量子计算与机器学习

Quantum Computing and Machine Learning

第4章 量子纠缠

中国科学技术大学
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本章大纲

- 量子纠缠
  - 贝尔不等式
- 纠缠交换
- 量子隐形传态
- 量子超密编码

01

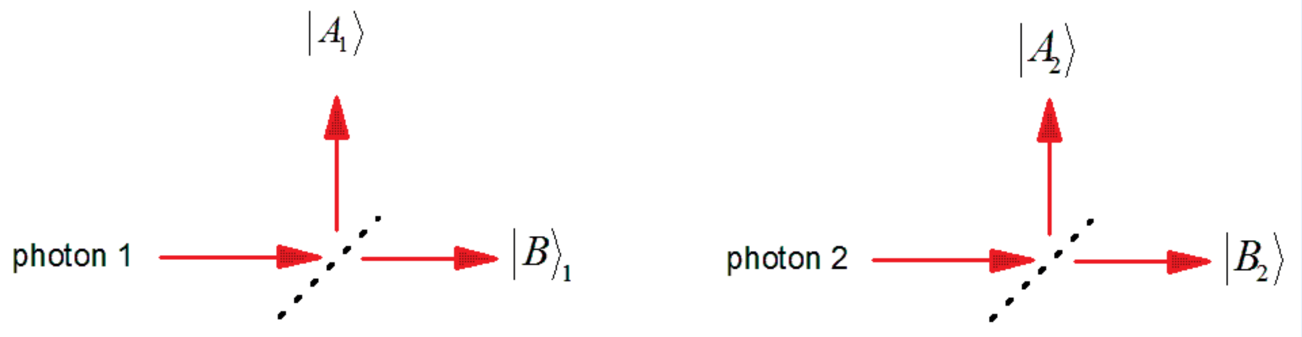
量子纠缠

Quantum Entanglement

量子纠缠

- 薛定谔提出一种情形：两个遥远的系统处于相关叠加态。

考虑两个光子和两个分束器：

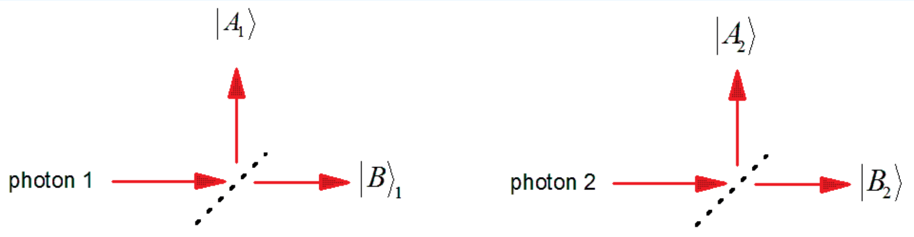


于是可以创造纠缠态：

$$|\psi\rangle = |A_1\rangle|A_2\rangle + |B_1\rangle|B_2\rangle$$

它们的路径完全相关。

量子纠缠



假设我们使用单光子检测器来测量光子1的路径：

然后发现它位于路径A1

那么对于光子2，他的路径一定是A2。

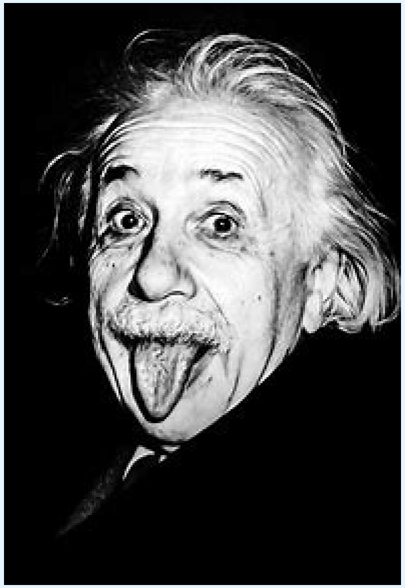
整个量子态坍塌成 $|\psi\rangle = |A_1\rangle|A_2\rangle$

对光子1的测量影响了光子2

不管两个光子的距离多远，这种影响都会立刻发生

量子纠缠

- EPR佯谬：



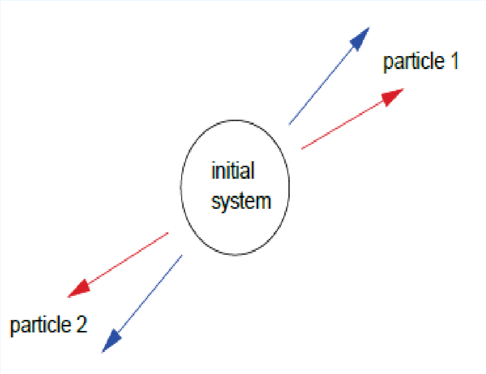
Albert Einstein

Boris Podolsky

Nathan Rosen

量子纠缠

- EPR佯谬



爱因斯坦：“如果一个物理理论对物理实在的描述是完备的，那么物理实在的每个要素都必须在其中有它的对应量，即完备性判据。当我们不对体系进行任何干扰，却能确定地预言某个物理量的值时，必定存在着一个物理实在要素对应于这个物理量，即实在性判据。”

对于这样一个量子态当测量一个量子的坐标时，总是能正确预测另外一个量子的坐标，当测量一个量子的动量时，总是能正确预测另一个量子的动量。因此坐标和动量都是实在性元素。然而海森堡不确定性原理表明这两个量无法同时被观测。另外，当两个量子态相隔较远时。对一个量子的测量依然可以影响到另外一个量子。从而导致对某种超距作用的承认。因此，爱因斯坦认为一个量子的动量或位置在被观测之前就是存在的。

Albert Einstein

Institute of Advanced Studies, Princeton

没有经过验证的电子邮件地址

Physics

关注

标题	引用次数	年份
Can quantum-mechanical description of physical reality be considered complete? A Einstein, B Podolsky, N Rosen Physical Review 47 (10), 777	25258	1935

Entanglement, local realism, and Bell inequalities

Realism: Objects possess definite properties prior to and independent of measurement

Locality: A measurement at one location does not influence a (simultaneous) measurement at a different location

Alice and Bob are in two separated labs

A source prepares particle pairs, say dice. They each get one die per pair and measure one of two properties, say color and parity

Bob

Alice

measurement 1:	color result:	A_1 (Alice), B_1 (Bob)
measurement 2:	parity result:	A_2 (Alice), B_2 (Bob)
possible values:	+1 (even / red) –1 (odd / black)	

$$A_1 (B_1 + B_2) + A_2 (B_1 - B_2) = \pm 2$$

$$A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 = \pm 2$$

$$\langle A_1B_1 \rangle + \langle A_1B_2 \rangle + \langle A_2B_1 \rangle - \langle A_2B_2 \rangle \leq 2$$

for all local realistic (= classical) theories

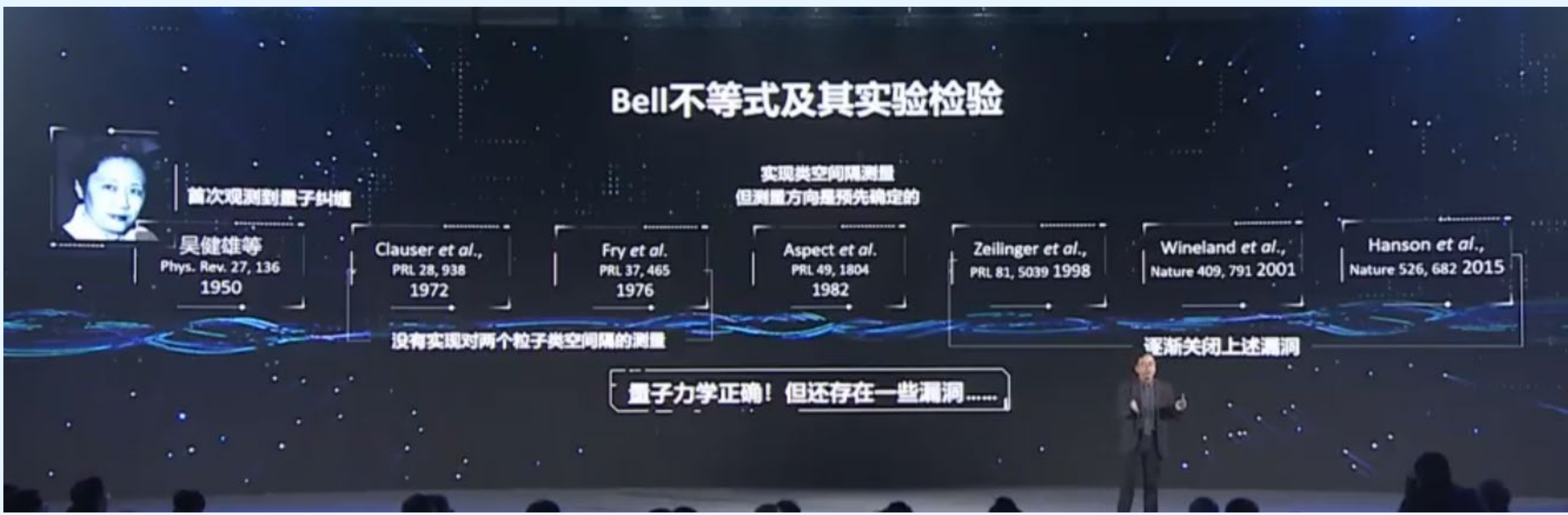
CHSH version (1969) of Bell's inequality (1964)

Entanglement, local realism, and Bell inequalities

- 量子纠缠中是否包含隐变量?

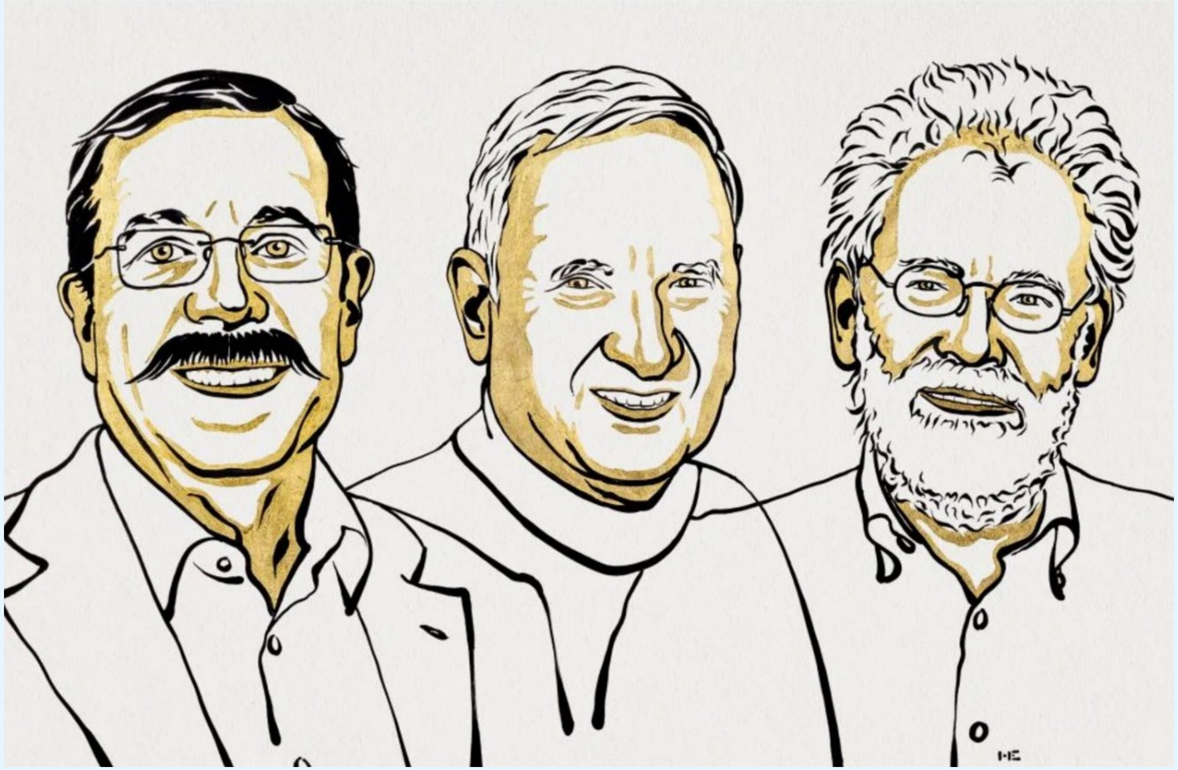
若隐变量理论成立，则贝尔不等式成立。

如果实验结果相悖于贝尔不等式，则隐变量理论不成立。



Entanglement, local realism, and Bell inequalities

Nobel Prize in Physics 2022



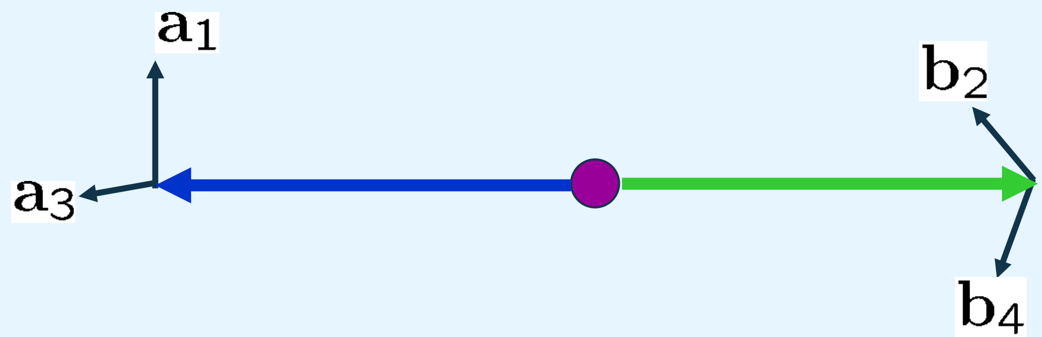
(From left) Alain Aspect, John F. Clauser, and Anton Zeilinger

Bell inequalities

Local hidden variables (LHV) and Bell inequalities

Bell entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$$S = C(\mathbf{a}_1, \mathbf{b}_2) + C(\mathbf{a}_3, \mathbf{b}_2) + C(\mathbf{a}_3, \mathbf{b}_4) - C(\mathbf{a}_1, \mathbf{b}_4)$$

LHV: $$|S| = \left|\left\langle \underbrace{\sigma_{\mathbf{b}_2}(\sigma_{\mathbf{a}_3} + \sigma_{\mathbf{a}_1}) + \sigma_{\mathbf{b}_4}(\sigma_{\mathbf{a}_3} - \sigma_{\mathbf{a}_1})}_{=\pm 2} \right\rangle\right| \leq 2$$

QM: $$S = 2\sqrt{2}$$

The quantum correlations cannot be explained in terms of local, realistic properties.

Bell inequalities

Charlie prepares a quantum system of two qubits in the state:

$$|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.$$

He passes the first qubit to Alice, and the second qubit to Bob.
They perform measurements of the following observables:

$$Q = Z_1 \qquad S = \frac{-Z_2 - X_2}{\sqrt{2}}$$

$$R = X_1 \qquad T = \frac{Z_2 - X_2}{\sqrt{2}}.$$

Simple calculations show that the average values for these observables, written in the quantum mechanical $\langle \cdot \rangle$ notation, are:

$$\langle QS \rangle = \frac{1}{\sqrt{2}}; \ \langle RS \rangle = \frac{1}{\sqrt{2}}; \ \langle RT \rangle = \frac{1}{\sqrt{2}}; \ \langle QT \rangle = -\frac{1}{\sqrt{2}}.$$

Thus, $\langle QS \rangle + \langle RS \rangle + \langle RT \rangle - \langle QT \rangle = 2\sqrt{2}.$

02

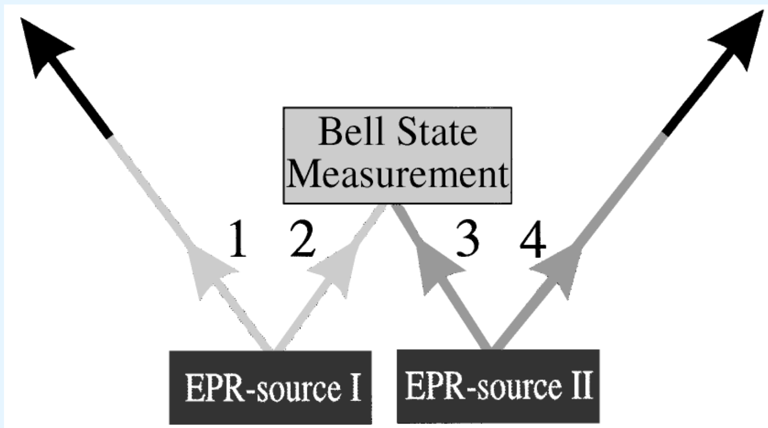
纠缠交换

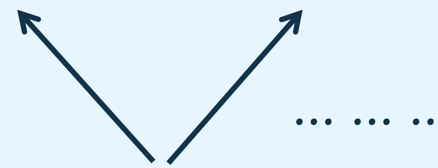
Entanglement Swapping

纠缠交换：两方情形

Idea: Zukowski *et al.* (1993)

First realization: Zeilinger group (1998)





"quantum repeater"

$$|\Psi\rangle_{1234} = \tfrac{1}{2}\left(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2\right) \times \left(|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4\right)$$

$$|\Psi\rangle_{1234} = \tfrac{1}{2}\left(|\Psi^+\rangle_{14}|\Psi^+\rangle_{23} + |\Psi^-\rangle_{14}|\Psi^-\rangle_{23} + |\Phi^+\rangle_{14}|\Phi^+\rangle_{23} + |\Phi^-\rangle_{14}|\Phi^-\rangle_{23}\right)$$

initial state factorizes into 1,2 x 3,4

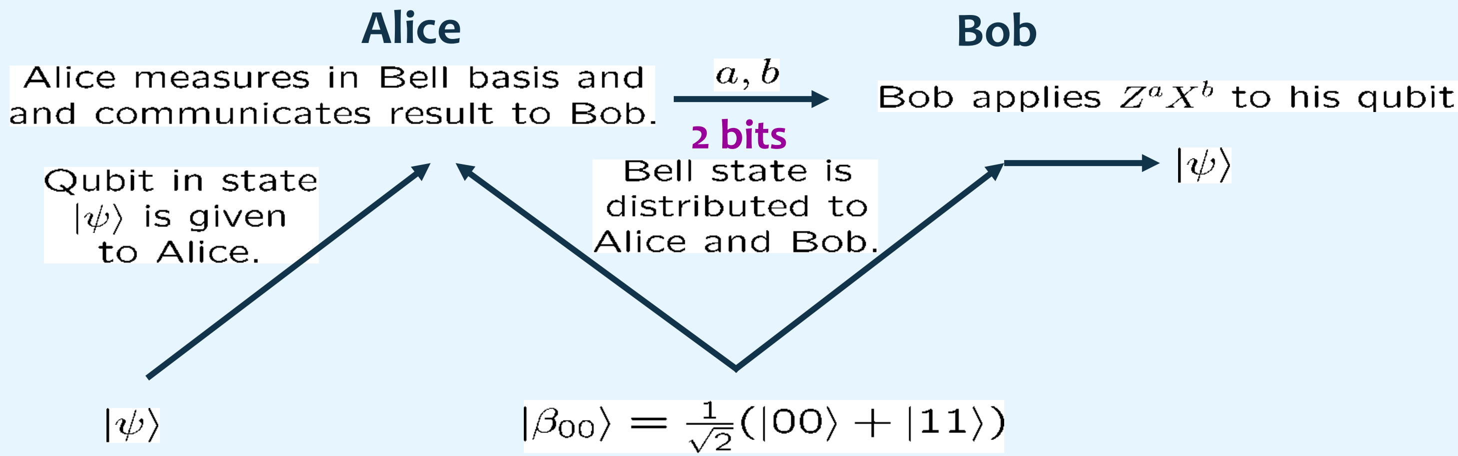
if 2,3 are projected onto a Bell state, then 1,4 are left in a Bell state

Picture: PRL 80, 2891 (1998)

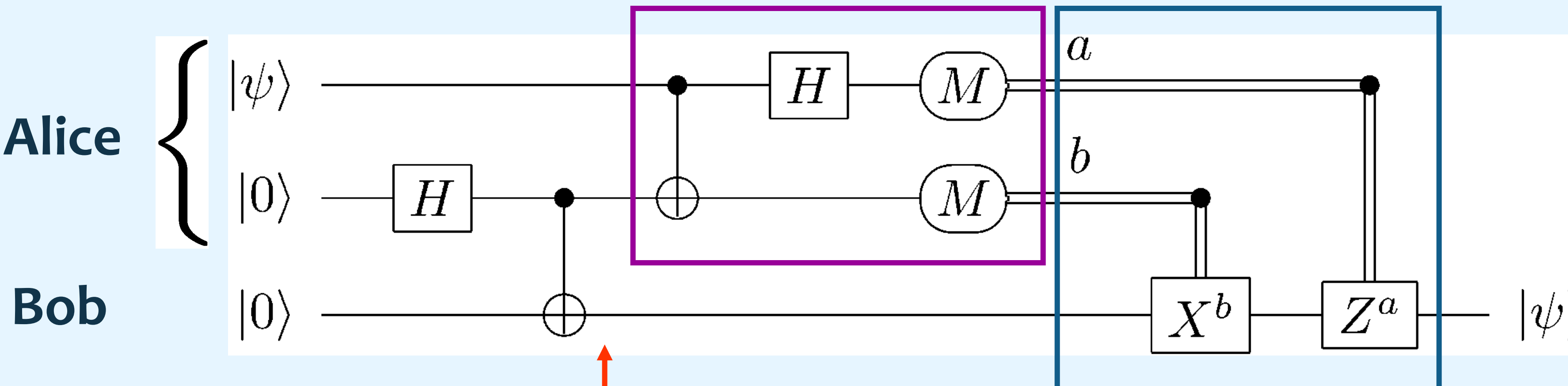
03

量子隐形传态

Quantum Teleportation



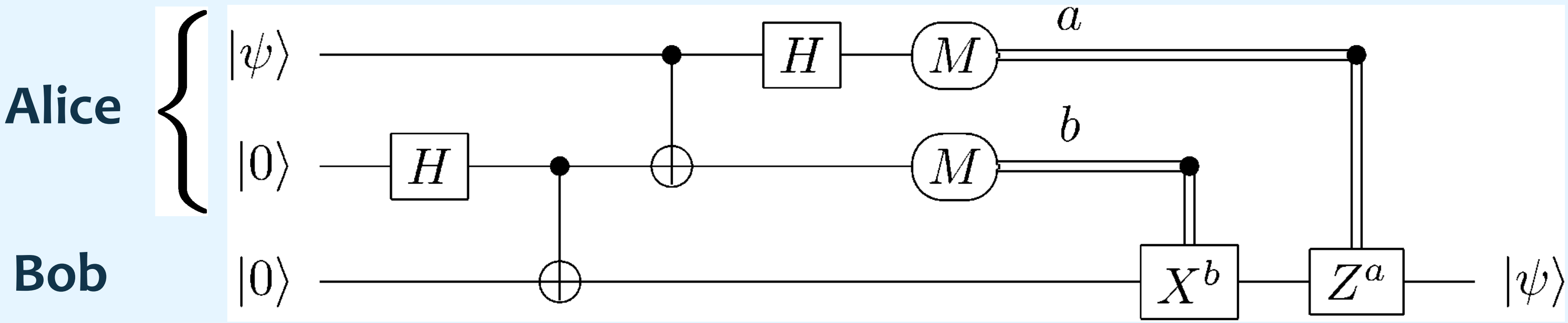
Alice measures in Bell basis.



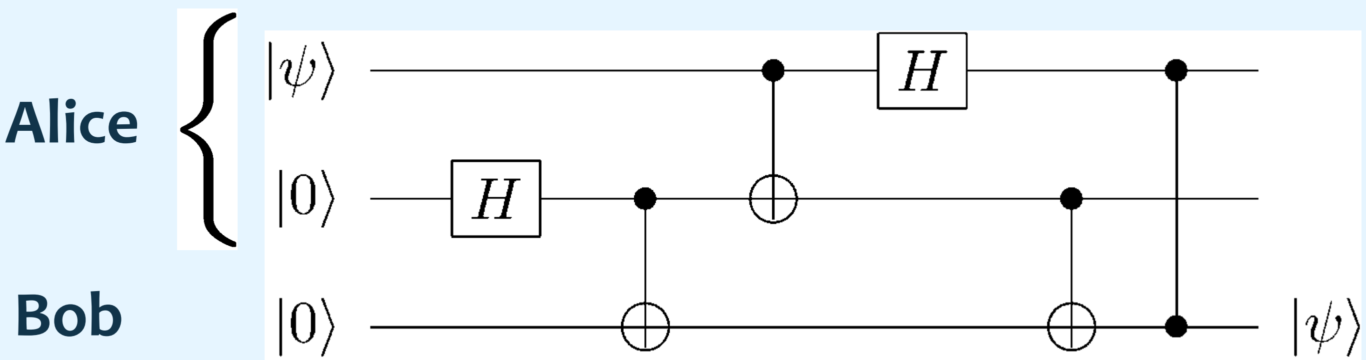
$|\psi\rangle \otimes |\beta_{00}\rangle$
Bell state is distributed to Alice and Bob.
Qubit in state $|\psi\rangle$ is given to Alice.

Alice communicates result to Bob.
Bob applies $Z^a X^b$ to his qubit.

Quantum teleportation



Standard teleportation circuit



Coherent teleportation circuit

量子隐形传态

1 qubit = 1 ebit + 2 bits

- 传输两个bit来达到传输一个qubit的目的。
- Step1：Alice和Bob共享一对贝尔态$|\beta_{00}\rangle=\frac{|00\rangle+|11\rangle}{\sqrt{2}}$。Alice手中有一个量子比特$|\Psi\rangle=\alpha|0\rangle+\beta|1\rangle$。

$$|\Psi\rangle_A|\beta_{00}\rangle_{AB}=\frac{\alpha}{\sqrt{2}}|00\rangle_A|0\rangle_B+\frac{\beta}{\sqrt{2}}|10\rangle_A|0\rangle_B+\frac{\alpha}{\sqrt{2}}|01\rangle_A|1\rangle_B+\frac{\beta}{\sqrt{2}}|11\rangle_A|1\rangle_B$$

$$=k\{[\alpha(|00\rangle+|11\rangle)+\alpha(|00\rangle-|11\rangle)]|0\rangle+[\beta(|01\rangle+|10\rangle)-\beta(|01\rangle-|10\rangle)]|0\rangle$$
$$+[\alpha(|01\rangle+|10\rangle)+\alpha(|01\rangle-|10\rangle)]|1\rangle+[\beta(|00\rangle+|11\rangle)-\beta(|00\rangle-|11\rangle)]|1\rangle\}$$

$$=k\Big[\frac{|00\rangle+|11\rangle}{\sqrt{2}}(\alpha|0\rangle+\beta|1\rangle)+\frac{|00\rangle-|11\rangle}{\sqrt{2}}(\alpha|0\rangle-\beta|1\rangle)+$$
$$\frac{|01\rangle+|10\rangle}{\sqrt{2}}(\alpha|1\rangle+\beta|0\rangle)+\frac{|01\rangle-|10\rangle}{\sqrt{2}}(\alpha|1\rangle-\beta|0\rangle)\Big]$$

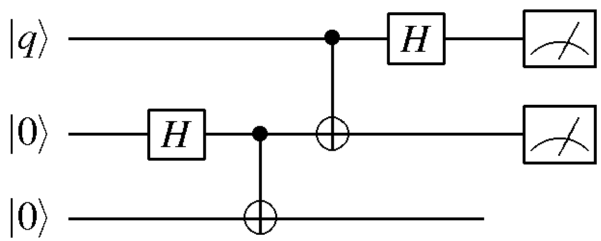
- Step2：Alice对手中两个量子比特进行Bell测量。得到四种结果，根据上式中的Bell态顺序得到2bit 00，01，10，11。并通过经典信道传输。

量子隐形传态

- Step3： Bob根据获得的2 bit（ab)对手中的量子比特进行Pauli Correction（实施 $X^a Z^b$）。 最后Bob一定获得 $\alpha|0\rangle + \beta|1\rangle$。

量子隐形传态也可以写成以标准基测量的形式：

Question 2. (Towards Teleportation) (See Handout III if you have problems answering this question.) Consider the following three qubit circuit that has as input an unknown qubit $|q\rangle$ and two zero states:



(a) With $|q\rangle = \alpha|0\rangle + \beta|1\rangle$, what is the output state before the measurements?

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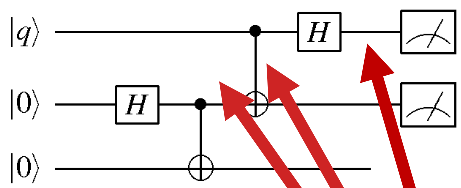


(a) With $|q\rangle = \alpha|0\rangle + \beta|1\rangle$, what is the output state before the measurements?

$$
\begin{aligned}
|q\rangle \otimes |\mathrm{EPR}\rangle &= (\alpha|0\rangle + \beta|1\rangle) \otimes \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\
&= \tfrac{1}{\sqrt{2}}(\alpha|0{,}00\rangle + \alpha|0{,}11\rangle + \beta|1{,}00\rangle + \beta|1{,}11\rangle) \\
&\mapsto \tfrac{1}{\sqrt{2}}(\alpha|0{,}00\rangle + \alpha|0{,}11\rangle + \beta|1{,}10\rangle + \beta|1{,}01\rangle) \\
&\mapsto \tfrac{1}{2}(\alpha|0{,}00\rangle + \alpha|1{,}00\rangle + \alpha|0{,}11\rangle + \alpha|1{,}11\rangle + \\
&\qquad \beta|0{,}10\rangle - \beta|1{,}10\rangle + \beta|0{,}01\rangle - \beta|1{,}01\rangle) \\
&= \tfrac{1}{2}|00\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) + \\
&\quad \tfrac{1}{2}|01\rangle \otimes (\alpha|1\rangle + \beta|0\rangle) + \\
&\quad \tfrac{1}{2}|10\rangle \otimes (\alpha|0\rangle - \beta|1\rangle) + \\
&\quad \tfrac{1}{2}|11\rangle \otimes (\alpha|1\rangle - \beta|0\rangle)
\end{aligned}
$$

量子隐形传态

- Q: 量子隐形传态超越光速了吗?

  A: 没有，因为经典信道传递的信息受光速限制。

- Q: 量子隐形传态传递了一个备份，是否违背不可克隆定理?

  A: 在隐形传态后，原始态因测量而坍塌，只有目标量子处于状态$|\Psi\rangle=\alpha|0\rangle+\beta|1\rangle$。

- 量子隐形传态中所传输的态可以是未知的，Alice 不需要获取量子态的任何信息。

04

量子超密编码

Quantum Superdense Coding

量子超密编码

技术基础：Bell态之间的转换

The four Bell states can be turned into each other using operations on only one of the qubits:

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$(X \otimes I)|\Phi_+\rangle = (X \otimes I)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle) = |\Psi_+\rangle$$

$$(Z \otimes I)|\Phi_+\rangle = (Z \otimes I)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = |\Phi_-\rangle$$

$$(ZX \otimes I)|\Phi_+\rangle = (ZX \otimes I)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(-|10\rangle + |01\rangle) = |\Psi_-\rangle$$

量子超密编码

2 bits = 1 qubit + 1 ebit

✓ 传输1个qubit来达到传输2个bit的目的

- Step1: Alice和Bob共享一个纠缠对 $|\Psi\rangle=\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ ，Alice持有第一个量子位，Bob持有第二个量子位。
- Step2: Alice根据想传的两个比特位，对自身量子比特作用相应的Pauli门。

Intended Message	Applied Gate	Resulting State $(\cdot\sqrt{2})$
00	I	$\|00\rangle + \|11\rangle$
10	X	$\|01\rangle + \|10\rangle$
01	Z	$\|00\rangle - \|11\rangle$
11	ZX	$\|10\rangle - \|01\rangle$

- Step3: Alice传送自己的量子比特给Bob， Bob对这两个量子比特进行Bell测量。根据测量结果推得Alice传输的经典比特。

量子超密编码

Initially: $|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Alice applies the following operator to her qubit: $Z^{b_2}X^{b_1}$

$$(Z^{b_2}X^{b_1} \otimes I)|\Phi_+\rangle$$

$b_1 = 0, b_2 = 0$ $\quad |\Phi_+\rangle$

$b_1 = 0, b_2 = 1$ $\quad (Z \otimes I)|\Phi_+\rangle = |\Phi_-\rangle$

$b_1 = 1, b_2 = 0$ $\quad (X \otimes I)|\Phi_+\rangle = |\Psi_+\rangle$

$b_1 = 1, b_2 = 1$ $\quad (ZX \otimes I)|\Phi_+\rangle = |\Psi_-\rangle$

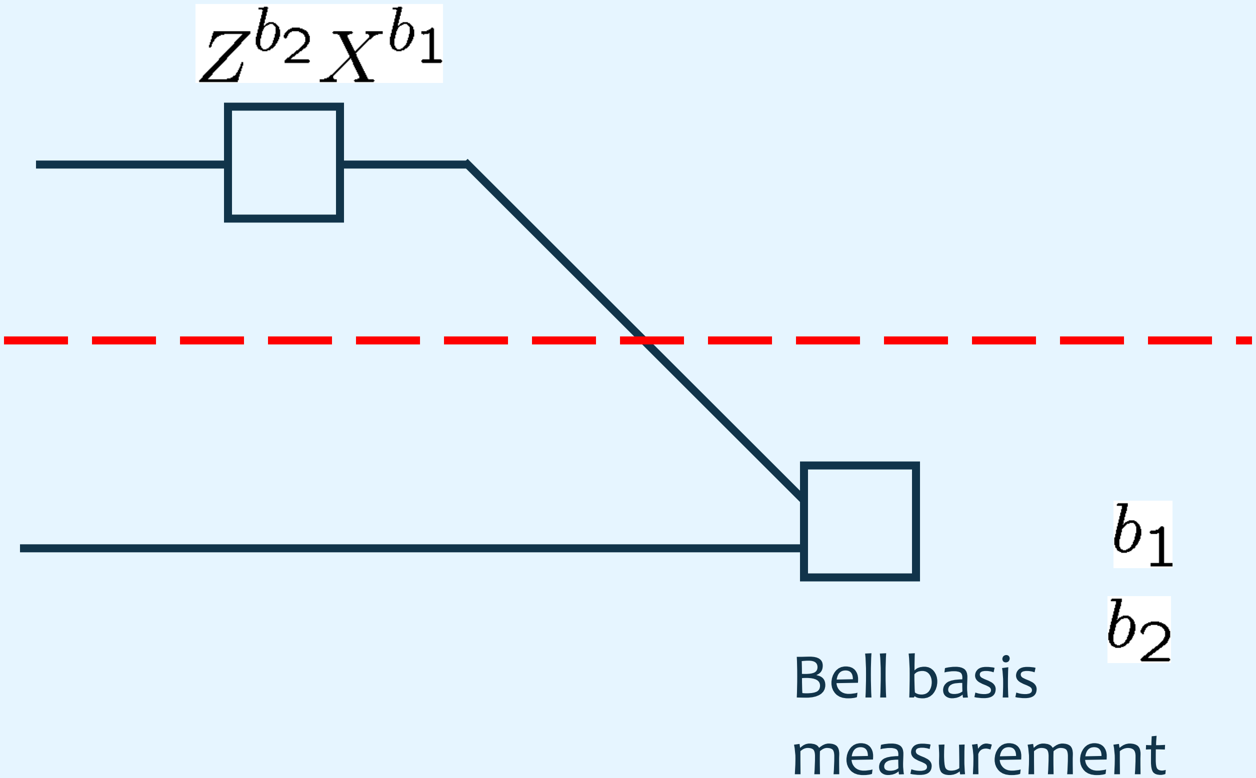
Bob can uniquely determine which of the four states he has and thus figure out Alice's two bits.

量子超密编码

b_1

b_2

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



量子超密编码

其他问题：

- 用1qubit传输3 bits被证明是不可能的。因此2bits/qubit的传输率是最优的。
- 超密编码不可能用经典方式模拟。
- 超密编码的实验已经成功实行。

[Zeilinger et al.,1996, Innsbruck, Austria]

- 我们可以做反向协议：使用A和B之间的经典通信发送量子信息。

Teleportation

$$\boxed{1 \text{ qubit} = 1 \text{ ebit} + 2 \text{ bits}}$$

Teleportation says we can replace transmitting a qubit with a shared entangled pair of qubits plus two bits of classical communication.

Superdense Coding

$$\boxed{2 \text{ bits} = 1 \text{ qubit} + 1 \text{ ebit}}$$

We can send two bits of classical information if we share an entangled state and can communicate one qubit of quantum information.

习题

4.1 证明贝尔态 $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ 可以等效表达为 $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|aa\rangle + |bb\rangle)$，其中$|a\rangle$和$|b\rangle$是任意一组正交归一基。

谢谢！