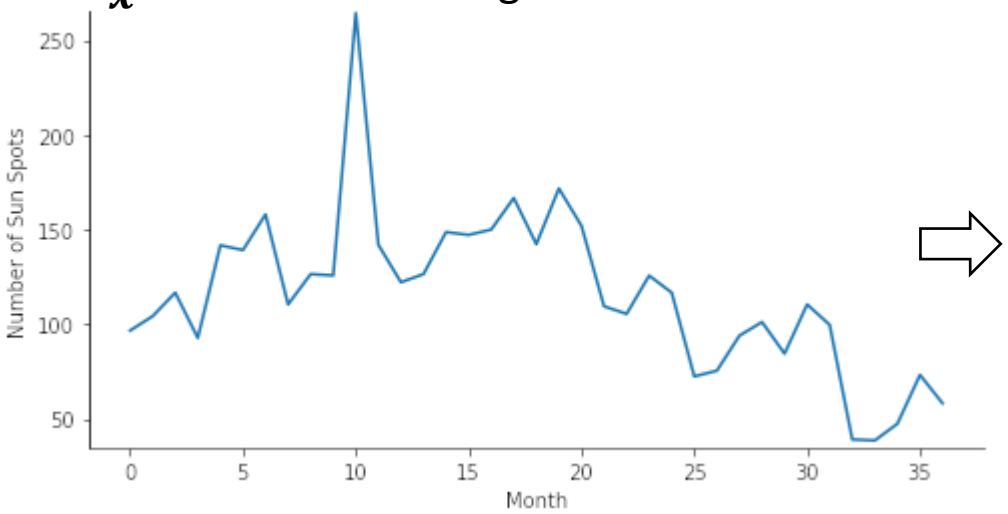


+



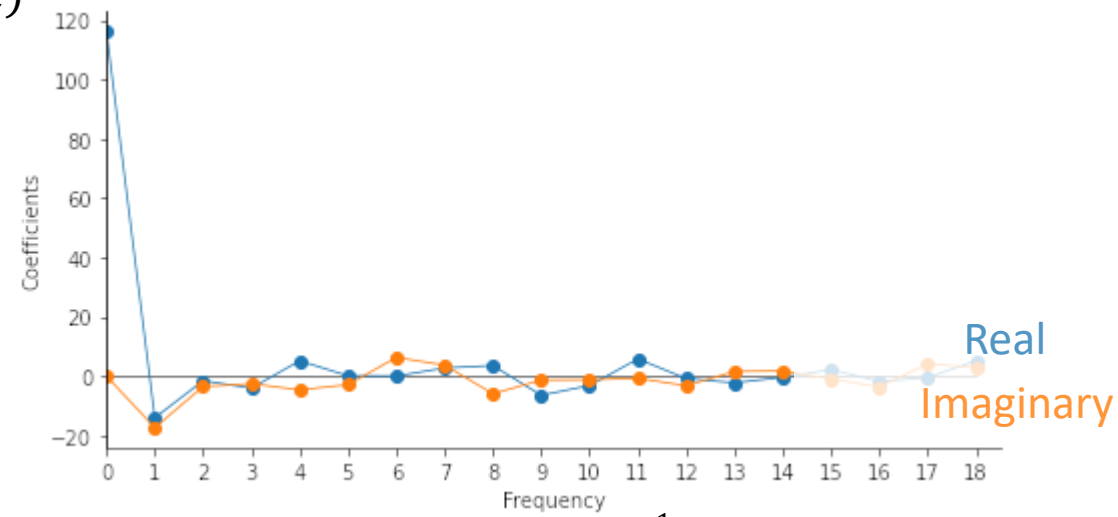
x

Original



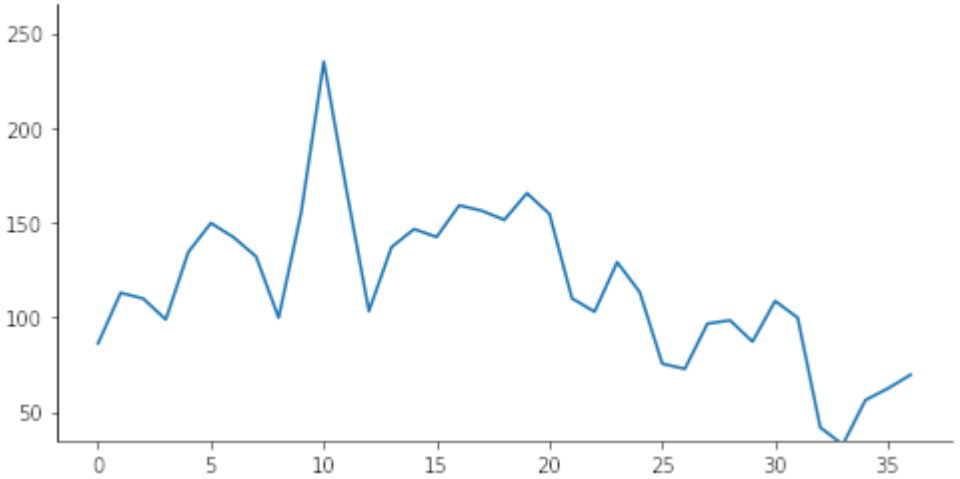
Discrete Fourier Transform

$F(x)$

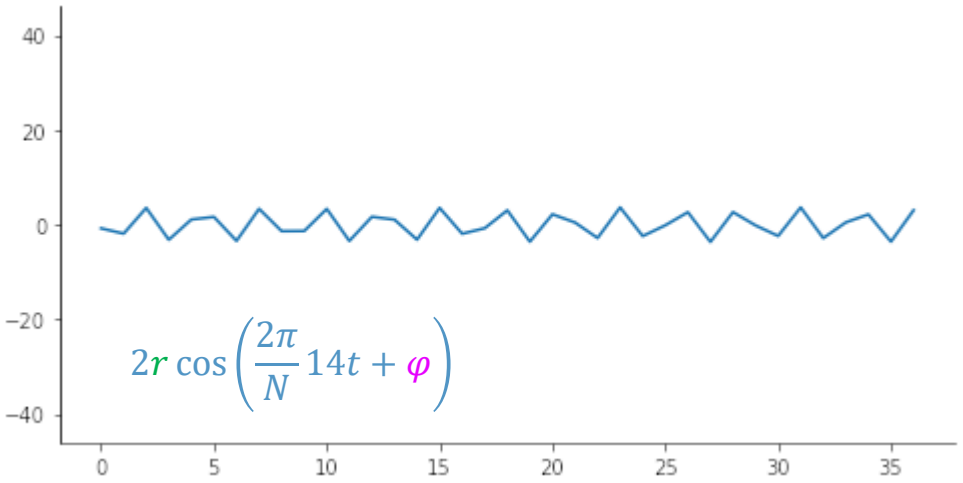


(series length^{-1})

Reconstruction

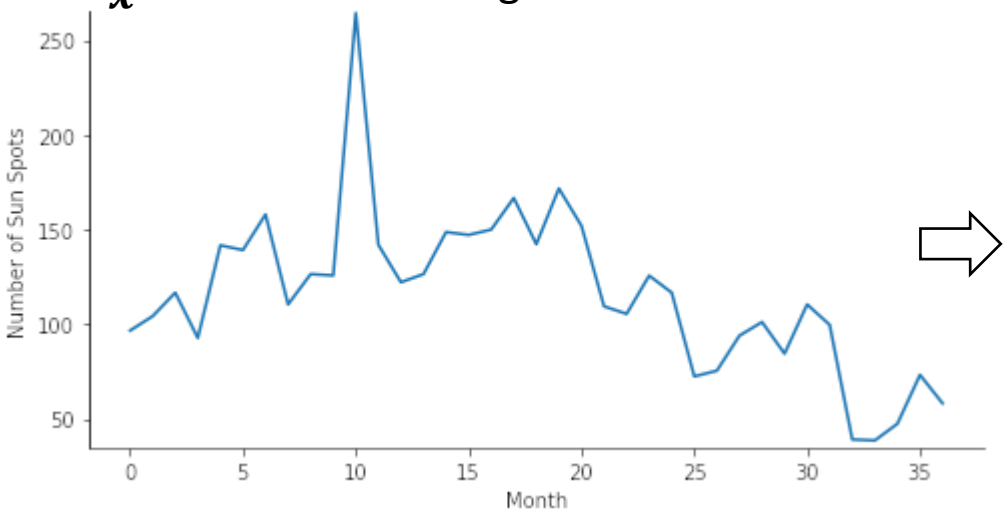


+



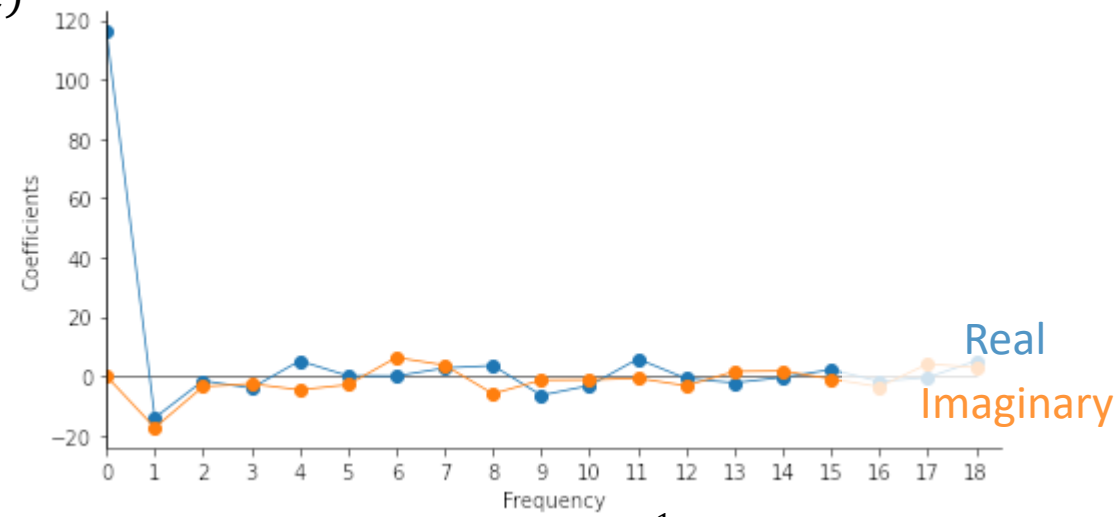
x

Original



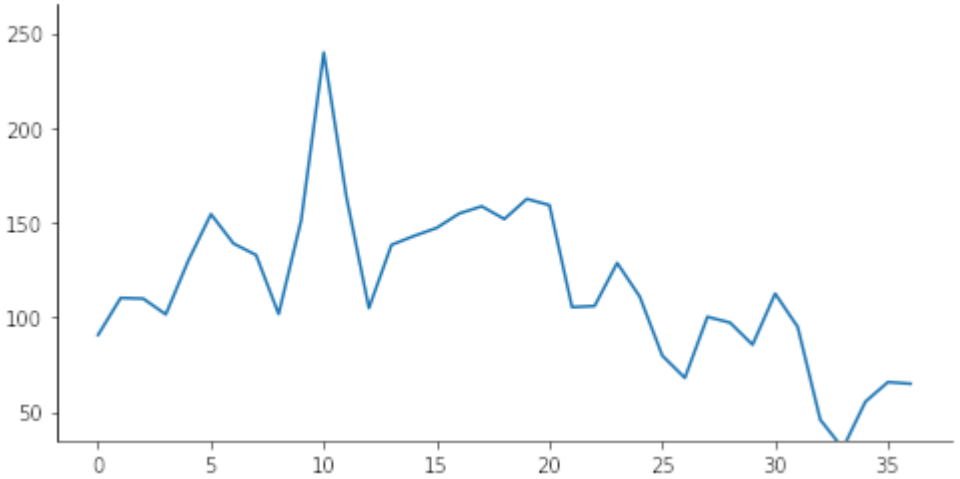
Discrete Fourier Transform

$F(x)$



(series length^{-1})

Reconstruction



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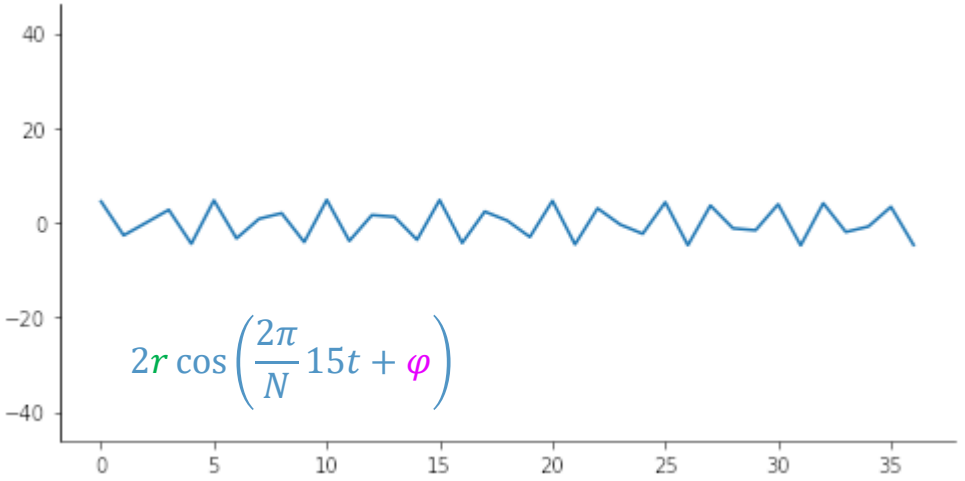
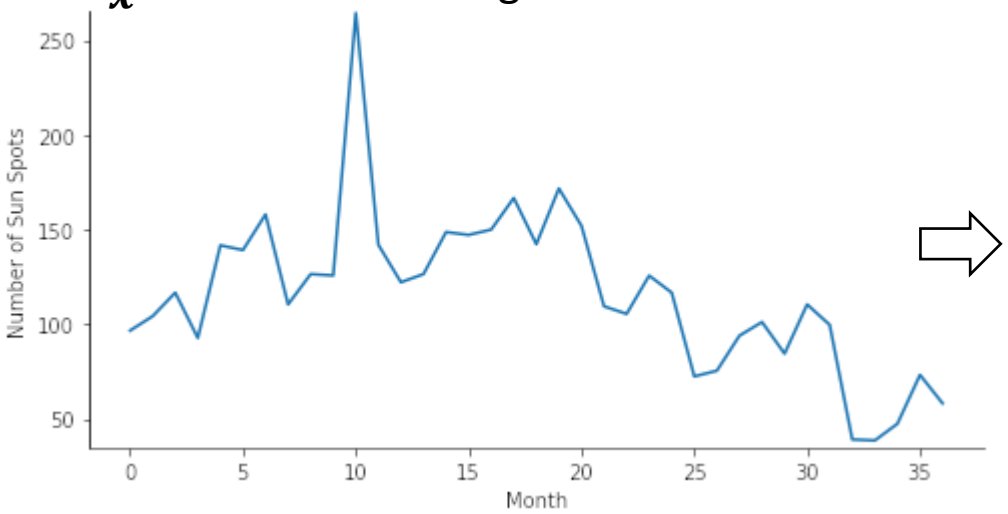


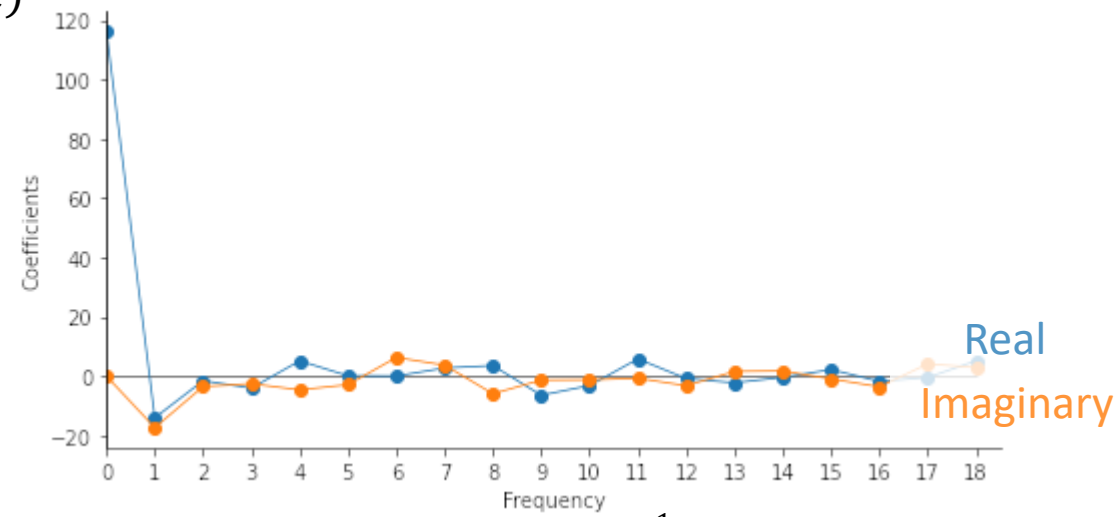
x

Original



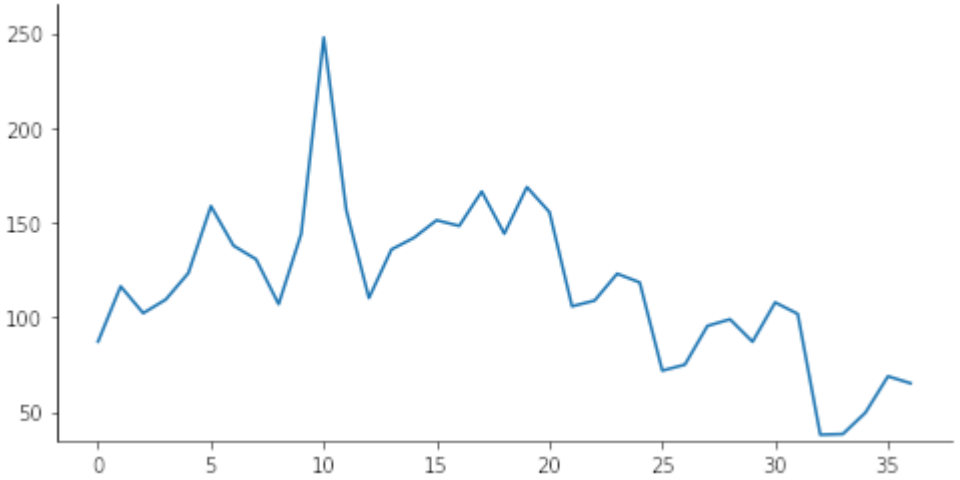
Discrete Fourier Transform

$F(x)$

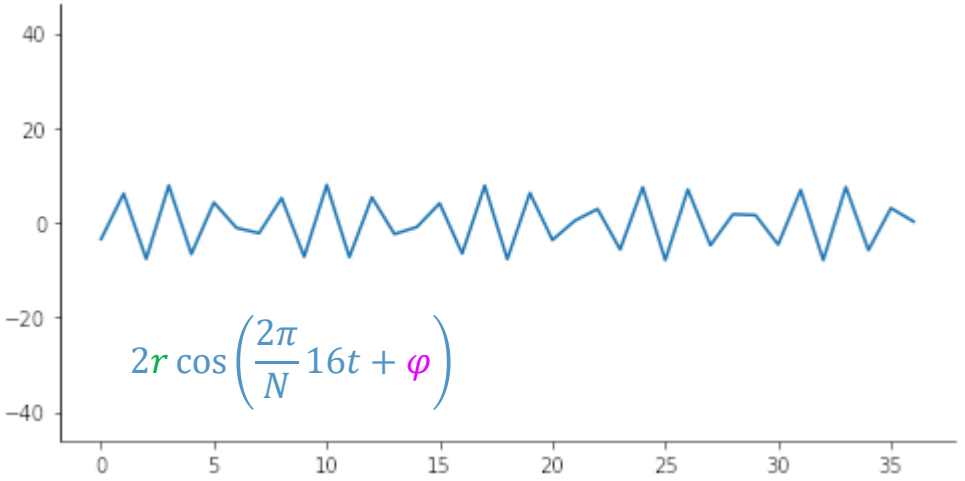


(series length^{-1})

Reconstruction

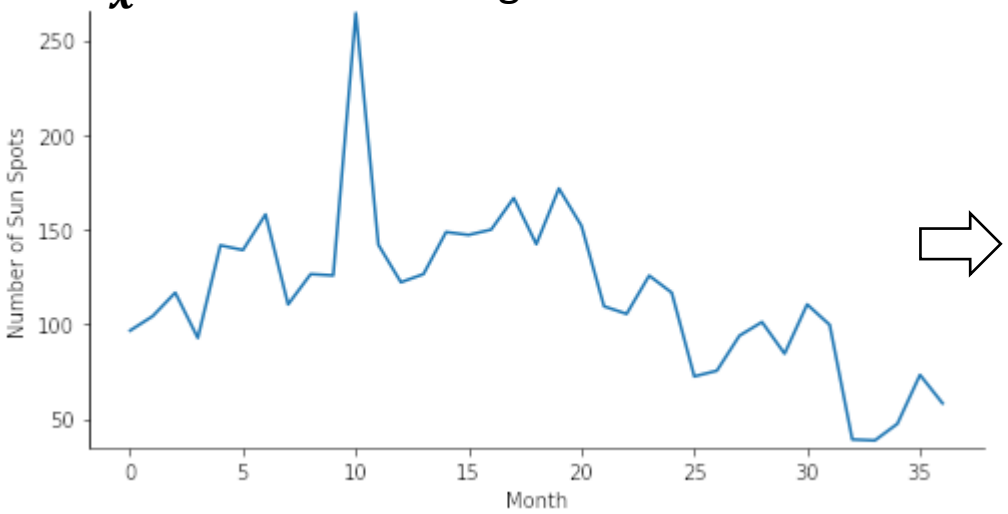


+



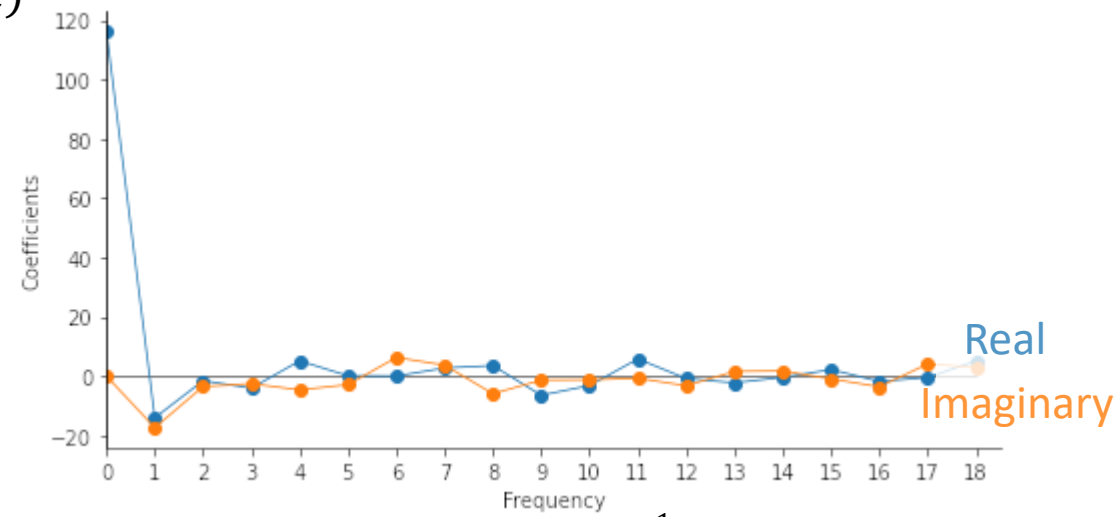
Original

x



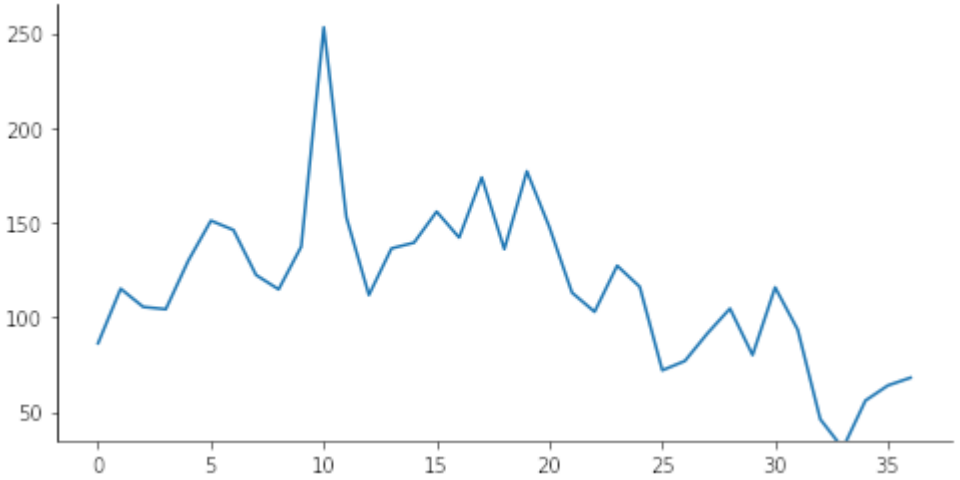
Discrete Fourier Transform

$F(x)$

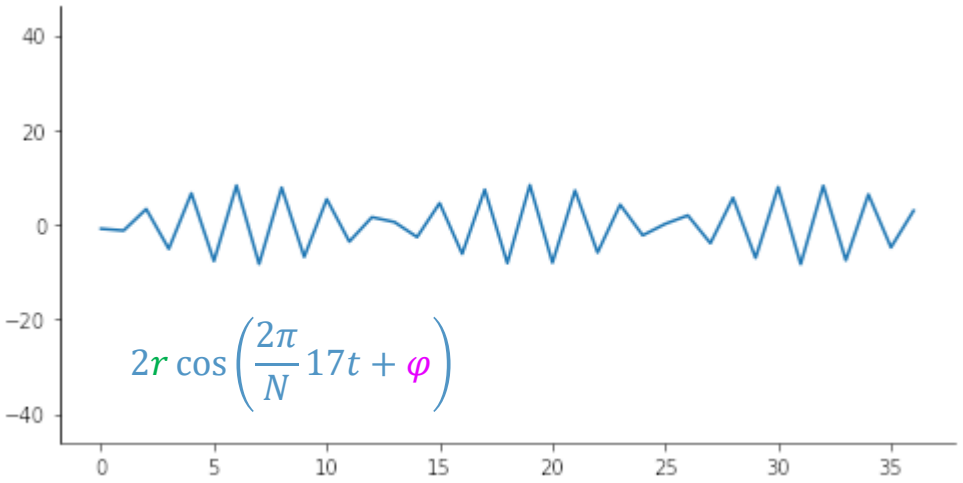


(series length^{-1})

Reconstruction

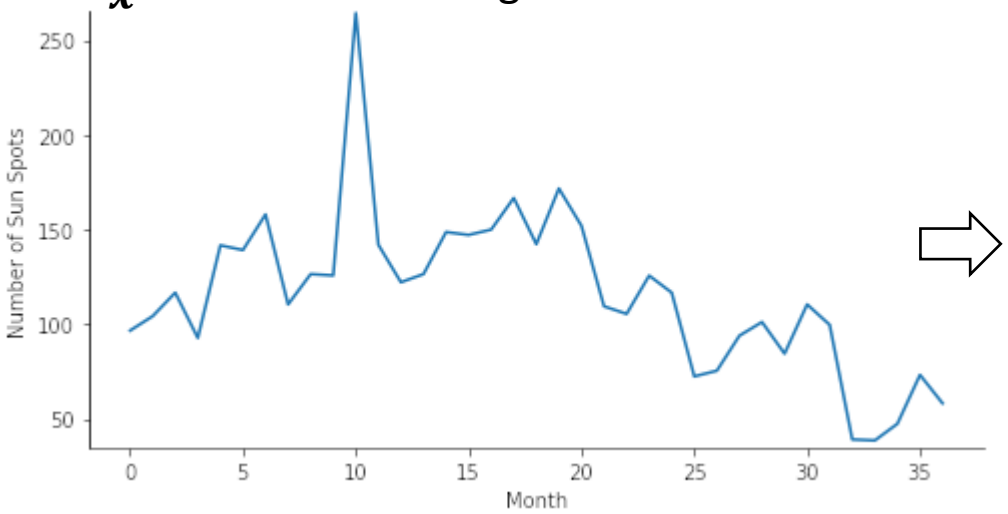


+



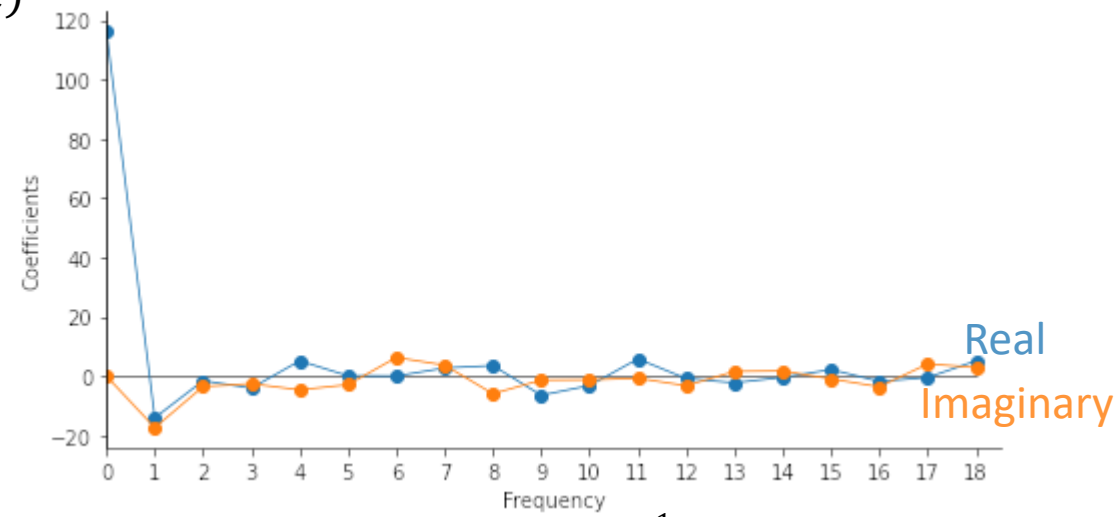
x

Original



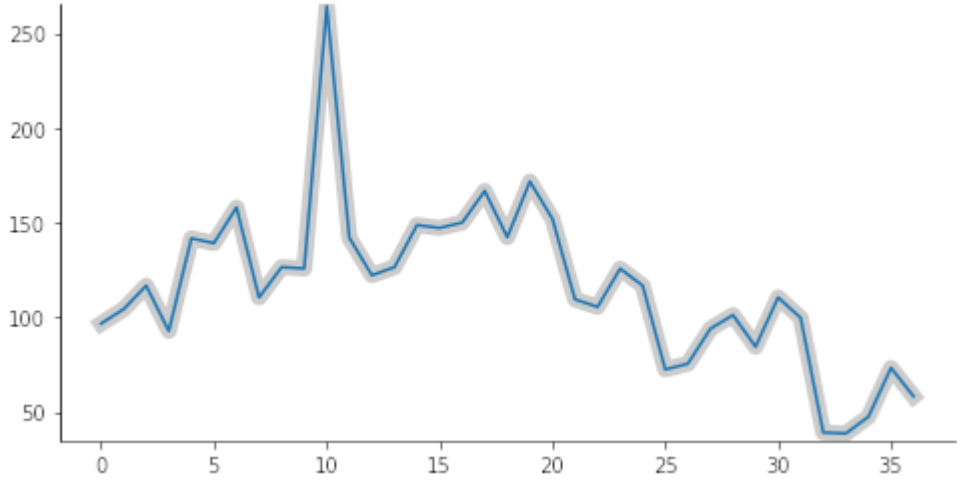
Discrete Fourier Transform

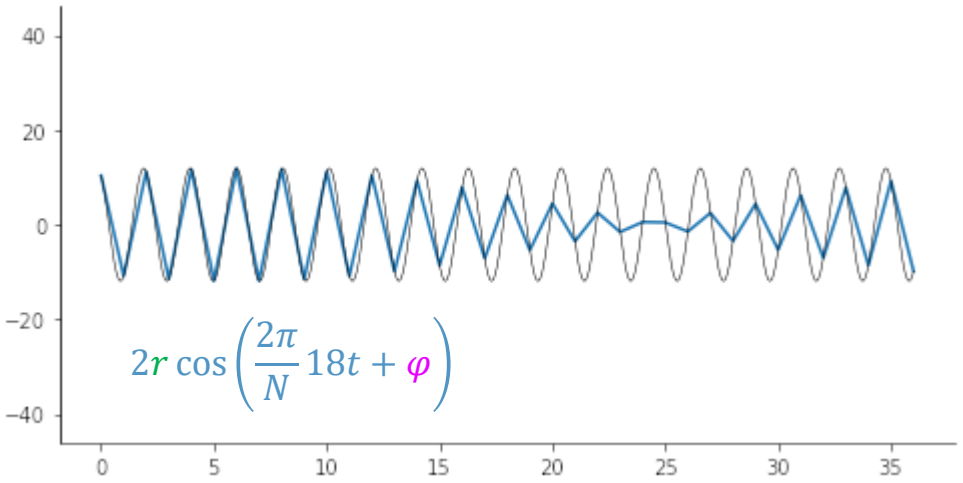
$F(x)$



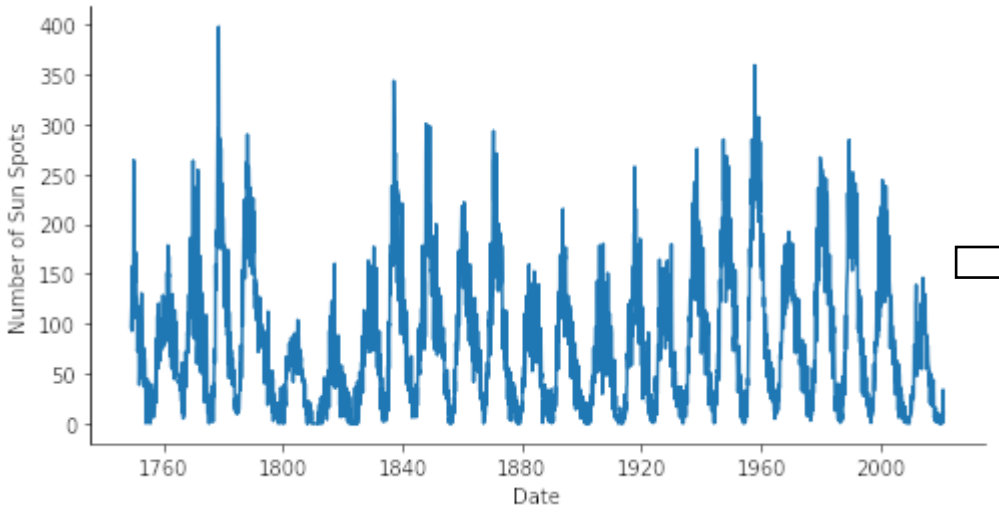
(series length^{-1})

$F^{-1}(F(x)) = x$ Reconstruction



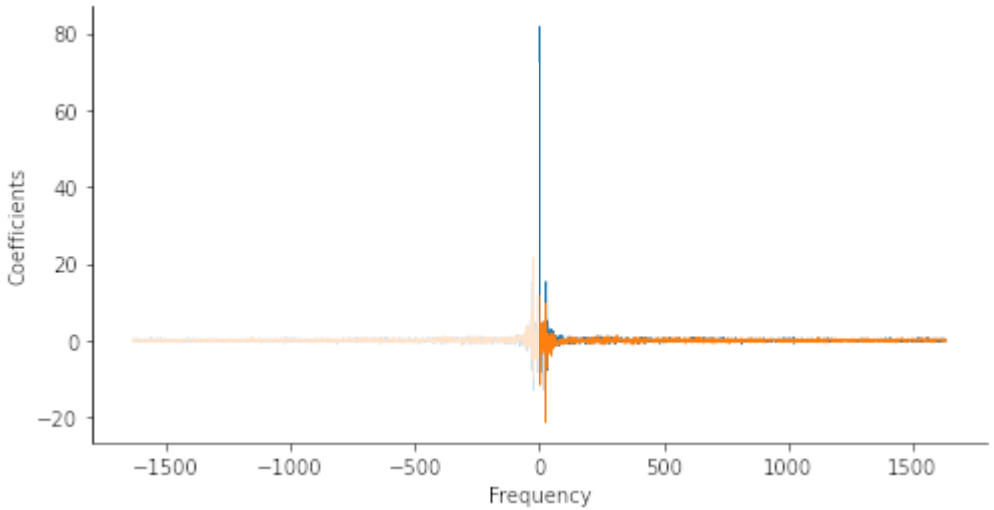


$\boldsymbol{x}$

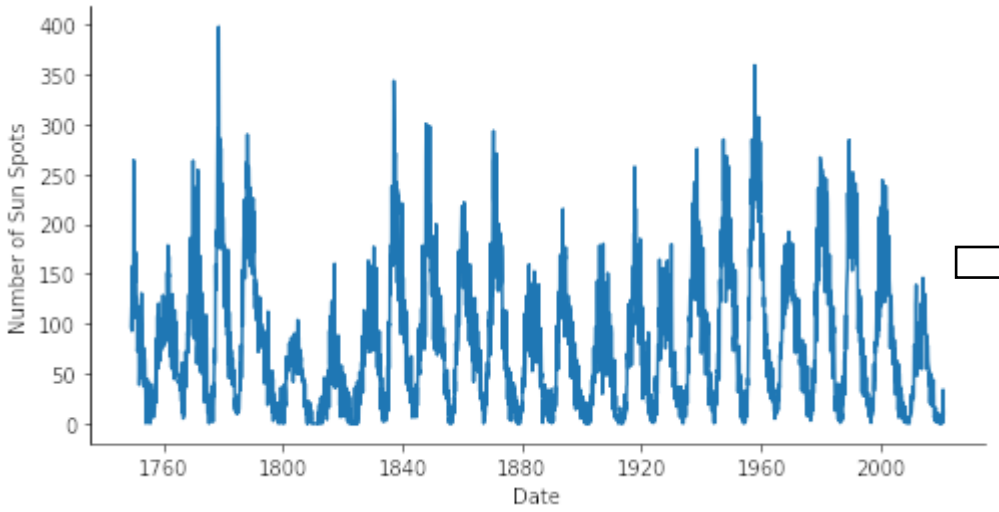


Discrete Fourier Transform

$F(\boldsymbol{x})$

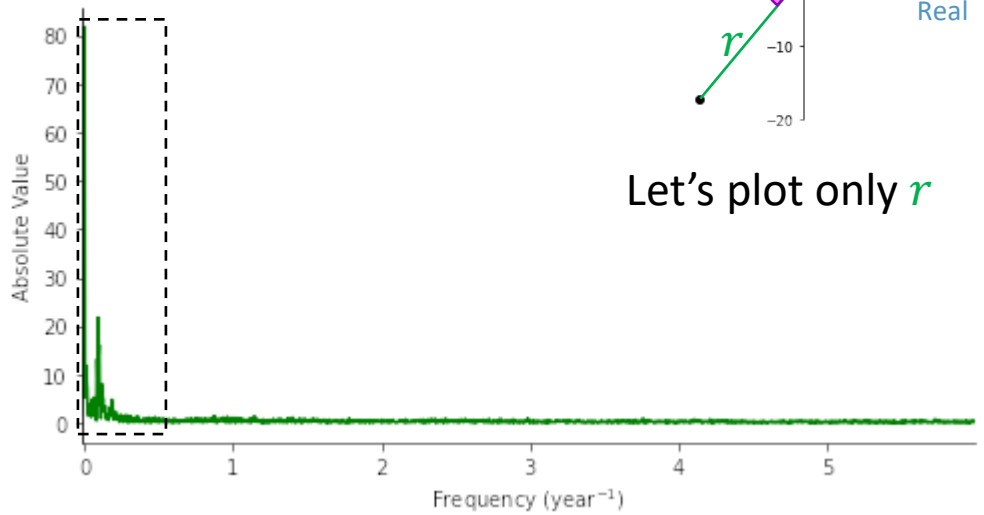


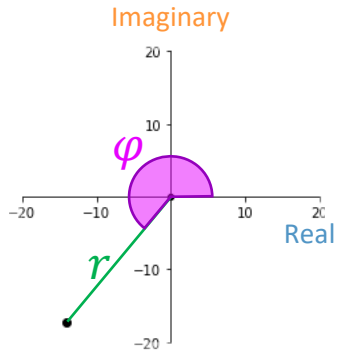
x



Discrete Fourier Transform

$r = |F(x)|$





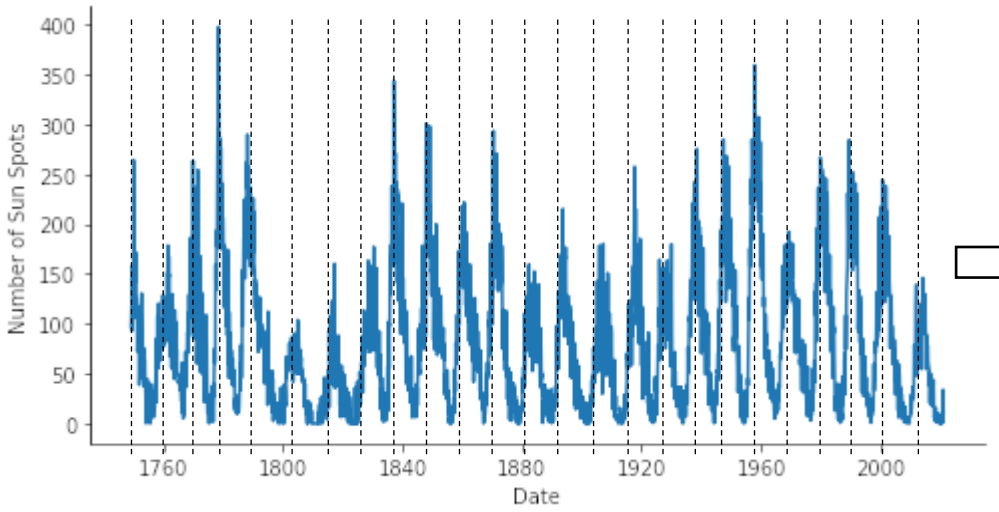
Let's plot only r

Repetitions per Sequence length

Repetitions per Year

Spectral density is $r^2 = |F(\boldsymbol{x})|^2$

$\boldsymbol{x}$



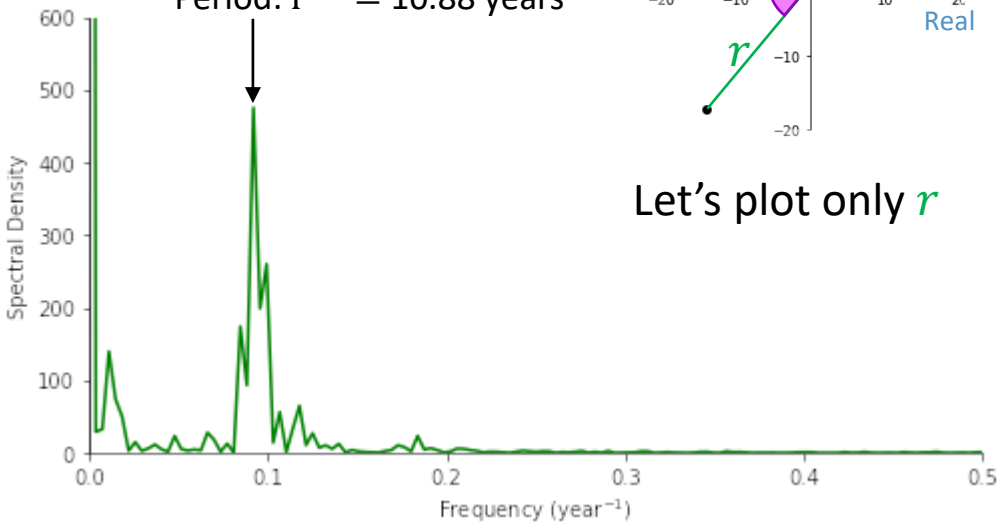
Discrete Fourier Transform

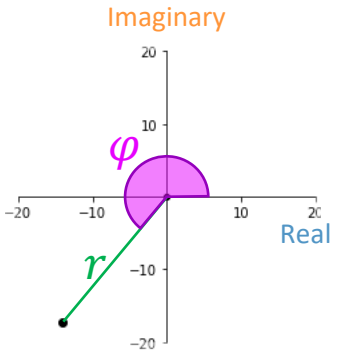
$r = |F(\boldsymbol{x})|$

Solar Cycle

Frequency: $\mathrm{f} = 0.092\ \mathrm{year}^{-1}$

Period: $\mathrm{f}^{-1} = 10.88$ years



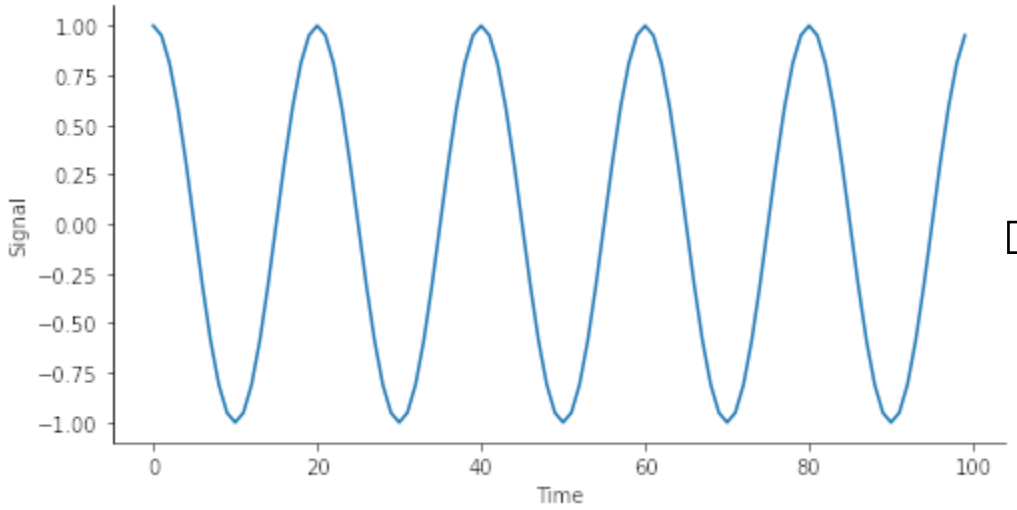


Let's plot only r

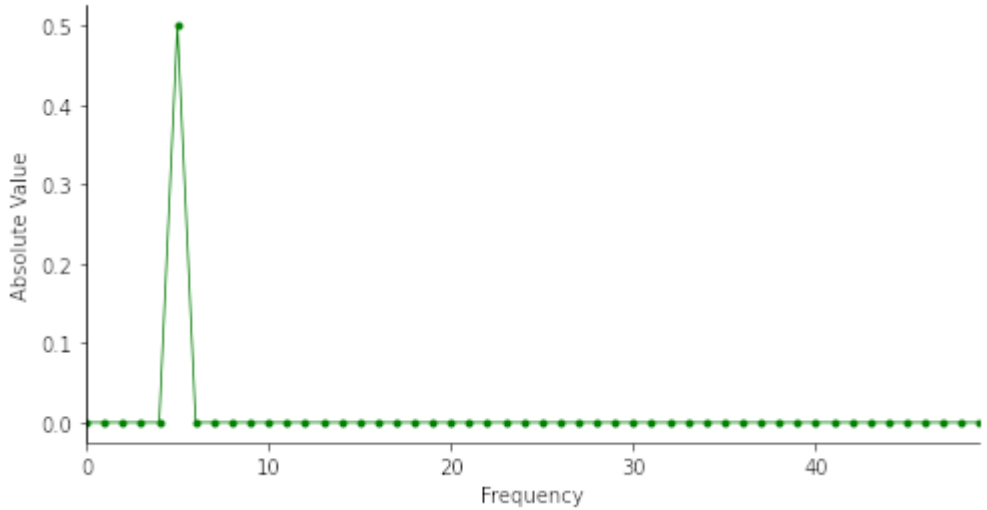
Repetitions per Sequence length ⇨ Repetitions per Year

Spectrum - Examples

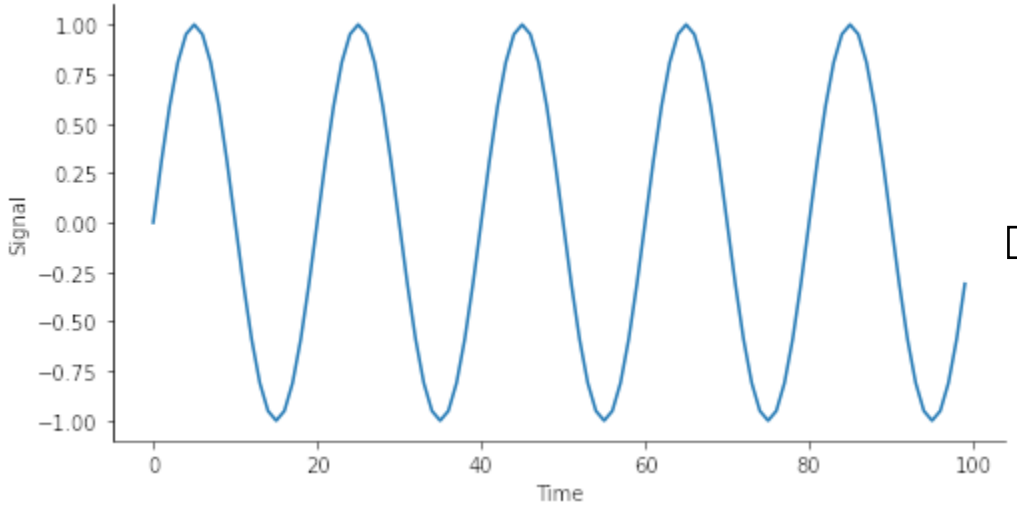
5 repetitions



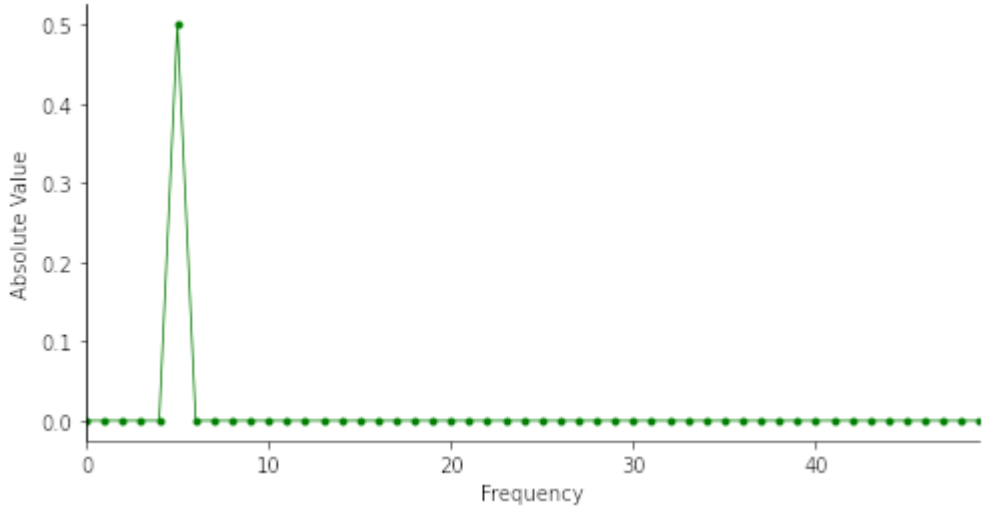
Discrete Fourier Transform



5 repetitions

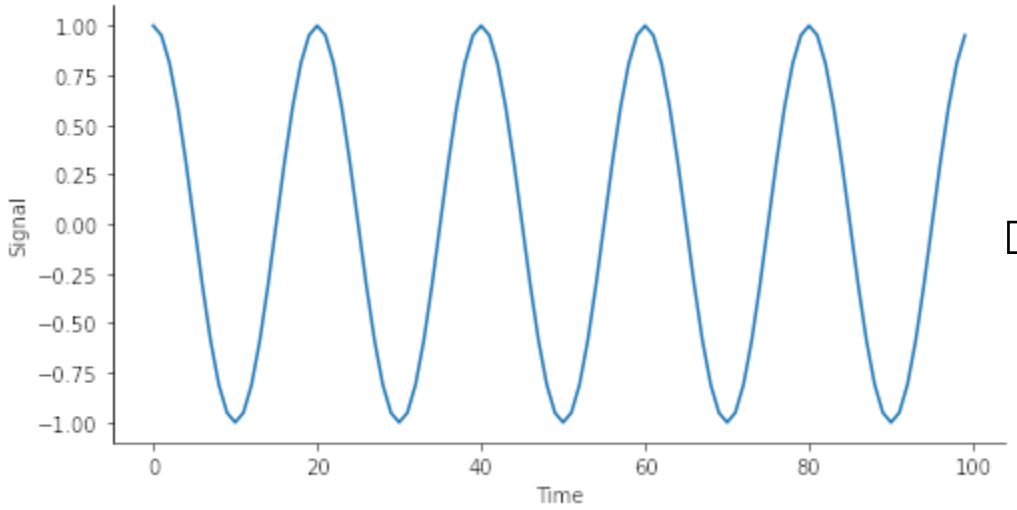


Discrete Fourier Transform

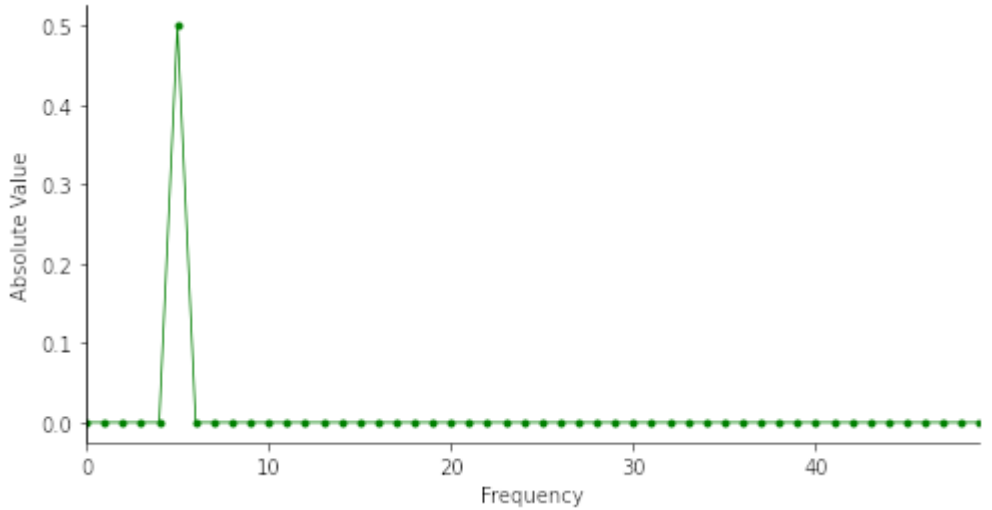


Spectrum - Examples

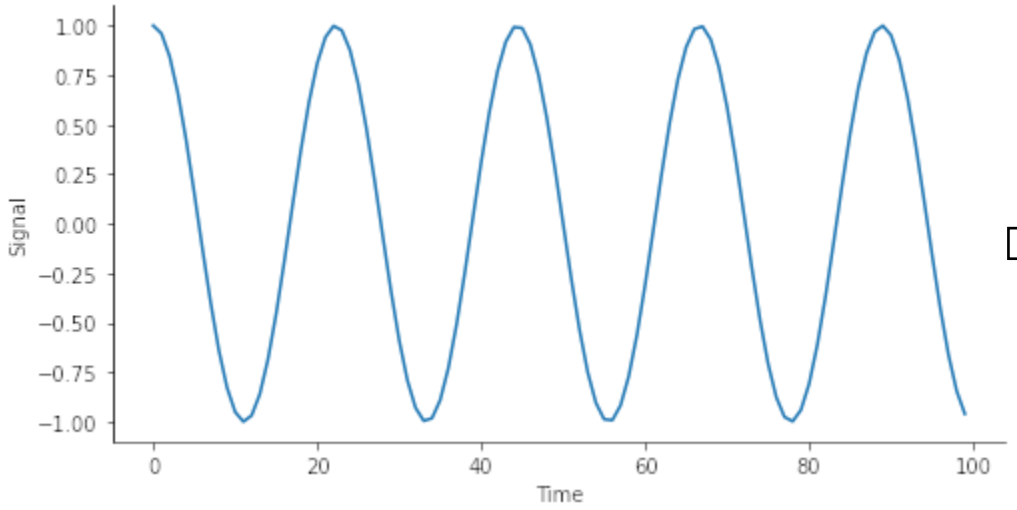
5 repetitions



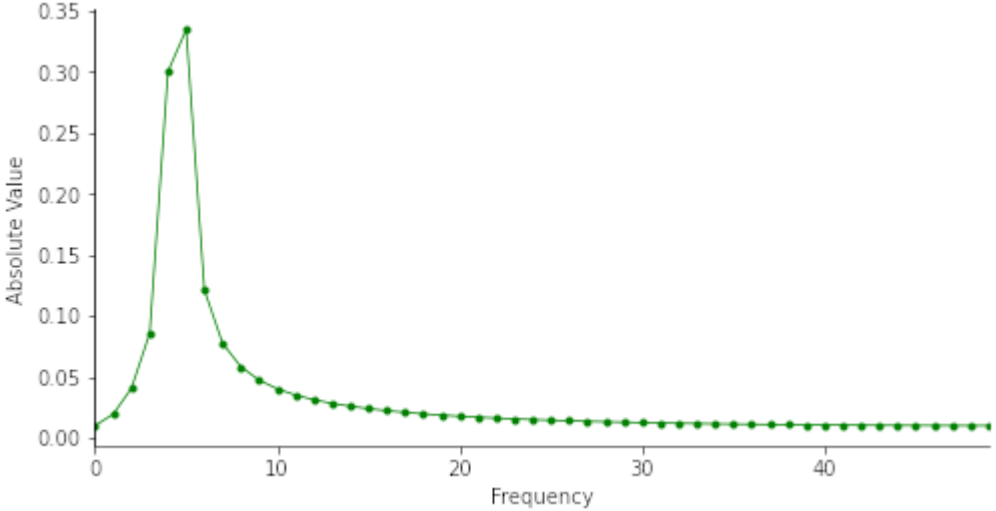
Discrete Fourier Transform



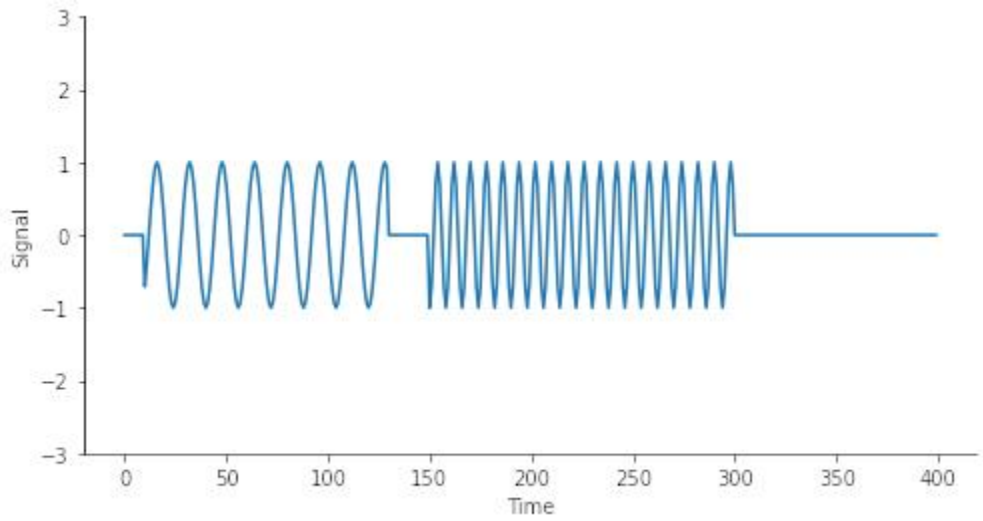
4.5 repetitions

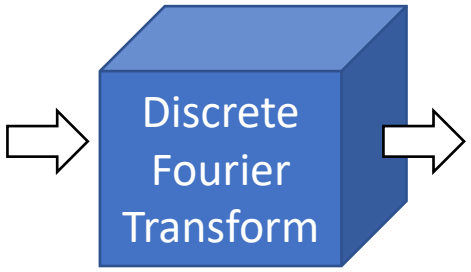


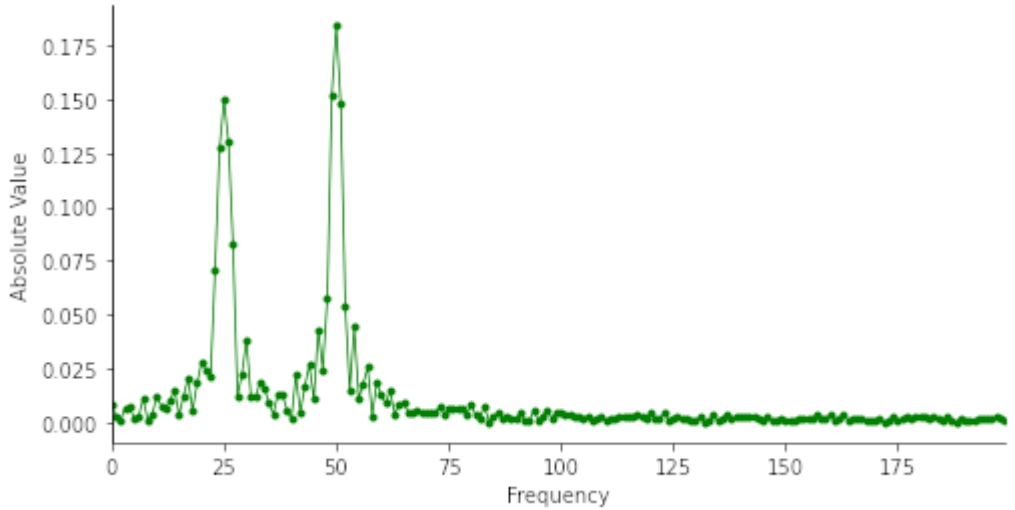
Discrete Fourier Transform

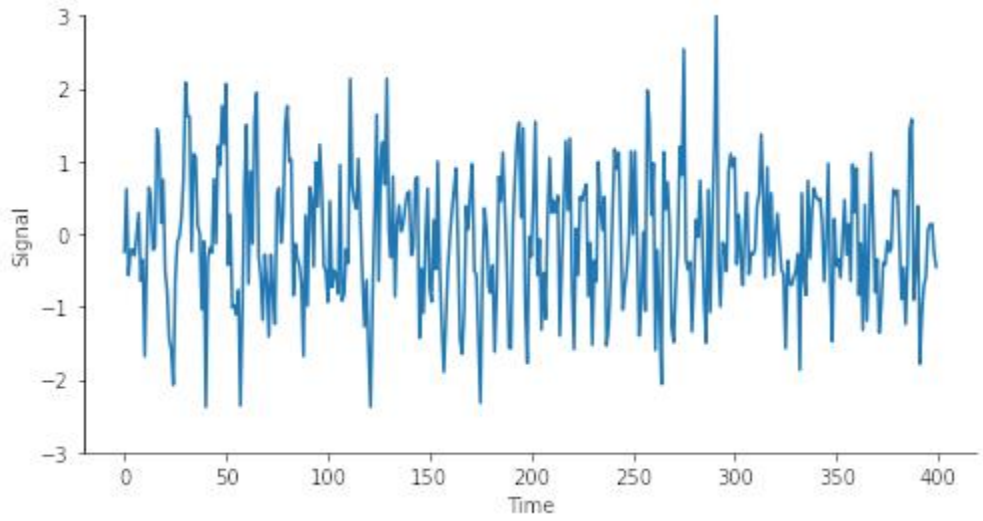


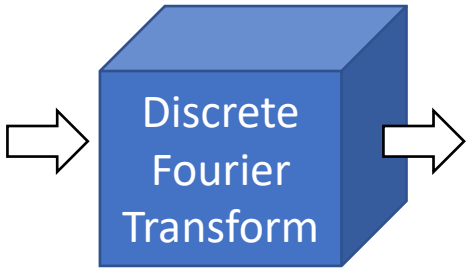
Spectrum - Examples

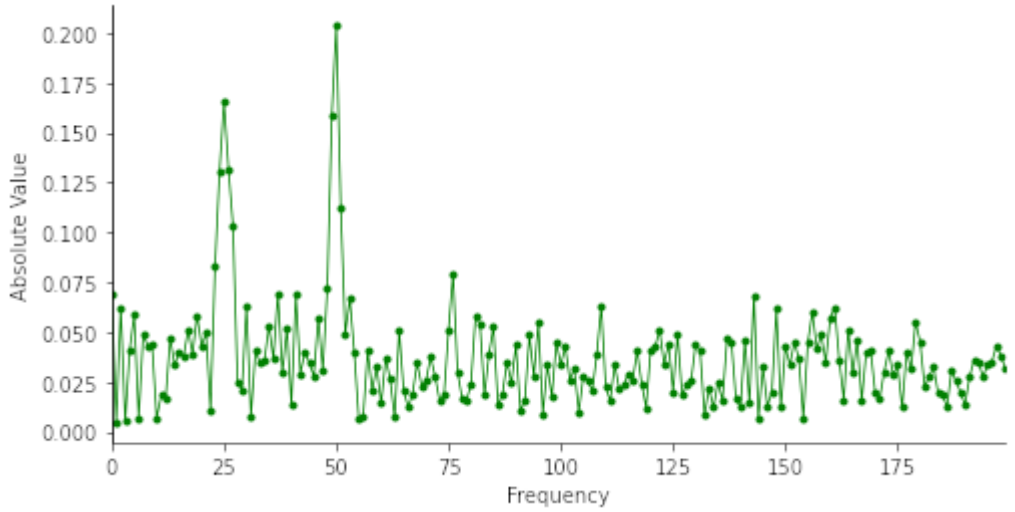






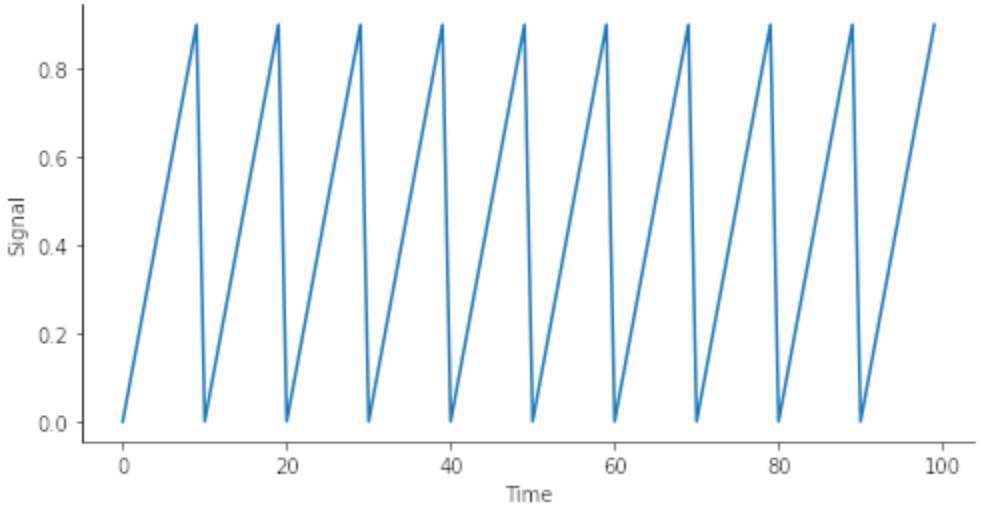




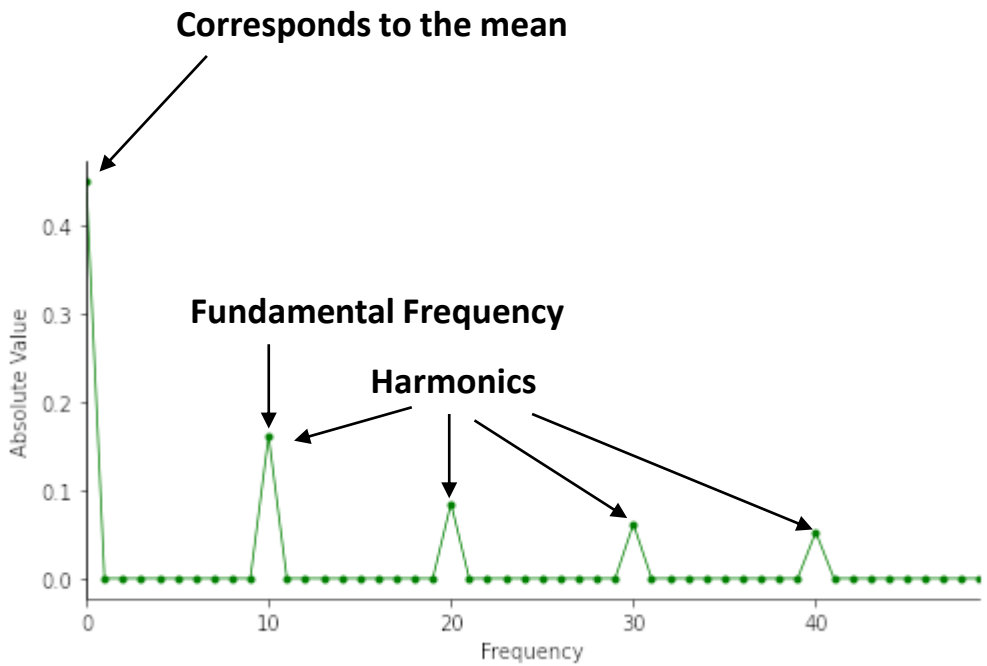


Spectrum - Examples

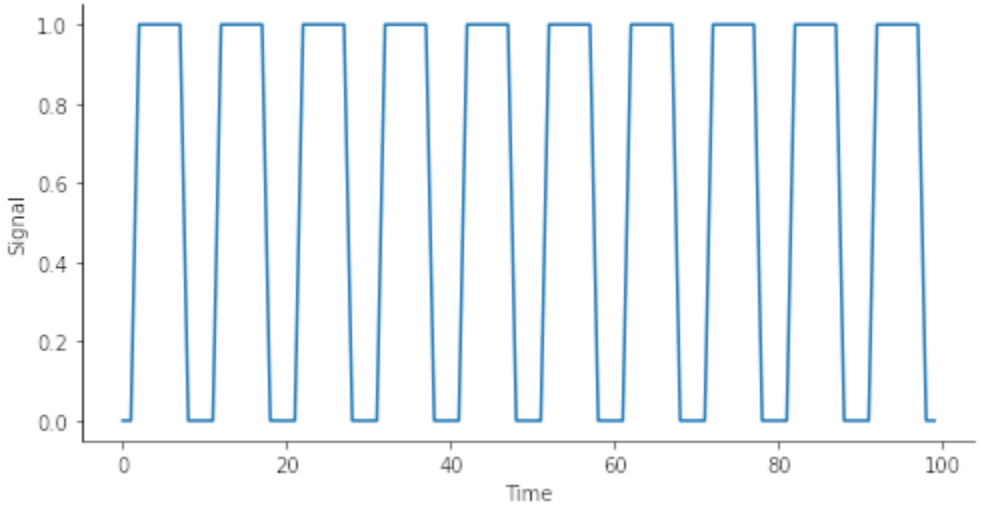
10 repetitions



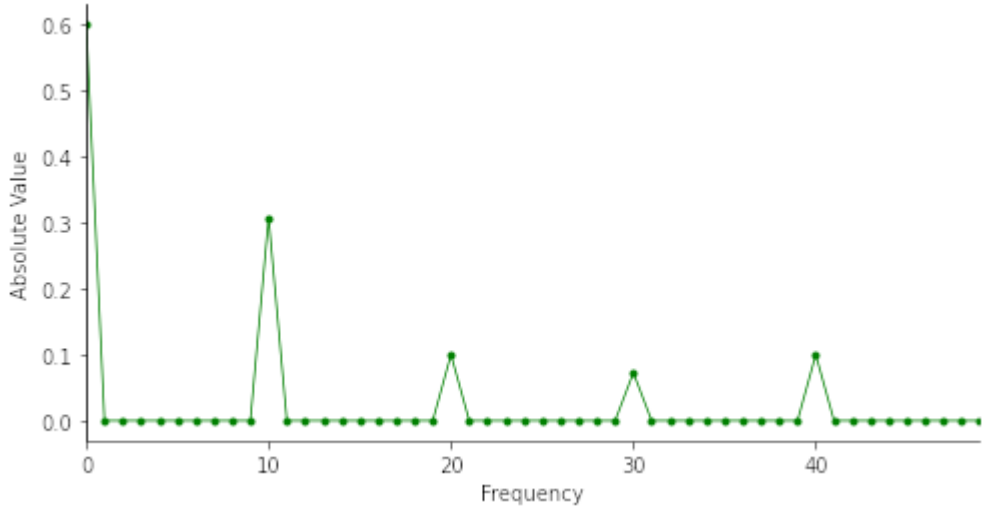
Discrete Fourier Transform



10 repetitions

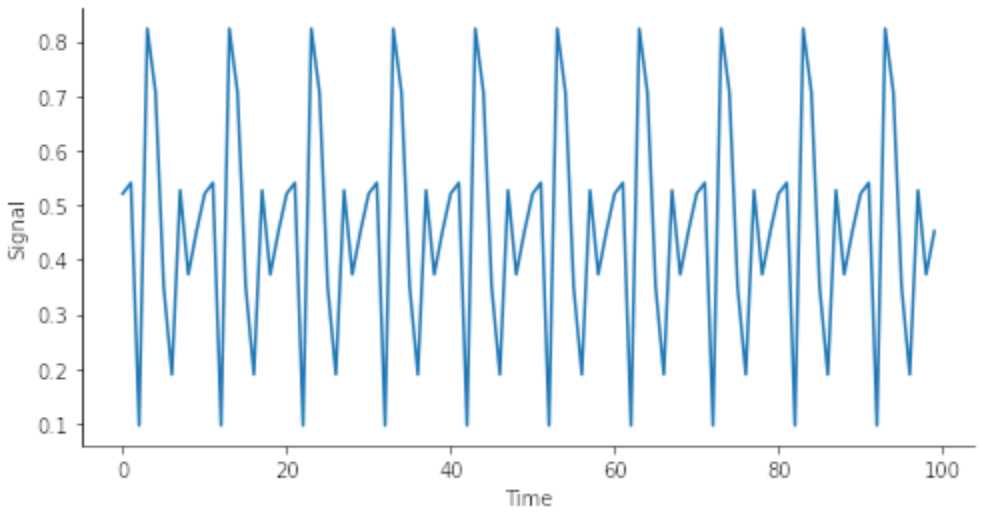


Discrete Fourier Transform

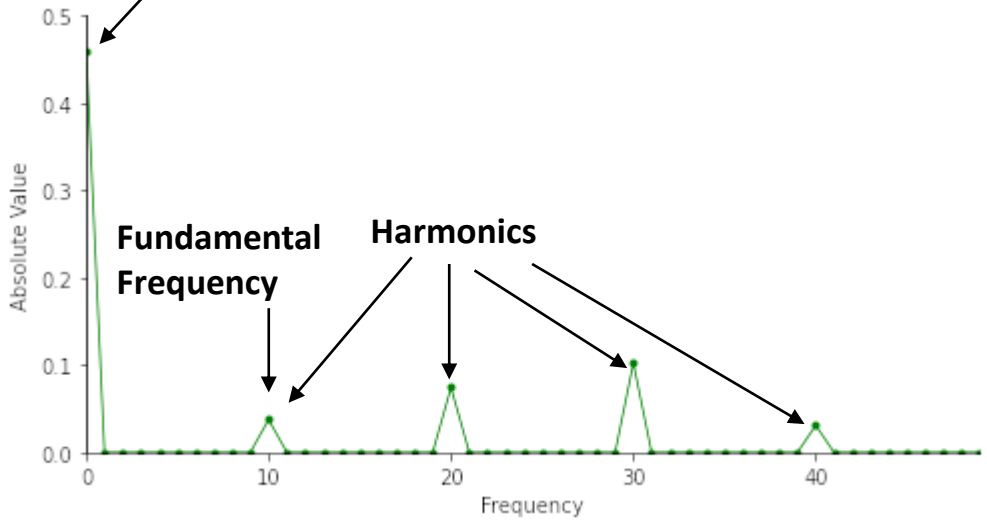


Spectrum - Examples

10 repetitions



Discrete Fourier Transform



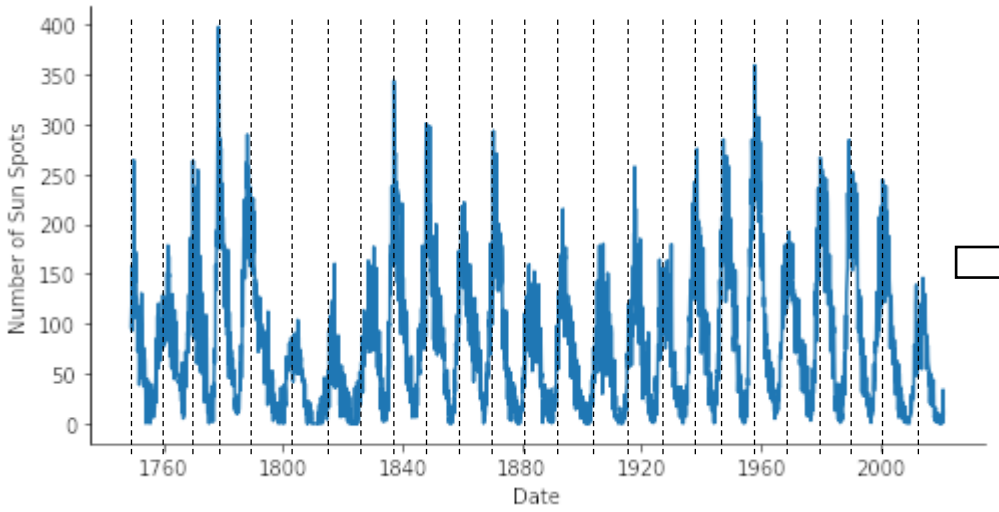
Corresponds to the mean DC component

Fundamental Frequency

Harmonics

Spectral density is $r^2 = |F(\boldsymbol{x})|^2$

$\boldsymbol{x}$



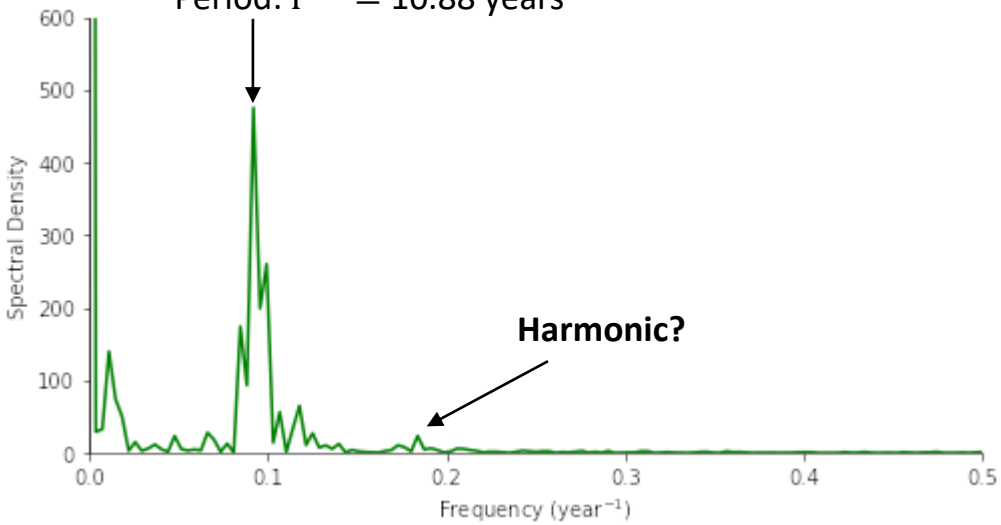
Discrete Fourier Transform

$r = |F(\boldsymbol{x})|$

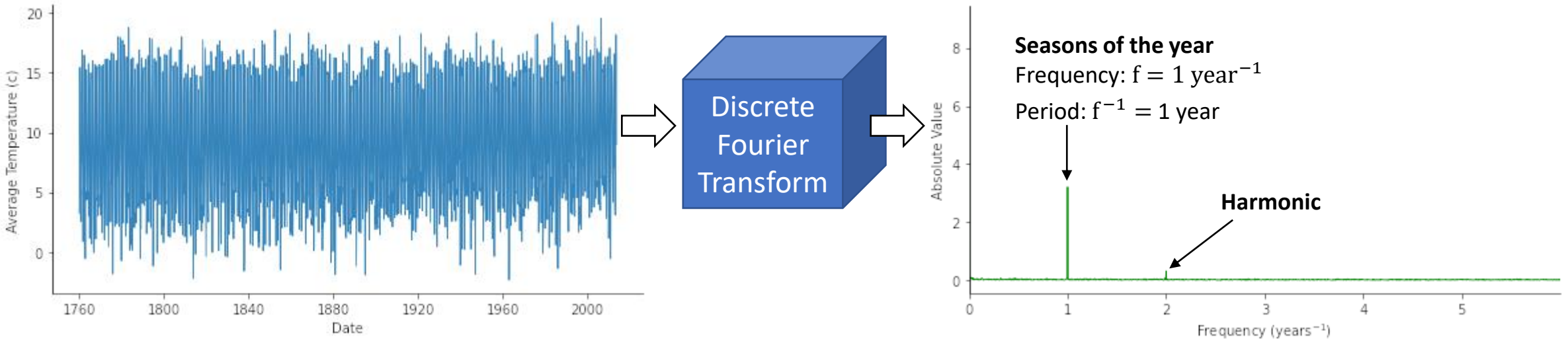
Solar Cycle

Frequency: $\mathrm{f} = 0.092\ \mathrm{year}^{-1}$

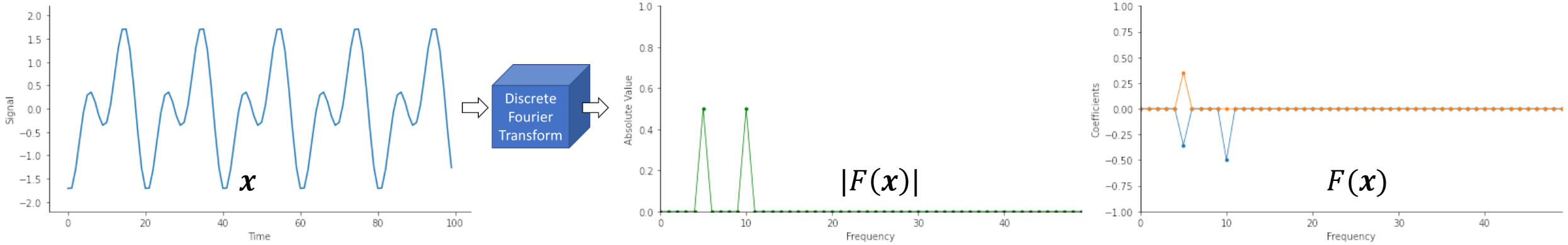
Period: $\mathrm{f}^{-1} = 10.88$ years



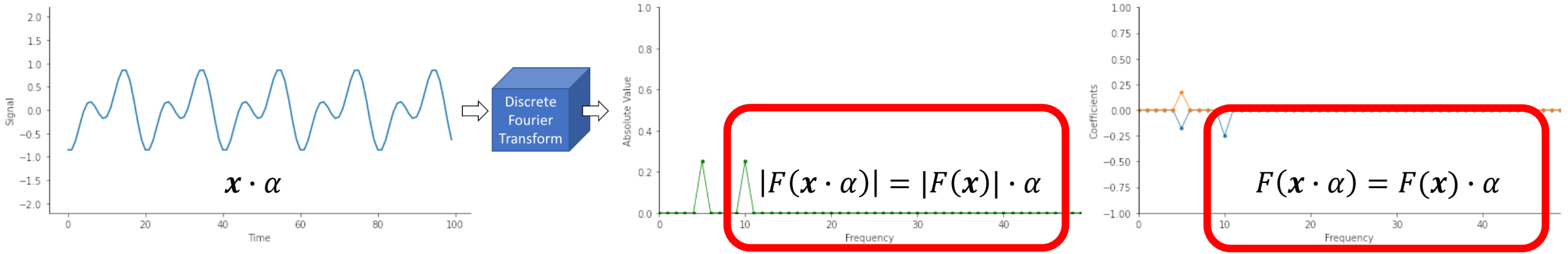
Repetitions per Sequence length ⇨ Repetitions per Year


Average Temperature (c)
20
15
10
5
0
1760
1800
1840
1880
1920
1960
2000
Date
Discrete Fourier Transform
Absolute Value
8
6
4
2
0
0
1
2
3
4
5
Frequency (years^{-1})
Seasons of the year
Frequency: $\mathrm{f} = 1\ \mathrm{year}^{-1}$
Period: $\mathrm{f}^{-1} = 1$ year
Harmonic

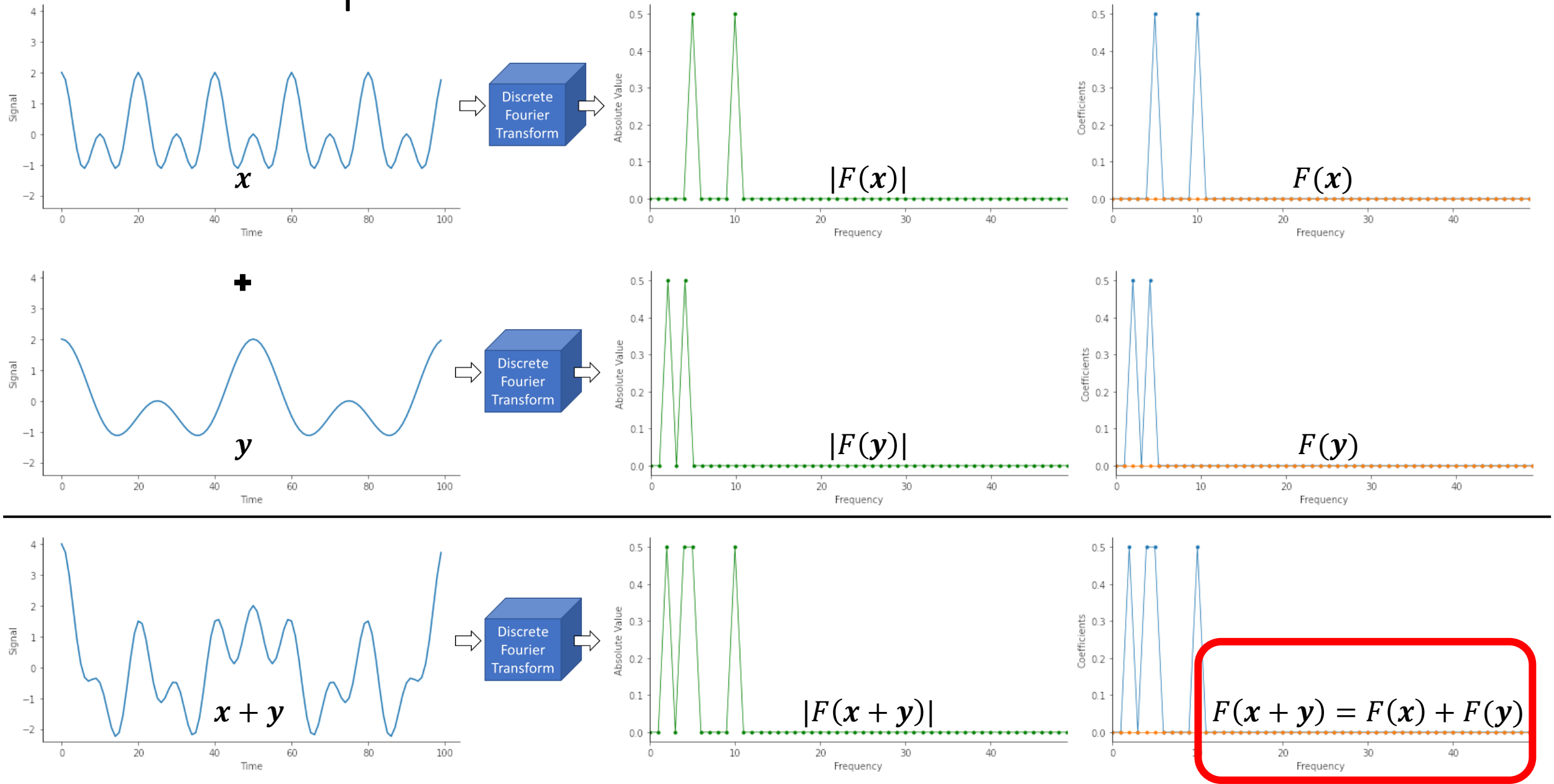
DFT - Properties



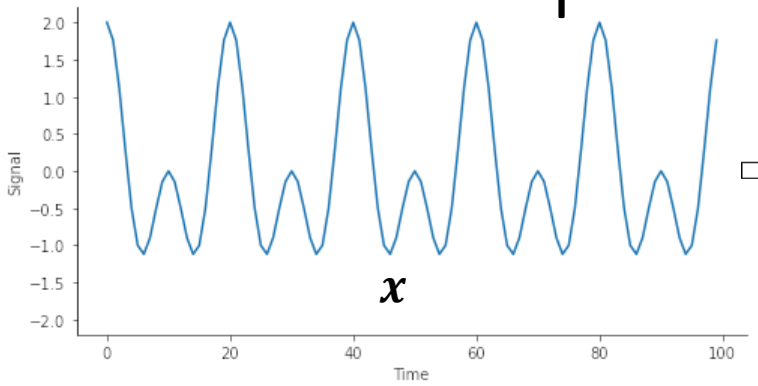
$\cdot \alpha \mid \alpha = 0.5$



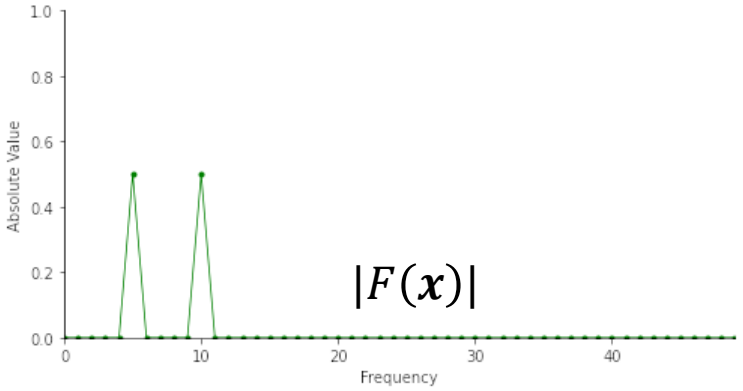
DFT - Properties



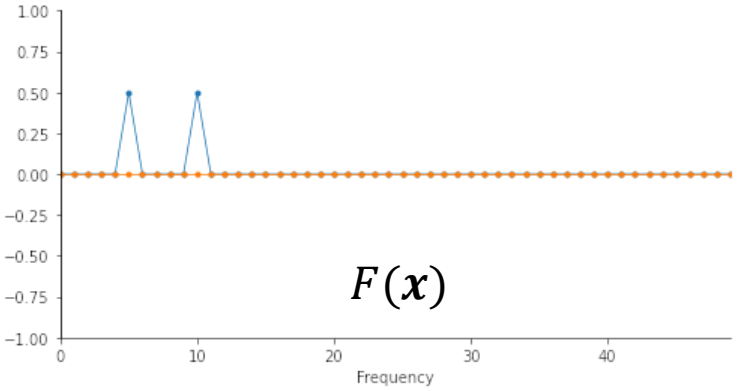
DFT - Properties



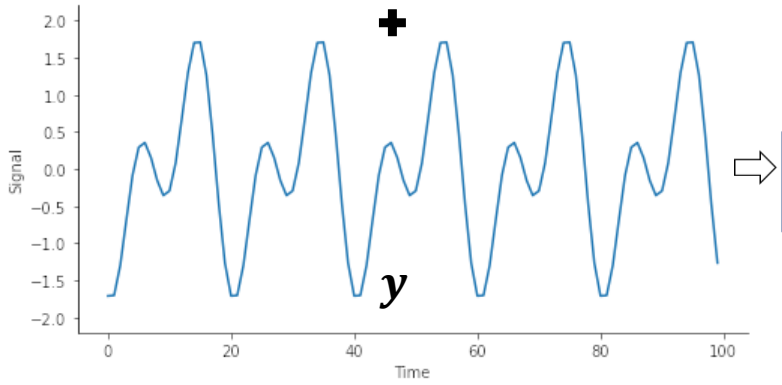
x



$|F(x)|$

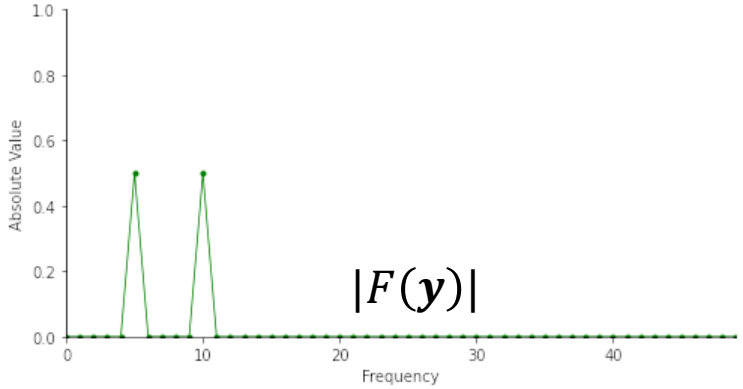


$F(x)$

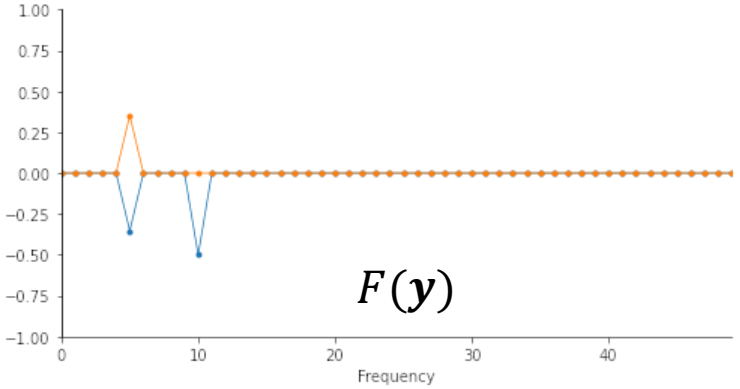


$+$

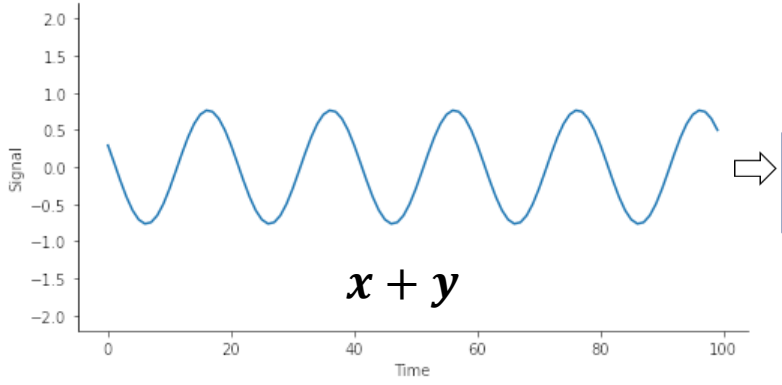
y



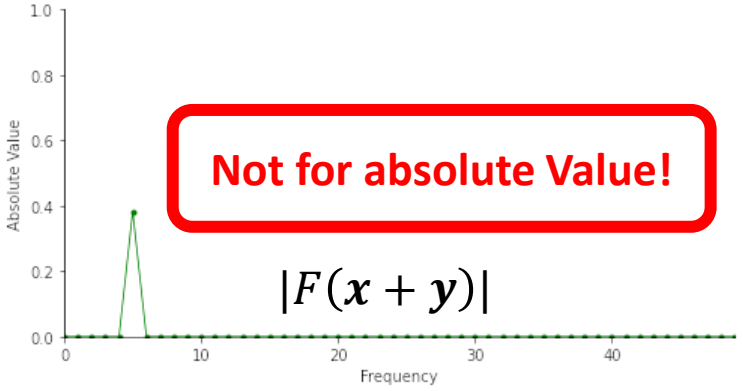
$|F(y)|$



$F(y)$

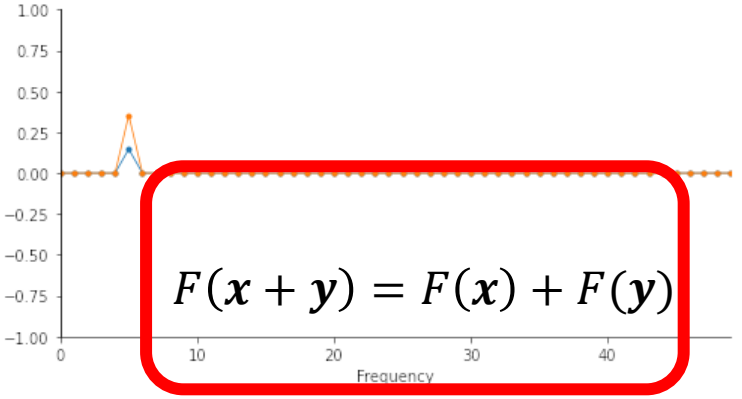


$x + y$



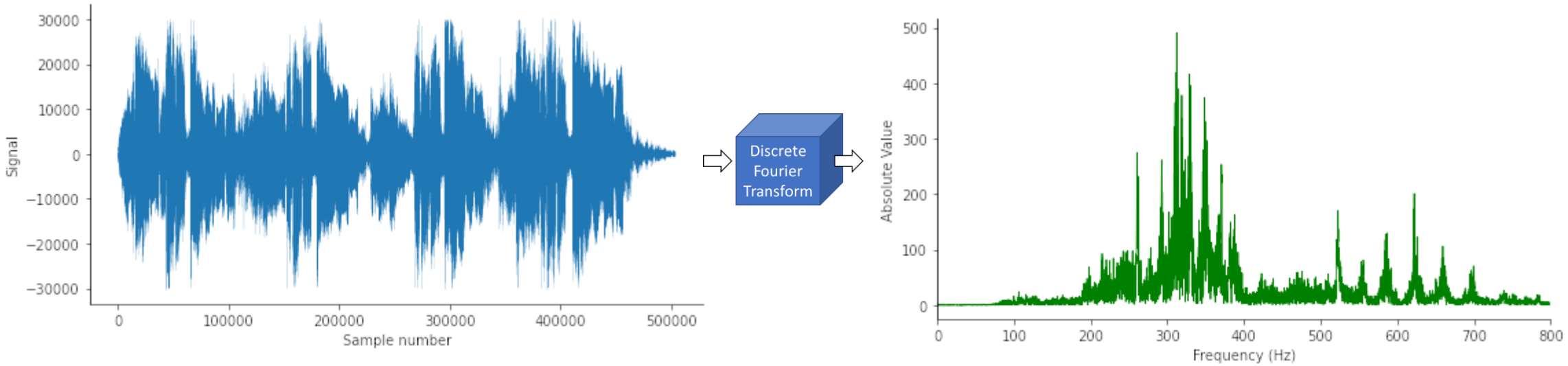
Not for absolute Value!

$|F(x + y)|$



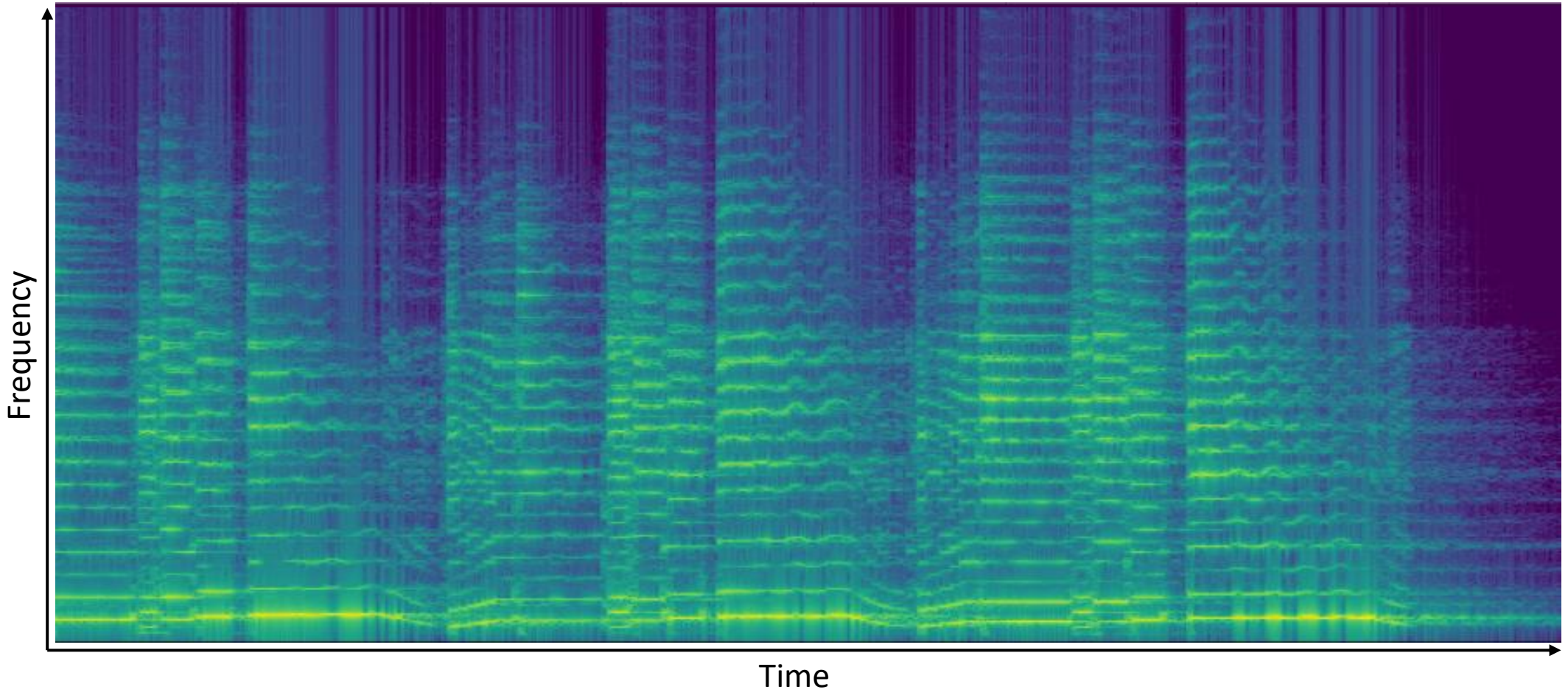
$F(x + y) = F(x) + F(y)$

Spectrograms

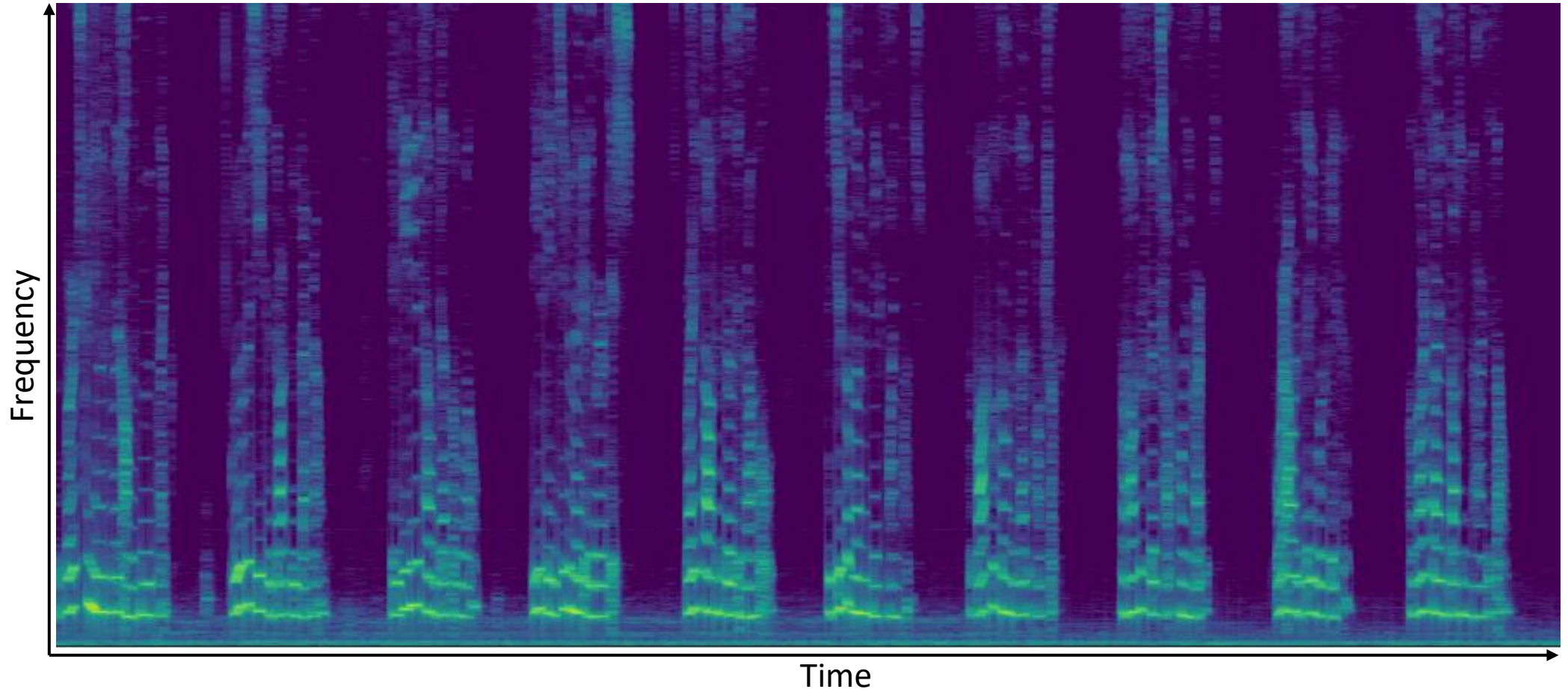


Spectrograms

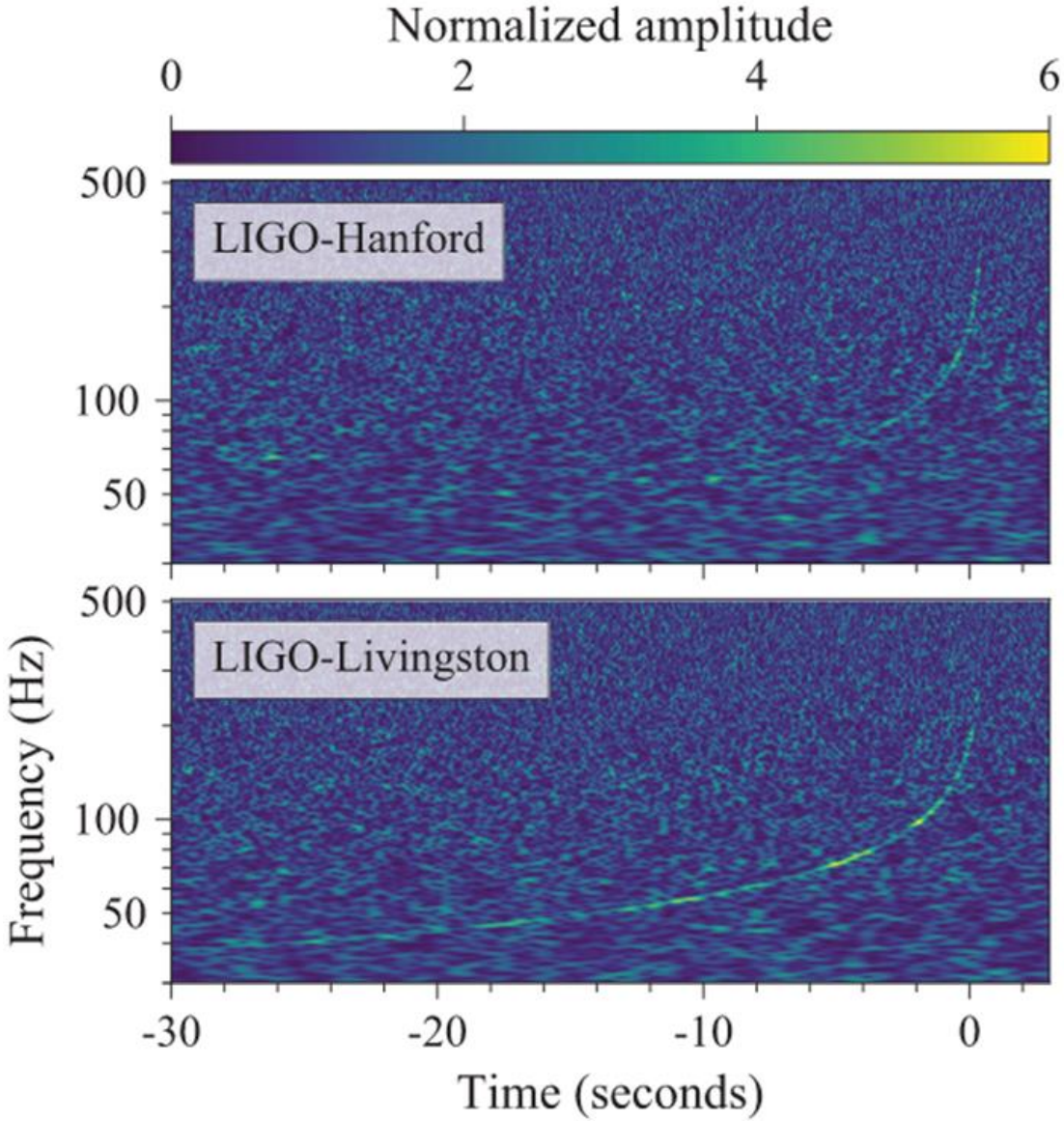
Take logarithm before mapping to colour.



Spectrograms



GW170817
Neutron Star Merger
Observed: 17 August 2017
Distance: 130 Mly



Summary

- Fourier Transform is map between
  - Time Domain
  - Frequency Domain
- Frequency Domain is complex:
  - Absolute value corresponds to amplitude.
  - Phase corresponds to shift of wave.
- We can use DFT to visualise/detect periodic patterns.
- Periodic patterns appear as peaks at:
  - Fundamental Frequency
  - Harmonics
- Spectrograms track frequencies over time.