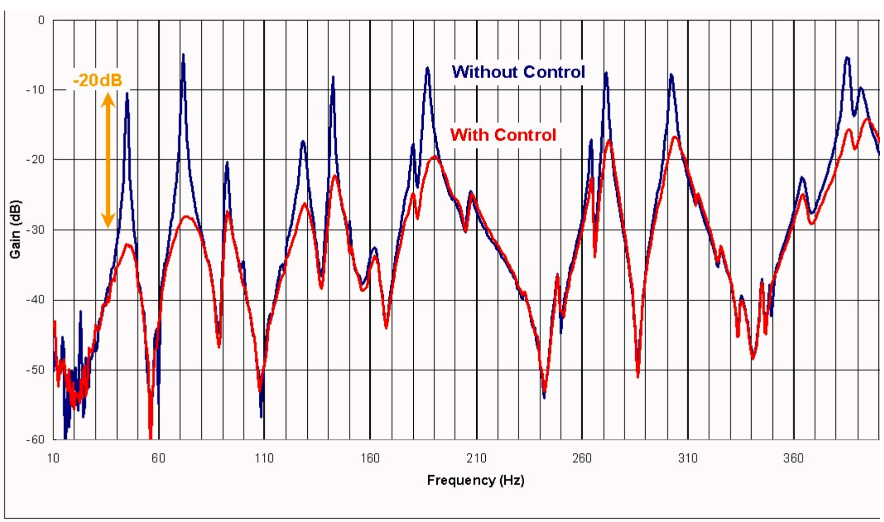
Advanced Dynamics Course

Vibration Isolation



Dr. Hassan HosseinNia

Is there any reference related to the topic?

Please download the part 10.6 of the book:
"Mechatronic Systems Design Methods, Models, Concepts"
Translation by Kristof Richmond

What is required before attending the class?

1. Study the Nichols plot to depict the frequency domain response:
https://en.wikipedia.org/wiki/Nichols_plot
2. Frequency domain stability analysis of the system:
Please read chapter 10.4, until page 651 (up to Elastic eigenmodes).

Do we need to bring computer?

Yes, we will practice to implement one example in Matlab. Please install Matlab in your laptop.

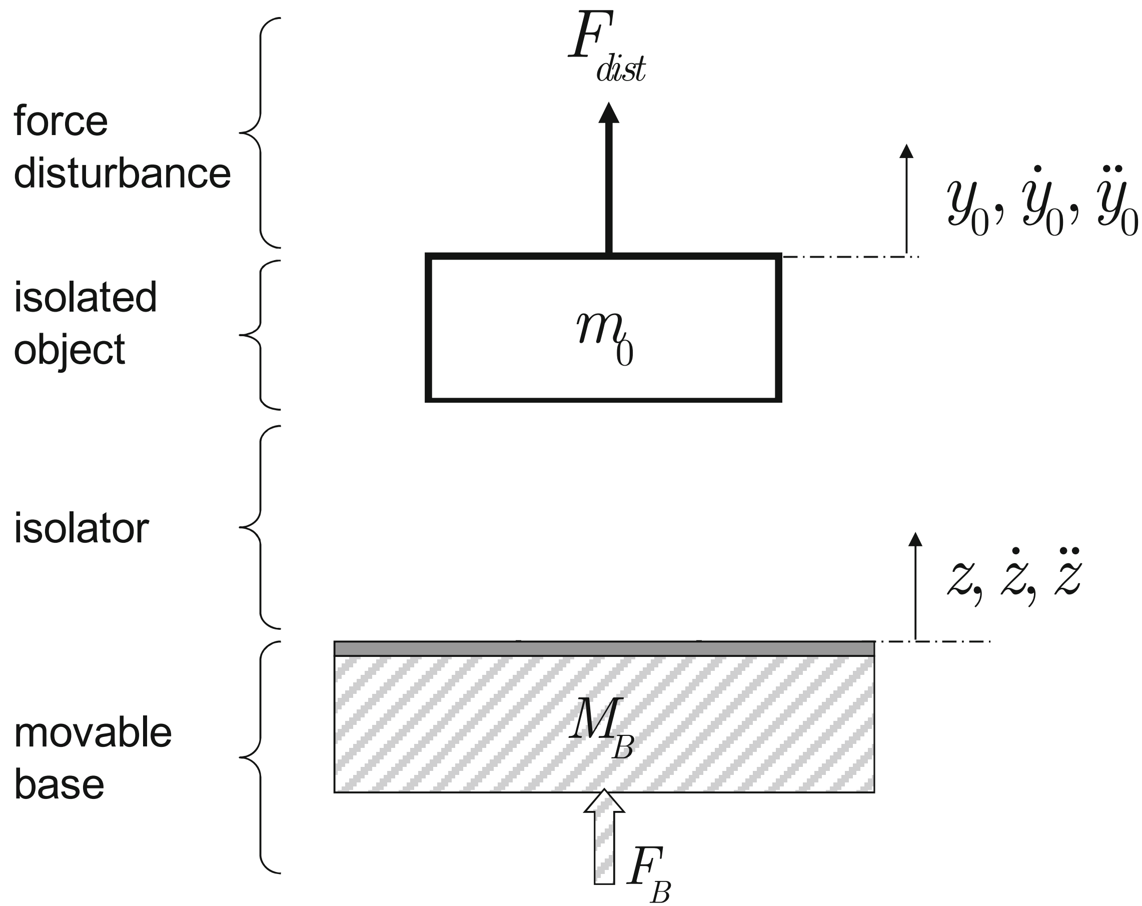
Contents

1. Passive vibration isolation and skyhook principal

2. Active Vibration isolation
    • Velocity feedback
    • Acceleration feedback

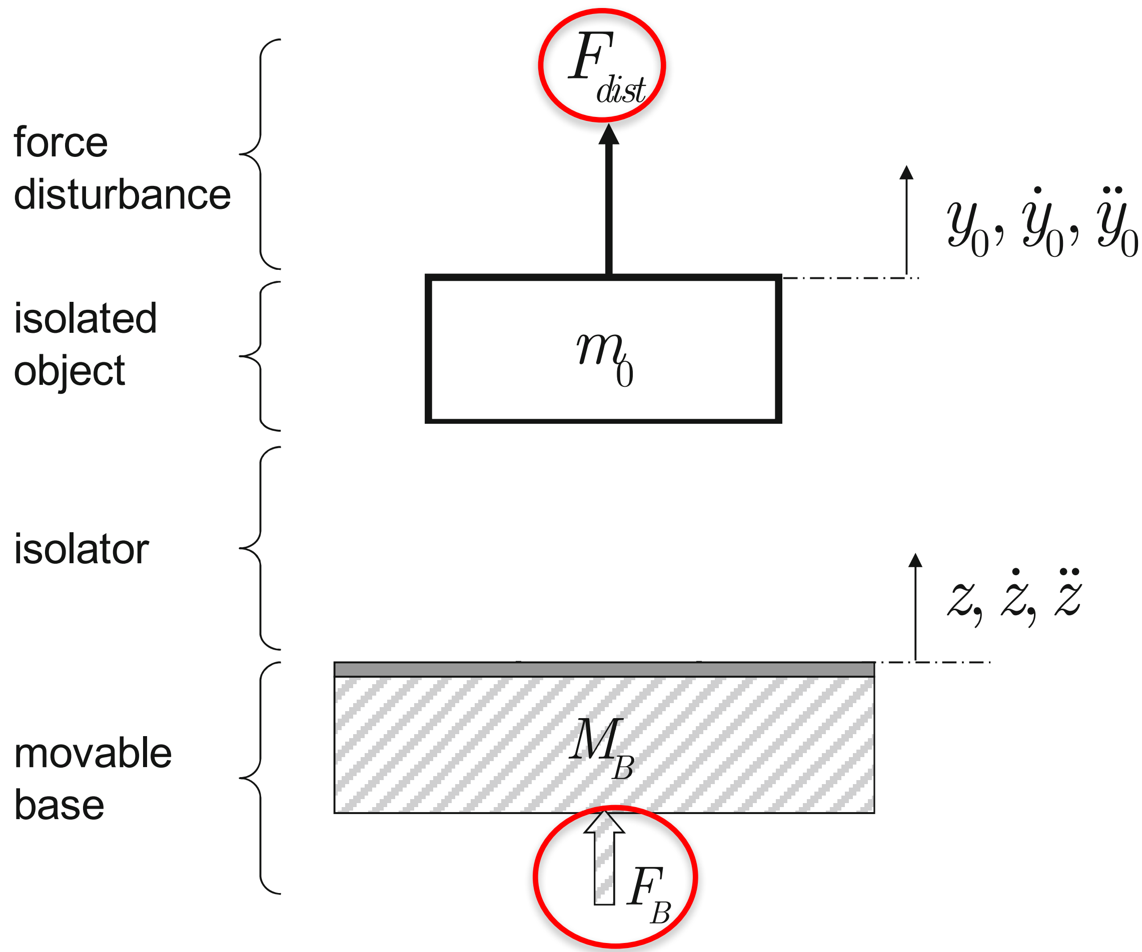
3. Practical example

Passive Vibration Isolation



Passive vibration isolator

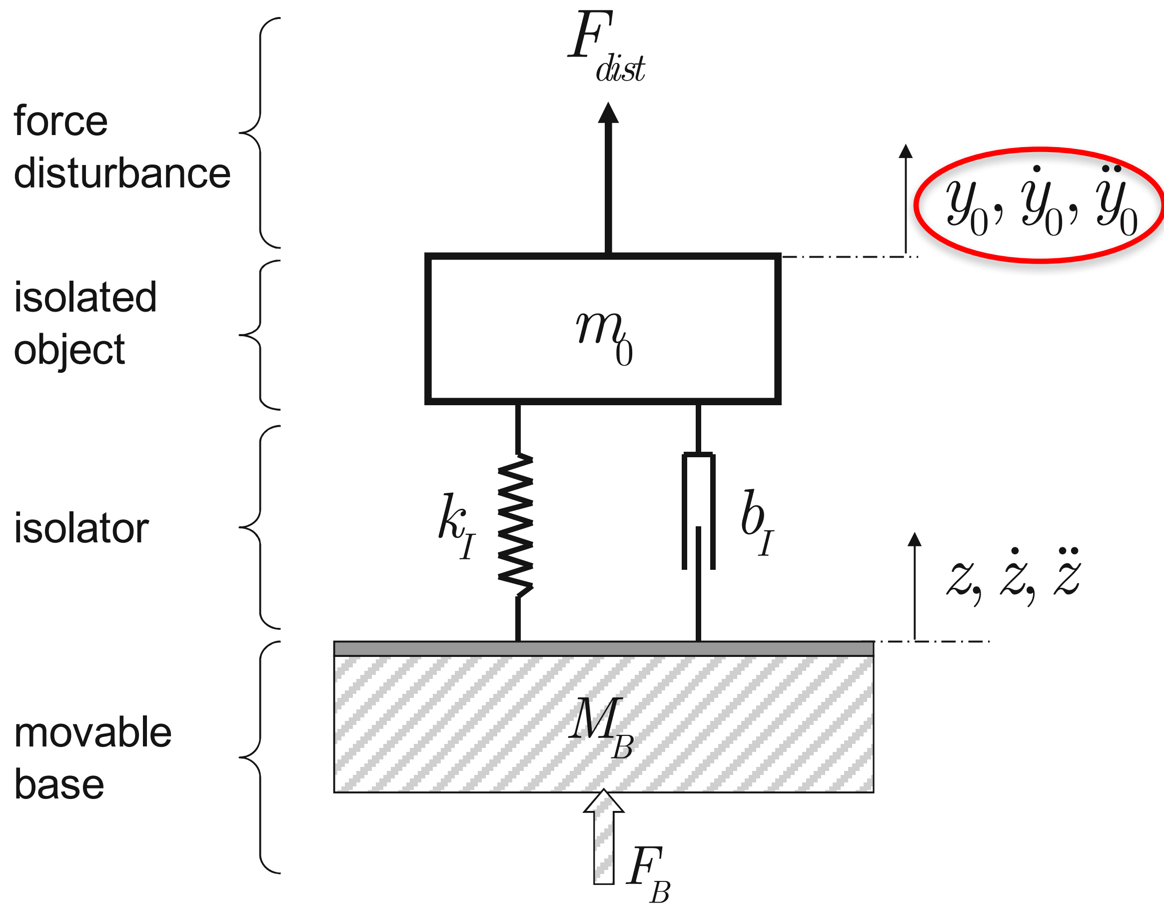
Passive Vibration Isolation



Passive vibration isolator

Passive Vibration Isolation

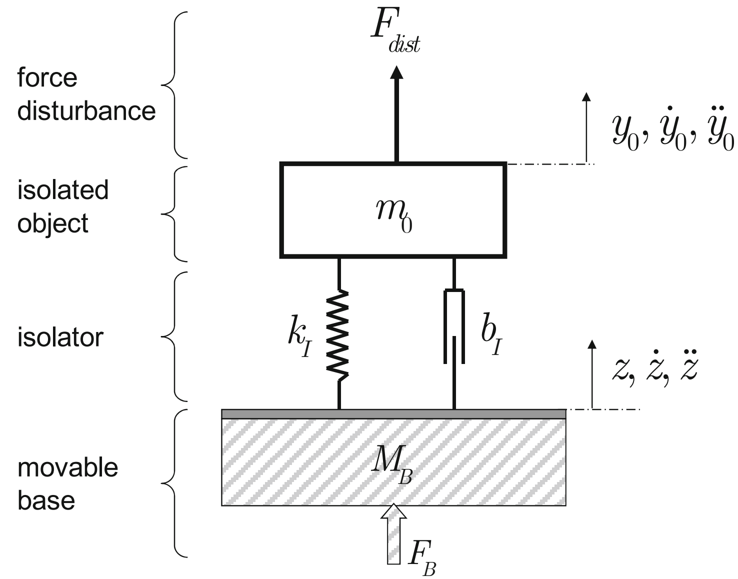
In both cases, the object **position** y_0 and its time **derivatives** $\dot{y}_0$, $\ddot{y}_0$ are to be affected as little as possible by the disturbance sources (for example, acceleration is highly relevant for the ride comfort of an automobile).



Passive vibration isolator

Passive Vibration Isolation

$$T_{isol}(s) = \frac{L\{y_0(t)\}}{L\{z(t)\}} = \frac{L\{\dot{y}_0(t)\}}{L\{\dot{z}(t)\}} = \frac{L\{\ddot{y}_0(t)\}}{L\{\ddot{z}(t)\}}$$



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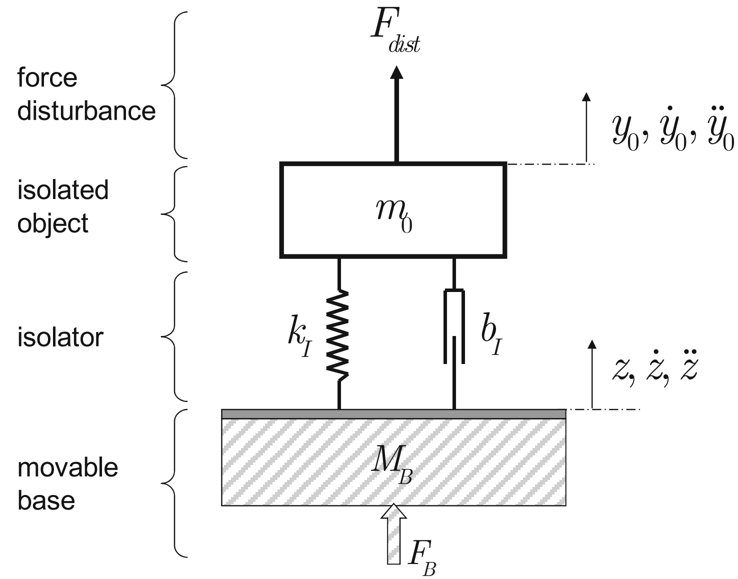
$$T_{isol}(s) = \frac{k_I + b_I s}{k_I + b_I s + m_0 s^2} = \frac{1 + 2d_0 \dfrac{s}{\omega_0}}{1 + 2d_0 \dfrac{s}{\omega_0} + \dfrac{s^2}{\omega_0{}^2}},$$

$$\text{where } \omega_0 = \sqrt{\frac{k_I}{m_0}}, \quad \frac{2d_0}{\omega_0} = \frac{b_I}{k_I}.$$



Passive vibration isolator

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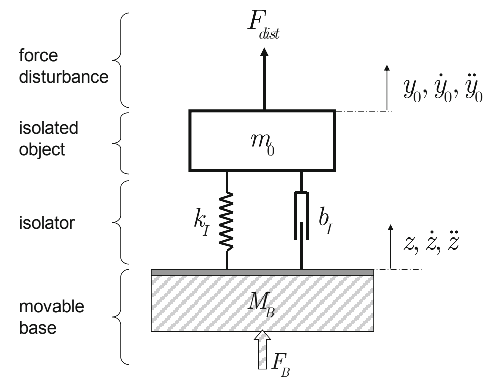
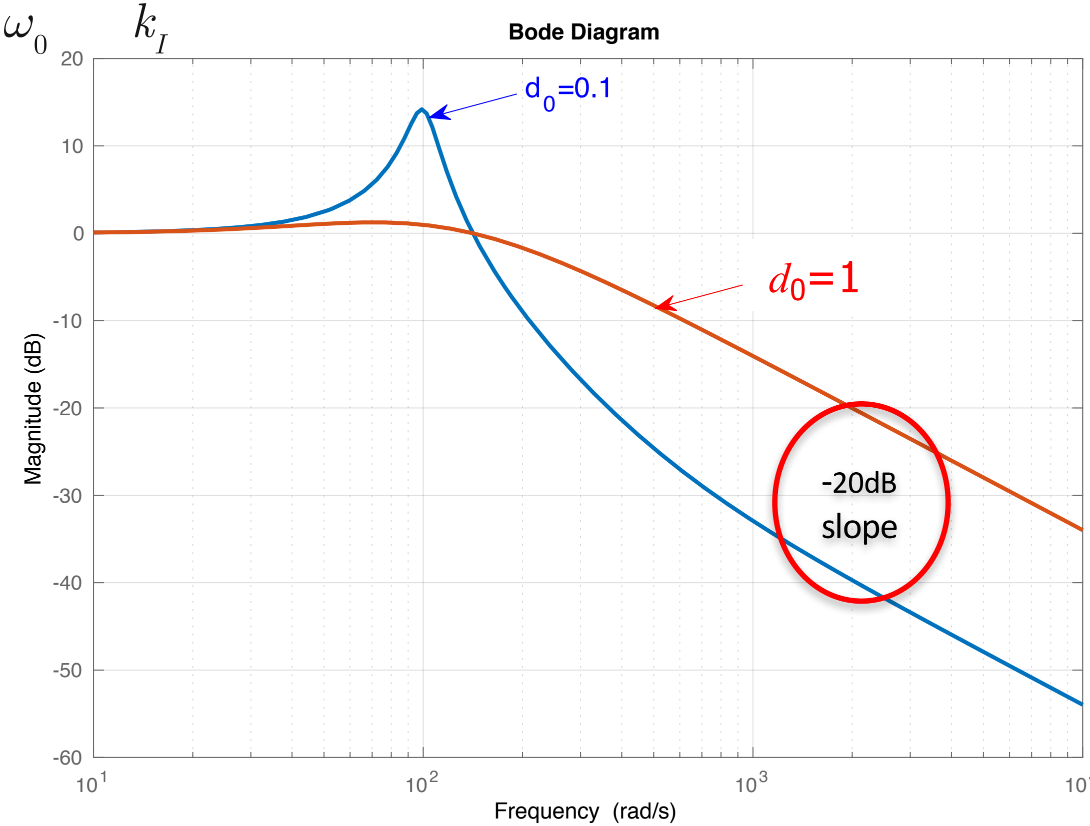


Passive vibration isolator

$$\text{where } \omega_0 = \sqrt{\frac{k_I}{m_0}}, \quad \frac{2d_0}{\omega_0} = \frac{b_I}{k_I} .$$



Passive Vibration Isolation

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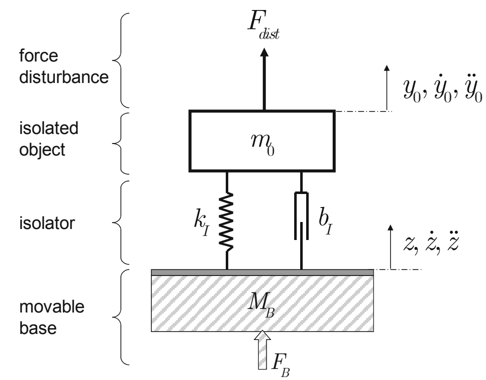

Passive vibration isolator

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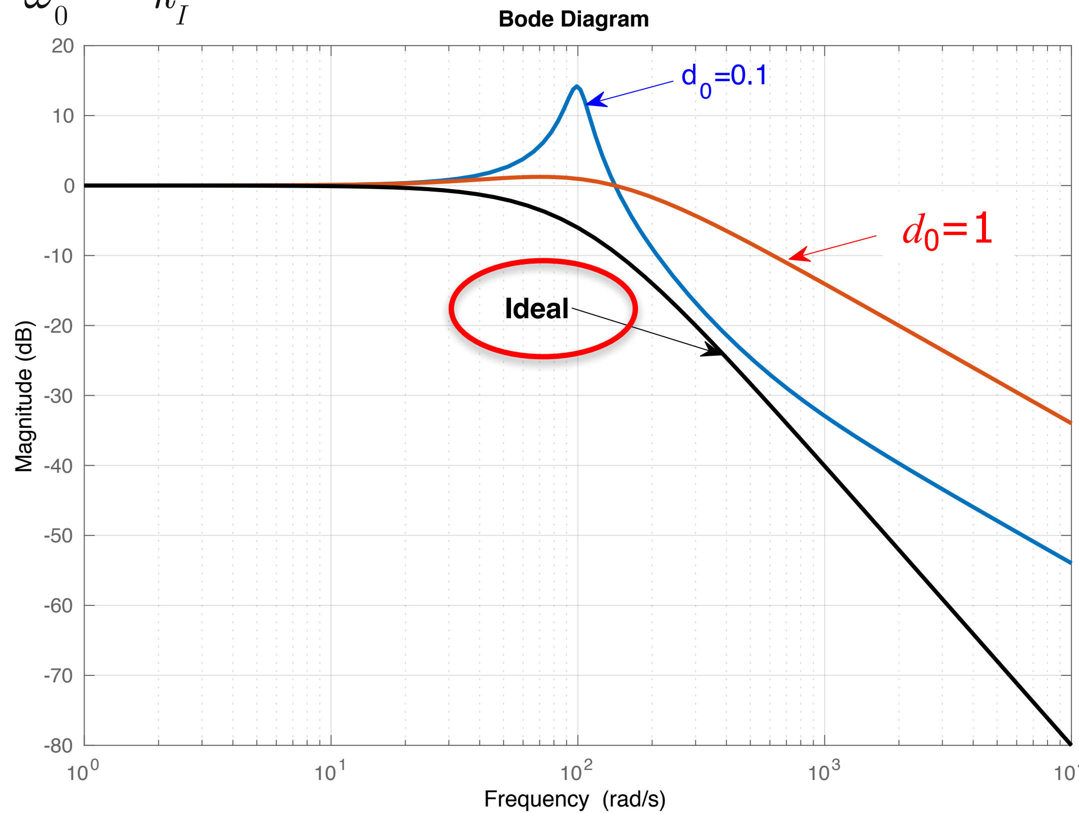
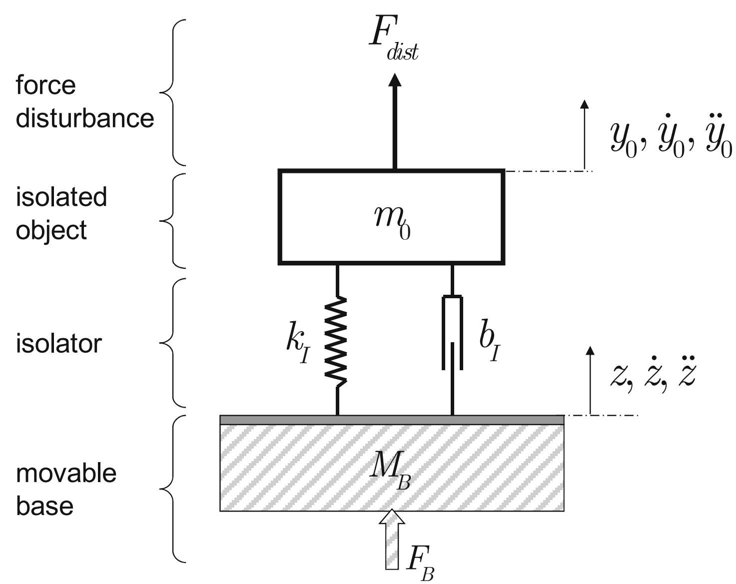


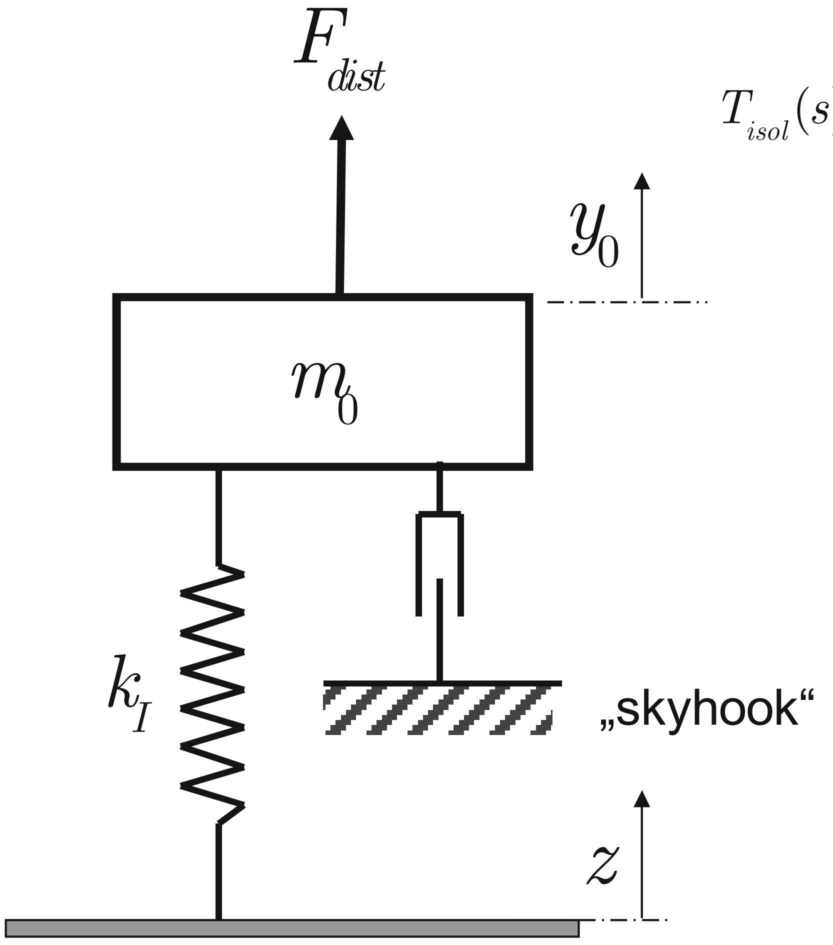
What is ideal mechanical solution?

$$T_{isol}(s) = \frac{k_I + b_I s}{k_I + b_I s + m_0 s^2} = \frac{1 + 2\cancel{d_0 \frac{s}{\omega_0}}}{1 + 2d_0 \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}},$$



Passive vibration isolator

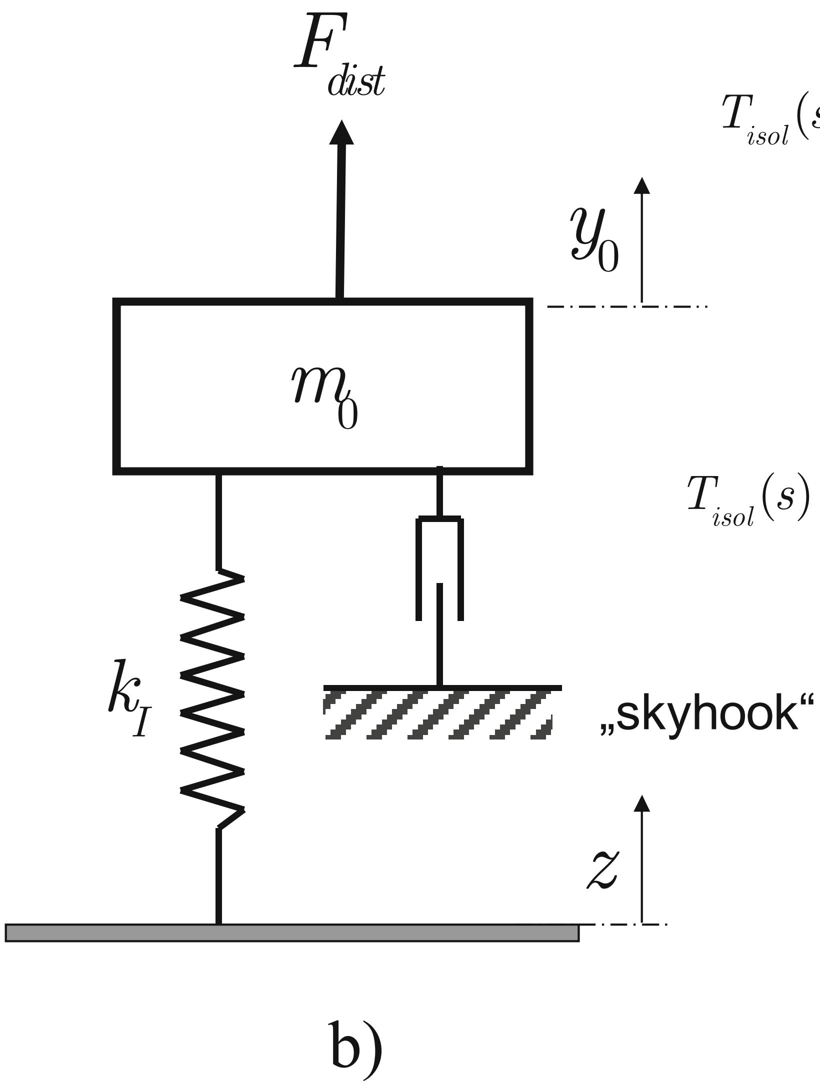
Active vibration isolation: the skyhook principle



b)

$$T_{isol}(s) = \frac{k_I + b_I s}{k_I + b_I s + m_0 s^2} = \frac{1 + 2d_0 \dfrac{s}{\omega_0}}{1 + 2d_0 \dfrac{s}{\omega_0} + \dfrac{s^2}{{\omega_0}^2}} \,,$$

Active vibration isolation: the skyhook principle



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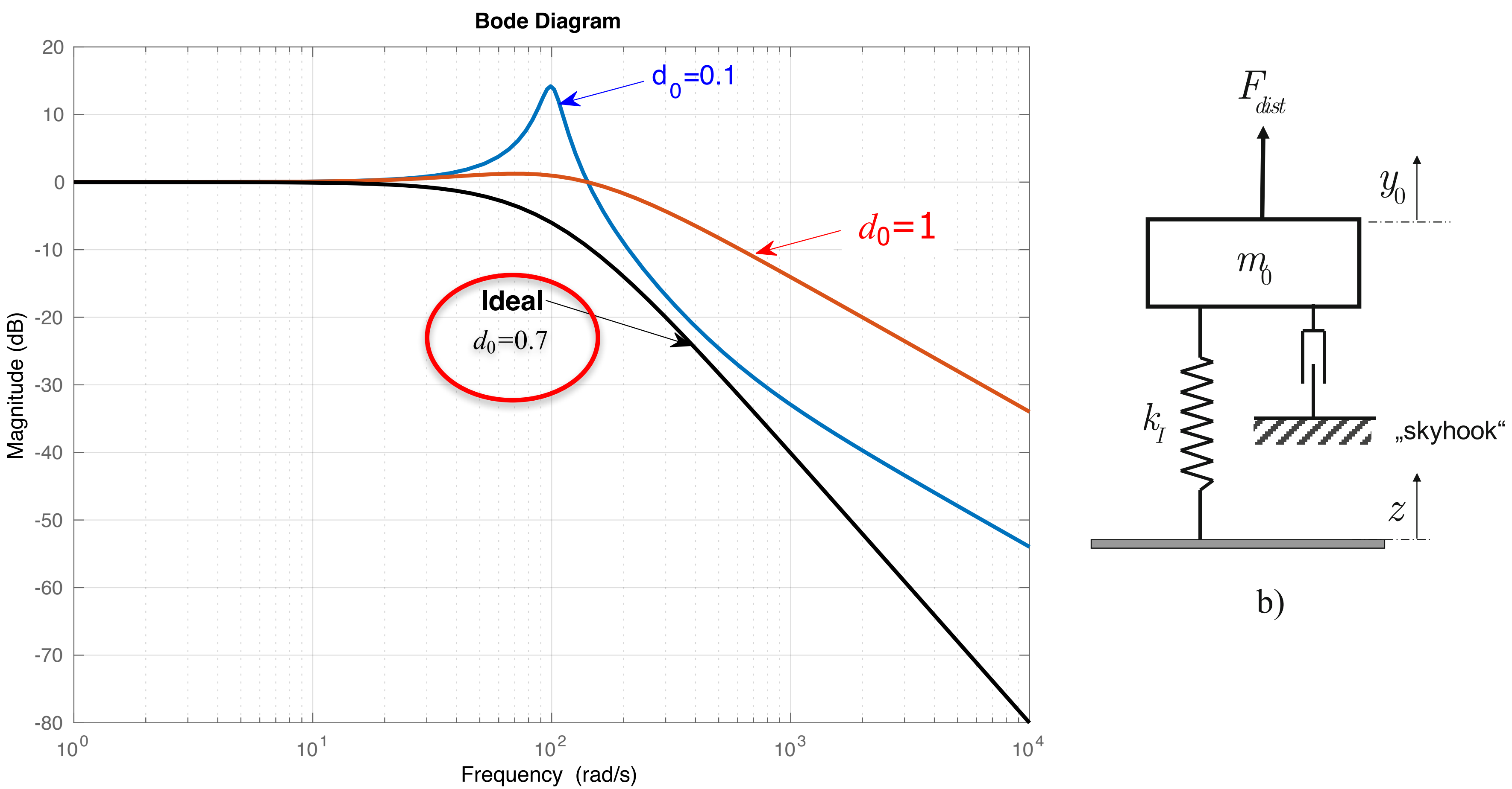
$$T_{isol}(s) = \frac{k_I}{k_I + b_I s + m_0 s^2} = \frac{1}{1 + 2d_0 \frac{s}{\omega_0} + \frac{s^2}{\omega_0{}^2}},$$

Active vibration isolation: the skyhook principle

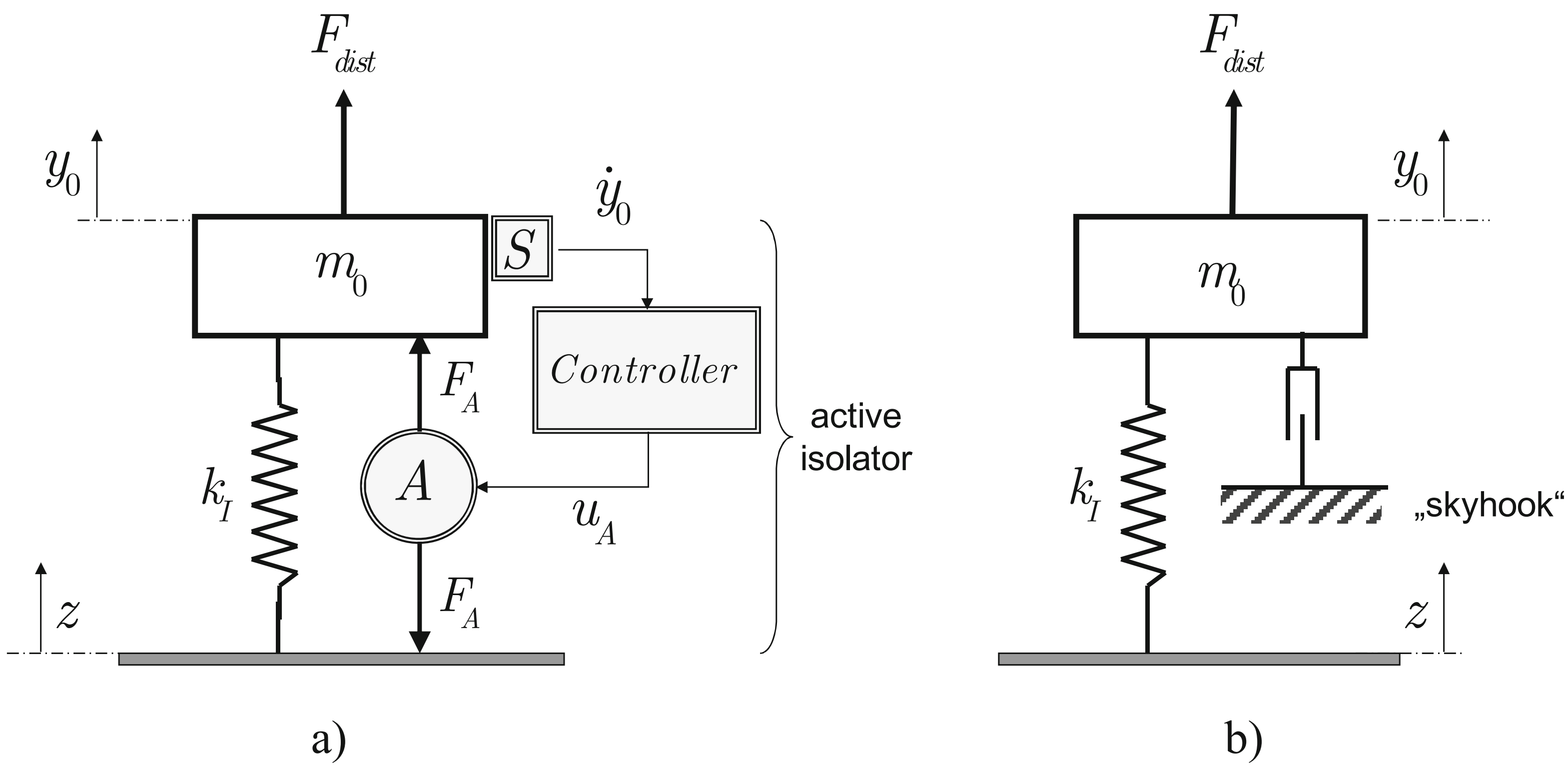


b)

The object is thus virtually "hooked" into inertial space (Fig. b) and damped as a function of absolute velocity

Active vibration isolation: the skyhook principle

- By measuring the inertial (absolute) velocity of the object to be isolated, a force proportional to it can be accurately generated in a suitable actuator (Fig a) and applied to the object.



What is the problem passive damping?

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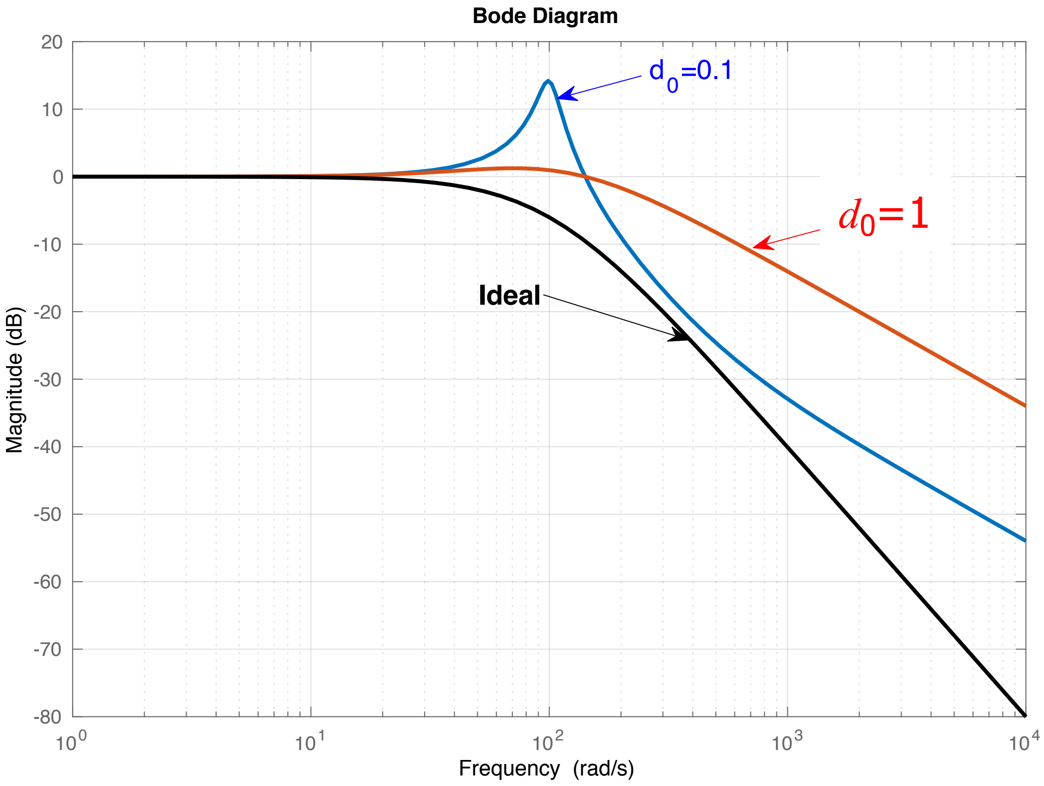


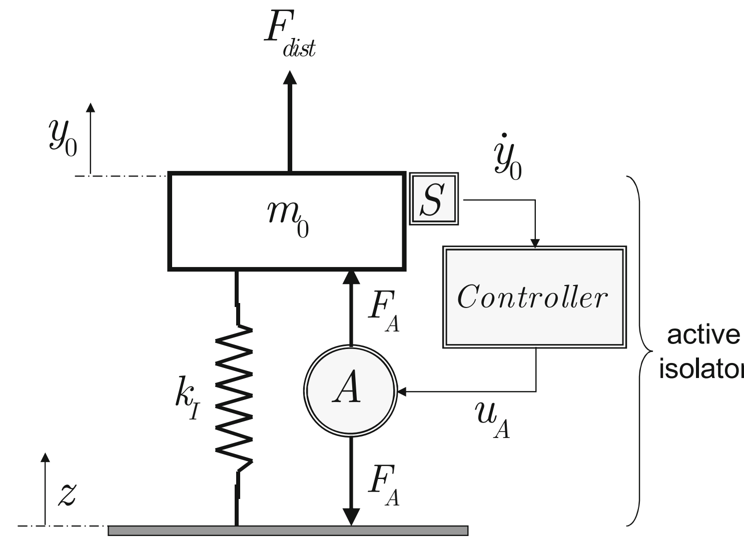
Active vibration isolation: Velocity feedback

The dynamic effect of velocity feedback for the skyhook arrangement:

MBS: $$Y_0(s) = \frac{1}{k_I + m_0 s^2}\left[F_{dist}(s) + F_A(s) + k_I Z(s)\right],$$

Actuator: $$F_A(s) = K_A \cdot u_A(s)$$

Controller: $$u_A(s) = -H(s) \cdot s Y_0(s)\,.$$



a)

Active vibration isolation: Velocity feedback

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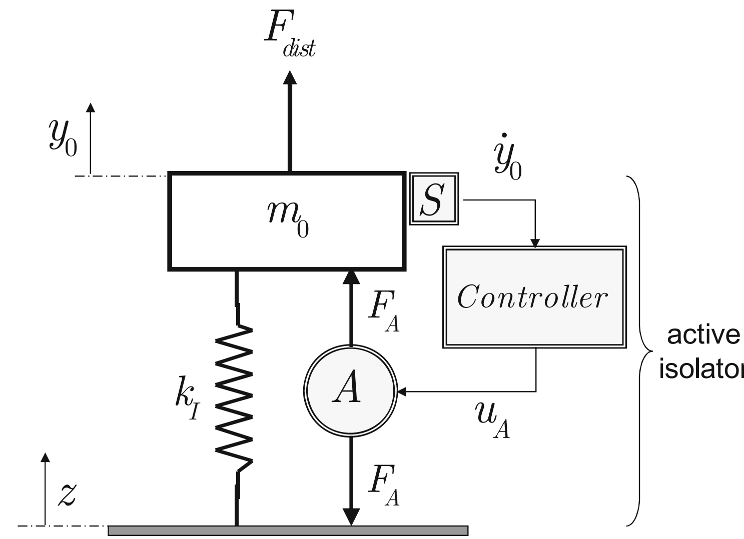

a)

For purely proportional feedback of the absolute velocity:

$$H(s) = K_H$$

Active vibration isolation: Ideal Velocity feedback

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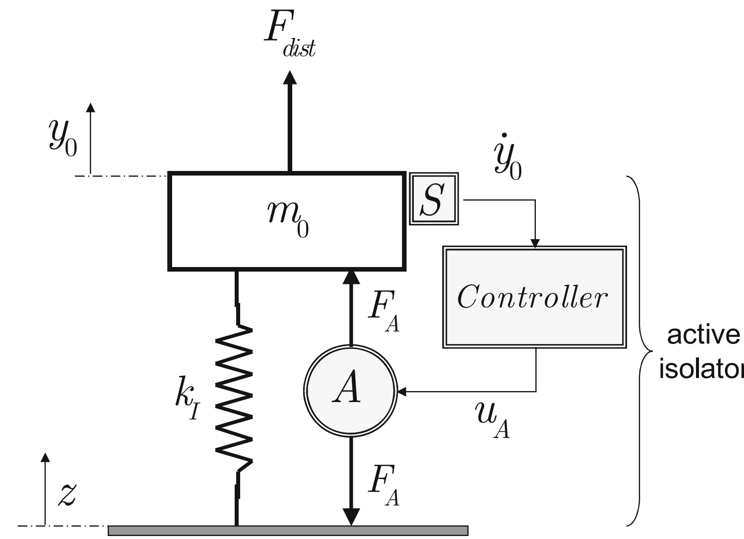


a)

For purely proportional feedback of the absolute velocity:

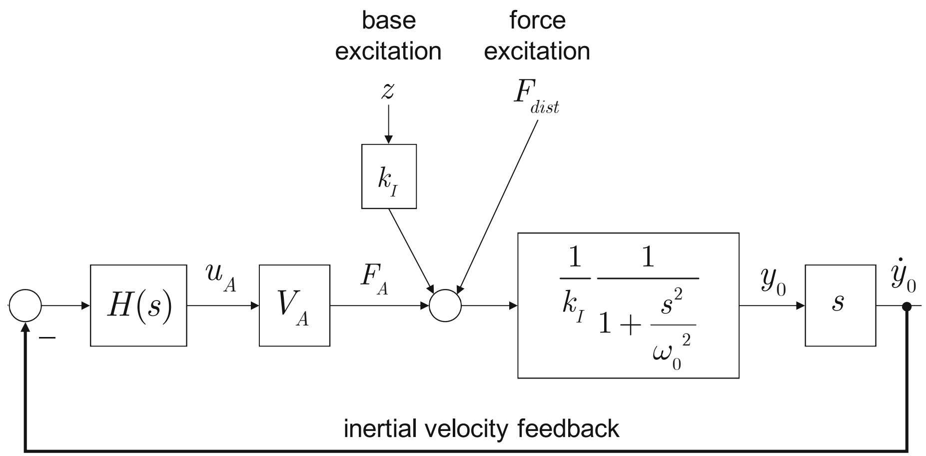
$$H(s) = K_H$$

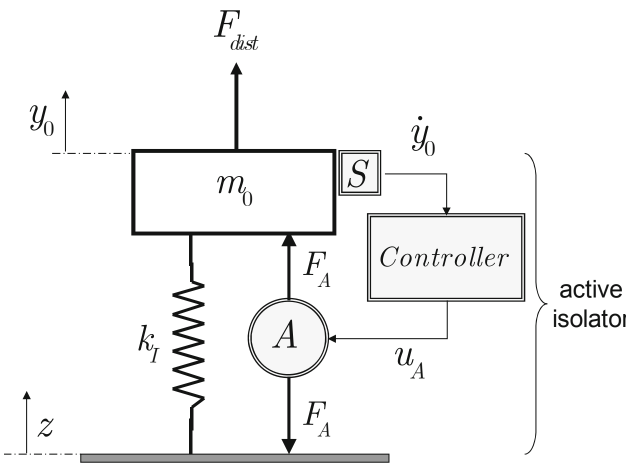
This gives for the closed control loop:

$$Y_0(s) = \frac{1}{\{\tilde{d}_0;\omega_0\}} Z(s) \quad + \quad \frac{1}{k_I}\frac{1}{\{\tilde{d}_0;\omega_0\}} F_{dist}(s)\ ,$$

$$\text{where}\ \ \omega_0 = \sqrt{\frac{k_I}{m_0}},\quad \tilde{d}_0 = \frac{\omega_0}{2k_I} K_A K_H \ \Rightarrow\ \mathrm{K}_H = \tilde{d}_0 \frac{2k_I}{\omega_0 K_A}\,.$$

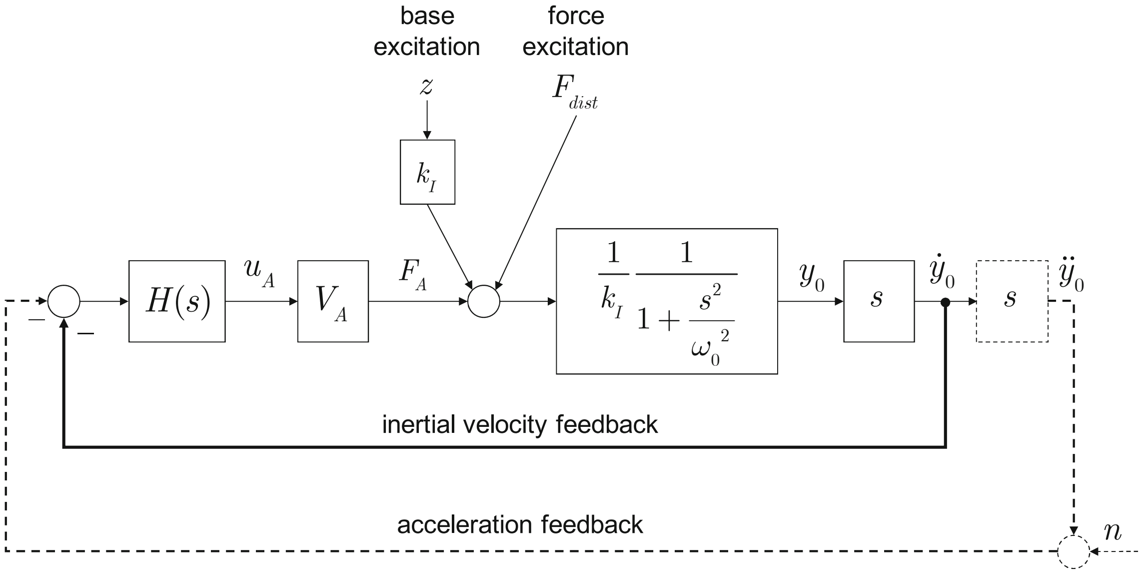
Active vibration isolation: Control loop for Velocity feedback

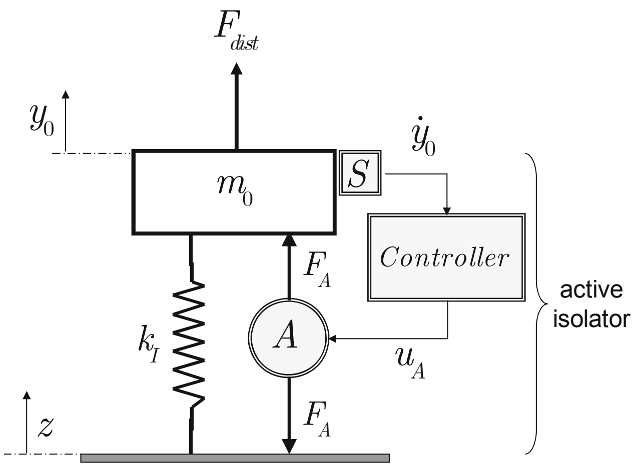




a)

Active vibration isolation: Control loop for Velocity feedback

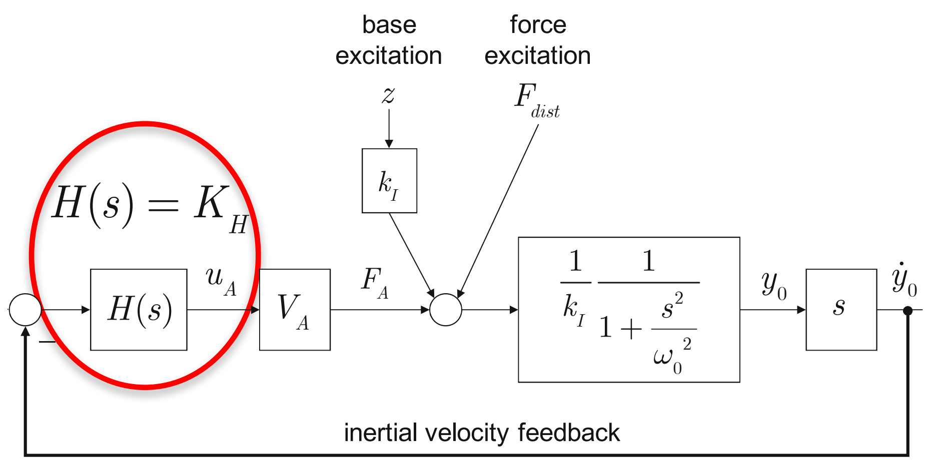


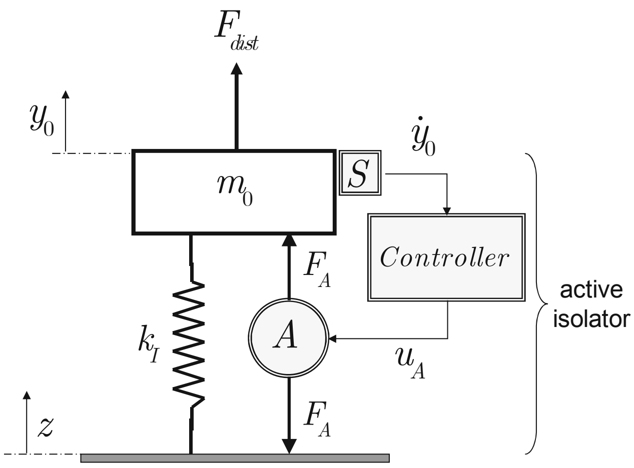


a)

A critical aspect for implementation is the physical **measurement of absolute velocity**, which requires so-called ***inertial sensors***.

Active vibration isolation: Control loop for Velocity feedback





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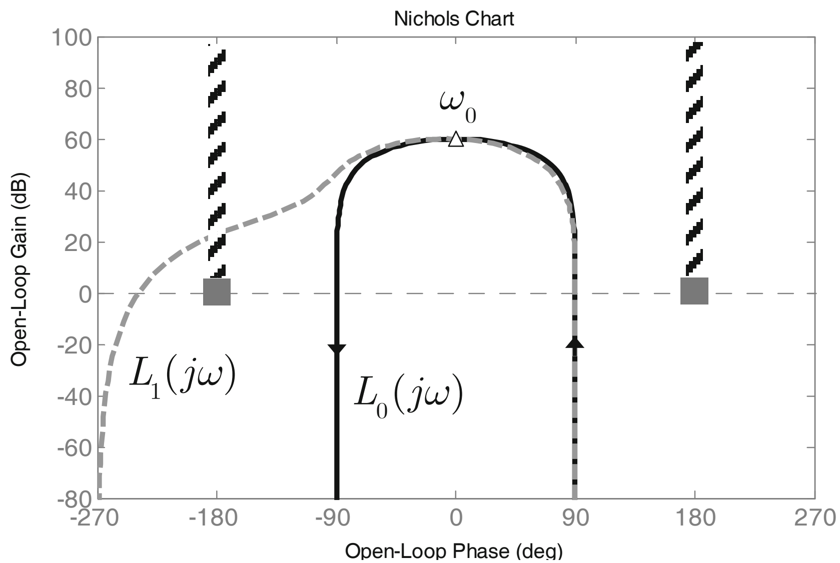
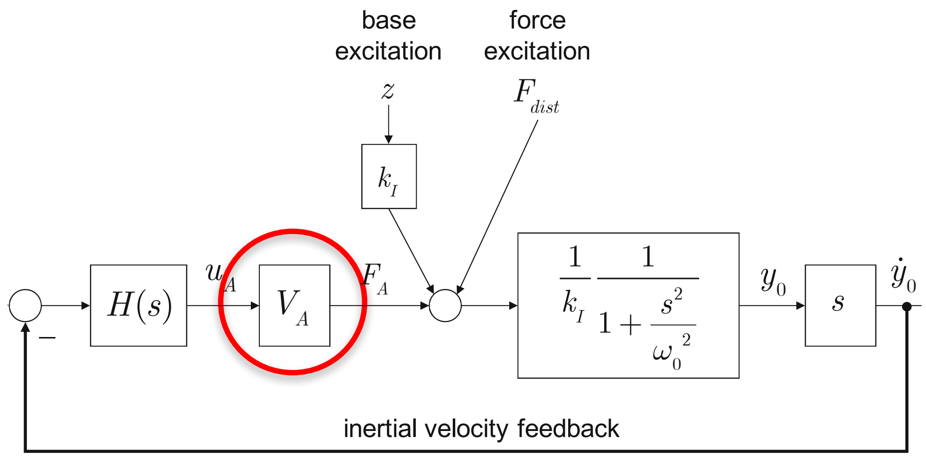
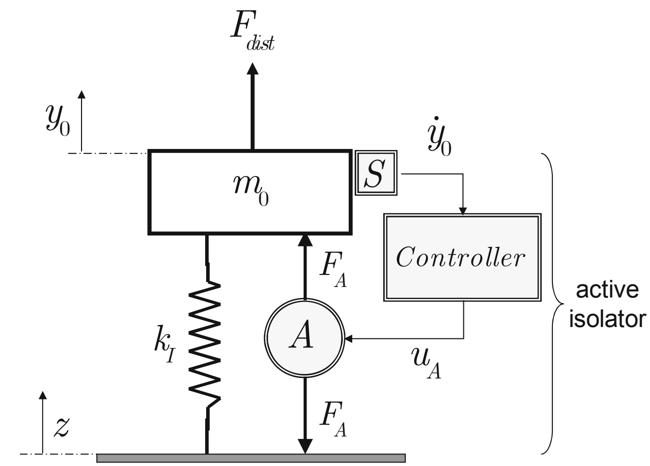
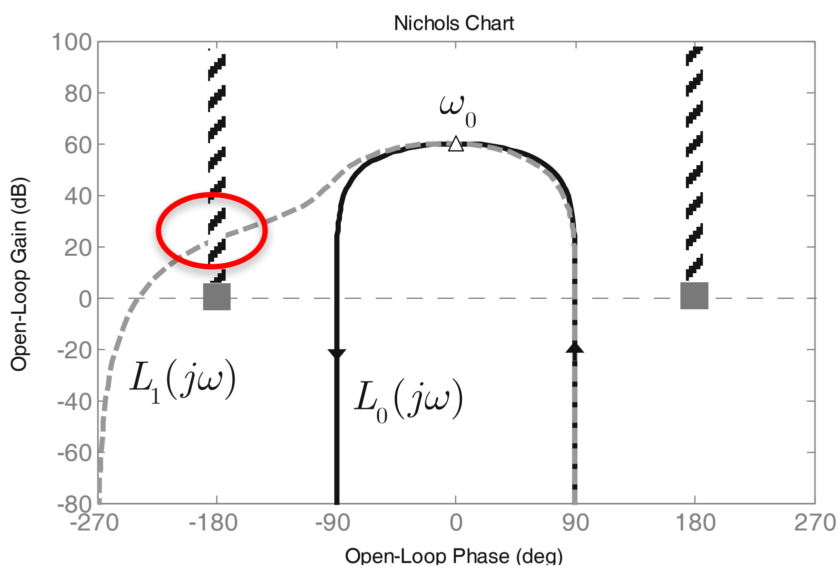


Active vibration isolation: Control loop for Velocity feedback





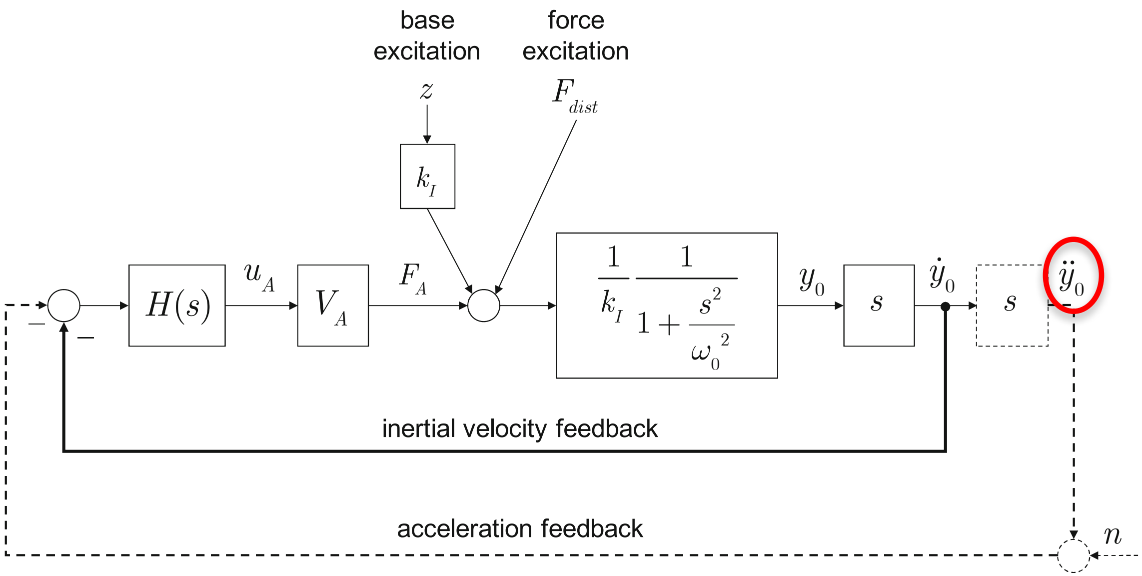
a)

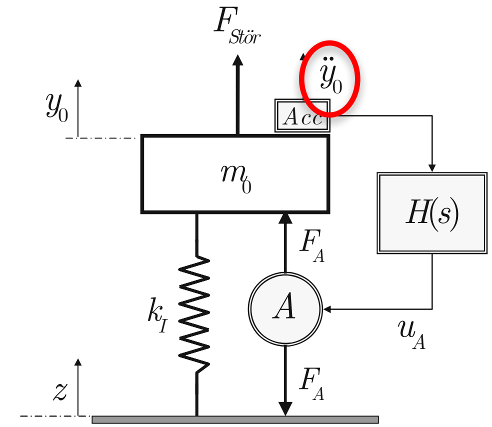


System become unstable including parasitic dynamic (dotted line). ☹

However, if additional parasitic dynamics (sensors, actuators, MBS structures) are considered, then stability problems and the limits of purely proportional feedback become clearly apparent. Thus, in such cases, a more complex controller design following the robust control strategies is required.

Active vibration isolation: Acceleration feedback

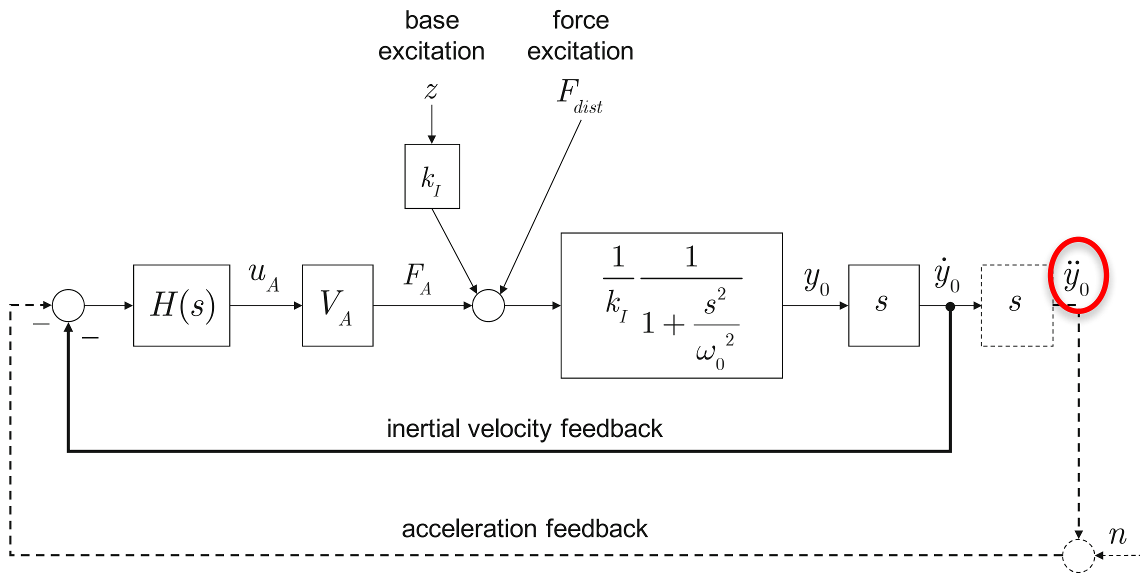


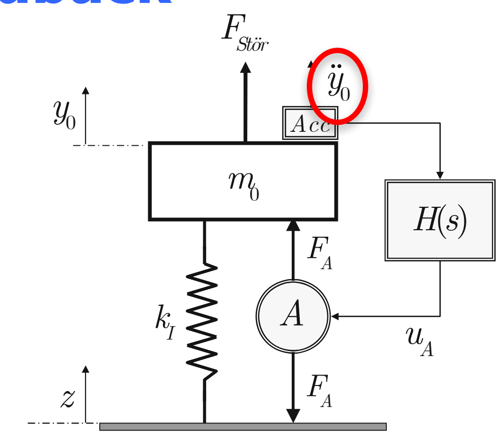


Velocity reconstruction: acceleration feedback

$$H(s) = \frac{K_H}{s}$$

Active vibration isolation: Acceleration feedback





Velocity reconstruction: acceleration feedback

$$H(s) = \frac{K_H}{s}$$

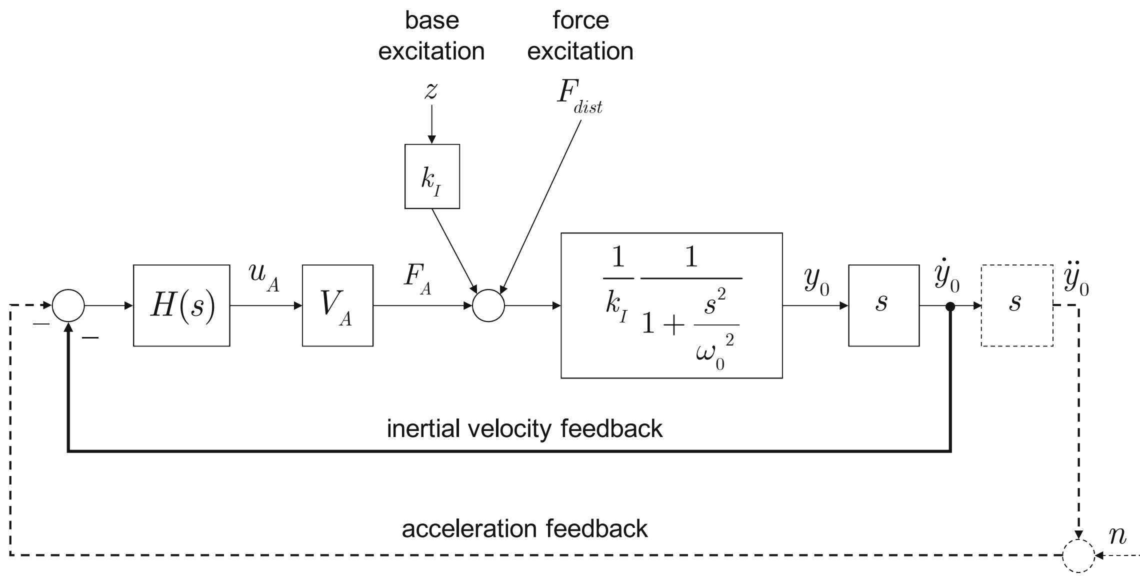
Upon closer inspection, this solution proves to be highly problematic

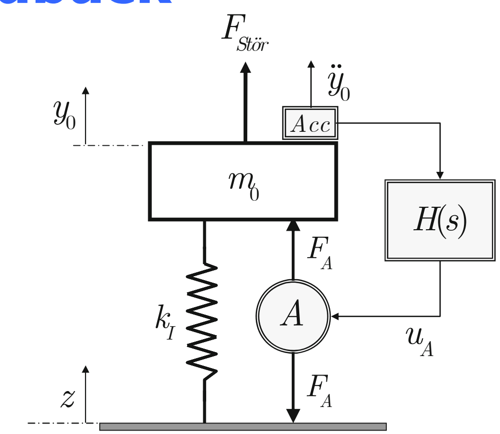
$$\Delta(s) = s\left(k + K_H s + m s^2\right)$$

an unstable root at the origin ☹

As can be easily verified, this root s=0 does not appear in the transfer functions $T_{y0/z}$, $T_{uA/z}$. The reason for this is that it cancels an equal term in the numerator (see observability problems in Sec. 10.7).
The stability problem only becomes apparent when considering the transfer functions $T_{uA/n}$ and $T_{y0/n}$ from the measurement noise n in the acceleration channel to the control variable u_A or the mass position y_0, where this unstable pole actually appears.

Active vibration isolation: Acceleration feedback



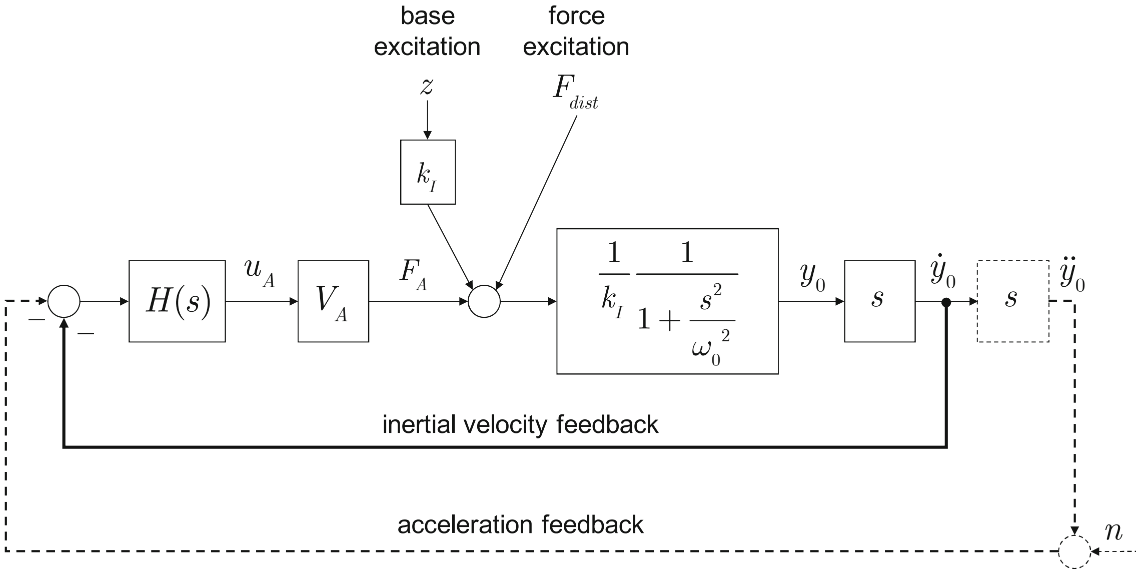


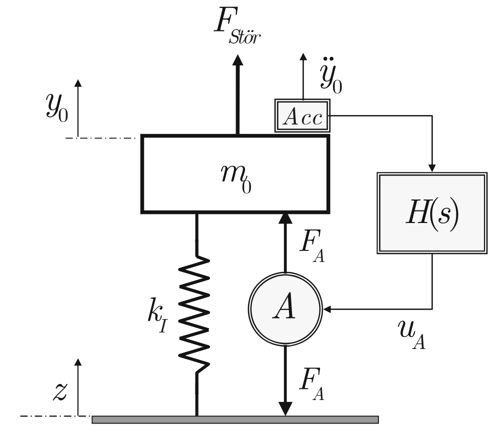
Stable acceleration feedback

One simple possibility to avoid this instability lies in the use of a high-pass filter at the output of the integrator to filter out constant sensor signal components:

$$H(s) = \frac{K_H}{s} \frac{\frac{s}{\omega_{HP}}}{1 + \frac{s}{\omega_{HP}}} = \frac{K_H}{\omega_{HP}} \frac{1}{1 + \frac{s}{\omega_{HP}}}.$$

Active vibration isolation: Acceleration feedback





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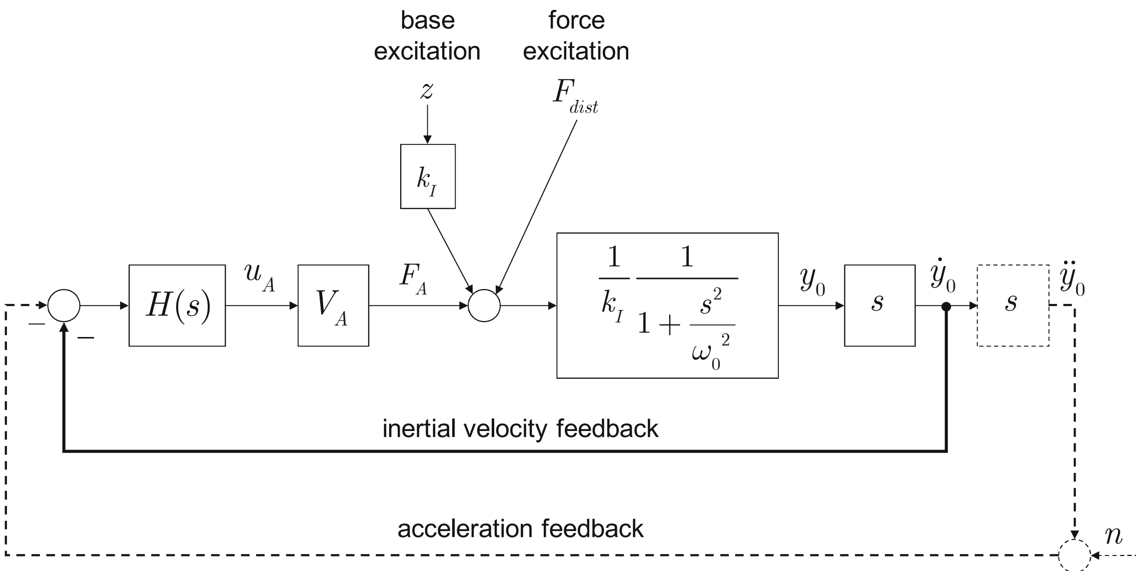
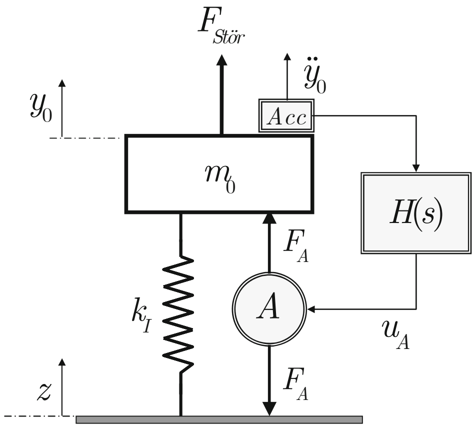
This series connection acts as a low-pass filter (LPF)

Active vibration isolation: Acceleration feedback





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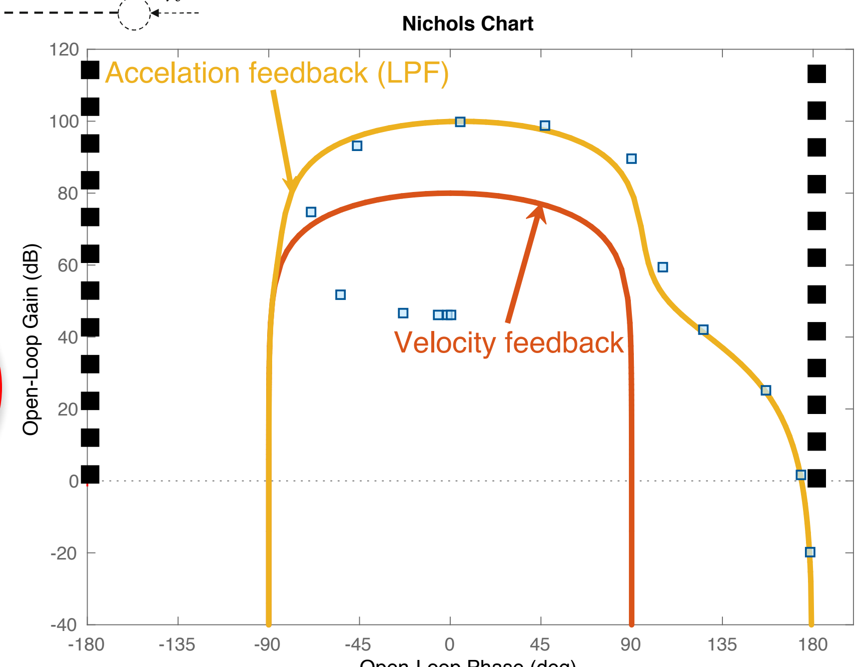
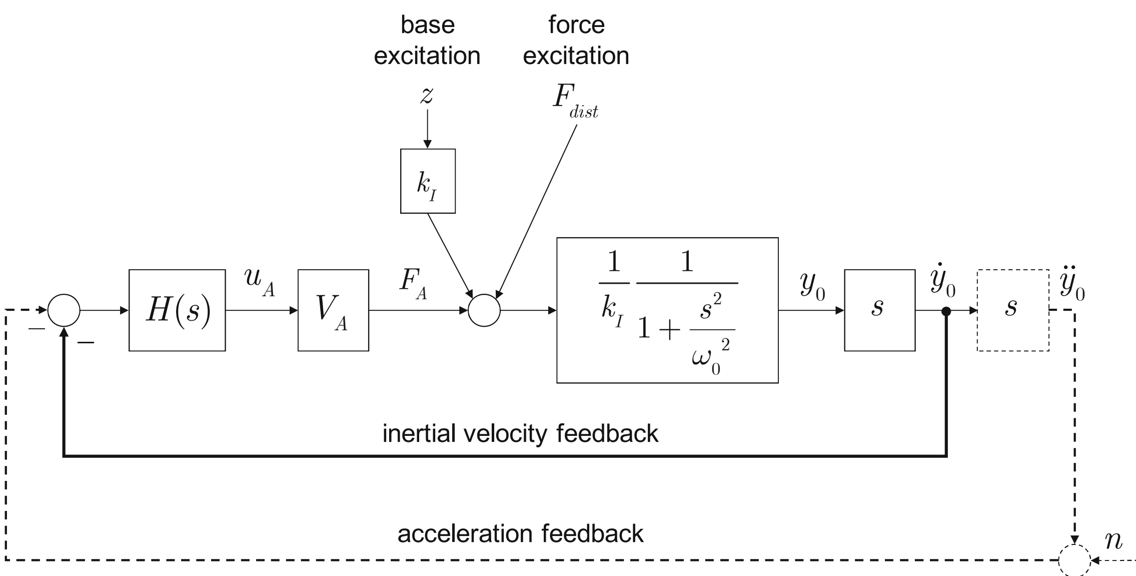
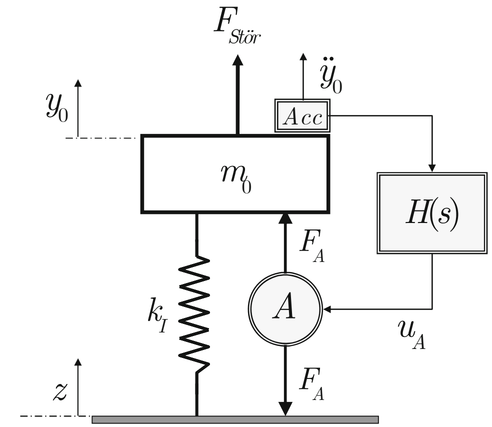


Active vibration isolation: Acceleration feedback





Stable acceleration feedback

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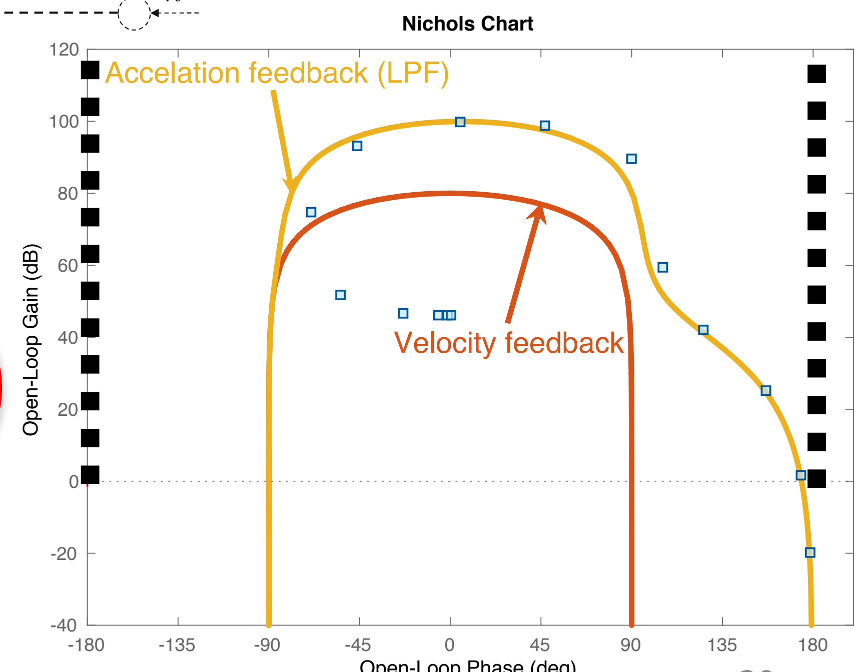
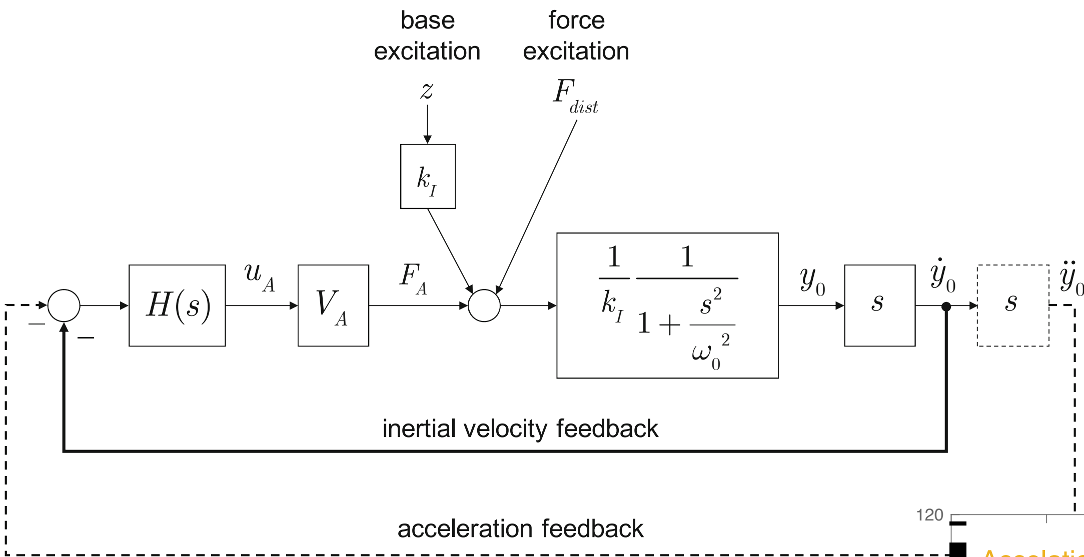
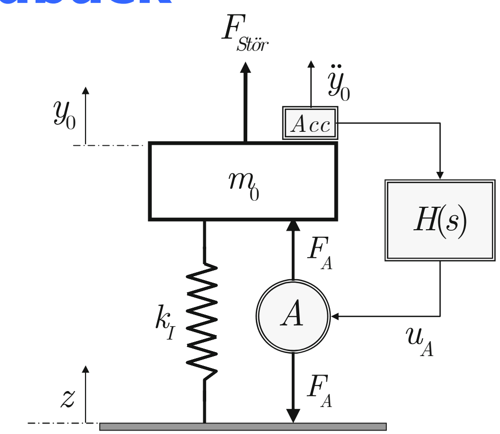


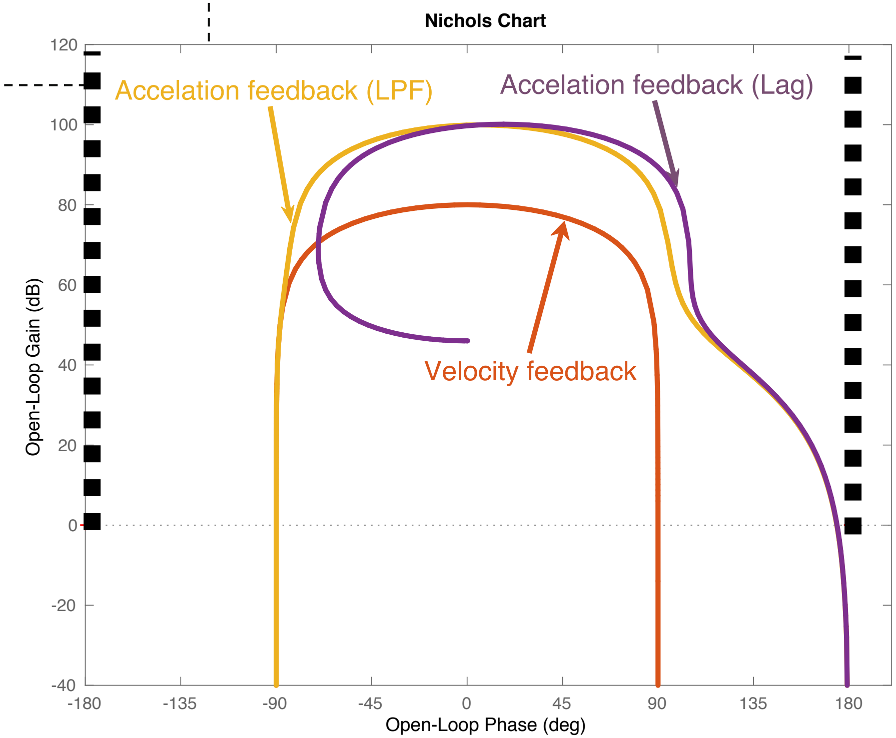
Given parasitic dynamic system become unstable. ☹

Active vibration isolation: Acceleration feedback





$$H_{lag}(s) = K_H \frac{1+\dfrac{s}{\omega_N}}{1+\dfrac{s}{\omega_D}}, \quad \omega_D < \omega_N$$



Design considerations for the skyhook principle

Design givens

Bounding frequency f_B, beyond which exciting disturbances $z(t)$ or $F_{dist}(t)$ are to be suppressed with a magnitude descent of -40 dB/decade.

Design considerations for the skyhook principle

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System configuration

- Suitable sensor and actuator
- Bearing K_I: the bounding frequency f_B naturally determines the bearing stiffness

$$\omega_B = \alpha \cdot \omega_0 = \alpha \cdot \sqrt{k_I / m_0}\,, \quad 0.1 \le \alpha \le 0.7$$

Active vibration isolation: Acceleration feedback

Design considerations for the skyhook principle

Skyhook-controller parameterization

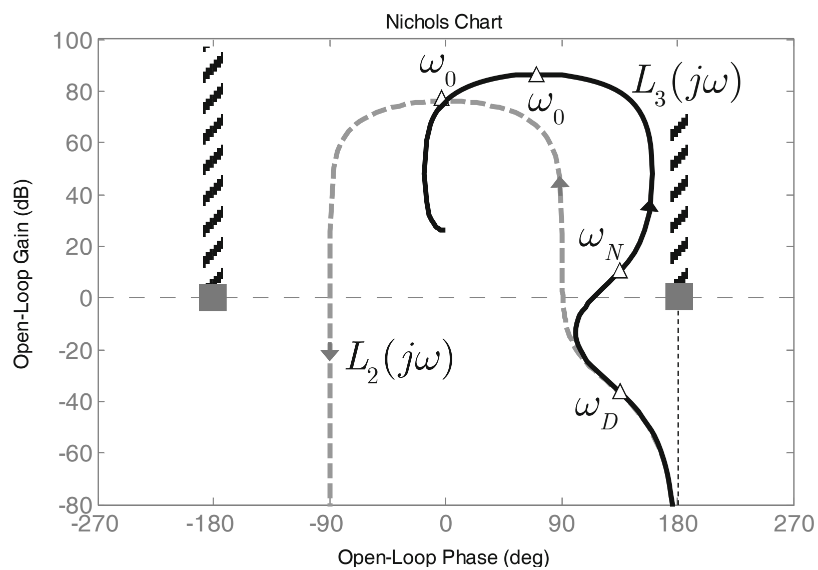
Low pass

$$\boxed{H_2(s) = K_H \frac{1}{1 + \frac{s}{\omega_D}}}$$

$$\boxed{K_H = \frac{k_I}{K_A\, \omega_0^{\,2}} \frac{25.8}{\left(\omega_B/\omega_0\right)^{2.7}}, \qquad \omega_D = 0.05\, \omega_0 \left(\omega_B/\omega_0\right)^{1.7}}$$

Lag element

$$\boxed{H_3(s) = K_H \frac{1 + \frac{s}{100\, \omega_D}}{1 + \frac{s}{\omega_D}}}$$



Active vibration isolation: Acceleration feedback

Design considerations for the skyhook principle

Skyhook-controller parameterization

Low pass

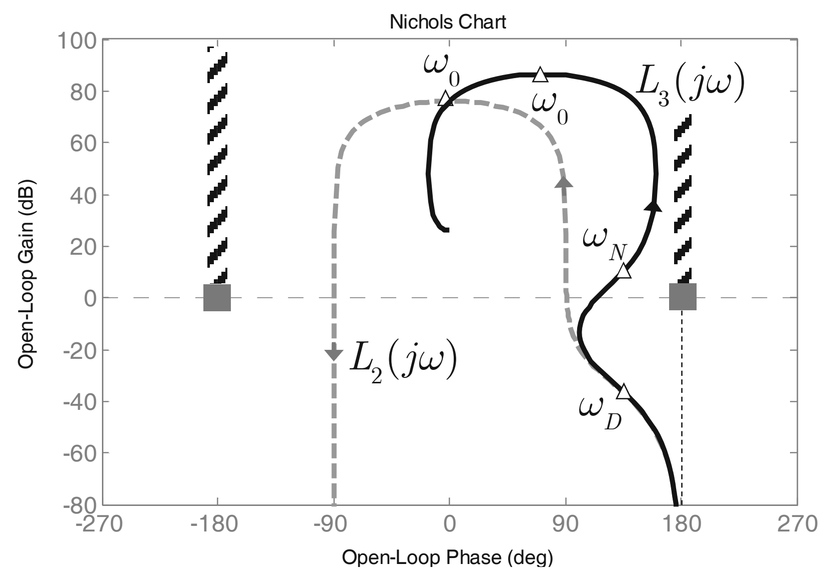
$$H_2(s) = K_H \frac{1}{1+\frac{s}{\omega_D}}$$

Lag element

$$H_3(s) = K_H \frac{1+\frac{s}{100\,\omega_D}}{1+\frac{s}{\omega_D}}$$

$$K_H = \frac{k_I}{K_A\,\omega_0^{\,2}} \frac{25.8}{\left(\omega_B/\omega_0\right)^{2.7}}, \qquad \omega_D = 0.05\,\omega_0 \left(\omega_B/\omega_0\right)^{1.7}$$

Actuator gain



Active vibration isolation: Acceleration feedback

Design considerations for the skyhook principle

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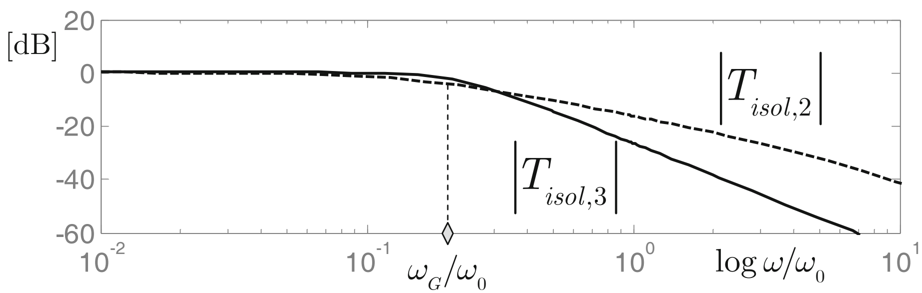
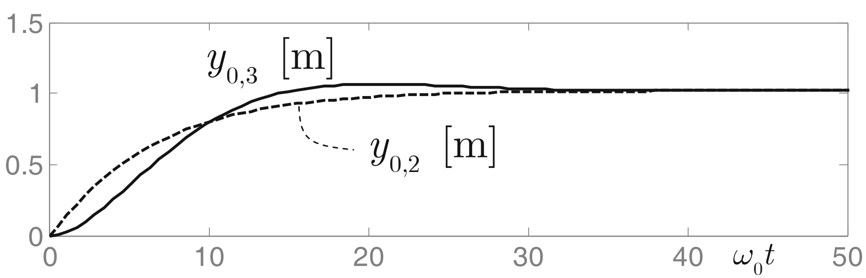
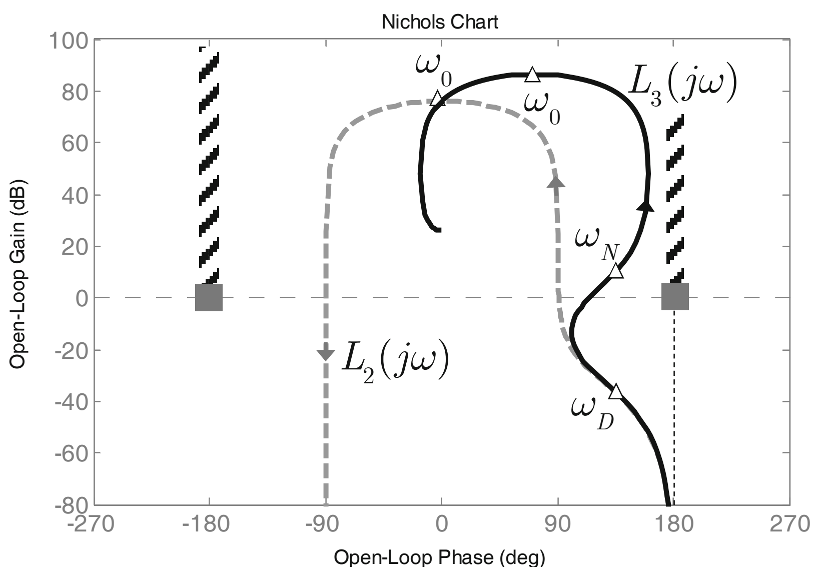
Actuator gain

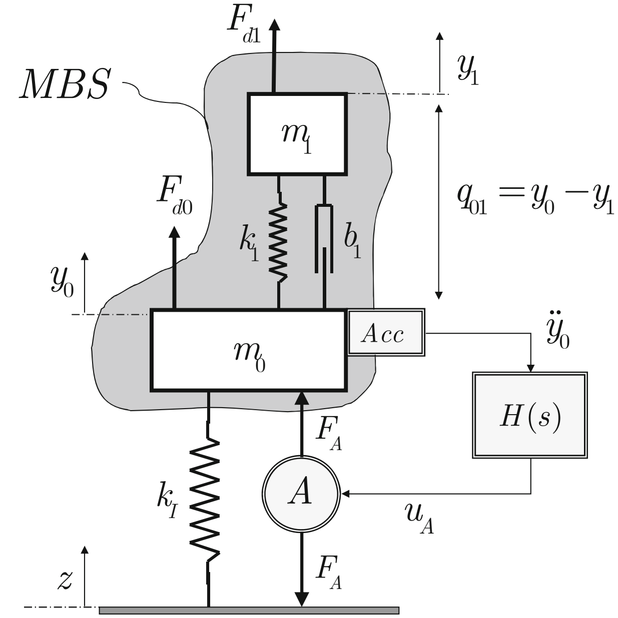






Example

System configuration: The multibody system shown in figure is to be decoupled from displacement excitations z(t) of the foundation with a bounding frequency f_B=1 Hz using active vibration isolation based on the skyhook principle. For this purpose, an accelerometer and a high- bandwidth actuator are available.



MBS: $m_0 = 50$ kg, $m_1 = 5$ kg, $k_1 = 40000$ N/m, $b_1 = 0.01$ Ns/m ,

Parasitic dynamics: $\tau_{par} = 1/\omega_{par} = 0.025$ s .

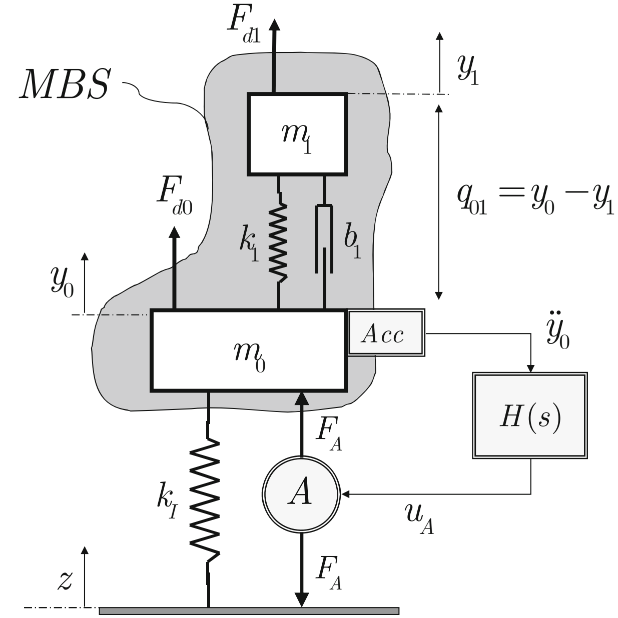
Choose: K_A=0.927 $\alpha = \omega_B / \omega_0 = 0.5$

Example

System Dynamics

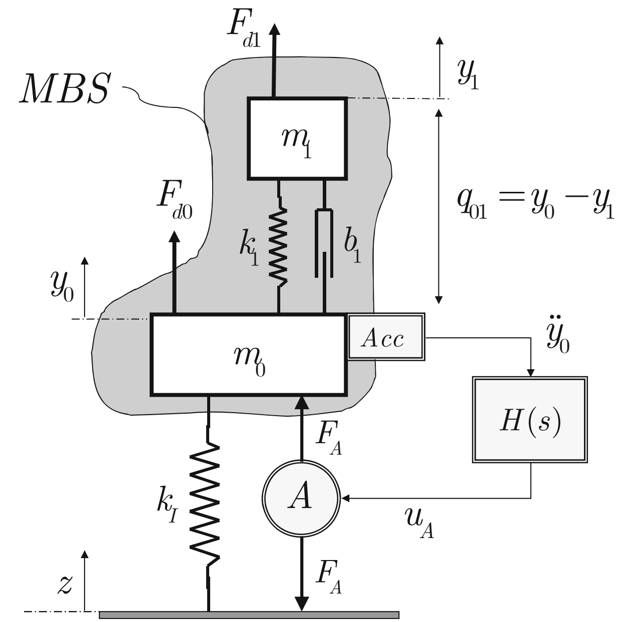
$$P(s) = \frac{1}{k_I} \frac{s^2}{\left\{d_{p0}; \omega_{p0}\right\}} \frac{\left\{d_z; \omega_z\right\}}{\left\{d_{p1}; \omega_{p1}\right\}} \frac{1}{\left[\omega_{par}\right]}$$

$$\omega_{p0} = 12.1 \text{ rad/s}, \ \omega_{p1} = 93.8 \text{ rad/s}, \ \omega_z = 89.4 \text{ rad/s}$$



Example

System Dynamics



$$P(s) = \frac{1}{k_I} \frac{s^2}{\{d_{p0}; \omega_{p0}\}} \frac{\{d_z; \omega_z\}}{\{d_{p1}; \omega_{p1}\}} \frac{1}{[\omega_{par}]}$$

$$\omega_{p0} = 12.1 \text{ rad/s}, \ \omega_{p1} = 93.8 \text{ rad/s}, \ \omega_z = 89.4 \text{ rad/s}$$

Now apply following control to suppress vibration

Low pass

$$H_2(s) = K_H \frac{1}{1 + \frac{s}{\omega_D}}$$

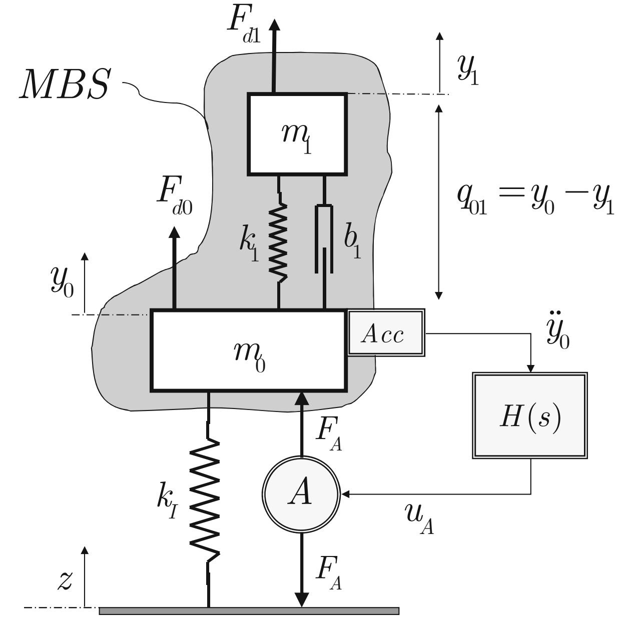
Lag element

$$H_3(s) = K_H \frac{1 + \frac{s}{100\,\omega_D}}{1 + \frac{s}{\omega_D}}$$

$$K_H = \frac{k_I}{K_A \omega_0^2} \frac{25.8}{(\omega_B/\omega_0)^{2.7}}, \qquad \omega_D = 0.05\,\omega_0 \left(\omega_B/\omega_0\right)^{1.7}$$

Actuator gain

Example



System Dynamics

$$P(s) = \frac{1}{k_I} \frac{s^2}{\{d_{p0}; \omega_{p0}\}} \frac{\{d_z; \omega_z\}}{\{d_{p1}; \omega_{p1}\}} \frac{1}{[\omega_{par}]}$$

$$\omega_{p0} = 12.1 \text{ rad/s}, \ \omega_{p1} = 93.8 \text{ rad/s}, \ \omega_z = 89.4 \text{ rad/s}$$

Control design

I-controller with high-pass: $H_1(s) = \dfrac{9160 \cdot 0.19}{\cancel{s}} \dfrac{\dfrac{\cancel{s}}{0.19}}{1 + \dfrac{s}{0.19}}$

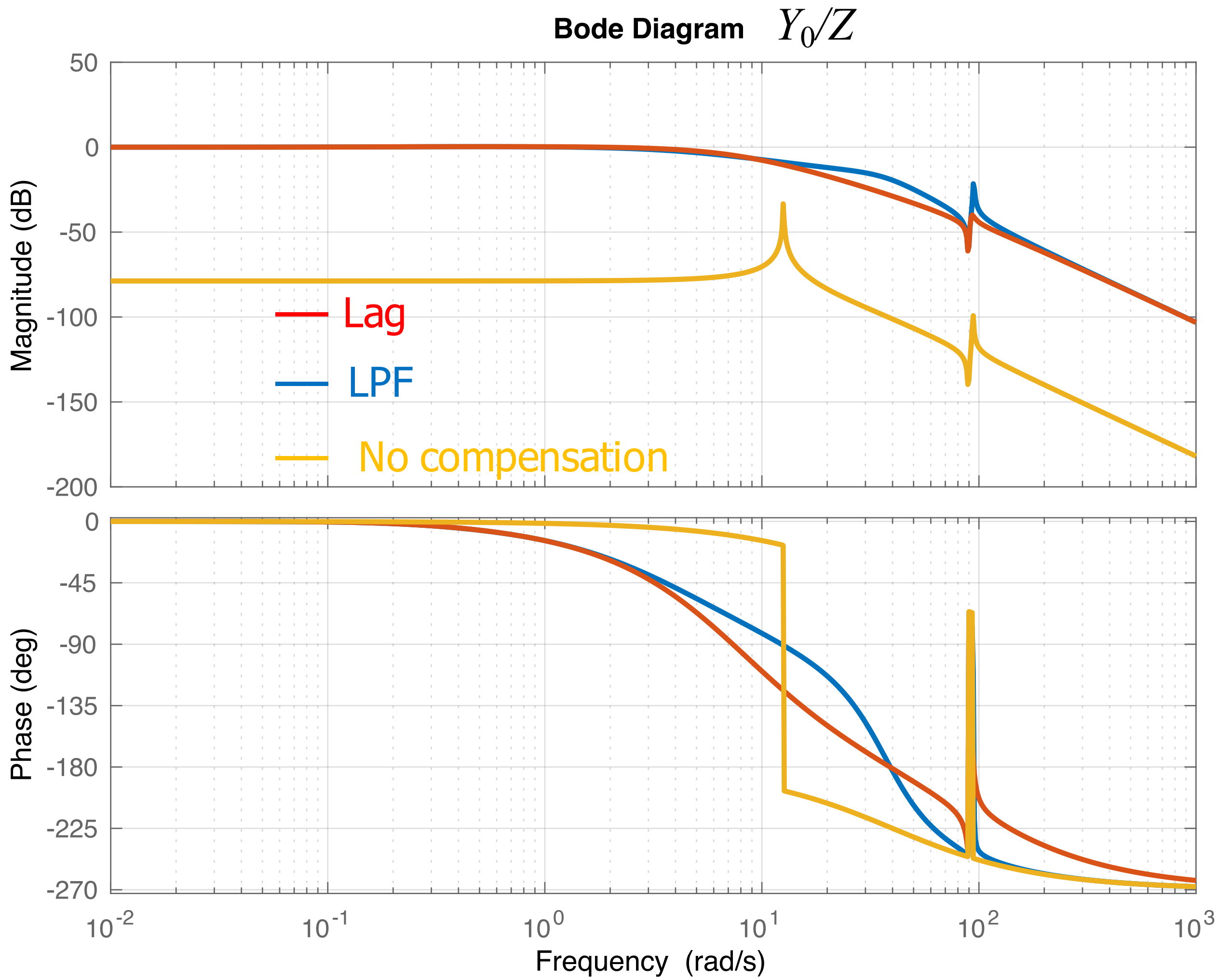
lag element: $H_2(s) = 9160 \dfrac{1 + \dfrac{s}{19}}{1 + \dfrac{s}{0.19}}$.

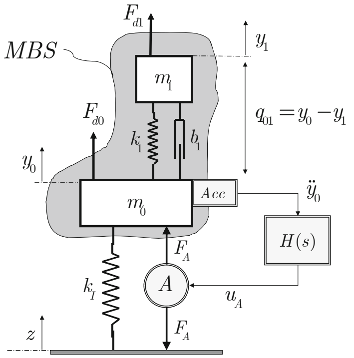
```
clc
clear
close all

%% system configuration
s=tf('s');
m0=50;m1=5;k1=40000;b1=0.01;
fB=1.0;a=0.5;wB=2*pi*fB;wpar=40;kI=(wB/a)^2*(m0+m1);
wp0=wB/a;wp1=93.8;wz=89.4;
dp0=1e-5;dp1=1e-5;dz=1e-5;
P=1/kI*s^2/(s^2/wp0^2+2*dp0*s/wp0+1)*1/(s/wpar+1)*(s^2/wz^2+2*dz*s/wz+1)/(s^2/wp1^2+2*dp1*s/w
p1+1);

%% controls
KA=0.927; % actuator gain
wD=0.05*a^0.7*wB;
KH=kI*25.8/(KA*wp0^2*a^2.7);
H1=KH/(s/wD+1); % LPF
H2=KH*(s/(100*wD)+1)/(s/wD+1); % Lag component

%% open loops
L1=P*H1;
L2=P*H2;

%% closed loops
T1=kI*P/(1+H1*P)/s^2;
T2=kI*P/(1+H2*P)/s^2;

% closed loop second body
Ts1=(b1*s+k1)/(m1*s^2+b1*s+k1)*T1-T1;
Ts2=(b1*s+k1)/(m1*s^2+b1*s+k1)*T2-T2;
```

```
%% Responses
figure(1)
w=logspace(-2,3,1000);
bode(T1,w), grid on, hold on %% Controlled system with LPF
bode(T2,w), grid on, hold on %% Controlled system with Lag
bode(P/s^2,w), grid on, hold on
set(findall(gcf,'type','line'),'linewidth',2)

figure(2) % Nichols diagram for stability test
nichols(L1), grid on, hold on
nichols(L2), grid on, hold on
set(findall(gcf,'type','line'),'linewidth',2)

figure(3) % step response of the system
step(T1,T2,5)
set(findall(gcf,'type','line'),'linewidth',2)

figure(4) % Effect of the controllers (LPF and Lag) on second body
step(Ts1,Ts2,5)
set(findall(gcf,'type','line'),'linewidth',2)
```

Example

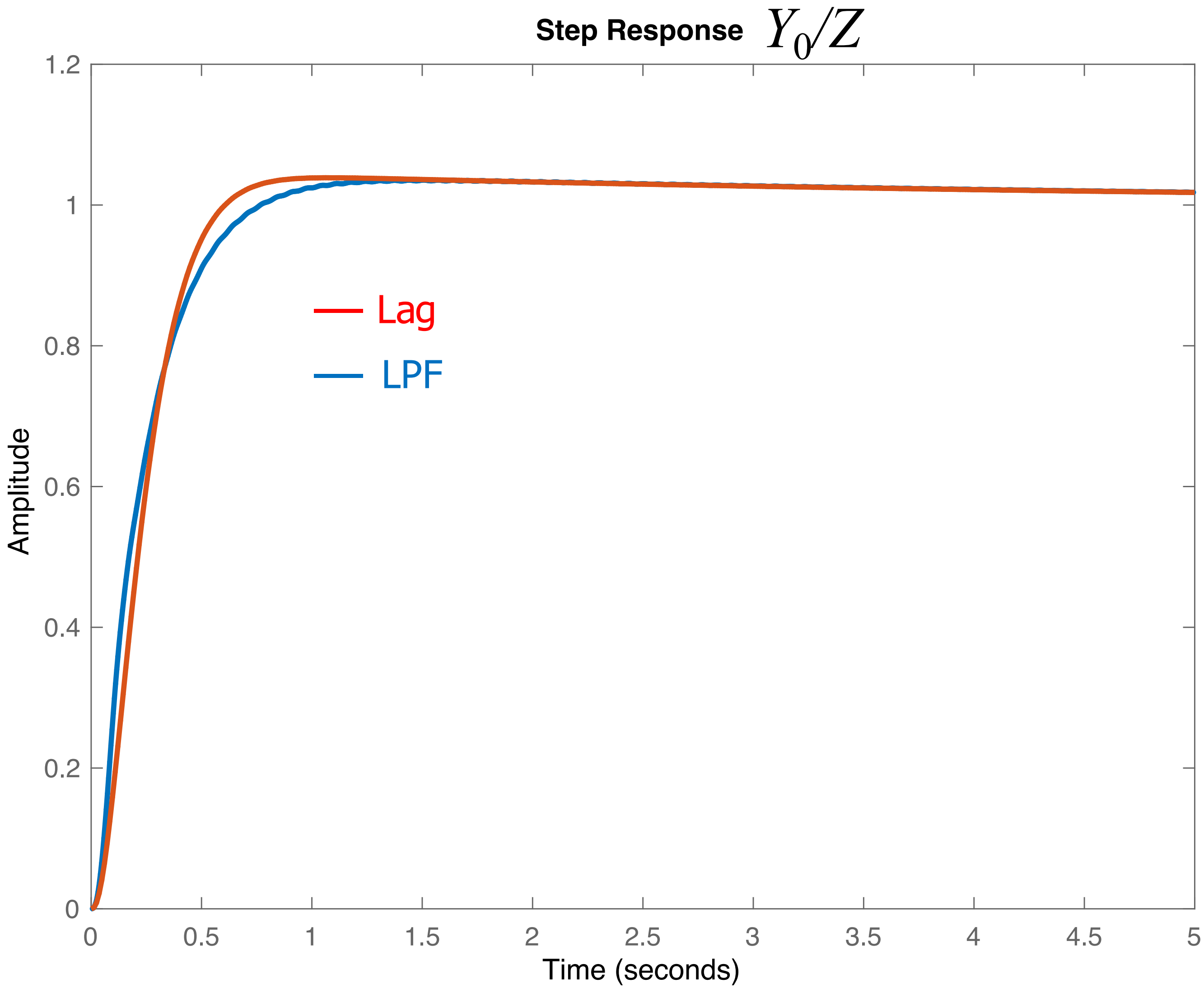
Responses

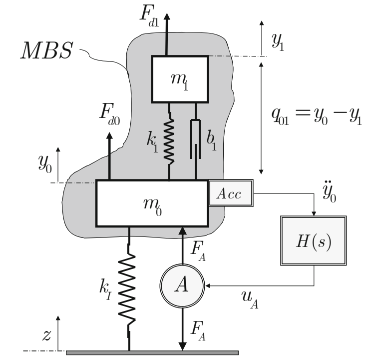




Example

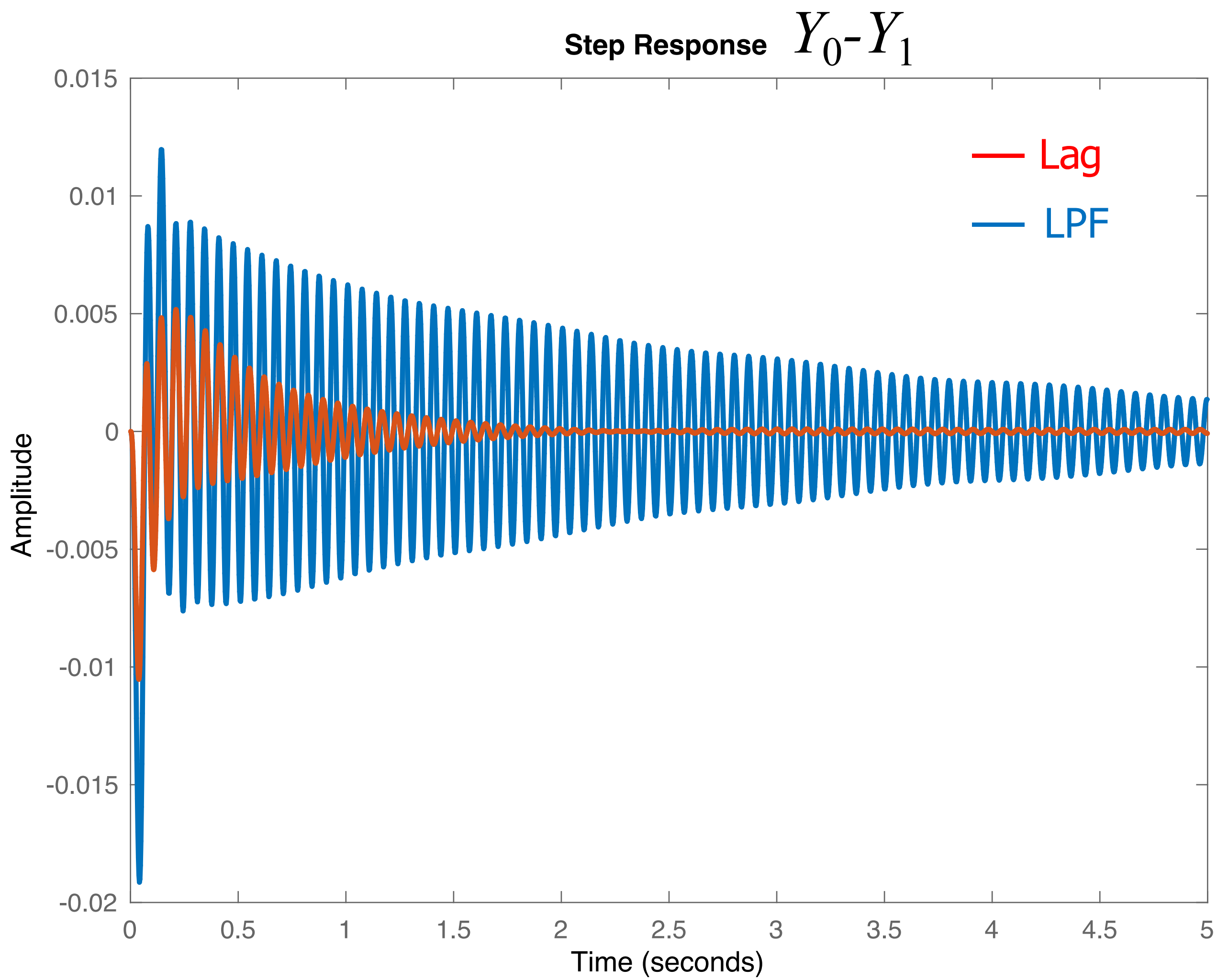
Responses

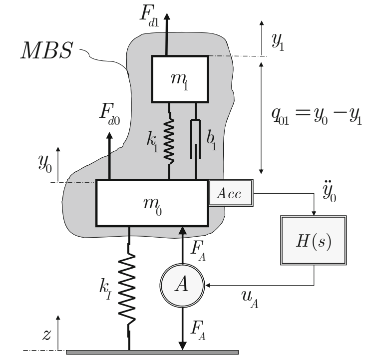




Example

Responses





Example

Responses