Q3.

20171213

PSET - 06 - Q-3

(a) $P(w_1) = 7/14 = 0.5$, $P(w_2) = 7/14 = 0.5$

(b) $\mu_{w_1} = \frac{1}{7}\Big((0,0) + (0,1) + (2,0) + (3,2) + (3,3) + (2,2) + (2,0)\Big)$

$= (12/7, 8/7) = (1.714, 1.143)$

& $\mu_{w_2} = \frac{(7,7) + (8,6) + (9,7) + (8,10) + (7,10) + (8,9) + (7,11)}{7}$

$= (7.714, 8.571)$

$\Sigma_1 = \begin{bmatrix} 1.571 & 0.881 \\ 0.881 & 1.476 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 0.571 & -0.645 \\ -0.645 & 3.62 \end{bmatrix}$

(c) equation for decision boundary

$P(w_1) P(x|w_1) = P(w_2) P(x|w_2)$

$\Rightarrow \frac{1}{2} \times \frac{1}{(\sqrt{2\pi})^2 |\Sigma_1|^{1/2}} e^{-\frac{1}{2}(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1)} = \frac{1}{2} \times \frac{1}{(\sqrt{2\pi})^2 |\Sigma_2|^{1/2}} e^{-\frac{1}{2}(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2)}$

$\Rightarrow \frac{1}{2}(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2) - \frac{1}{2}(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1) = \ln\left|\frac{\Sigma_1}{\Sigma_2}\right|^{1/2}$

$\Rightarrow$ Putting values,

$x^2 - 2.14y^2 + 3.05xy - 9.16x + 45.7y - 205 = 0$

(As $h = -\frac{3.05}{2}$, $a=1$, $b = -2.14$ $\Leftrightarrow$ $h^2-ab>0$)

∴ hyperbola

class w_2

class w_2

class w_1

(e) Let the penalties for each misclassification are different for the 2 classes, w_1 & w_2, & let $\dfrac{penality(w_1)}{penality(w_2)} = k$,

Now, $p(w_1|k) = \dfrac{P(x|w_1)\, P(w_1)}{P(x)} \times \left(\dfrac{k}{k+1}\right)$

$P(w_2|x) = \dfrac{P(x|w_2)\, P(w_2)}{P(x)} \times \left(\dfrac{1}{k+1}\right)$

Now, $P(w_1|x) = P(w_2|x)$ and $P(w_1) = P(w_2) = \frac{1}{2}$

$\Rightarrow P(x|w_1) \times k = P(x|w_2)$

$\Rightarrow (x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2) - (x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1) = \ln\left|\dfrac{|\Sigma_1|}{|\Sigma_2|}\right|$

Hence, the new decision boundary will change according to k, when k increases, the decision boundary moves towards the region of less penalty, and vicaversa.