Transmission and Mile Per Gallon (MPG)

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Summary

The report explored the relationship between transmission types and miles per gallon (MPG). Figure 1 in appendix showed that an manual transmission trend to be better for MPG, considering other factors. Then, we performed t-test to verify the hypothesis. To quantify the MPG difference between automatic and manual transmissins, we fitted the MPG (outcome) and a set of variables (regressors) in to multiple linar models. We selected tansmission (am), number of cylinders (cyl), gross horsepower (hp), and weight (wt) as repressors.

Results

The mean of automatic transmission: 17.1473684 miles per gallon.
The mean of manual transmission: 24.3923077 miles per gallon.

Model Selection
We used nested model search. The detailed process of model selection is shown in **Appendix**.
We set mpg as the outcome and transmission (am), number of cylinders (cyl), gross horsepower (hp), and weight (wt) as repressors and fitted the linear model: $MPG_i = \beta_0 + \beta_1 am(manual) + \beta_2 cyl(6) + \beta_3 cyl(8) + \beta_4 hp_i + \beta_5 wt_i + \epsilon_i$. Assume $\epsilon_i \sim N(\mu, \sigma^2)$.
The coefficients of the linear model are showed below:

```
##      (Intercept) factor(am)manual     factor(cyl)6     factor(cyl)8
##      33.70832390       1.80921138      -3.03134449      -2.16367532
##               hp               wt
##      -0.03210943      -2.49682942
```

Interpretion
The intercept β_0: the expected mpg of automatic transmission when there is 4 cylinders, 0 gross horsepower, and none weight.
The β_1: the expected change in mpg comparing those in manual to those in automatic transmission.
The β_2: the expected change in mpg comparing those with 6 cylinders to those with 4 cylinders.
The β_3: the expected change in mpg comparing those with 8 cylinders to those with 4 cylinders.
The β_4: the expected change in mpg for every 1 gross horsepower increase.
The β_5: the expected change in mpg fot every 1000 lbs increase in the weight of cars.

T-test

Assumption: Unequal variance. Detailed test results is shown in **Appendix**.

H_0: $\mu_{auto} = \mu_{manual}$.

H_a: $\mu_{auto} < \mu_{manual}$.

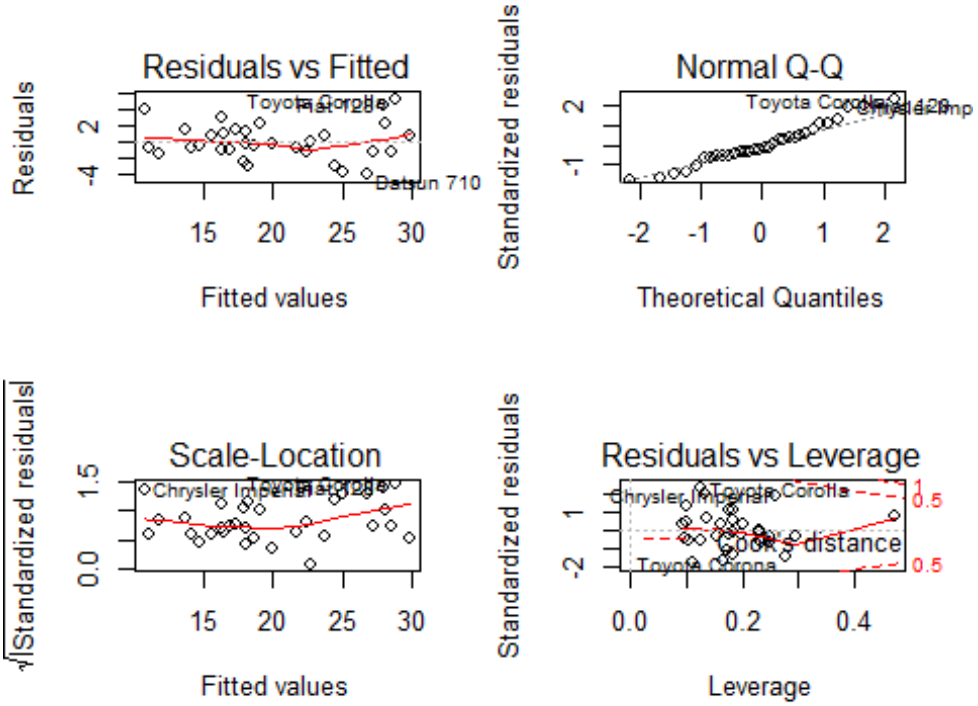
The p-value: 6.868191710^{-4}. The type I error α = 0.05, then we rejected H_0 and concluded that a manual transmission is better for MPG.

Quantify the MPG Difference

$\mu_{manual} - \mu_{auto}$ = 7.2449393.

Adjustment: $MPG_{manual} - MPG_{auto}$ = abs(β_1) = 1.8092114.

Diagnostics



The mean of residuals is 2.688821410^{-17}, approximating to zero.

“Residual vs Fitted”: residuals are independent. “scale-location”: the homoscedasticity of residuals. “Normal Q-Q”“: all residuals were approximately stardard normal distributed.”Residual vs Leverge“: within 0.5, no outliers.

The uncertainty

We assumed other variables not included in the linear model were completely randomized. The error ϵ in the linear model followed normal distribution. However, we are uncertain whether there is a better linear model with other combination of diferent regressors.

Appendix

Figure

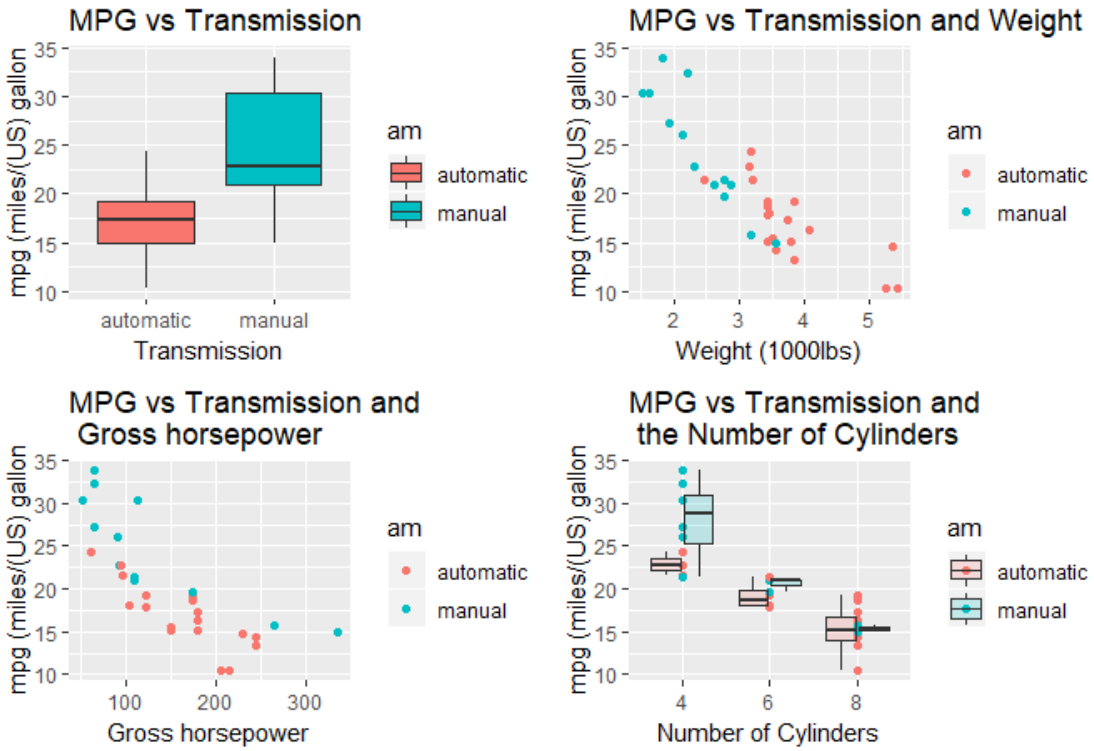


Figure 1. The relationship between MPG and a set of variables

Model Selection

```
## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
##   Res.Df   RSS Df Sum of Sq      F   Pr(>F)
## 1     30 720.9
## 2     28 264.5  2     456.4 24.158 8.01e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + disp
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1     30 720.90
## 2     28 264.50  2    456.40 26.7353 3.956e-07 ***
## 3     27 230.46  1     34.04  3.9875   0.05601 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
```

```
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
##   Res.Df   RSS Df Sum of Sq       F   Pr(>F)
## 1     30 720.9
## 2     28 264.5  2     456.4 31.2446 9.43e-08 ***
## 3     27 197.2  1      67.3  9.2141 0.005266 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
## Model 4: mpg ~ factor(am) + factor(cyl) + hp + drat
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1     30 720.90
## 2     28 264.50  2    456.40 30.4123 1.557e-07 ***
## 3     27 197.20  1     67.30  8.9686  0.005961 **
## 4     26 195.09  1      2.11  0.2807  0.600747
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
## Model 4: mpg ~ factor(am) + factor(cyl) + hp + wt
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     30 720.90
## 2     28 264.50  2    456.40 39.286 1.388e-08 ***
## 3     27 197.20  1     67.30 11.585  0.002164 **
## 4     26 151.03  1     46.17  7.949  0.009081 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
## Model 4: mpg ~ factor(am) + factor(cyl) + hp + wt
## Model 5: mpg ~ factor(am) + factor(cyl) + hp + wt + qsec
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1     30 720.90
## 2     28 264.50  2    456.40 39.6232 1.772e-08 ***
## 3     27 197.20  1     67.30 11.6849  0.002166 **
## 4     26 151.03  1     46.17  8.0172  0.009017 **
## 5     25 143.98  1      7.04  1.2230  0.279293
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
## Model 4: mpg ~ factor(am) + factor(cyl) + hp + wt
## Model 5: mpg ~ factor(am) + factor(cyl) + hp + wt + factor(vs)
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
```

```
## 1     30 720.90
## 2     28 264.50  2    456.40 39.7065 1.737e-08 ***
## 3     27 197.20  1     67.30 11.7095  0.002146 **
## 4     26 151.03  1     46.17  8.0341  0.008954 **
## 5     25 143.68  1      7.35  1.2782  0.268968
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
## Model 4: mpg ~ factor(am) + factor(cyl) + hp + wt
## Model 5: mpg ~ factor(am) + factor(cyl) + hp + wt + factor(gear)
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1     30 720.90
## 2     28 264.50  2    456.40 36.5938 5.143e-08 ***
## 3     27 197.20  1     67.30 10.7916  0.003124 **
## 4     26 151.03  1     46.17  7.4043  0.011916 *
## 5     24 149.67  2      1.36  0.1091  0.897096
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ factor(am) + factor(cyl)
## Model 3: mpg ~ factor(am) + factor(cyl) + hp
## Model 4: mpg ~ factor(am) + factor(cyl) + hp + wt
## Model 5: mpg ~ factor(am) + factor(cyl) + hp + wt + factor(carb)
##   Res.Df    RSS Df Sum of Sq       F   Pr(>F)
## 1     30 720.90
## 2     28 264.50  2    456.40 32.9605 3.33e-07 ***
## 3     27 197.20  1     67.30  9.7201 0.005206 **
## 4     26 151.03  1     46.17  6.6691 0.017370 *
## 5     21 145.39  5      5.63  0.1627 0.973489
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

T-test Results

```
##
##  Welch Two Sample t-test
##
## data:  mpg by factor(am)
## t = -3.7671, df = 18.332, p-value = 0.0006868
## alternative hypothesis: true difference in means is less than 0
## 95 percent confidence interval:
##       -Inf -3.913256
## sample estimates:
## mean in group automatic    mean in group manual
##                17.14737                24.39231
```