CMSC 412: HW #1

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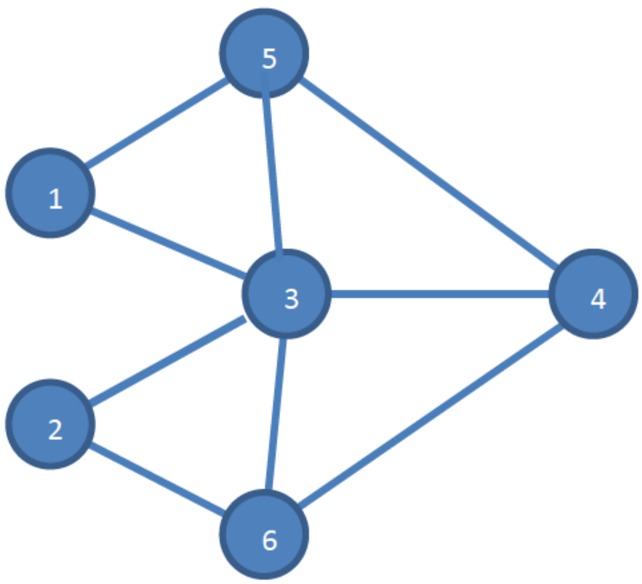
1. Compute the (normalized degree centrality, (normalized) betweenness centrality, and (normalized) closeness centrality of all nodes in the following graph.



Normalized Degree Centrality:

Node	# Edges	Weight
1	2	0.4
2	2	0.4
3	5	1
4	3	0.6
5	3	0.6
6	3	0.6

Normalized Betweenness Centrality:

Node i	Nodes (j,k)	$\delta_{j,k}$	$\delta_{j,k}(i)$	$\frac{\delta_{j,k}}{\delta_{j,k}(i)}$	Node i	Nodes (j,k)	$\delta_{j,k}$	$\delta_{j,k}(i)$	$\frac{\delta_{j,k}}{\delta_{j,k}(i)}$
1	(2, 3)	1	0	0	2	(1, 3)	1	0	0
1	(2, 4)	2	0	0	2	(1, 4)	2	0	0
1	(2, 5)	1	0	0	2	(1, 5)	1	0	0
1	(2, 6)	1	0	0	2	(1, 6)	1	0	0
1	(3, 4)	1	0	0	2	(3, 4)	1	0	0
1	(3, 5)	1	0	0	2	(3, 5)	1	0	0
1	(3, 6)	1	0	0	2	(3, 6)	1	0	0
1	(4, 5)	1	0	0	2	(4, 5)	1	0	0
1	(4, 6)	1	0	0	2	(4, 6)	1	0	0
1	(5, 6)	1	0	0	2	(5, 6)	1	0	0

Node 1: 0 Node 2: 0

Node i	Nodes (j,k)	$\delta_{j,k}$	$\delta_{j,k}(i)$	$\frac{\delta_{j,k}}{\delta_{j,k}(i)}$	Node i	Nodes (j,k)	$\delta_{j,k}$	$\delta_{j,k}(i)$	$\frac{\delta_{j,k}}{\delta_{j,k}(i)}$
3	(1, 2)	1	1	1	4	(1, 2)	1	0	0
3	(1, 4)	2	1	0.5	4	(1, 3)	1	0	0
3	(1, 5)	1	0	0	4	(1, 5)	1	0	0
3	(1, 6)	1	1	1	4	(1, 6)	1	0	0
3	(2, 4)	2	1	0.5	4	(2, 3)	1	0	0
3	(2, 5)	1	1	1	4	(2, 5)	1	0	0
3	(2, 6)	1	0	0	4	(2, 6)	1	0	0
3	(4, 5)	1	0	0	4	(3, 5)	1	0	0
3	(4, 6)	1	0	0	4	(3, 6)	1	0	0
3	(5, 6)	2	1	0.5	4	(5, 6)	2	1	0.5

Node 3: $\frac{4.5}{10} = 0.45$ Node 4: $\frac{0.5}{10} = .05$

Node i	Nodes (j,k)	$\delta_{j,k}$	$\delta_{j,k}(i)$	$\frac{\delta_{j,k}}{\delta_{j,k}(i)}$	Node i	Nodes (j,k)	$\delta_{j,k}$	$\delta_{j,k}(i)$	$\frac{\delta_{j,k}}{\delta_{j,k}(i)}$
5	(1, 2)	1	0	0	6	(1, 2)	1	0	0
5	(1, 3)	1	0	0	6	(1, 3)	1	0	0
5	(1, 4)	2	1	0.5	6	(1, 4)	2	0	0
5	(1, 6)	1	0	0	6	(1, 5)	1	0	0
5	(2, 3)	1	0	0	6	(2, 3)	1	0	0
5	(2, 4)	2	0	0	6	(2, 4)	2	1	0.5
5	(2, 6)	1	0	0	6	(2, 5)	1	0	0
5	(3, 4)	1	0	0	6	(3, 4)	1	0	0
5	(3, 6)	1	0	0	6	(3, 5)	1	0	0
5	(4, 6)	1	0	0	6	(4, 5)	1	0	0

Node 5: $\frac{0.5}{10} = .05$ Node 6: $\frac{0.5}{10} = .05$

Normalized Closeness Centrality:

Node	Sum of shortest Paths	C_c
1	8	0.75
2	8	0.75
3	5	1
4	7	0.71
5	7	0.71
6	7	0.71

2. A k-regular undirected network is a network in which every vertex has degree k.

   (a) Show that the vector 1 = (1, 1, 1, ..., 1) is an eigenvector of the adjacency matrix with eigenvalue k.

   Suppose G is a k-regular, undirected network. Let A be the n x n adjacency matrix of G. By matrix multiplication, each row of eigenvector x will give the sum of row k since each matrix has degree k, where $\lambda \cdot x$ is simply k, thus $\lambda = k$

   $$Ax = \lambda x$$

   (b) By making use of the fact that eigenvectors are orthogonal, show that there is no other eigenvector that has all elements positive. Discussion: The Perron-Frobenius theorem says that the eigenvector with the largest eigenvalue always has all elements non-negative, and hence the eigenvector 1 gives, by definition, the eigenvector centrality of our k-regular network and the centralities are the same for every vertex.

Since eigenvectors are orthogonal, any other eigenvector $y = (y_1, y_2, ..., y_n)$, times the adjacency matrix will have k the largest, non-negative eigenvalue of A by the Perron-Frobenis theorem.

(c) Name a centrality measure that could give a different centrality value for different vertices in a regular network. Give an example network to demonstrate that.

Katz Centrality of G is given by vector

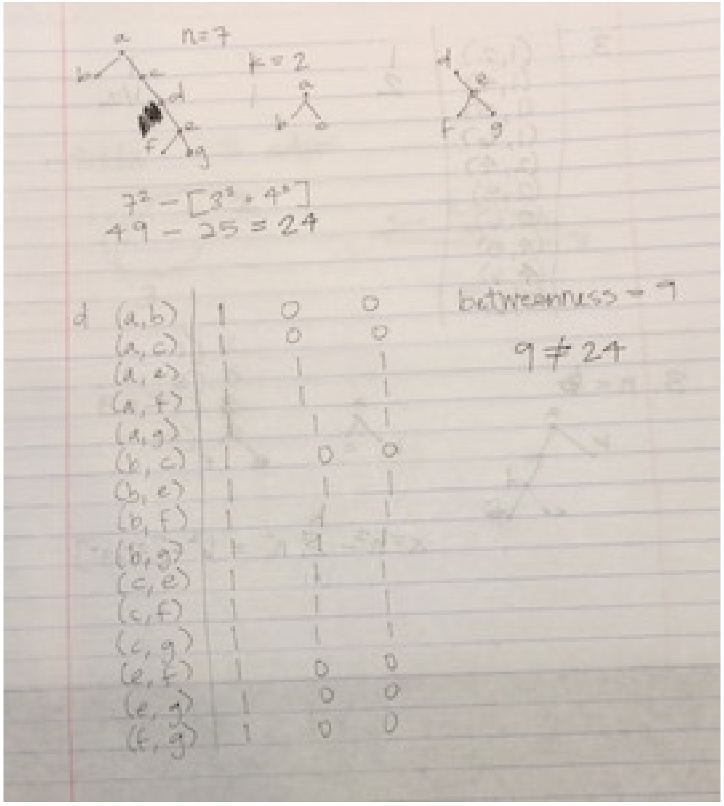
$$\vec{v} = (I \ - \ \alpha A)^{-1} \ 1$$

where $\alpha > 0$ and $\alpha \neq \frac{1}{k}$

3. Consider an undirected (connected) tree of n vertices. Suppose that a particular vertex in the tree has degree k, so that its removal would divide the tree into k disjoint regions, and suppose that the sizes of those regions are $n_1, n_2, n_3, ..., n_k$. Show that the un-normalized betweenness centrality x of the vertex is

$$x \ = n^2 \ - \ \sum_{i=1}^{k} n_i^2$$

By definition, the un-normalized C_b = the number of shortest paths between all other nodes in the network that go through node x. By removing some vertex x with degree k, we can create k disjoint trees. Each of these sub-trees with nodes i and j on the same region no longer have shortest paths through x anymore and the above equation does not hold. See image below for counter-example:



4. Calculate the (un-normalized) closeness centrality of the i^{th} vertex from the end of a "line graph" of n vertices. Here, a line graph on n vertices 1, 2, 3, ... , n has exactly $n-1$ edges that connect vertices i and $i+1$, for $i = 1, ..., n-1$.

$$C_i = [\frac{i(i-1) + (n-i)(n+i+1)}{2}]^{-1}$$

5. Program in UndirectedGraph.java using graph.txt as input arguments, and outputs to file wdegree.txt