Intersections between Ancient China and Early Greece — Philosophy of Yin-yang Diagram and Fibonacci Sequence

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To What Extent Does a Comparative Analysis of the Fibonacci Sequence and Yin-yang Diagram Reveal the Philosophical Intersections between Ancient China and Early Greece?

This paper aims to discover the connections between ancient Chinese and early Greek philosophy through the lens of the Yin-yang diagram and the Fibonacci sequence. Following a brief literature review and an explanation of the methodology, the paper first provides an overview of the history of the two diagrams and the relevant philosophical ideologies of the two ancient civilizations. The findings are therefore discussed by conducting a quantitative analysis of Fibonacci numbers. Subsequently, an extended inquiry into existing properties of Fibonacci numbers and their relations to the Yin-yang diagram and a further comparison of the two philosophies are discussed. The findings mainly reveal significant underlying connections, including the sexadecimal calendar counting system and the five classical elements. Differences, such as correlative or causal thinking mode and unified or autonomous epistemology, are found, indicating the fundamental distinctive origins of cosmology. The limitation draws on the potentially biased data selection criteria and limited existing scholarly frameworks to be referenced.

Keywords: ancient civilizations, ancient China, early Greece, mathematics, Fibonacci sequence, Yin-yang diagram, Pythagoras, Confucius.

1 Introduction

One of the most effective means to study and establish the relationships between different civilizations is by investigating mathematics (Neugebauer 1969), which contains profound philosophical ideas (Suppes 2011) (Horsten 2023). The Yin-Yang diagram is an iconic symbol representing balanced and harmonious natural forces that are at the heart of Chinese philosophy (Lin and Chen 2007). The Fibonacci sequence, a mathematical series where each number is the sum of the preceding two, is found to be originated mainly from ancient Greece, a period in which philosophy had been illustrated into mathematics by Pythagoreans (Kalman and Mena 2003). Ancient Chinese and early Greece philosophies had flourished simultaneously in the Axial Age (8th-3rd centuries B.C.), with intellectuals such as Confucius (551-479 B.C.) and Pythagoras (570-490 B.C.) creating transcendental visions in constructing the worldview of the country (Hill 2006) (Bellah 2011) and have strongly influenced the shape of the Eastern and Western culture up to the present day (Heit 2005). Similar to ancient Greek philosophers and astronomers who believed that the entire cosmos is a structure of related bodies (Neugebauer 1969), ancient Chinese philosophers also believed that everything is all unified (Rošker 2021). Additionally, in the beginning of the 20th century, several anthropologists and ethnologists suggested that all cultures could be traced back to a common source (Martin et al. 2017).

Therefore, this paper aims to investigate the philosophical intersections between ancient China and early Greece by examining the Yin-yang diagram and the Fibonacci sequence, with the scope of the study focusing on the Axial Age. In addition, this paper hypothesizes that the Yin-yang diagram and the Fibonacci sequence share some degree of similarity in numerical features, as well as intrinsic philosophical connections. Hence, the research question to be scrutinized is:

To what extent does a comparative analysis of the Fibonacci sequence and Yin-yang diagram reveal the philosophical connections between ancient China and early Greece?

Through the quantitative analysis and comparative discussion, this paper unmasks the philosophical commonalities between ancient China and early Greece while noting their essential epistemological distinctions. Specifically, the similarities are mainly shown in the sexadecimal calendar system, five classical elements, while the differences are more fundamental: ancient Chinese tended to think in correlative, unified, pragmatic ways, and

ancient Greece were leaned to think in causational, autonomous, logical ways, which allude to the concords or disconcords between political, social classes, and intellectual inquiry.

The rest of the paper is organized as follows: Section 2 briefly explains the definitions of the terms used in this paper; Section 3 summarizes the previous literatures; Section 4 outlines the methodology approach. Then, section 5 provides historical overviews of the Yin-yang diagram and the Fibonacci sequence as well as their philosophical significances. Section 6 conducts a quantitative analysis of Fibonacci numbers based on the framework of Yin-yang cosmology. Hereby, section 7 detaily discusses the findings, along with the extended inquiry based on the existing Fibonacci properties and the broader comparative discussion of the philosophy in two countries. Finally, after noting the limitation and further studies in section 8, section 9 concludes by highlighting key philosophical findings as well as the significance of studying philosophical connections utilizing interdisciplinary approaches between East and West.

2 Definitions

This section provides a listed definition of frequently used terms in this paper, including the Fibonacci sequence, the golden ratio, the Pisano Period, the Yin-yang diagram, and *Hetu*.

***Fibonacci Sequence*:** The Fibonacci sequence or Fibonacci series, denoted as $F(n)$, is a series of numbers where each number is the sum of the previous two (Kalman and Mena 2003), starting by

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34\ldots \text{ (OEIS A000045).}$$

Each Fibonacci number can be expressed as

$$F(n) = F(n-1) + F(n-2),$$

with the seed values:

$$F(0) = 0 \text{ and } F(1) = 1.$$

***Golden Ratio*:** The golden ratio, also called the golden mean, the golden proportion, and the golden section, is denoted by the Greek letter Φ or φ to represent the irrational number

$$\frac{1+\sqrt{5}}{2} \approx 1.61803398...,$$

In specific, two positive numbers x and y (assuming $x > y$) are in the golden ratio if the ratio between the sum of two numbers and the larger one has the same value as the ratio between the larger one and the smaller one (Choi 2023). That is:

$$\varphi = \frac{x+y}{x} = \frac{x}{y}.$$

Euclid referred to the golden ratio as "division in extreme and mean ratio," and it can be derived into the golden rectangle and the golden spiral (Markowsky 1992).

***Pisano Period*:** The Pisano Period, denoted as $\pi(m)$, is defined by

> *"determining the length of the period of recurring series obtained by reducing a Fibonacci series by a modulus m"* (Wall 1960, p.525).

In other words, a sequence repeats periodically in each modulo m. When $m = 1, 2, 3, 4, 5, 6,...$the Pisano Period is

$$\pi(m) = 1, 3, 8, 6, 20, 24,... \text{ (OEIS A001175).}$$

For instance, the sequence of Fibonacci numbers modulo 4 starts as:

$$0, 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, 1, 1, 2,... \text{ (OEIS A079343).}$$

The pattern, $0, 1, 1, 2, 3, 1$, repeats, meaning the period is 6. Thus, $\pi(4) = 6$, or $F(n) \bmod 4$, where

$$F(n) = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34 \text{ (OEIS A000045).}$$

Moreover, Lagrange noted the property of the Pisano Period's remainder in 1774 (Livio 2003). Specifically, when $\pi(10) = 60$, the last digits of the Fibonacci sequence repeat at the periodicity of 60 for integers. Subsequently, the final two digits repeat at the cycle length of 300, and the final three digits have a period of 1500. Later, mathematicians

proved that 15,000 is the period for the final four digits, 150,000 for the last five digits, and the pattern continues (Wells 1997; Meisner 2018). Thus, when the final n digits $= 1, 10, 100, 1000$, the corresponding length of the repeating cycle of n in the Fibonacci sequence is listed as

$$60, 300, 1500, 15000, 150000, 1500000,\ldots \text{ (OEIS A096363).}$$

***Yin-yang Diagram*:** The Yin-yang diagram, or Tai-Chi diagram (*Taijitu* 太极图), is a symbolic representation to show the dynamic relationships between Yin and Yang (Lin and Chen 2007), illustrating the significance of various numbers in Chinese classical texts such as *I-Ching*.

***Hetu*:** *Hetu*, the Yellow River Map, or the River Chart, originated from ancient China, is one of the two oldest diagrams in the world (Wang 2012). It illustrates the formation of the Yin-yang diagram, which shows its inherent composition as number patterns, representing Yin with black dots and Yang with white dots.

3 Literature Review

There are a number of papers written in the field of the fibonacci sequence and Yin-yang diagram, individually. Unfortunately, few scholarly works have been done to investigate the connections between the fibonacci sequence (and the golden ratio) and the Yin-yang diagram. Some discussed the possible, vague relations between *Hetu* and the golden Rectangle (Yao and Lin 2014); some innovatively shown the close ties between the arrangements of five elements and two key number series of Fibonacci sequence modulo 5, which is the center number of *Hetu* (Chen 2015). In addition, some scholars perceived from the perspective of philosophical connection, where Pythagoras and Plato believed that equilibrium created the universe, which is highly consistent with typical Chinese Yin-yang Bipolar Equilibrium theory (Zhang 2011). Others approached it from the realm of statistical inference: the dually flat manifold, which is, to some extent, the composition of the generalized Pythagorean theorem, could be applied to Yin-yang machines (Amari 2010).

Moreover, it was proven mathematically that the golden ratio existed in the visual of the Yin-yang diagram, and the Fibonacci spiral matched the formation of Yin-yang by taking time and space into account (Inchauspe and Arakaki 2023). Some created the *Luoshu* (a parallel or derivative of Hetu)-Fibonacci Diagram by claiming the profound mathematical tie between the two to disclose the scientific similarity between Chinese medicine and modern biology (Chen et al. 2013). Other newer works have attempted to hypothesize the three-dimensional Yin-yang diagram in quantum mechanics (Leong 2024).

Although each previous work mentioned above approached the topic from specific and distinct aspects, the gap that explores the number system of Yin-yang concerning number patterns in the Fibonacci series remains open. Additionally, it could be perceived that there is a polarized focus on being either mathematical or philosophical in different works, indicating the lack of understanding or emphasis on the connection between mathematics and philosophy and their relations to the epistemology of various civilizations. This paper aims to fill these gaps by using an exploratory approach. Moreover, some existing properties of the Fibonacci series are referred to in this paper as well, including the hidden name of the god in number 216 (Khan 2013), the spinning seeds at the angle of 0.48 (Naylor 2002), Binet's formula (Kilic 2008), as well as the remainder properties of repeating last digits in Pisano Period, noted by Lagrange in 1774 (Livio 2003).

4 Methodology

This paper employs a quantitative analysis to investigate the philosophical connections between ancient China and early Greece. The research follows a descriptive statistical analysis approach, collecting data using the existing Fibonacci series and Pisano period, summarizing and interpreting the statistics through the lens of Yin-yang cosmology, and providing a philosophical, comparative study of the trends and characteristics of the findings.

Data Selection

The data used in this research is mainly attributed to the Pisano Period (henceforth $\pi(m)$). The criteria for selecting data is based on the significant numbers mentioned in *I-Ching* and *Hetu*, which together comprise the Yin-yang diagram.

Research Design

The first part of the quantitative analysis uses the modulo of the Pisano Period, filtering data by using significant numbers to construct the Yin-yang diagram. Hence, four groups of modulo are statistically analyzed: (1) According to the numbers of eight trigrams, $mod\ 4$, $mod\ 8$ are summarized; (2) According to the numbers of *Hetu*, $mod\ 5$, $mod\ 6$, $mod\ 9$ are summarized; (3) According to the number of the Great Expansion, $mod\ 50$ is summarized; (4) According to the number of Heaven and Earth, $mod\ 25$, $mod\ 30$, $mod\ 55$ are therefore summarized.

The second part of the quantitative analysis is based on the remainder of the properties of the Pisano Period noted by Lagrange. This paper replicates the process of generating the first repetitive cycle of the last digits $\pi(10) = 60$, supported by its ratio to the golden ratio and the visual of the Fibonacci Spiral. The remaining properties of the final digits are referred to as proven properties and are discussed in relation to the important number 60 in the ancient Chinese calendar system.

Comparative Analysis

After the quantitative analysis, a comparative analysis is therefore discussed in three steps. Firstly, based on the numerical patterns found in the quantitative analysis, their respective philosophical significance in the two contexts is discussed in four groups. Secondly, an extended inquiry into existing properties of Fibonacci numbers and their relations to the Yin-yang diagram is thus briefly discussed, including the hidden name of the god in number 216, the spinning seeds at the angle of 0.48, and Binet's formula. Finally, a broader comparison is made between ancient Greek and Chinese philosophy.

5 Foundations

5.1 Yin-yang Diagram in Early China Philosophy

The philosophical concept of Yin-yang occurred much earlier than the Yin-yang diagram. Hence, to fully comprehend the Yin-yang diagram, when and how the Yin-yang cosmology was developed shall first be understood, considering that the origin of Yin-yang signifies the beginning of China's civilization. In addition, this paper notes that the philosophical significance of Yin-yang builds the foundations of the Yin-yang diagram, so the philosophy of Yin-yang is subsequently introduced. Finally, the history of the Yin-yang diagram is explained.

5.1.1 Yin-yang's Time Periods

The cosmology of Yin-yang was initially discovered and organized by the first authorized emperor of China, Fuxi (伏羲), during the Three Sovereigns period, which is China's earliest known prehistoric period recorded by the *Annuls of the Three Sovereigns* written by the historian Sima Zhen (679-732). Similar to other pre-literate civilizations, the prehistorical records of China are primarily oral history that cannot be verified by current archeological data (Liu and Xu 2007). Hence, the exact periods when Chinese civilization began are quite controversial. Some indicated that the early China periods could be traced back to 2100-200 B.C., from Three Sovereigns and Five Emperors to the Xia, Shang, and Zhou Dynasties (Liu 2009). Meanwhile, some elucidated that China appeared in the Neolithic Age (10000-2200 B.C.), a period that indicates significant prehistory accounts of humans (Keightley 2022). Disregarding to identity of the exact years to define the beginning of China, it is clear that the Three Sovereigns period specifies the known beginning of Chinese civilizations, parallel to the appearance of Yin-yang.

In *the Book of History* (*Shangshu* 尚书), Confucius and his fellow students (Fei 2017) first organized the history of Three Sovereigns in the Eastern Zhou dynasty (771-256 B.C.). Confucius is regarded as the most influential Chinese philosopher (Confucius 2023). His creation of the Confucianism school has been characterized as an ethical and social philosophy, a way of life, a theory of government, or traditions (Yao and Yao 2000), rather than a religion (Freedman 1962), and most importantly, as guiding principle of finding the

“middle ways” of yin and yang (Baghos 2017). Moreover, it shall be considered that Confucius was born in the period of Hundred Schools of Thought (zhuzi baijia 诸子百家), where numerous schools and philosophies emerged, covering from the 6th century BC to 221 BC (Wei and Rui 2019). Various well-known philosophers were active in this period, including Confucius, Mencius (372-289 B.C.), and Laozi (571-? B.C.). Regarded as the Golden Age of Chinese philosophy, the knowledge legacy left by Hundred Schools of Thought has remained its central position in China's long history to shape people’s ways of living, which “characterizes the behavior of individuals composing a social group collectively” (White 1959, cited in Sapir 1917, p. 2).

The further connection between Fuxi and Confucius, the two most significant figures across centuries, is through the book *I-Ching* (*the Book of Changes* 易经). In *I-Ching*, the wide-spreading cosmology of Yin-yang was first fully introduced and became universally prevalent. In the book, Yin and Yang are represented as divided and undivided lines, further composing eight trigrams (three lines) and sixty-four hexagrams (six lines) (Feng 1948). *I-Ching* is deemed as the ancient Chinese philosophy's progenitor (Zhang 1968) and divination to tell the oracle and connect heaven and earth (Baynes 2001). Particularly, the earliest author of *I-Ching* was attributed to Fuxi. In addition, the author of this earliest Commentary, *Yi Zhuan,* was accredited to Confucius, who expanded the original *I-Ching*. His comments became the only officially canonized Commentary in history (Redmond and Hon 2014), which has been reputed as "the Classic of Classics" by Confucians in the Han Dynasty (Ritsema et al. 2018).

Although the concept of Yin-yang was prevalently indicated in *I-Ching*, contributed by Fuxi and Confucius, the Yin-yang graph was not formulated until the Song dynasty (960–1279) by collective scholars, ranging from Daoism and Confucianism (Hu 1977). Thus, concerning the wide time range that Yin-yang covers, this paper mainly draws the research realm from Three Sovereigns, the Eastern Zhou dynasty, until the Song dynasty.

5.1.2 Philosophy of Yin-yang

Yin-yang is the most essential and influential way of thinking for the Chinese, as ontology, cosmology, and outlook in life (Jiang 2013). Specifically, Yin-yang is considered a traditional and indigenous way of thinking and is one type of Chinese philosophy (Huang

2016). Depending on the contexts, there are many different ways to regard Yin-yang: as an epistemology (Li 2014), as a representative symbol (Fenichel, 2014), as a dialectical logic system (Munduate et al. 1999), as well as the main framework of Chinese medicine (Sun et al. 2013). Most importantly, the Yin-yang worldview has remained central to Confucianism and Daoism (Cheng 1977).

In *Discussion of States* (*Guoyu Zhouyu* 国语·周语) compiled in 4 B.C, Yin-yang was first mentioned. The chapter describes an earthquake that happened in 780 B.C. by stating that:

> *"When the Yang is concealed and cannot come forth, and when the Yin is repressed and cannot issue forth, then there are earthquakes"* (Zhou Yü, 1.10).

Clearly, Yin and yang were referred to as two natural forces that shaped the earth's formation. In addition, Yin-yang was also referred to in the oldest Chinese medical treatise, *The Yellow Emperor's Inner Classic* (黄帝内经), which covered basic health conditions and the highest levels of being, whereby stated that the "true person" (Zhenren 真人) is able to

> *"carry and support heaven and earth and grasp and master Yin-yang"* (Zhang 2002, cited in Wang 2012, p. 2).

As seen, Yin-yang became the abstract natural spirit that ancient human beings could practice. Furthermore, *I-Ching* refers to Yin-yang representing two opposed forces that are interdependent and complementary of each other (Wu et al. 2015), having the component of dualistic antagonism and harmony at the same time. Examples of Yin-yang include heaven and earth, man and woman, black and white, ascending and descending, internal and external, dark and light, and materials and spirits (Sun et al., 2013).

Therefore, considering that Yin-yang has a wide range of philosophical meanings, this paper discusses Yin-yang cosmology with reference to the classical texts of Confucianism and Taoism, including *Analects* and *Tao Te Ching*, the most important of which is the *I-Ching*.

5.1.3 History of Yin-yang Diagram

It was during the Song dynasty that the Yin-yang diagram started to be formulated. Zhu Zheng (1072-1138), followed by a Qing dynasty scholar Hu Wei (1633-1714), summarized the development of the Yin-yang graph in three trends (Hu 1977): Liu Mu's (1011-1064) deduction of *Hetu* and *Luoshu* (River Diagrams and Luo Writings); Shao Yong's (1011-1077) reorganization of *Xiantian Tu* (The Diagram Before Heaven) and *Houtian Tu* (The Diagram After Heaven); and Zhou Dunyi's (1017–1073) creation of *Taijitu* (The Diagram of the Great Ultimate). It was due to their contributions that the first Yin-yang symbol appeared in the early Ming dynasty in Zhang Huang's *Gu Taiji Tu (*Ancient Diagram of Great Ultimate) and Zhao Huiqian's *Tiandi Ziran Hetu* (River Diagram of the Spontaneous Process of Heaven-and-Earth) (Fig. 1). Nonetheless, some scholars argue that Yin-yang diagram was originated from Daoism, and Zhou Dunyi borrowed it to categorize it in the realm of Confucius. In contrast, others debated that Zhou Dunyi discovered the diagram first, and Daoists later borrowed it (Li 1993). Indeed, Confucianism and Daoism, as leading schools of Chinese philosophy, though taking different approaches to practices, share the commonality of their core values in the Yin-yang graph.

Figure 1. River diagram of the Spontaneity of Heaven and Earth. (Zhao, Weiqian 赵扔谦. *The Essence of the Six Books* 六书本义. Siku Quanshu, 1929).

It shall be noted that the formation of the first Yin-yang symbol was inherited from *Hetu* (Fig. 2), which is believed to be one of the two oldest diagrams in the world (Wang 2012). As “a divinely revealed chart in mythical times” (Louis 2003), *Hetu* was recorded to have two functions in early texts: one as a divined object to obtain heaven’s approval on the legitimacy of human decisions, second as the early chart of eight trigrams in *I-Ching* (Saso 1978). In ancient times, *Hetu* was used for the mediation purposes of Daoism and Buddhism (Louis 2003). Later, Liu Mu re-composed it in the Song Dynasty to emphasize its mathematical enchantment as a representation of the image of Heaven, as the direct representation of the number patterns described in *I-Ching*. According to the *Appendices* by an anonymous author in *I-Ching*, the universe's origin could be found in numbers, in which all odd numbers are Yang (Heaven), and all even numbers are Yin (Earth). Specifically, it is stated that

> *"To Heaven belongs (the number) 1; to earth, 2; to heaven, 3; to earth, 4; to heaven, 5; to earth, 6; to heaven, 7; to earth, 8; to heaven, 9; to earth, 10. The numbers belonging to Heaven are five, and those belonging to Earth are (also) five. The numbers of these two series correspond to each other (in their fixed positions), and each one has another that may be considered its mate. The heavenly numbers amount to 25, and the earthly to 30. The numbers of Heaven and Earth together amount to 55. Through these, changes and transformations are effected, and spirit-like agencies are kept in movement. The numbers of the Great Expansion, (multiplied together), make 50, of which (only) 49 are used (in divination)…"* (Legge 2019, p. 482).

In *Hetu*, the black dots represent Yin (2, 4, 6, 8, 10), which indicates the heavenly numbers; the white dots represent Yang (1, 3, 5, 7, 9), which indicates the earthly numbers. Later, Yin-yang School successfully connected *Hetu* with the Five Elements (Feng 1983), stating that

> *"One, the number for Heaven, produces Water, and six, the number for Earth, completes it. Two, the number for Earth, produces Fire, and seven, the number for Heaven, completes it. Three, the number for Heaven, produces Wood, and eight, the number for Earth, completes it. Four, the number for Earth, produces Metal, and nine, the number for Heaven, completes it. Five, the number for Heaven, produces Soil, and then, the number for Earth, completes it"* (Feng 1948, p. 140).

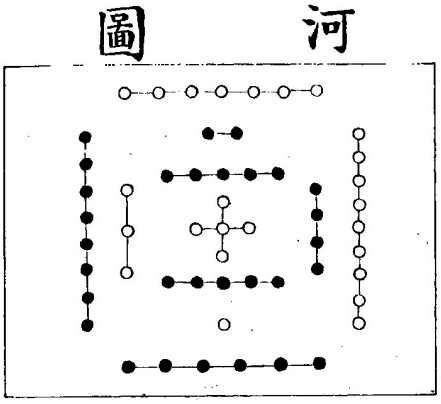

Figure 2. *Hetu* (the River Chart). (Zhu, Xi, 朱熹. *Zhouyi Benyi* 周易本義. Siku Quanshu edition.1929).

Moreover, in Liu Mu's book *Yishu Gouyintu*, he creatively deducted the formation of *Hetu*, which entirely matches another mathematical pattern in I-Ching, as stated in *Appendices*. In contemporary days, Ji Bin and Guan Mu, the authors of *Shensheng Taiji* (1999), have deducted the numbers of heaven and earth 55, the numbers of the Great Expansion 50, and the actual number of each eight trigrams (Fig. 3) from *Hetu*.

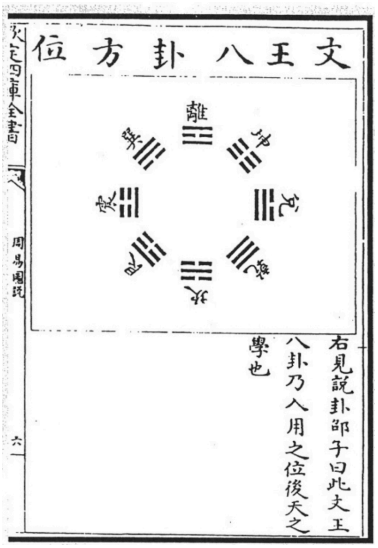

Figure 3. The King Wen arrangement of the Eight Trigrams. (Zhu Xi, 朱熹. *Zhouyi benyi* 周易本義. Siku Quanshu edition.1929).

In sum, it could be stated that the Yin-yang diagram is originated from *Hetu*, which is closely attached to the number patterns described in *I-Ching*, and later revealed, deducted, and organized by numerous scholars in the Song dynasty, such as Zhou Dunyi and Zhao

Huiqian, until recent days such as Ji Bin and Guan Mu. Therefore, this paper conducts research based on these scholars' significant contributions to discovering the number patterns in the Yin-yang diagram, mainly focusing on the number of heaven and earth, the number of the Great Expansion, and the actual numbers of eight trigrams, the Grand Terminus, as well as the relationships between each number in *Hetu*.

5.2 Fibonacci Sequence in Early Greek Philosophy

To comprehend the philosophical meanings embedded in the Fibonacci sequence, understanding its history is necessary. This section provides an overview of Fibonacci sequence's roots in *Elements*, as well as further ontological significances of mathematics raised by the ancient Greek philosophers, led by Pythagoreans.

5.2.1 History of Fibonacci Sequence

The Fibonacci sequence was discovered by the medieval Italian mathematician Leonardo Pisano Bigollo (1175-1250) in his book *Liber Abaci* (Book of Calculation) written in 1202 (Fibonacci and Sigler 2002). This famous sequence has resulted in various impacts in the fields of mathematics, arts, physics, architecture, computer science, biology, astronomy, and so forth (Yayenie 2011).

It should be noted that the mathematical concept of the Fibonacci sequence was not new in history; its idea was known sixteen centuries before to ancient Greeks as the golden ratio (Debnath 2011). In fact, in Leonardo's early life, scholars recorded that he was intrigued by Greek arithmetic, geometry, and algebra. It was right after his visit to Greece, he published the book *Liber Abaci* where Fibonacci sequence first appeared. Specifically, the concept of the golden ratio was first mentioned in the most successful mathematics textbook, *Elements* (Ratner 2009), written by the great Greek mathematician, the Father of Geometry, Euclid of Alexandria (300 B.C.). Besides, Foutakis (2014) identified that ancient Greeks applied and used golden ratio in their constructions of towers, altars, and tombs.

While the prevailing view is that the Fibonacci sequence originated in ancient Greece, some scholars believe that it originated from other countries, such as ancient India (Singh 1985) and ancient Egypt (Debnath 2011). As the heated debate continued, while taking

account of the complex history of its origin from different points of view, this paper considers its origin from the period of ancient Greece for the purpose of discussion.

5.2.2 Roots of Mathematics in Ancient Greek Philosophy

It is worth mentioning that many of the theorems that Euclid discussed in *Elements* were not discovered by himself, including the golden ratio. It was reported that he arranged works from earlier Greek mathematicians, such as Hippocrates(450-380 B.C.), Theaetetus (417-369 B.C.), and Pythagoras. In modern times, the golden ratio was mathematically proven to are highly connected to the golden ratio (Fowler 1991) (Markowsky 1992). Moreover, it was proven that any four consecutive Fibonacci numbers could combine to form a Pythagorean triple (Pagni 2001) (Kalman and Mena 2003). In Proclus's (410-485 B.C.) commentary on Euclid's proof work of Pythagorean theorem in *Elements*, he reported that

> "*If we listen to those who wish to investigate ancient history, it is possible to find them referring this theorem back to Pythagoras and saying that he sacrificed an ox upon its discovery.*" (Euclid 1482, cited in Huffman 2024, 5)

Pythagoras lived in the late sixth and early fifth centuries; he was born and raised on the island of Samos, on the coast of Asia Minor near Turkey, and near Miletus, the birthplace of Greek philosophy, according to Xenophanes (570-475 B.C.) and Heraclitus's (500-? B.C.) references. Many reports show that he widely traveled around countries in the Near East, such as Babylonia, Egypt, and Phoenicia (Huffman 2019). According to the references of Aristoxenus (335 B.C.), it was evident that

> "*Pythagoras went to Babylon and learned from Zaratas that Light and Darkness were the male and female principles from which the world was created; within the world, there were two gods, one celestial, the other terrestrial*" (West 1971, p. 32),

It was supported by concrete evidence that the Pythagorean Theorem was discovered by Babylonian mathematicians at least 1000 years earlier (Ratner 2009) and by Egyptians who used the theorem to build their pyramids (Maor 2019), considered that high similarities between mathematics and astronomy in Babylonia, Egypt, and Greece (Neugebauer 1969).

Nevertheless, there were a number of ancient Greek intellectuals who significantly contributed to the schools of philosophy, such as Thales (620-545 B.C.), Plato (427-347

B.C.), Hippocrates, and Aristotle (384-322 B.C.) (Graham 2011). It is hard to distinguish who discovered which theorems, considering the loss or ambiguity of the early texts. However, great thinkers of ancient Greece undoubtedly inspired and influenced each other during the 5th, 6th, and 7th periods B.C. and produced pivotal intellectual inspirations and achievements in the fields of Philosophy, Mathematics, Astronomy, Music, etc. It happened that the name Pythagoras is the most familiar of them all, to whom many early Greek scholars accredited, and the only one who stood for a determinate moment (Long 2011). Therefore, to unveil the philosophy behind the golden ratio by Euclid or the Fibonacci series by Leonardo, understanding Pythagoras is inevitably the key to unlocking the doors of ancient Greek philosophy.

Despite Pythagoras' most well-known identity as a mathematician, he was also regarded as a great philosopher. He was the first to use the term "philosophy" (Long 2011), and Aristotle referred to him as the "founder of a way of life" (Huffman 2024). Several prominent Greek philosophers, such as Plato and Aristotle, were also categorized as Pythagoreans. Some indicated that Plato, Aristotle, and other great Greek philosophers plagiarized Pythagoras' ideas, which were considered the source of all true philosophy (Huffman 2024). Most convincingly, many of Plato's successors in the Acamdey regarded Pythagoras had anticipated Plato's central theses (Burkert 1960; Dillon 1977).

Pythagoras elucidated that our world could be purely generated by mathematics (West 1971). He attributed numbers to not only the natural philosophical characterization that every number has its element in the natural world but also the metaphysical characterization that numbers create everything (Huffman 2024). His most famous Pythagorean Theorem is defined as

> *"in a right-angled triangle the square of the hypotenuse is equal to [the sum of] the squares of the other two sides"* (Diog 8, cited in Riedweg 2012, p. 41).

In fact, the discovery was based on his discovery of musical basic harmonies, three scales, diatonic, chromatic, and enharmonic, which transferred into the series of the first number, *tetraktýs*, where Pythagoras considered it as a basic construction of the whole universe (Riedweg 2012). Worshipped as the master, Pythagoras was regarded as the only mortal human who could hear this cosmic music, who

> *"perceived the overall harmony of the spheres and of the stars that move within them, which we do not hear because our nature is limited"* (VPyth 30, cited in Riedweg 2012, p. 29).

According to Aristotle's references, Pythagoras and his fellow school found a structural analogy between all things and numbers, in which the universe can be condensed into simple mathematical operations and

> *"the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number"* (Ross 1941, cited in Riedweg 2012, p. 80).

Pythagoreans regard that *Tetraktýs* contains the consonant intervals of the octave, the fifth, and the fourth, with the corresponding ratio 2: 1, 3: 2, and 4: 3, which is together equivalent to the "perfect" number ten (Dekad) and could be represented in a perfect triangle (Fig. 4). Plato's philosophical proposes seamlessly aligned with the doctrine of the old Pythagoreans (Riedweg 2012). In the final chapter of *The Republic* (2002), as a concluding myth, *Tetraktýs* represented the movements of stars:

> "*third in swiftness appeared to move according to the law of this reversed motion the fourth; the third appeared fourth and the second fifth*" (p. 480).

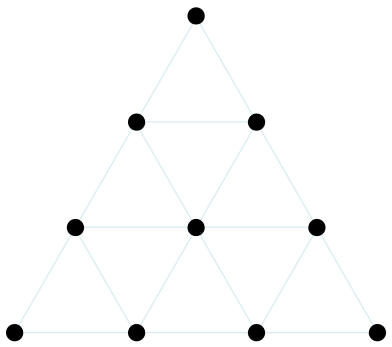

Figure 4. Tetraktýs perfect triangle. (Hemenway, Priya. *Divine Proportion*, 63. Sterling Publishing, 2006).

Starting from Pythagoras, the numbers have become the key to explaining the cosmos for Pythagoreans, as the pivot point for ancient Greek philosophers to base their thoughts, and as a fundamental starting point for the flourishment of natural sciences. In addition, his pondering of the world was regarded as the central point that the Renaissance revived on, as his ideas were manifested in various fields such as art, music, literature, and science (Burkert 1972).

Thus, it can be concluded that along the long history of mathematics and philosophy in the Western world, the Fibonacci series first appeared as the golden ratio in Euclid's *Elements*. In *Elements*, it was recorded that Pythagoras refined the laws of numbers from nature to explain the construction of the entire universe, which built the foundations of the ancient Greek philosophy. Hence, this paper regards that the ancient Greek period mostly shaped the philosophical formation of the Fibonacci series, led by Pythagoreans.

5.3 Axial Age: Linking Ancient China and Greek Philosophy

The Axial Age, defined as approximately the middle centuries of the first millennium before the Common Era (ranging from the 8th to 3rd centuries B.C.), is the most significant transformation in world history, as suggested by many prominent social scholars in the twentieth century, including Karl Jaspers, Eric Voegelin, Shmuel N. Eisenstadt, and Max Weber (Wittrock 2005), where societies from different major civilizations went through series of revolutions and had reached their peaks in the realm of ideas, creating irreversible effects on human history (Bellah 2005; Eisenstadt 2012).

Scholars such as Bellah (2011) identified ancient China and early Greece as two of the leading countries that flourished in the Axial Age. The primary wave of axial age existed in which a new type of intellectual elite had a transcendental vision, "a kind of standing back and looking beyond" (Schwartz 1975, cited in Eisenstadt 2012, p. 2), to newly explain and construct the world. Revolutions occurred when those visions were successfully institutionalized and culturally transmitted, societies were re-ordered, and the dynamics of human history changed (Eisenstadt 2012).

Upon discussion in the above sections, among all other intelligent elites in ancient China and early Greece, Confucius and Pythagoras were some of the most well-recognized leading figures, coincidently during the period of the Axial Age, where their philosophical ideas prospered and created the most significant impacts. Although the creation of the Yin-yang graph and the Fibonacci Sequence did not happen until later centuries, the original philosophical meanings of each could be traced to those two figures during the Axial Age. Confucius, whose life was devoted to reviving the ancient wisdom of prehistoric China, composed his commentary, *Yi Zhuan*, on the *I-Ching*, where the concepts of yin and yang first

proliferated. Pythagoras created philosophy and discovered mathematics from music, and the original concept of the Fibonacci sequence can be found in Euclid's *Elements,* with emphasis on the golden ratio and Pythagorean Theorem.

This paper, therefore, examines the philosophical ideas put forward by two figures as well other intellectuals who broadly constructed the ideologies of ancient China and early Greece centered on the Axial Age.

6 Quantitative Analysis

The first part of the quantitative analysis mainly involves the statistical summary of several individual modulo of the Fibonacci sequence (Table 1), based on the definition of the Pisano period, which is divided into four groups of numbers.

The first group of the Pisano period is attributed to eight trigrams, so the modules being calculated are $mod\ 2$, $mod\ 4$, and $mod\ 8$. As shown in the table, when modulo is 4, the number pattern of $0, 1, 1, 2, 3, 1$ repeated itself, the length of this sequence is $\pi\ (4) = 6$, and the sum equals 8. When the modulo is 8, the repeating cycle length is 12, summing up to 32. As summarized in Table 2, in terms of modulo 2, 4, and 8, the trend of changing in sum is subsequently 2, 8, 32, each following sum is four times larger than the previous one; the trend of changing in the length of the repetitive cycle is from 3, 6, 12, in which each subsequent term is two times bigger than the previous one.

The numbers of the second group are composed of *Hetu*. Therefore, modulo 5, modulo 6, and modulo 9 are calculated, as shown in Table 1. The length $F\ (n)\ mod\ 5$ is 20, with a count of four zeros, which is the number that occurs the most in the sequence. Both mean and median are 2, and the sum equals 40, a multiple of 8. $Mod\ 6$ and $mod\ 9$ share many similar statistics, including not only their lengths of the repeated cycle ($\pi\ (6) = \pi\ (9) = 24$) but also the mode ($= 1$) and count of zeros ($= 4$). Additionally, their sums are one more digit of their original number of belonging modulo ($\sum mod\ 6 = 66$, $\sum mod\ 9 = 99$).

The third significant number is the Great Expansion number 50, so the statistics of $F(n) \; mod \; 50$ are calculated (Table 1). Although the cycle length now equals 300 ($\pi(50) = 300$), the total number of zeros still remains at the count of 4. The median is 25; the mean of 24.67 can be approximated to 25 if only the whole digit is taken. Moreover, it is shown that all modules calculated above contain mode 1 or 0.

The fourth group covers the numbers of Heaven and Earth. Therefore, modulo 25, modulo 30, and modulo 55 (Table 1) are statistically summarized. The cycle length of $mod \; 25$ is $\pi(25) = 100$, and that of $mod \; 30$ is $\pi(30) = 120$. The length of $F(n) \; mod \; 55$ is 20, the same as $mod \; 5$.

As summarized in the statistics of the first 55 Pisano periods in Table 2, several patterns are easily identified: (1)The product always remains 0; (2) The number of zeros is either 1, 2, or 4 (expect $F(n) \; mod \; 1$); (3) the greatest common divisor is always 1 (expect $F(n) \; mod \; 1$); (4) expect $F(n) \; mod \; 3$, the length of the repetitive cycle is always an even number ($m > 2$, then $\pi(m) = 2n$).

The second part of the quantitative analysis involves the analysis of the first 50 Fibonacci numbers based on their feature as Great Expansion numbers. Its relation to the golden ratio is checked, and its golden spiral is shown as an approximate visual graph. Table 3 shows the statistical summary, where the median equals 60696.5. Figure 5 shows the ratio of the first 50 Fibonacci numbers to the golden ratio, where the first ten numbers “oscillate” in a great range, and the remaining suddenly becomes highly aligned with the golden ratio $1.618...$. Therefore, in Figure 6, a primary Fibonacci spiral is formed based on the number input of the first 50 Fibonacci numbers.

The last part of the quantitative analysis is a replication of exploring the final digits based on the discovery from Lagrange in 1774, where each Fibonacci number is divided by ten, and the remainder of each is the last digit. In Table 4, as a known property, it is once again shown that the value of the subsequent final digit is an addition of the previous two final digits, which is precisely the same feature of how the Fibonacci sequence is constructed. In other words, the modulo of Fibonacci sequence has the same additive property as the typical Fibonacci sequence. Most importantly, the final digit has its first repetitive cycle in the length of 60. Similarly, F (n) $mod \; 10$ (Table 1) shows that the length equals 60.

7 Comparative Discussion

This section begins with a discussion of the results of the quantitative analysis, followed by an extended study based on the existing properties of the Fibonacci series. Finally, an extensive comparison is made between ancient Chinese and Greek philosophy in relation to numbers.

7.1 Discussion of Findings

Firstly, from closely examining the results from quantitative analysis, this paper considers the Pisano period of modulo 4 and modulo 8 (Table 1), the statistical summary of the first 55 Pisano periods (Table 2,), and the final digits of the cycle length of 60 (Table 4) as one group, for the purpose of discussion. The summation of $mod\ 2$, $mod\ 4$, and $mod\ 8$ are subsequently 2, 8, and 32, each multiplied by 4; their cycle lengths are 3, 6, and 12, each multiplied by 2; and if multiplying these two multipliers two times 4, 8 is therefore produced. This pattern of highlighting numbers two and four is rooted in emphasizing the evolvement of natural changes in *Appendics* of *I-Ching*:

> *"Therefore in (the system of) the Yi there is the Grand Terminus, which produced the two elementary Forms. Those two Forms produced the Four emblematic Symbols, which again produced the eight Trigrams"* (Legge 2019, p. 486).

In Daoism, the same idea is stressed by Laozi in the *Tao Te Ching* (2008),

> *"The Tao produced One; One produced Two; Two produced Three; Three produced All things"* (p. 78).

Similarly to Pythagoreans in ancient Greece, the number three is regarded as the first and perfect number, and the creation of all things is completed by it (Riedweg 2005). It symbolizes harmony (Triad), comprised of virtue, health, friendship, universal good, and God, so everything owes its preservation to harmony (Guthrie and Fideler 1987). It highlights the nature of changes and the critical holder of nature. Moreover, it appeared as the means of classifying Plato as the good, the true, the beautiful, and the unification of time, space, and causality, and the counts of three animals in Greek and Rome sacrificial rites (Schimel 1993). However, numbers four and eight indicate justice and harmony; since oath is justice, then Zeus is the God of Oaths (Guthrie and Fideler 1987). Most importantly, it is associated by

Pythagoreans with *Tetraktys* ("fourthness") on the musical scale of producing harmonious sound and as the indication of the Kosmos (Tetrad). Four is also considered the most material number, representing the first known world order from nature to the beginnings of civilization, hence the existence of fire, air, water, and earth as the main material components of the world (Schimel 1993). Interestingly, it seems like the understanding of numbers three and four between ancient Chinese and Greeks overlaps, that they all regarded three as the end of the discovery of the universe's essence and four as the beginning evolution of the material world. The difference lies in that numbers are treated individually by their properties from the perspective of Pythagoreans. At the same time, their meanings continuously evolve after certain thresholds from the perspectives of ancient Chinese sages. Specifically, multiples of four, such as the eight, thirty-two, and sixty-four, are not of central importance in ancient Greek philosophy, which means their properties as numbers are still mathematically vital to study, but their philosophical significance no longer persists. However, in ancient China, four extends to eight, and eight trigrams extends to sixty-four hexagrams, which cover the expansion of everything in *I-Ching*, as it said in *Appendices* that

> *"The eight trigrams served to determine the good and evil (issues of events), and from this determination was produced the (successful prosecution of the) great business (of life)"* (Legge 2019, p. 486).

Moreover, based on the property discovered by Lagrange in 1744, the quantitative analysis replicates the process to generate the final digit by dividing ten by each Fibonacci number (Table 4) and found out that one cycle repeats up until the Pisano period equals 60, $\pi\,(10) = 60$ (Table 1). According to the further discovery by Lagrange, when each Fibonacci number is separately divided by $10, 100, 1000, 10000...$, the cycle lengths have the pattern of

$$60, 300, 1500, 15000, 150000...$$ (OEIS A096363).

As seen, between the lengths that have noticeable changes, 60, 300, and 1500, 5 is the standard multiplier; both the length of $mod\ 5$ and $mod\ 55$ equals 20. If the first and second cycle lengths are added, the result is 360 ($60 + 300 = 360$); multiplying the first two is 18000 ($60 \times 300 = 18000$). It has been discovered that there is a pattern of evolution from numbers 20, 60, to 180, and, separately, 360. The significance of these numbers is deeply rooted in the way ancient Chinese divided the time of the universe. Drawing from the system of Yin-yang in *I-Ching* and according to the observation of the rotation of the solar system's

planets, ancient Chinese divides the cycle of time into a system called “San Yuan Jiu Yun” (Three Cycles Nine Periods 三元九运), that there are nine small cycles (Yun) in total with three big cycles (Yuan). In the solar system, Saturn and Jupiter meet every 20 years, so each Yun consists of 20 years; Saturn, Jupiter, and Mercury meet every 60 years, so each Yuan consists of 60 years; the nine significant planets meet with each other in one line every 180 years, so the most extensive cycle is 180 years. Three Yun form one Yuan, and three Yuan form one big cycle; together, it completes the most significant cycle of 180 years. It is believed that at nine different Yun of “San Yuan Jiu Yun,” the fortune of the earth is dominated by one star in the universe, and together every 180 years, would change an utterly different fortune because ancient Chinese believed that there is a corresponding relationship between the laws of natural phenomena and personnel fortune on earth (Steele 2011). Additionally, in *Appendices* of *I-Ching*, 360 is “corresponding to the days of the year” (Legge 2019, p. 483). In Greek Philosophy, 60 is considered the round number. The fourth power of sixty ($60^4 = 12960000$) is regarded as Plato’s “marriage number,” which is the length of one world year. Plutarch (46-119) added that 60 is the first measure of heavenly appearances (Schimel 1993). More broadly speaking, many solid pieces of evidence proved that ancient Greek astronomers directly used Babylonian arithmetical methods and astronomical systems (Evans 1998), while Babylonians adopted the hexadecimal system of counting, based on factors of 60, where the ancient 12 Zodiac system fits well, from the Sumerians in 3500 B.C.. Later, a few Babylonian astronomers discovered that

> *“The division of the circle into 360 degrees, and the division of degrees into 60 and 3600 parts”* (Wells 1997, p. 127),

In which people still use the division of one hour as 60 minutes and that of one minute as 60 seconds, and so forth. In sum, the number 60 and its beyond factors represent the system of counting time in both ancient China and Greece. However, it is clear that ancient Greeks adopted the sexadecimal system from more ancient civilizations in the Near East, in which they later directly assigned the philosophical meanings to the numbers, leaving little space to discuss in the realm of Greek philosophy. Nevertheless, ancient Chinese discovered it in a relatively systematic and comprehensive way, involving the early observation of the operation of the universe, as well as the oracle to tell different fortune periods of the earth.

In the statistic table of the first 55 Pisano Period (Table 2), the counts of zeros are either 1, 2, or 4; all product quals to one, and the greatest common divisor (GCD) equals 1

(these patterns not simply apply to the first 55 Pisano Period, but to all Pisano Period). It is also worth noting that whichever modulo in the residue table is studied, a new round of repetition is triggered every time the positions of the numbers 0 and 1 appear. Hereby, it is clear that the emphasis draws on the numbers 0, 1, and 2. In ancient Chinese philosophy, the appearance of the number zero is explicit but implicitly implied as a cosmological symbol of the origin of all. In *I-Ching*, zero is the "Yi" of "the Grand Terminus... (that) produced the two elementary Forms" (Legge 2019, p. 486); in *Tao Te Ching*, zero is "the Tao produced One"(Laozi and Legge 2008, p. 78). Zero is *Wuji* (the Ultimate Void 无极), which is the prerequisite for starting the state of *Taiji* (the Grand Terminus 太极), aligning with Chen Tuan's *Wujitu*. In other words, *Wujitu* accentuates the state of unified origin. In contrast, *Taiji* highlights the transition from the unitary source into a diversified world, so that nature, lives, and materials started to appear (Wang 2012). From *Wuji* (zero), *Taiji* (one) appears, hence two, three, and everything. Number two, as known, usually represents the two opposite but complementary Forms, Yin and Yang, and two is the previous state of producing "the Four emblematic Symbols" (Legge 2019, p. 486). Alternatively, in the view of Pythagoreans, one is not considered a number at all, since by Euclid's definition, the number is an aggregate composed of units. They believed that one, geometrically represented by the point, is both even and odd, both female and male, although it contains more of the male principle (Wells 1997). One is the primordial number that permeates every other number, a fundamental unity (*Monad*) that comprises relations, wholesome as a nonpolarized existence. One is God; it holds a quite divine position that stands behind other numbers and does not fall under any ordering system to which 2, 3, or 4 belongs (Schimel 1993). The *Monad* is the absolute intelligence, for which the *Dyad*, number two, sees according to. Two, the first even number, and the first prime and the only even prime number. To ancient Greeks, two is not considered a natural number either: when they observed it, it consisted of the beginning and the end, but there was no middle. Specifically, they usually regard the operation of multiplication as generating more remarkable results than addition; however since $2 + 2 = 2 \times 2$, they considered two as an exceptional case. To Pythagoreans, numbers represent not simply quantitative signs, whereas each is "a qualitative, archetypal essence, possessing a distinct, living personality" (Guthrie and Fideler 1987, p. 321). Philosophically speaking, two is unlimited, indefinite, inequality, and fittingly science; all proofs and persuasions are parts of science. In Alexander's (356 - 323 B.C.) *Successions of Philosophers,* he reported the doctrines of Pythagorean's memoirs of primary numbers. The *Monad* is the beginning of all things; the Infinite *Dyad* arises from it, so naturally, the *Dyad* is subordinate to the *Monad*.

From *Monad* and *Dyad*, "numbers" arise; from "numbers" arise "points"; from points generate lines, and from lines generate plane figures. Subsequently, plane figures derive solids; solids derive sensible bodies with four elements: fire, water, earth, and air. The world is made from a combination of these elements, with life and intelligence, in a spherical shape, with the earth in the center, which is also spherical and inhabited everywhere. The earth also has antipodes; what is below for us is above for them (Guthrie and Fideler 1987). From the above order $(0), 1, 2, 3, 4$, cosmology is built for each ancient civilization, which shares many similarities— from the origin of abstract points to the evolution of the material world. The differences lie in the perspective that in ancient Chinese, zero is the pre-existence of one, and from the absolute emptiness, *Wuji*, arises the very beginning, *Taiji*; in early Greece, the *Monad* and the *Dyad* play similar roles. It can be concluded that, for two countries, the ontology of the universe's origin is highly closed, and the divergences occur as the number increases.

Regarding the numbers of *Hetu*, the quantitative analysis of the Pisano period of modulo 5, modulo 6, and modulo 9 (Table 1) is individually conducted. In $mod\ 5$, the cycle length $\pi\ (5) = 20$ has the same mean as the median, equal to 2; its total value equals 40. The cycle length of $mod\ 6$ and $mod\ 9$ is the same ($\pi\ (6) = \pi\ (9) = 24$); their sums are the repeated number of themself, which are 66 and 99. In the eyes of ancient Chinese intellectuals, five, nine, and six bears almost the most critical position in the numbers of all, manifested in *Hetu* (Fig. 2). Only the emperor is entitled to “the honor of Nine-Five.” Moreover, in *I-Ching*, each sixty-four hexagram contains six lines (六爻), each line is either Yin or Yang. The Yin line is referred to as “six + its position,” and the Yang line is called “nine + its position.” For instance, in the hexagram Song, the first position containing a Yin line (broken) is called “the first SIX,” whereas the fourth position containing a Yang line (unbroken) is called “the fourth NINE” (Legge 2019). This use of description permeates the whole book of *I-Ching*. Therefore, as the famous Poet Su Shi (1037- 1101 C.E.) commented in his *Collection of Dongpo*, nine is considered the number of “Old Yang” (老阳) and six as the number of “Old Yin” (老阴). Moreover, the cycle length of $mod\ 5$ and $mod\ 55$ (Table 1) is 20, which is again a “Yun” in the system of “San Yuan Jiu Yun.” The sum of $mod\ 5$ is 40, which is the sum of values of eight trigrams (Ji and Guan 1999):

$$1 + 6 + 7 + 2 + 8 + 3 + 4 + 9 = 40;$$

Reversely, 40 is the product of five and eight, which are again the significant numbers in the Yin-yang system, which is

$$5 \times 8 = 40.$$

Furthermore, the summations of mod 6 and mod 9 are individually 66 and 99, which stresses the importance of numbers six and nine. Considering $\pi(6) = \pi(9) = 24$, 24 is the total sum of "Old Yin"; the common multiplier is also six because one hexagram consists of six lines. Number 24 is also "twenty-four qis" (二十四节气), an ancient Chinese calendar system to divide one year (360 days) into different stages of natural change that was being used after 600 B.C., according to the observation of the movement of the sun and the moon, starting at the winter solstice. One cycle is 360 days, with 15 days for each qi, and 24 stages run in rotation (Sôma et al. 2004). Ancient emperors used 24 qis to regulate the country; farmers used it to predict when to plow their fields and to harvest. It is worth noting that 24, the division of days of the year, appears to be the sum value for mod 6 and mod 9, which are the "Old Yin" and "Old Yang." Most importantly, the significance of number five in ancient Chinese and Greek philosophy should be noted. In *Hetu* (Fig. 2), By examining the downward, leftward, upward, rightward, and middle number compositions in *Hetu*, it is noticeable that the differences between the two numbers in the outside and inside circles are all equal to five, that is

$$6 - 1 = 5,$$
$$8 - 3 = 5,$$
$$7 - 2 = 5,$$
$$9 - 4 = 5,$$
$$10 - 5 = 5.$$

Five is being positioned at the center, giving special visual attention. In *Appendics* of *I-Ching*, it indicates that

> *"The numbers belonging to heaven are five, and those belonging to earth are (also) five"* (Legge 2019, p. 482).

Like Pythagorean's four elements that construct the world (or sensible bodies), five elements (Wuxing 五行) remain its central position in the world epistemology in ancient China: fire, wood, Earth, water, and metal. Represented by the corresponding eight trigrams in *I-Ching*, each element has the relationship of generating or overcoming each other, which had become the basis of traditional Chinese medicine. In the long river of Chinese history, derived from

the system of Wuxing, each of the five elements has its correspondents in five virtues, five colors, five musical notes, five viscera, five tastes, five emotions, and so on (Dechar 2006). Earth is of the central position of all, standing for the central void Earth (中央虛土), symbolized as the number five. On the other hand, unlike ancient Chinese philosophers who had a unified opinion on the importance of the elements, early Greek philosophers had been fiercely debating which element held the most primordial position among the four elements: fire, air, water, and Earth. Thales regarded fire as the origin of all, Anaximenes (d. 528 B.C.) favored air, and Heraclitus (535-475 B.C.) championed fire as the one (Russell 2004). However, some philosophers did not admit the four elements at all, suggesting that the world comprises much smaller substances. For instance, Anaxagoras (500-428 B.C.) claimed the world is composed of an infinite number of particles, or "seeds" (*spermata*), while Democritus (460-370 B.C.) believed the soul is composed of atoms, forming the well-known atomic theory nowadays (Berryman 2023). Besides, a few ancient Greek philosophers argued that it reserved a sacred, hidden fifth element called aether among the existing four classical elements. Aether is invisible, void, quintessence, and the immortal soul, which was asserted by Heracleides (390-322 B.C.) and the Pythagoreans that

> *"each world of the stars is air and aether surrounding earth in the infinite aether"* (Guthrie and Fideler 1987, p. 309).

It was claimed by Parmenides (born c. 515 B.C.) that the "ethereal nature and all the signs" could be found in aether (Graham 2011, p. 221). Supplementarily, commenting on the discussion of the soul, Dicaearchus (c. 320 B.C.) identifies four fundamental Pythagorean doctrines:

> *"(1) that the soul is immortal; (2) that it transmigrates into other kinds of animals; (3) that after certain intervals the things that have happened once happen again, so that nothing is completely new; (4) that all animate beings belong to the same family"* (Porphyry 1965, cited in Huffman 2024).

In Plato's *Timaeus*(1925), he mentioned that

> *"so likewise of air, there is the most translucent kind which is called by the name of aether"* (58d),

in which aether is modeled as dodecahedron in Platonic Solids. Platonic Solids are five polyhedrons with different faces; each polyhedron is assigned one of the four elements: cube with Earth, octahedron with air, icosahedron with water, and tetrahedron with fire. The remaining dodecahedron, the quintessence, is used for "the universe as a whole" (Long 2011,

p. 227) since it nearly approaches the shape of a sphere (Zeyl 2023). Geminus (10 B.C. - 60 A.D.) reported that three of the five solids are due to the Pythagoreans, which are tetrahedron, cube, and dodecahedron. In Pacioli's (1445-1517) *De Divina Proportione* (1509), he attributed the second half of the book to Platonic Solids, indicagting that God brought the fifth essence as the celestial virtue, and four solids, earth, air, water, and fire, were herein created; he explains further that mankinds' sacred proportion, or heaven, is given the shape of dodecahedron. Later, Kepler (1571-1630) proposed a model of the solar system based on Platonic Solids (Fig. 8), where he assigned the cube to Saturn, the tetrahedron to Jupiter, the dodecahedron to Mars, the icosahedron to Venus, and the octahedron to Mercury (Olsen et al. 2021). In conclusion, the concept of number five has vital significances in ancient Chinese and Greek philosophy. In China, it mainly refers to the element Earth, which represents the void similar to *Wuji*; in ancient Greece, it refers to the less-known fifth element, aether, which comes from the heaven and differs from the remaining terrestrial four elements, and in the form of dodecahedron in Platonic Solids. The former one is more explicitly spread as common sense, and the latter one is more implicitly known in a small number of philosophers; the former one is more culturally encompassed in the formation of the system of Yin-yang, without any further debate, but the latter is more controversial due to the nature of Greek philosophy is highly based on science and the arts of persuasion.

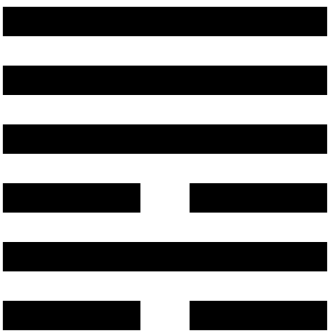

Figure 7. Song hexagram 讼卦. (Finney, Ben. "I-Ching hexagram 06: ䷅ 'Arguing'." 2002).

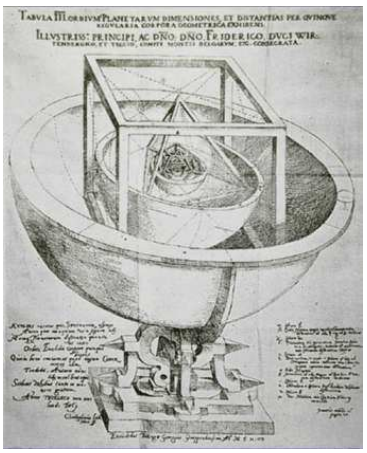

Figure 8. Kepler Nested Model of Solar System. (Kepler, Johannes. *Mysterium Cosmographicum*. Tübingen, 1596).

Last but not least, in the quantitative analysis of the number of Heaven and Earth, fifty-five is composed of the heavenly number 25 and the earthly number 30. Thus, the Pisano Period of $mod\ 25$, $mod\ 30$, and $mod\ 55$ (Table 1) is each statistically summarized. The repetitive lengths of the cycle of the heavenly and earthly numbers are individually $\pi\ (25) = 100$ and $\pi\ (30) = 120$. if dividing each by 10, the numbers 10 and 12 remain, which are the core numbers of the Chinese sexagenary cycle. Namely, ten Heavenly Stems (十天干) (Fig. 9) and twelve Earthly Branches (十二地支) (Fig. 10), each stem or branch has one of the five elements. It is due to the combination of matching characters from both cycles that the big cycle "Yuan" (60 years) is composed in "San Yuan Jiu Yun." In other words, Heavenly Stems and Earthly Branches are the fundamental basis of the "San Yuan Jiu Yun," or the essentials of the Chinese calendar system (Smith 2011). In ancient Greece, the perfect number is ten because it is the summation of musical harmonious numbers, such as

$$1 + 2 + 3 + 4 = 10,$$

in the viewpoints of Pythagoreans, whereas the number 12 is the basis of time sexagenary counting system, adapted from Babylonians, perfectly fitting 12 Zodiac signs (Schimel 1993). In addition, the number 25 is closely associated with the Pythagorean Theorem that

$$25 = 3^2 + 4^2 = 5^2,$$

and it follows the sum of odd numbers:

$$25 = 1 + 3 + 5 + 7 + 9.$$

The importance of number 55 draws on the fact that there are only 55 sets of integers a, b, c, and d, for which every integer is in the form of

$$ax^2 + by^2 + cz^2 + du^2,$$

and it is among the few triangular Fibonacci numbers 0, 1, 3, 21, and 55 (Wells 1997). Furthermore, according to the deduction of *Shensheng Taiji* (1999), the summation of two elementary Forms ($= 10$) and the values of eight trigrams ($= 40$) equals the number of the Great Expansion ($10 + 40 = 50$). As results show (Table 1), the median of the first 50 Fibonacci numbers is 60696.5, emphasizing the importance of numbers six and nine. If approximating to the whole digit, the mean and median of $F(n)\ mod\ 50$ equal 25, which is the heavenly number in *I-Ching*. To Greeks, 50 appears as an indefinite round number (Schimel 1993) and is the smallest number that could be summed in squares in two different ways (Wells 1997):

$$50 = 5^2 + 5^2 = 7^2 + 1.$$

In short, it seems that in terms of numbers 12, 25, and 55, mathematics and philosophical meanings are relatively disconnected compared to a more systematic Chinese Yin-yang thinking mode; therefore, no further epistemological comparison could be made between the two countries.

	1	2	3	4	5	6	7	8	9	10
Modern Graph	甲	乙	丙	丁	戊	己	庚	辛	壬	癸
Pinyin	jia	yi	bing	ding	wu	ji	geng	xin	ren	gui
Shang Graph (ca. 1200 BC)										

Figure 9. Ten Heavenly Stems (Smith, Adam. *Calendars and years II: astronomy and time in the ancient and medieval world*, 2. 2011)

	1	2	3	4	5	6	7	8	9	10	11	12
Modern Graph	子	丑	寅	卯	辰	巳	午	未	申	酉	戌	亥
Pinyin	zi	chou	yin	mao	chen	si	wu	wei	shen	you	xu	hai
Shang Graph (ca. 1200 BC)												

Figure 10. Twelve Earthly Branches (Smith, Adam. *Calendars and years II: astronomy and time in the ancient and medieval world*, 2. 2011)

7.2 Extended Inquiry

Besides the quantitative analysis conducted in this paper, there are various discoveries on the pattern of the Fibonacci sequence by modern scholars that might have a potential link to the Yin-yang diagram. This section will briefly discuss number 216, angle 0.48, and Binet's Formula.

In *The 216 Letter Hidden Name of God - Revealed*, authored by Lucien Khan (2013), he discovered that in the Pisano period, divided by ten, the last digit repeats every 60 numbers ($\pi\,(10) = 60$), the $216th$ number is:

$$619220451666590135228675387863297874269396512$$

If all of the digits in this number are added, the sum equals 216. He notes that the secret or hidden name of God is believed to contain 216 characters. He added that Plato knew the cubes of 3, 4, and 5 in the Pythagorean triple equal 216, that is:

$$3^3 + 4^3 + 5^3 = 216,$$

and the year of completing a Zodiac Age is also 2160. His most important finding is he found a parallel between the secret name of God, which contains 216 characters, and 216 Numbers found inside the Metatrons Cube (Fig. 11), and concluded that the secret name of God is hidden in the Metatrons Cube (which is a polyhedron composed of five Platonic Solids). From the perspective of ancient Chinese, 216 contains the divine meaning as well. In *Appendics* of *I-Ching*,

> *"The numbers (required) for Qian (or the undivided line) amount to 216; those for Kun (or the divided line), to 144. Together they are 360, corresponding to the days of the year"* (Legge 2019, p. 483).

Specifically, it states that the numbers for Yang amount to 216 in total, and together with 144 numbers for Yin, they construct the days of the year, 360. As discussed before, the world of ancient China was constructed out of two forces, Yin and Yang, as the minimum unit; in *I-Ching*, it is explained that the total number of Yin and Yang is the cycle length of the year. Thus, the divine significance of the number 216 in ancient Greek and Chinese philosophy is shared in different ways.

216 Number Pattern (Each column adds to perfect 60)

1	**1**	**2**	**3**	**5**	**8**	**3**	**1**	**4**	**5**	**9**	**4**	**3**	**7**	**7**	**7**	**4**	**1**
4	9	3	2	5	7	2	9	1	1	1	2	3	5	8	3	1	4
9	4	3	7	7	7	4	1	5	6	1	7	8	5	3	8	1	9
1	1	2	3	5	8	3	1	4	5	9	4	3	7	7	7	4	1
6	1	7	8	5	3	8	1	9	9	9	8	7	5	2	7	9	6
9	4	3	7	7	7	4	1	5	6	1	7	8	5	3	8	1	9
9	9	8	7	5	2	7	9	6	5	1	6	7	3	3	3	6	9
6	1	7	8	5	3	8	1	9	9	9	8	7	5	2	7	9	6
1	6	7	3	3	3	6	9	5	4	9	3	2	5	7	2	9	1
9	9	8	7	5	2	7	9	6	5	1	6	7	3	3	3	6	9
4	9	3	2	5	7	2	9	1	1	1	2	3	5	8	3	1	4
1	6	7	3	3	3	6	9	5	4	9	3	2	5	7	2	9	1
60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60

Figure 11. 216 Name Pattern. (Khan, Lucien. *The 216 Letter Hidden Name of God - Revealed*. CreateSpace, 2013.)

Furthermore, the golden ratio is said to be found in plants such as sunflowers. When seeds in the center grow, it pushes other seeds outward. Each seed rotates to a certain degree relative to the previous seed, and together, they create a spiral pattern (Takaki et al. 2003). If the meristem in the seedhead is rotated by an angle of 0.5, two arms happen to appear. When the angle is set to 0.48, it results in 25 radial arms or 12 complete rotations (Fig. 13); two arms form the shape of a yin-yang diagram (Naylor 2002). As discussed, number 12 is the number of twelve Earthly Branches in the Chinese calendar counting system; number 25 is the heavenly number; one step forward, the cycle length $mod\ 25$ is proved to be 100 ($\pi\,(25) = 100$), which corresponds to the Heavenly Stems if dividing by 10. Additionally, the unit degree of the octagonal is around 0.471404, which is approximated to 0.48.

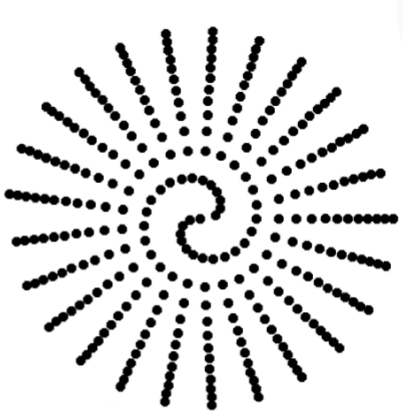

Figure 13. Sead head rotation, angle $= 0.48$. (Naylor, Michael. "Golden, and π flowers: A spiral story." *Mathematics Magazine*, 2002).

In 1843, French mathematician Jacques Binet discovered a formula, namely Binet's Formula (Fig. 14), for the usual Fibonacci Sequence F (n), which could generate Fibonacci numbers by inputting any numbers (Kilic 2008). As seen, the formula is characterized by two natural numbers, two and five, which emphasizes the importance of 2 and 5 one more time. To summarize briefly, in ancient Greek philosophy, number five is the hidden fifth element aether, the marriage number (because it is composed of the first odd number 3 and the first even number 2), and total number of Platonic Solids (Wells 1997); in ancient Chinese philosophy, it represents the essential five elements, and sacred number of *Hetu*, namely the number of unchanging, not to mention Greek's unlimited number 2 and the basic forms of the Yin-yang universe.

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1-\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right] = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

Figure 14. Binet's Formula. (Lee et al. "The Binet formula and representations of k-generalized Fibonacci numbers." *Fibonacci Quarterly* 39, 2001).

7.3 Further Comparison

By thoroughly discussing the findings from quantitative analysis and the existing literature, it is rational to conclude that numbers allude to the fundamentals of understanding, knowing, and constructing the world; philosophy is a powerful vehicle for carrying and assembling these reflections. Differences in ways of knowing are thus reflected in the construction of figures.

Number five is essential in both countries, but their approach differs significantly. Specifically, they derived the system of classical elements in both places: Chinese' Wuxing and Greek's four elements (and aether as the hidden fifth). In ancient China, five stands for more than simple elements, while more broadly, it is the only unchanged (不变) among all the changes (变), the main principle written in *I-Ching*. If following along the narratives of the discussion above, it can be summarized as a system of how the unchanged and the change interact and influence each other. By observing nature in the realm of time, space, season

changing, and the universe, ancient Chinese organized the laws of natural operation into the broad idea of Yin-yang, which is divided into two central interconnected systems in *I-Ching*. The first system follows the principle of constantly changing everything, mainly in the aspects of time (时) and position or order (位), represented by the numbers 4, 8, 10, 12, 24, 20, 60, 360, and 180. There are four forms, eight trigrams, ten Heavenly Stems and twelve Earthly Branches, twenty-four qis, "San Yuan Jiu Yun," and days of the year, all with a common theme in time. It represents the first successful attempt to construct a calendar, phases of the season changing, and time changing of a day. As even numbers, they are derivative, generative, and unlimited, considering the essence of time is constantly changing. The second system follows the principle of unchanging, where ancient Chinese thinkers strived to find unchanging things while acknowledging the constant changing of everything. The representative numbers are 5, 6, 9, 25, 30, and 55, which are numbers of *Hetu* and that of Heaven and Earth. Numbers from the constantly changing system one are indeed the evolution of addition, subtraction, multiplication, and division based on the unchanging system two, whereas the evolution, in turn, reflects each other in their respective ways, for all laws have the same root. Beginning by recognizing the nature of nature, ancient Chinese sages then applied such magical natural laws to real life, such as divination, oracle, or revelations of wisdom. In other words, by generating unlimited changing things from the pure, only unchanging natural principle, it goes from essence (体) to application (用), as always referred to in old Chinese texts. The archetypal number is the number of Great Expansion 50, which represents fifty small sticks as the tool to use for divination for the first time to communicate between heaven and human beings.

The system of unchanges in changes is universally presented in ancient Chinese philosophical fundamental beliefs. Speaking of how the two approach philosophy differently, ancient Greeks approached it initially from natural philosophy, the theory of knowledge, whereas ancient Chinese approached it from the perspectives of politics, economics, and other secular realms (Reding 2019). In Confucianism, the discussions are highly bound to the empirical experience rather than speculating about non-practical issues (Yu 2008). This is supported by three famous quotes by Confucius in *Analects*:

> "*While you do not know life, how can you know about death?*" (Legge 1930, p. 142);
>
> "*The subjects on which the Master did not talk, were: extraordinary things, feats of strength, disorder, and spiritual beings*" (Legge 1930, p. 87);

> "*To give one's self earnestly to the duties due to men, and, while respecting spiritual beings, to keep aloof from them, may be called wisdom*" (Legge 1930, p. 73).

Confucius proposed that in this secular world, this human community should be a good person (Yu 2008); however, he did not deny the existence of the divine world and spiritual beings. Indeed, the pursuit of spiritual self-actualization is not absent in ancient China at all. In *I-Ching*, it says that

> *"There is no thought and no action. It is still and without movement; but, when acted on, it penetrates forthwith to all phenomena and events under the sky"* (Legge 2019, p. 484).

The exact meaning is referred to in Mencius' *Jin Xin I*, where he states that "All things are already complete in us," (Legge 1930, p. 935) and in *Gong Sun Chou I*, he said that "I attained to an unperturbed mind" (Legge 1930, p. 523). Similarly, in *Tao Te Ching*, Laozi states that

> *"The Tao in its regular course does nothing (for the sake of doing it), and so there is nothing which it does not do."* (Legge 2008, p. 67).

As seen, the repeated word is the concept of nothingness or no action, which contradicts what is proposed as a virtual, good gentleman in the secular world proactively practicing political values and socio-economic orders. Nothingness, or the state of void like the element Earth in Wuxing proposed, is simply the manifestation of natural principles (*Dao* 道) in human beings, the way of living that is back to the origin, to the ultimate truth, to recall the memory of our spirit; in other words, it is the Grand Terminus, or *Wuji* as later scholars called, that remains unchanging, across time and space, heaven and earth, lasting forever. The practices of this greatest *Dao* are firstly through to be the best, relational self in the relations with different people, family members, students and teachers, friends, societies, economics, and politics (Yu 2008) in a harmonious way, which constitutes the secular, changing part of Chinese philosophy. The unchanging *Dao* is the axis, the destination, and all the changing things, such as the relational self, time, calendar, and numbers, which thus have directions to rotate around. Because of the common consensus of the greatest *Dao* across different social classes, such a systematic, harmonious way of Yin-yang thinking could be continuously carried, generated, and practiced in the long history of Chinese civilizations.

On the other hand, Ancient Greece had a completely different way of knowing the world; harmony was of central importance concerning the gods in ancient Greece. If ancient

Chinese emphasized the people's responsibility to carry on the *Dao* in generating harmony between heaven, humans, and earth, ancient Greeks then strived to prove the actual existence of harmony, their hidden fifth element aether, in the heavenly bodies (Lloyd 1996) and in numbers and musical notes for Pythagoreans. The essence of harmony is subject to tension and opposites. Heraclitus highlighted the fundamentals of harmony:

> *"They do not understand how that which differs with itself is in agreement: harmony consists of opposing tension, like that of the bow and the lyre"* (Freeman 1983, p. 28).

He proposed that harmony can be hidden; what is hidden is a natural harmony, and only wise people like himself could see the value of it, which highly aligns with the fifth sacred element, aether, agreed by Pythgoreans, Plato, Heracleides, and other great philosophers. To Heraclitus, fire ranks first among the other four elements, which have the characteristics of constantly renewing and fluid all the time. Harmony has taken its course of unity of two opposite forces against the way of constant change and universal absence of permanence (Li 2008). Philolaus' (470-385 B.C.) understanding of harmony is quite similar to Heracleides'; he reflects that

> *"the things which were like and related needed no harmony; but the things which were unlike and unrelated and unequally arranged are necessarily fastened together by such a harmony, through which they are destined to endure in the universe"* (Freeman 1983, p.74).

Therefore, it is no wonder why there are various, irreconcilable philosophical disagreements about which element owns the primordial position among the remaining classical elements in natural philosophy; the disagreement extends to other later Greek philosophical propositions such as seeds or atoms in metaphysics, the utilization of different lunisolar calendar system in each city-state (Lloyd 1996), and even the governmental system as Democracy. It is simply through the constant disagreements and oppositions that ancient Greeks reached and maintained ever-lasting harmony.

Philolaus argues that the universe is composed of two forms, the Limited and Unlimiteds. The Unlimiteds are not subject to any specific structure or quantity, mainly shown in four elements (earth, air, fire, and water) and space and time. The Limiteds result from the set limits, such as shapes for the unlimited; two fit together in a “Harmonia” way (Huffman 2024), expressed mathematically. As one of three prominent figures in

Pythagoreans, he followed the tradition that harmony is realized through musical notes in music, specifically, octaves. He writes that

> "*the content of the Harmony (Octave) is the major fourth and the major fifth; the fifth is greater than the fourth by a whole tone; for from the highest string (lowest note) to the middle is a fourth, and from the middle to the lowest string (highest note) is a fifth*" (Freeman 1983, p. 74).

Aether, which corresponds to the number five, represents the immortal soul that rests in harmony. Aristotle referenced the Pythagorean theory of the soul as

> "*a kind of harmony, for harmony is a blend or composition of contraries, and the body is compounded out of contraries*" (Mckeon 1973, p. 160).

Moreover, in *Timaeus*, Plato further states that

> "*... the ratios of their numbers, motions, and other properties, everywhere God, as far as necessity allowed or gave consent, has exactly perfected and harmonized in due proportion.*" (Jowett 1949, p. 39)

Indeed, the structure of the ratio itself represents harmony by comparing two numbers; without the parallel of another number, harmony could not be formed. In realizing harmony, Pythagoreans believed in recognizing the embedded rational order (Kosmos) in harmony; it is through pondering deeply the Kosmos that the soul would fall under order (Guthrie 1962; Li 2008). The nature of the octave is that it is a system with a fixed, predetermined interval, as Plato and Pythagoreans agreed that harmony is set to comply with the pre-set, perfect, universal order. In Homeric poems, there were already implications that even the king of the gods could not control certain events. In *Heogony*, a poem composed by Hesiod (700 B.C.) describing the genealogies of the Greek gods, it can be identified that a hidden hand controlled the destiny of the kings of the gods. Subsequently, Cronus, the first generation of Titans, replaces Uranus; herein, Zeus replaces Cronus, and Zeus faced further replacement by the unseen higher power that oversees all things (Sedley 2010). It is argued that the Greek's tendency of determinism may arise from their tendency to think about preconditions of the future following the rules of logical implications, whereas the Chinese' pondering about fate is more pragmatic and flexible, without a rigid, fixed constrain (Raphals 2013). Either way, the idea of harmony is linked to the moral governance of heaven and earth (Lloyd 1996). Through the lens of finding harmony in music, ancient Chinese and Pythagoreans approached it in different means: The Chinese musical model was structured in

the way that different sounds could respond to one another, whereas Pythagoreans explored sounds in a linear, sequential model (Li 2008). Wong (2020) made the analogy that ancient Greeks tended to think of the world as a linear model of billiard balls hitting one another in a setting motion; ancient Chinese saw the world more as a compound of plural substances constituted by the relationships between each other.

Because of ancient Chinese and Greeks' different ways of understanding harmony, their ways of thinking naturally differ as well. As Lloyd (1996) discussed, ancient Greek philosophers developed theories of causation, devoting much effort to elaborate explicit theories, defending their ideas, and persuading their competitors in a highly logical manner. To them, the definition of cause and effect also lacked agreement. In contrast, early Chinese intellectuals set up schemas of correlations on a contextual basis to practice wisdom embodied in the greatest *Dao* rather than arguing about what *Dao* was, mainly because the comprehensive, unified worldview had already been consensed at a very early stage. The Chinese practice of the greatest *Dao* was highly dependent on the social order of the rulers and the ruled, which, due to rulers have the most significant responsibility and capacity to carry and inherit *Dao* in the realm of human beings, which is characterized by the natural principle of Yin and Yang. The political, social, and *Dao* were strongly coherent and interdependent. Alternatively, Greeks never formed an agreement on political ideals. Although Plato proposed the idea of the philosophical king who could only best rule the city-states by studying philosophy to gain absolute wisdom, it can be seen that ancient Greeks tended to prioritize their ideal above all, which held the autonomous, superior position. This is similar to the atomic philosophy proposing that behind the four elements of the material; there are unsplittable atoms as the most minor units and the Pythagorean theory of numbers as the fundamentals of all things (Lloyd 1996).

8 Limitations and Further Studies

The paper approaches the quantitative analysis by inputting the Fibonacci sequence; choices of modulo are made based on the broad system of Yin-yang. This is mainly due to the fact that there is no set Yin-yang sequence or list of numbers to analyze, whereas it provides a systematic context, philosophical references, and various schemas behind the simple

Yin-yang diagram. Alternatively, the nature of Fibonacci numbers determines they could be easily statistically coded and analyzed, while their underlying metaphysical meanings rooted in ancient Greek philosophy are relatively scattered and divided, not forming a unified and coherent epistemological system like in ancient China, which to some extent because the ancient Greeks borrowed the counting system from the Babylonians, Egyptians, and Sumerians, leaving no spaces to develop their own's. Therefore, one of the limitations of this paper is that using the pre-set system of Yin-yang in quantitative analysis may result in a biased finding due to the original skewed data collection criteria. However, this paper shall argue that this approach is reasonable for explanatory, comparative, and discussion purposes, considering that the evolution of Western mathematics, since Pre-socrates, has had a growing tendency to be separated from philosophy, which leaves less space to form a contextualized cosmology linking different disciplines like ancient China did. Pythagoras himself did not leave any books or writings, and those Pythagorean philosophical meanings were agitated and noted later by his followers, such as Plato and Heraclitus. Later, western mathematicians contributed more efforts, specifically in the evolvement of mathematics itself, which explains why natural sciences and other disciplines that follow the implications of logic are so developed and advanced in the Western world. In other words, early Greeks, unlike ancient China, did not form a deep sense of cohesion between political parties and scientists, astronomers, philosophers, and mathematics; which had no consensus on how these various disciplines could contribute to that cohesion or whether they needed a whole-rounded cohesion at the first place (Lloyd 1996).

Secondly, the lack of literature on the research topics this paper focuses on should be considered. Some papers explored the mathematical model of *Hetu* or *Luoshu* about the Fibonacci sequence. However, the philosophical thickness and historical origins behind the two were not sufficiently discussed and compared. Alternatively, there are also a significant number of scholarly works that have been researched on the ancient Chinese and early Greek philosophical comparisons, but they emphasize the metaphysical levels without involving the statistical analysis of numbers. The most important reason might be the language barrier in translating old Chinese texts to modern English, let alone that ancient Chinese is accounted under such distinct discourse systems and cultural contexts, given its rich historical background that is entirely different from the West, which maintains its center of academic voice nowadays. Despite these reasons, the dearth of scholarly works on ancient Chinese is a pity, living in this so-called multicultural world. Thus, this paper could only use an

experimental methodology to explore the relations between the two, considering its highly interdisciplinary nature and lack of guidance from previous works, which indicates having enough literature space to do more future works.

Overall, the methodology of this paper could be renewed in future scholarly works based on more advanced techniques or more inclusive, nonbiased data collection criteria. Moreover, if tracing to the beginning places of the two origins, there are more profound and ancient topics that remain unexplored, for instance, the potential link between the Pythagorean theorem and *the Gougu* theorem. Greek mythology, which laid the foundation of Greek philosophy, and the ancient civilizations in the Mediterranean region, such as Babylon and Ancient Egypt, which highly influenced the period of prosperity of ancient Greece, remain undiscussed. If digging deeply into the starting points of all civilizations, no matter in the realms of antiquity sciences, mathematics, philosophy, theology, mythology, or even oral history, there may be more potential connections between them that nowadays seem polarized. It might also discover from the extraordinary civilizational origins more of the ultimate problems of mankind, which present-day advanced science is attempting to explore innovatively and experimentally, such as where mankind came from and where the universe is going. The early investigations of ancient civilizations were the acquisition of cognitive development, which is knowing how to believe, explain, and construct the human world (Lloyd 1996). Ancient civilizations have been shrouded in mysticism, labeled as primitive, superstitious, ignorant, and too far away from today's lives. It shall be noted that every moment of the future is made up of tiny movements in the past and that the reason mankind can stand today to build the future is that the entire origin of mankind has gone through centuries and centuries of history. Thus, future scholarly works that combine different disciplines could conduct more research on topics such as the above.

9 Conclusion

In a nutshell, this paper investigates the potential philosophical connections between ancient China and early Greece through the lens of Fibonacci numbers and the Yin-yang diagram. By firstly providing historical and philosophical contexts of the Yin-yang diagram

and Fibonacci sequence, this paper conducts a quantitative analysis based on the Yin-yang cosmology and Pisano Peirod. After closely discussing the philosophies behind the findings, it can be concluded that there are both philosophical similarities and differences between the two countries.

According to the discussions, the similarities between ancient China and early Greek philosophy are shown in the sexadecimal calendar system as the constructive way to count time and space as well as the five classical elements as the means to categorize the fundamental natural principles in the world. Compared to the similarities found in mathematics, there are more philosophical differences that exist, which are attributed to the epistemological distinctions of knowing and explaining the early world. Ancient Chinese preferred to think in the correlative and pragmatic modes with a unified epistemology, while ancient Greeks tended to think in the causational and logical modes with an autonomous epistemology. Due to different modes of thinking, the ancient Chinese favored consistency between politics, social class, and metaphysical inquiry, while the ancient Greeks took the opposite inconsistent position. Either way, in the early stages, numbers bear more metaphysical meanings in reflecting the first attempts of ancient civilizations to understand the universe and to construct the ontological values of each.

It shall be noted that there might have been more communication between the broader Eastern and Western intellectual worlds before the 21st century. As Norden (2017) indicated, in the 17th century, European philosophers who encountered Chinese thought stated it belonged to the course of philosophy immediately. It was the Jesuits, who were professionally trained in philosophy, first translated the *Analects* authored by Confucius into a European language and titled it *Confucius the Chinese Philosopher* (1687). In 1897, Leibniz (1646-1716) was greatly inspired by the structure of Yin and Yang as broken and unbroken lines in *I-Ching,* which is highly parallel to his discovery of binary code (Perkins 2004) in modern times. Scholars also discovered that the Pythagorean theorem and the Chinese Gougu theorem have an uncanny resemblance to each other (Martzloff 2006; Gustafson 2012) despite their completely different origins and geographic locations. In short, the seemingly polarized worlds of East and West, as exemplified by the roots of the Fibonacci sequence and the yin-yang diagram in philosophy explored in this paper, may have more similarities than premised, and the deeper underlying reasons for these collisions of ideas are

yet to be further explored by applying novel methodologies and interdisciplinary perspectives.

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