HW 6 - SMAI ROLL_NUMBER - 20171213

Q3. (d,e) plots on page 2

20171213

PSET-06 - Q-3

(a) $P(\omega_1) = 7/14 = 0.5$, $P(\omega_2) = 7/14 = 0.5$

(b) $\mu_{\omega_1} = \frac{1}{7}\left((0,0)+(0,1)+(2,0)+(3,2)+(3,3)+(2,2)+(2,0)\right)$

$= (12/7, 8/7) = (1.714, 1.143)$

& $\mu_{\omega_2} = \frac{(7,7)+(8,6)+(9,7)+(8,10)+(7,10)+(8,9)+(7,11)}{7}$

$= (7.714, 8.571)$

$\Sigma_1 = \begin{bmatrix} 1.571 & 0.881 \\ 0.881 & 1.476 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 0.571 & -0.645 \\ -0.645 & 3.62 \end{bmatrix}$

(c) equation for decision boundary

$P(\omega_1)\, P(x|\omega_1) = P(\omega_2)\, P(x|\omega_2)$

$\Rightarrow \frac{1}{2} \times \frac{1}{(\sqrt{2\pi})^2 |\Sigma_1|^{1/2}} e^{-1/2(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1)} = \frac{1}{2} \times \frac{1}{(\sqrt{2\pi})^2 |\Sigma_2|^{1/2}} e^{-1/2(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2)}$

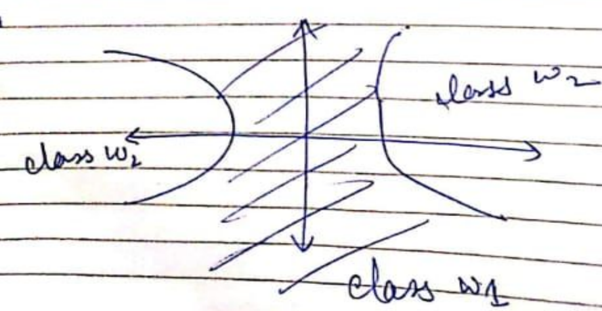
$\Rightarrow \frac{1}{2}(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2) - \frac{1}{2}(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1) = \ln\left|\frac{\Sigma_1}{\Sigma_2}\right|^{1/2}$

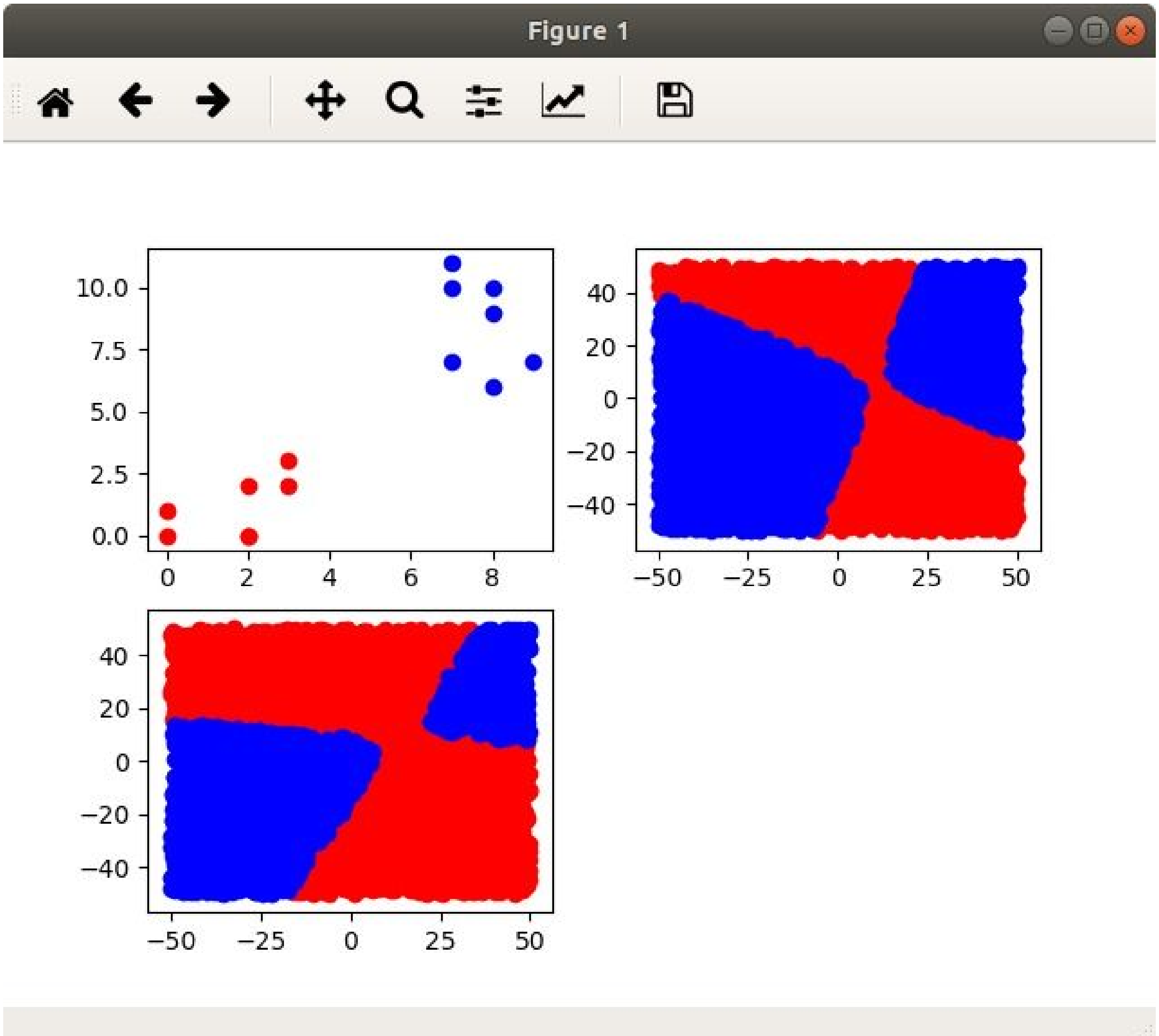
$\Rightarrow$ Putting values,

$x^2 - 2.14y^2 + 3.05xy - 9.16x + 45.7y - 205 = 0$

(As $h = -\frac{3.05}{2}$, $a = 1$, $b = -2.14$ $\Leftrightarrow$ $h^2 - ab > 0$)

$\therefore$ hyperbola





(e) Let the penalities for each misclassification are different for the 2 classes, w_1 & w_2, & let $\frac{\text{penality}(w_1)}{\text{penality}(w_2)} = k$,

Now, $P(w_1|x) = \frac{P(x|w_1)\,P(w_1)}{P(x)} \times \left(\frac{k}{k+1}\right)$

$P(w_2|x) = \frac{P(x|w_2)\,P(w_2)}{P(x)} \times \left(\frac{1}{k+1}\right)$

Now, $P(w_1|x) = P(w_2|x)$ and $P(w_1) = P(w_2) = 1/2$

$\Rightarrow P(x|w_1) \times k = P(x|w_2)$

$\Rightarrow (x-\mu_2)^T \Sigma_2^{-1} (x-\mu_2) - (x-\mu_1)^T \Sigma_1^{-1} (x-\mu_1) = \ln \left| \frac{|\Sigma_1|}{|\Sigma_2| k} \right|$

Hence, the new decision boundary will change according