

HW 6 - SMAI ROLL_NUMBER - 20171213

Q2.

20171213

Ans-2) HW6

$$(a) \frac{P(\theta|B_1)P(B_1)}{P(B_1)} = \frac{P(\theta|F_1)}{P(F_1)}$$

$$\frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(\theta-\mu_1)^2}{2\sigma_1^2}\right) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(\theta-\mu_2)^2}{2\sigma_2^2}\right)$$

Given $\sigma_1 = \sigma_2 \therefore -(\theta-\mu_1)^2 = -(\theta-\mu_2)^2$

$$\therefore (\theta-\mu_1)^2 - (\theta-\mu_2)^2 = 0$$

$$(\theta-\mu_1 - \theta + \mu_2)(\theta-\mu_1 + \theta - \mu_2) = 0$$

$$(\mu_2 - \mu_1)(2\theta - (\mu_1 + \mu_2)) = 0$$

$\therefore \mu_1 \neq \mu_2 \therefore \boxed{\theta = \frac{\mu_1 + \mu_2}{2}}$

$$(b) P(\theta|B_1) + P(B_1) = P(\theta|F_1) * P(F_1)$$

$$\frac{P(B_1)}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(\frac{\mu_1 + \mu_2}{2} - \mu_1)^2}{2\sigma_1^2}\right) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(\frac{\mu_1 + \mu_2}{2} - \mu_2)^2}{2\sigma_2^2}\right) * P(F_1)$$

$$\therefore \frac{P(B_1)}{\sigma_1} e^{-\frac{(\mu_1 - \mu_2)^2}{8\sigma_1^2}} = \frac{1}{\sigma_2} e^{-\frac{(\mu_1 - \mu_2)^2}{8\sigma_2^2}} P(F_1)$$

$$\therefore e^{-\left[\frac{(\mu_1 - \mu_2)^2}{8} \left[\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2}\right]\right]} = \frac{\sigma_1}{\sigma_2} \cdot \frac{P(F_1)}{P(B_1)}$$

let $\boxed{\sigma_1 = k\sigma_2}$

$$e^{-\left[\frac{(\mu_1 - \mu_2)^2}{8} \left(\frac{1}{k^2\sigma_2^2} - \frac{1}{\sigma_2^2}\right)\right]} = k \frac{P(F_1)}{P(B_1)}$$

then $\boxed{\frac{P(F_1)}{P(B_1)} = \frac{1}{k} e^{-\frac{(\mu_1 - \mu_2)^2}{8\sigma_2^2} [1 - k^2]}}$

$\boxed{k > 0}$



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$$(c) \quad P(BG) * \frac{1}{\sqrt{2\pi} \sigma_1} e^{-\frac{(\theta^* - \mu_1)^2}{2\sigma_1^2}} = P(FG) * \frac{1}{\sqrt{2\pi} \sigma_2} e^{-\frac{(\theta^* - \mu_2)^2}{2\sigma_2^2}}$$

$$\text{Given: } P(BG) = 4 * P(FG), \mu_1 = 100, \mu_2 = 200 \text{ and } \sigma_1 = \sigma_2 = \sigma$$

$$\therefore \quad 4 * P(FG) * \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\theta^* - 100)^2}{2\sigma^2}} = P(FG) * \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\theta^* - 200)^2}{2\sigma^2}}$$

$$\ln 4 - \frac{(\theta^* - 100)^2}{2\sigma^2} = -\frac{(\theta^* - 200)^2}{2\sigma^2}$$

$$\therefore \quad -(\theta^* - 200)^2 + (\theta^* - 100)^2 = 2\sigma^2 \ln 4$$

$$\cancel{2\theta^{*2}} \Leftarrow (\theta^* - 200 - \theta^* + 100)(\theta^* - 200 + \theta^* - 100) = 2\sigma^2 \ln 4$$

$$100(-300 + 2\theta^*) = 2\sigma^2 \ln 4$$

$$300 + \frac{2\sigma^2 \ln 4}{100} = 2\theta^*$$

$$\theta^* = 150 + \frac{2\sigma^2 \ln 4}{100}$$

$$\boxed{\theta^* = 150 + 0.0138\sigma^2}$$

