CMSC 412: Social Network Analysis & Cybersecurity Risks

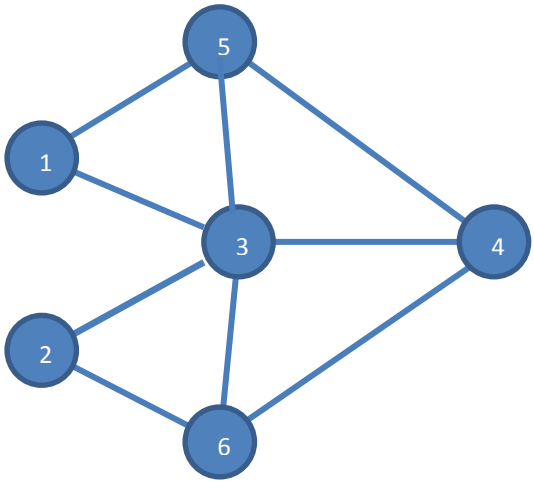
Fall 2016

Homework #1

Assigned: 09/26/2016

Due on: 10/10/2016

1. Compute the (normalized) degree centrality, (normalized) betweeness centrality, and (normalized) closeness centrality of all nodes in the following graph.



2. A k-regular undirected network is a network in which every vertex has degree k.
    a. Show that the vector $\mathbf{1} = (1, 1, 1, \ldots, 1)$ is an eigenvector of the adjacency matrix with eigenvalue k.
    b. By making use of the fact that eigenvectors are orthogonal, show that there is no other eigenvector that has all element positive. Discussion: The Perron-Frobenius theorem says that the eigenvector with the largest eigenvalue always has all elements non-negative, and hence the eigenvector 1 gives, by definition, the eigenvector centrality of our k-regular network and the centralities are the same for every vertex.
    c. Name a centrality measure that could give different centrality value for different vertices in a regular network. Give an example network to demonstrate that.

3. Consider an undirected (connected) tree of n vertices. Suppose that a particular vertex in the tree has degree k, so that its removal would divide the tree into k disjoint regions, and suppose that the sizes of those regions are $n_1, n_2, n_3, \dots, n_k$. Show that the unormalized betweeness centrality x of the vertex is

$$x = n^2 - \sum_{i=1}^{k} n_i^2$$

4. Calculate the (unormalized) closeness centrality of the i^{th} vertex from the end of a "line graph" of n vertices. Here a line graph on n vertices $1, 2, 3, \dots, n$ has exactly $n - 1$ edges that connect vertices i and $i + 1$, for $i = 1, \dots, n - 1$.

5. Write a program in your preferred programming language to compute *weighted* degree centrality of an *undirected* graph. The weighted degree centrality of a node v is the sum of all the weights on the edges that are incident to v. The program will read the graph from a file called "graph.txt" and output the degree centrality of nodes to a file called "wdegree.txt".

   The file "graph.txt" includes multiples lines in which the first line contains two integers n and m that correspond to the number of nodes and edges in the graph. Each of the following m lines contain three integers u, v, and w, separated by one space, to denote an edge from u to v of weight w. Nodes are numbered from 1 to n.

   The output file "degree.txt" contains exactly n lines in which the k^{th} line is the (unnormalized) weighted degree centrality of node k.

   Sample input/output:

graph.txt	**wdegree.txt**
3 2 1 2 4 1 3 7	11 4 7

   Your submission must include

   - The source file(s)
   - The sample input/output
   - A README file that describes the compile and running instruction