# Yang Xiao

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# 1. Introduction to the Analysis of Used Car Market Dynamics:

This study delves into the intricacies of used car pricing, a vital aspect of the automotive industry with significant economic implications. By examining factors like make, model, year, mileage, and more, the research aims to uncover the key determinants of a used car's market value. The significance of this research lies in its potential to enhance market transparency and inform strategic decision-making in the future

# 2. EDA

### **EDA** for overview



# Analysis:

- 1.Selling Price: The distribution shows a high frequency of cars in the lower price range, indicating a market dominated by budget-friendly options.
- 2.Kilometers Driven: Most cars have lower kilometers, suggesting a prevalence of relatively less used vehicles. Higher km driven cars are fewer, possibly due to decreased value or desirability.
- 3. Fuel Type: Petrol cars outnumber diesel, reflecting consumer preference or market availability. Other fuel types are significantly less common.
- 4. Transmission Type: Manual transmission cars are more prevalent than automatic, possibly due to lower cost or higher availability in the used car market.

These trends provide valuable context for understanding consumer preferences and market dynamics in the used car sector.

# 3. Model

# 3.1 Bayesian Regression Model

# 3.1.1 Modeling

The code uses a Gaussian loss function, indicating the model minimizes squared errors, assuming normally distributed residuals. The estimators are the beta coefficients for predictors and sigma for residual variation, estimated via MCMC sampling with Stan's NUTS algorithm. Predictors likely include car attributes like year and km\_driven. The approximation method, MCMC, approximates the posterior distributions, providing a range of plausible values for parameters, capturing uncertainty instead of single-point estimates, which is a key advantage of Bayesian methods.

```
print(fit,digits=4)
## Inference for Stan model: anon model
## 4 chains, each with iter=2000; warmup=1000; thin=1; ## post-warmup draws per chain=1000, total post-warmup draws=4000.
                                            2.5%
                                                         25%
                 mean se mean
                                                                    50%
## beta[1]
               0.2304 0.0001 0.0042
                                          0.2221
                                                     0.2276
                                                                 0.2304
                                                                            0.2331
## beta[2]
               0.0554 0.0003 0.0152
                                          0.0262
                                                     0.0450
                                                                 0.0556
                                                                            0.0652
## beta[3]
## beta[4]
              -0.0052
                       0.0000 0.0005
                                          -0.0063
                                                     -0.0056
                                                                -0.0052
                                                                            -0.0049
                       0.0000 0.0017
                                          0.0032
                                                     0.0054
                                                                 0.0066
                                                                            0.0078
               0.0066
## beta[5]
               -0.0653
                       0.0000 0.0023
                                          -0.0698
                                                     -0.0669
                                                                 -0.0653
                                                                            -0.0638
## beta[6]
                       0.0000 0.0007
               0.0027
                                          0.0014
                                                     0.0023
                                                                 0.0027
                                                                            0.0032
## sigma
## lp__
               0.0519
                       0.0000 0.0006
                                          0.0508
                                                     0.0516
                                                                 0.0519
                                                                            0.0523
           97.5% n_eff
## beta[1]
                       2429 0.9998
               0.2387
               0.0848
## beta[2]
## beta[3]
                       1896 1.0017
               -0.0042
                       4139 1.0001
## beta[4]
               0.0101
                       2967 0.9999
                       2307 0.9998
## beta[5]
               -0.0609
## beta[6]
               0.0041
                       4129 1.0004
## sigma
               0.0530
                       2302 1.0001
## lp__
##
           10667.1954
                       1634 1.0042
## Samples were drawn using NUTS(diag_e) at Mon Dec 11 01:20:12 2023.
## For each parameter, n eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

# (1) Analysis of model fitting results

- 1. Year (beta[1] = 0.2304): Newer cars command higher selling prices, emphasizing the depreciation effect. The significant positive coefficient indicates a strong relationship between a car's age and its market value.
- 2. Kilometers Driven (beta[3] = -0.0052): Cars with higher mileage are priced lower, showcasing wear and tear's impact on valuation. This negative coefficient reflects the common consumer preference for less-used vehicles.
- 3. Fuel Type (beta[4]): The coefficient for fuel type (assuming it corresponds to beta[4] = 0.0066) indicates a slight effect on the selling price.
- 4. Seller Type, Number of Owners, Transmission: These factors' coefficients (e.g., beta[5], beta[6]) suggest varying impacts on price, though specific interpretations depend on the reference categories used.
- 5. Model Reliability and Precision: The model's low sigma value (0.0519) indicates precise predictions with minimal error variability. Rhat values near 1 and substantial effective sample sizes (n\_eff) suggest good convergence, lending credibility to the model's estimates.

### 3.1.2 Sensitivity Analysis Modeling

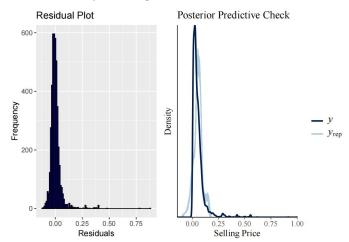
```
print(fit_new_prior, digits = 4)
## Inference for Stan model: anon_model.
                                          warmup=1000; thin=1;
    4 chains, each with iter=2000;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
                                                                                   50%
## beta[1]
                  0.2303
                            0.0001 0.0042
                                                   0.2219
                                                                 0.2275
                                                                               0.2303
                                                                                             0.2331
## beta[2]
## beta[3]
                 0.0553
                             0.0003 0.0148
                                                   0.0263
                                                                 0.0454
                                                                               0.0552
                                                                                             0.0655
                                                   -0.0062
                                                                -0.0056
                                                                                            -0.0049
                             0.0000 0.0005
                                                                              -0.0052
## beta[4]
## beta[5]
## beta[6]
                  0.0067
                             0.0000 0.0017
                                                   0.0035
                                                                 0.0055
                                                                               0.0066
                                                                                             0.0078
                  -0.0653
                             0.0000 0.0022
                                                   -0.0696
                                                                 -0.0668
                                                                               -0.0654
                  0.0027
                             0.0000 0.0007
                                                   0.0014
                                                                 0.0023
                                                                               0.0027
                                                                                             0.0032
## sigma
## lp__
                   0.0520
                             0.0000 0.0006
                                                   0.0509
                                                                 0.0516
                                                                               0.0519
                                                                                             0.0523
                                                                          10664.8737 10665.9896
              10664.5693
                             0.0463 1.8946
                                              10659.9637 10663.5347
##
                    97.5% n eff
                                     Rhat
## beta[1]
## beta[2]
                  0.2386
0.0841
                            2334 1.0057
1955 1.0031
## beta[3]
## beta[4]
## beta[5]
                  -0.0042
                             4289 0.9994
                  0.0101
                             3083 1.0011
                            2528 1.0030
                  -0.0610
## beta[6]
## sigma
                  0.0041
0.0531
                            4161 0.9998
1913 0.9994
## lp__
              10667.2161
                            1675 1.0007
## Samples were drawn using NUTS(diag_e) at Mon Dec 11 01:21:17 2023.
## For each parameter, n_eff is a crude measure of effective sample size, ## and Rhat is the potential scale reduction factor on split chains (at
## and Rhat is the potent
## convergence, Rhat=1).
```

### **Conclusion for Sensitivity**

- (1) The posterior distribution of parameter beta: The mean and quantile of parameter beta for the two runs are very close, indicating that the model has a certain degree of stability in estimating these parameters. Different posterior samples are also within the 95% confidence interval, indicating that the parameter estimation is consistent.
- (2) The posterior distribution of parameter sigma: The mean and quantile of parameter sigma from the previous and subsequent runs are also very close, and the 95% confidence interval overlaps, indicating that the model's estimation of standard deviation is also consistent.
- (3) Rhat vale: The Rhat values of both previous and subsequent runs are close to 1, indicating that the model has achieved reasonable convergence between different chains.

In summary, the results of the two runs are very similar, and the model has a certain degree of stability and consistency for different posterior sampling. This indicates that the parameter estimation of the model is reliable and not easily affected by initial conditions.

# 3.1.3 EDA for Bayesian Regression Model



### Graphic analysis

The residual plot indicates a right-skewed distribution, suggesting the model un derpredicts for some observations. A concentration of residuals near zero suggests accurate predictions for many cases, but the long tail points to significant errors for others, potentially due to outliers or unmodeled factors.

The PPC plot reveals good model fit around the data's central tendency but poor fit at the tails. This mismatch indicates that the model might not capture the full variability of the data, especially for higher selling prices.

Overall, the model performs well for typical values but needs refinement to handle the full range of the selling price distribution, possibly by including additional predictors, investigating outliers, or introducing non-linear terms. Further model diagnostics are recommended to enhance its predictive power.

# 3.2 Hierarchical Bayesian Model

# 3.2.1 Modeling

The model employs a Gaussian family, implying a squared-error loss function to assess the fit between predicted and actual selling prices.

The predictors include both numerical (year, km\_driven) and categorical (fuel, seller\_type, transmission, owner) variables. The estimators are the regression coefficients for these predictors.

The model also accounts for random effects due to fuel type. For parameter estimation, the model uses MCMC with increased iterations and higher adapt\_delta for convergence, reflecting a robust Bayesian inference approach. The priors for the coefficients and intercept are normally distributed, while the prior for the standard deviation is Cauchy-distributed, encapsulating prior beliefs about these parameters' distributions.

```
# Print the mos
summary(model)
                              odel summary
 ## Warning: There were 4 divergent transitions after warmup. Increasing
## adapt_delta above 0.98 may help. See
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
## Family: gaussian
## Links: mu = identity; sigma = identity
## Links: mu = identity; sigma = identity
## Formula: selling_price - year + km_driven + seller_type + transmission + owner + (1 | f

## Data: car_data (Number of observations: 4340)
## Draws: 3 chains, each with iter = 1000; warmup = 500; thin = 1;
## total post-warmup draws = 1500
 ## Group-Level Effects:
 ## -fuel (Number of levels: 5)
## Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## add(Intercept) 0.46 0.31 0.16 1.28 1.00 411 479
##
## ## Population-Level Effects:
## Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
## Intercept 1.29 0.26 0.73 1.77 1.00 500
** "Oar" 0.26 0.01 0.23 0.28 1.00 1793
** "Oar" -0.05 1.00 1795
 ## year
## km_driven
                                                                          -0.08
                                                                                                0.01
                                                                                                                  -0.11
                                                                                                                                      -0.05 1.00
                                                                                                                                                                      1795
## seller_typeIndividual
## seller_typeIndividual
## seller_typeTrustmarkDealer
## transmissionManual
## ownerFourth&AboveOwner
## ownerSecondOwner
## ownerTestDriveCar
## ownerTestDriveCar
                                                                        -0.11
                                                                                                0.03
                                                                                                                 -0.17
                                                                                                                                      -0.06 1.00
                                                                                                0.08
                                                                                                                    0.13
                                                                                                                                      0.45 1.00
                                                                                                                0.13
-1.58
-0.16
-0.13
-0.07
-0.16
                                                                                                                                    0.45 1.00
-1.43 1.00
0.16 1.00
-0.02 1.00
0.63 1.00
0.02 1.00
 ## ownerThirdOwner
                                                                          -0.07
                                                                                                0.05
                                                                 Tail ESS
```

```
## Intercept 516
## year 1347
## km_driven 1100
## seller_typeIndividual 1218
## seller_typeTrustmarkDealer 1122
## transmissionManual 1315
## ownerFourth&AboveOwner 1046
## ownerFourth&AboveOwner 1106
## ownerFestbriveCar 1216
## ownerThirdOwner 1338
## Family Specific Parameters:
## Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma 0.74 0.01 0.72 0.75 1.00 2352 1187
##
## Travs were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

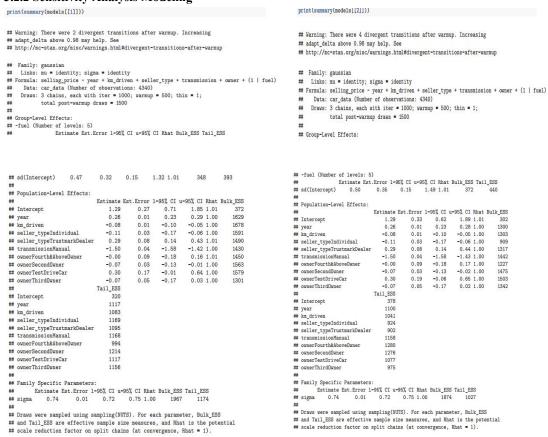
#### Analysis of model fitting results

Convergence: The Rhat value approaches 1, indicating that the model has converged well.

Effective sample size: Bulk\_ESS and Tail\_The high value of ESS indicates good posterior sampling efficiency.

Predictive ability: Through the posterior prediction test (PPC) chart, it can be seen that the model's predictions match the distribution of actual data quite well.

### 3.2.2 Sensitivity Analysis Modeling



### **Conclusion for Sensitivity**

# Group Level Effects

The estimated value of SD (Intercept) has slightly changed, but the 95% confidence interval has a large span, especially the upper bound. This may indicate that the model exhibits a certain sensitivity in prior selection for random intercepts of fuel types.

Population Level Effects

The estimated values of Intercept and year remain stable in two rounds of fitting, indicating that the

model is not sensitive to prior selection of these parameters. Km\_ Driven, seller\_ TypeIndividual, seller\_ The estimated values of typeTrustmarkDealer, transmissionManual, and owner categories are also relatively stable, indicating that these fixed effects estimates of the model are robust under prior changes.

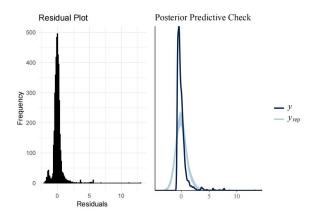
# Family Specific Parameters

The estimation value of sigma is very stable under both priors, indicating that the model is not sensitive to the selection of priors in estimating the standard deviation of residuals.

#### Model diagnosis

The Rhat value is equal to 1 on all parameters, indicating that the model converges well. Bulk\_ESS and Tail\_ESS is high enough for most parameters, indicating that the posterior distribution has sufficient effective sample size for reliable estimation. Overall, the sensitivity analysis of the model indicates that the posterior estimate is relatively insensitive to the selection of priors, which increases confidence in the model results.

# 3.2.3 EDA for Hierarchical Bayesian Model



### Graphic analysis

Residual plot: The residuals are mainly concentrated around 0, but there seems to be a slight right deviation. In an ideal situation, the residuals should be symmetrically distributed around 0 without any obvious skewness or outliers. This may indicate slight shortcomings in certain aspects of the model.

PPC chart: Black lines represent the density of actual data, while blue lines represent the density of data predicted by the model. The degree of overlap between these two indicates that the model predictions are consistent with the actual observed values, but also suggests the possibility of overfitting.

Reasoning and Evaluation

# 4. Conclusions

### (1) Key Findings:

The study revealed significant insights into how various factors influence the pricing of used cars. Variables such as the car's age, mileage, fuel type, and seller type play crucial roles in determining its market value. The modeling approach demonstrated the importance of both quantitative and qualitative factors in the used car market.

# (2) Implications:

The research provides valuable insights for potential buyers and sellers in the used car market, offering a deeper understanding of what factors most significantly affect car prices.

# 5. Reference

- [1] Franke, Michael, and Timo B. Roettger. "Bayesian regression modeling (for factorial designs): A tutorial." (2019).
- [2] Kemp, Charles, Andrew Perfors, and Joshua B. Tenenbaum. "Learning overhypotheses with hierarchical Bayesian models." Developmental science 10.3 (2007): 307-321.

# 6. Code

```
```{r setup, include=FALSE}
knitr::opts_chunk$set(echo = TRUE)
library(readr)
library(ggplot2)
library(dplyr)
library(tidyr)
library(scales)
library(rstan)
library(patchwork)
library(tidyverse)
library(bayesplot)
library(brms)
# 2. EDA
```{r,echo=FALSE, results='hide',message=FALSE}
# Load the dataset
car data orginal <- read csv("C:/Users/xiaoy/Desktop/578/project/CAR DETAILS FROM CAR
DEKHO.csv")
# Check for missing values
missing values <- sapply(car data orginal, function(x) sum(is.na(x)))
# Transforming categorical variables using one-hot encoding
car_data <- car_data_orginal %>%
  mutate at(vars(fuel, seller type, transmission, owner), as.factor) %>%
  mutate_if(is.factor, as.numeric)
# Normalizing numerical columns
normalize <- function(x) {
  return ((x - min(x)) / (max(x) - min(x)))
car_data$year <- normalize(car_data$year)</pre>
car_data$selling_price <- normalize(car_data$selling_price)</pre>
car_data$km_driven <- normalize(car_data$km_driven)</pre>
# Create the individual plots
p1 <- ggplot(car_data, aes(x=selling_price)) +
  geom_histogram(bins=30, fill="blue", alpha=0.7) +
  labs(title="Distribution of Selling Price", x="Selling Price", y="Frequency")
p2 <- ggplot(car_data, aes(x=km_driven)) +
  geom histogram(bins=30, fill="green", alpha=0.7) +
  labs(title="Distribution of Kilometers Driven", x="Kilometers Driven", y="Frequency")
```

```
p3 <- ggplot(car_data, aes(x=factor(fuel))) +
  geom bar(fill="orange", alpha=0.7) +
  labs(title="Count of Cars by Fuel Type", x="Fuel Type", y="Count") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
p4 <- ggplot(car data, aes(x=factor(transmission))) +
  geom_bar(fill="purple", alpha=0.7) +
  labs(title="Count of Cars by Transmission Type", x="Transmission Type", y="Count") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
# Combine the plots
combined_plot \leftarrow p1 + p2 + p3 + p4
# Display the combined plot
combined plot
#3. Model
## 3.1 Bayesian Regression Model
### 3.1.1 Modeling
```{r,echo=FALSE, results='hide',warning=FALSE}
# Define the model in Stan
stan model <- "
data {
  int<lower=0> N; // number of data items
  int<lower=0> K; // number of predictors
  matrix[N, K] x; // predictor matrix
  vector[N] y;
                   // outcome vector
}
parameters {
  vector[K] beta;
                         // coefficients for predictors
  real<lower=0> sigma; // standard deviation
model {
  y \sim normal(x * beta, sigma);
}
# Exclude 'name' column and ensure all predictors are numeric
x matrix <- as.matrix(car data[,!(names(car data) %in% c("selling price", "name"))])
# Data for Stan model
```

```
stan_data <- list(
  N = nrow(car data),
  K = ncol(x_matrix),
  x = x_matrix,
  y = car data$selling price
)
# Fit the model
fit <- stan(model code = stan model, data = stan data, iter = 2000, chains = 4)
print(fit,digits=4)
### 3.1.2 Sensitivity Analysis Modeling
```{r,echo=FALSE, results='hide',warning=FALSE}
new_prior <- "
data {
  int<lower=0> N; // number of data items
  int<lower=0> K; // number of predictors
  matrix[N, K] x; // predictor matrix
  vector[N] y;
                    // outcome vector
parameters {
  vector[K] beta;
                          // coefficients for predictors
  real<lower=0> sigma; // standard deviation
model {
  beta \sim normal(0, 1);
  sigma \sim cauchy(0, 5);
  y \sim normal(x * beta, sigma);
fit new prior <- stan(
  model code = new prior,
  data = stan_data,
  iter = 2000,
  chains = 4
print(fit_new_prior, digits = 4)
### 3.1.3 EDA for Bayesian Regression Model
"\"\r,echo=FALSE\
posterior draws <- extract(fit)$beta
# Compute the predicted values (y_pred) by multiplying the predictors with the posterior draws of beta
# Note: 'x matrix' should be the matrix of predictors you used when fitting the model
```

```
y_pred <- as.matrix(x_matrix) %*% t(posterior_draws)</pre>
# Calculate the residuals
residuals <- as.vector(car_data$selling_price) - rowMeans(y_pred)
# Residual Plot
 residual plot<-ggplot(data = NULL, aes(x = residuals)) +
  geom histogram(binwidth = 0.01, color = "black", fill = "blue") +
  xlab("Residuals") +
  ylab("Frequency") +
  ggtitle("Residual Plot")
if (ncol(y_pred) != nrow(x_matrix)) {
    y pred <- t(y pred)
# Ensure that 'car_data$selling_price' is a vector
selling price vector <- as.vector(car data$selling price)
# PPC plot
ppc plot <- ppc dens overlay(y = selling price vector, yrep = y pred) +
  xlab("Selling Price") +
  ylab("Density") +
  ggtitle("Posterior Predictive Check")
combined plot <- residual plot + ppc plot
combined plot
## 3.2 Hierarchical Bayesian Model
### 3.2.1 Modeling
```{r,echo=FALSE, results='hide',warning=FALSE}
# Read the data
car_data <- car_data_orginal
# Encode categorical variables
car data$fuel <- as.factor(car data$fuel)
car_data$seller_type <- as.factor(car_data$seller_type)</pre>
car data$transmission <- as.factor(car data$transmission)
car_data$owner <- as.factor(car_data$owner)</pre>
# Normalize numerical variables
car data$year <- scale(car data$year)</pre>
car_data$selling_price <- scale(car_data$selling_price)</pre>
car_data$km_driven <- scale(car_data$km_driven)</pre>
# Define the model with adjusted settings
model <- brm(
  selling price ~ year + km driven + seller type + transmission + owner + (1|fuel),
  data = car data,
```

```
family = gaussian(),
  prior = c(
    set_prior("normal(0,5)", class = "b"),
    set prior("normal(0,5)", class = "Intercept"),
    set prior("cauchy(0,2)", class = "sd")
  ),
  chains = 3,
  iter = 1000, # Increased total iterations
  warmup = 500, # Increased warmup iterations
  control = list(adapt_delta = 0.98) # Increased adapt_delta for better convergence
# Print the model summary
summary(model)
### 3.2.2 Sensitivity Analysis Modeling
```{r,echo=FALSE, results='hide',warning=FALSE,message=FALSE}
priors list <- list(
  set_prior("normal(0, 2)", class = "b"),
  set_prior("normal(0, 10)", class = "b"))
models <- list()
for (i in seq_along(priors_list)) {
  models[[i]] <- brm(
    formula = selling_price ~ year + km_driven + seller_type + transmission + owner + (1|fuel),
    data = car data,
    family = gaussian(),
    prior = priors_list[[i]],
    chains = 3,
    iter = 1000,
    warmup = 500,
    control = list(adapt_delta = 0.98),
    seed = 123
  )
  print(summary(models[[i]]))
### 3.2.3 EDA for Hierarchical Bayesian Model
"\"\r,echo=FALSE\
residuals data <- residuals(model)
residual plot <-ggplot() +
```

```
geom_histogram(aes(x = residuals_data), binwidth = 0.1, fill = "blue", color = "black") +
theme_minimal() +
labs(x = "Residuals", y = "Frequency", title = "Residual Plot")
posterior_predictive <- posterior_predict(model)
selling_price_vector <- as.vector(car_data$selling_price)
ppc_plot<-ppc_dens_overlay(y = selling_price_vector, yrep = posterior_predictive) +
labs(title = "Posterior Predictive Check")
combined_plot <- residual_plot + ppc_plot</pre>
```