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Abstract

Given f, g, and H such that $f + H \in \mathrm{MU}(1)$, $g + H \in \mathrm{MU}(1)$. Then

$$\forall h \in H, \ (H - h) + \{f, g\} \text{ is satisfiable}$$

Proof. For $n = 1$, it is clearly true.
Then by induction and the disjoint splitting propery of MU(1) formulas

Suppose $\Phi := Q\varphi$ with $\varphi = H + \{f, g\}$ with the above property. Then (conjecture) Φ is true iff both $Q(H + f)$ and $Q(H + g)$ are ture

Need the following property.
$F \in$MU(1). $f, g \in F$, a path from f to g
$f_1 = f, \cdots, f_n = g$ and $L_1, \cdots, L_n$ such that
$L_1 \in f_1$, $\neg L_i, L_{i+1}$ are in f_{i+1} for $i = 1, \cdots n - 1$, $\neg L_n \in f_n$.

Claim: Let $f, g \in F$, π_1, π_2 are two paths from f to say to g_1 and g_2. Then the two path are compatable, that is, they do not contain complementary literals.
Proof. For $n = 1$ clearly. For $n > 1$ by using disjoint splitting.
So, in this case, QMU(2) is solvable in polynomial time.
Suppose F is lean $d(F) = 2$.
Case 1. $F - \{f\} \in \mathrm{MU}(1)$, and for all $h \in F - \{f\}$, $F - \{h\}$ is satisfiable.
Case 2. $F - \{f\} \in \mathrm{MU}(1)$, and there is $g \in F - f$, $F - g$ is in MU(1) (then this case)
Case3. $F' \subseteq F \in$MU(1) such that $F - F'$ has more than one clauses.
Case 4. $F \in \mathrm{MU}(2)$

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1 General Structure of LEAN(2)

Suppose $F \in \mathrm{MLEAN}(2)$ unsatisfiable, but $F \notin \mathrm{MU}(2)$.

Then F must contain a MU(1) subformula. So, let $G \subseteq F$ be a subformula in MU(1). Let $\theta = F - G$.

Plaese note that F is matching lean, the formula θ' obtained from θ by omitting variables occuring in G must be mlean and has deficiency 1.

Case 1. For any clause $g \in G$ such that $(G - \{g\}) + \theta$ is satisfiable. That is, G is the only MU(1) subformula of F.

We suppose it is not case 1.

Case 2. For some $g \in G$ such that $(G - \{g\}) + \theta$ is unsatisfiable. That is there is $G' \subseteq F$ such that $G' \neq G$ and $G' \in \mathrm{MU}(1)$.

Because $G' = G' \cap G + G' \cap \theta$ and G' has deficiency 1, it must be that $\theta \subseteq G'$.

Let $H = G \cap G'$, $\eta = G' - H$. That is, F can be written as follows

$$F = \theta + H + \eta \quad \text{with } \theta + H \in \mathrm{MU}(1) \text{ and } \eta + H \in \mathrm{MU}(1)$$

Let θ^- (resp. η^-) be the formula obtained from θ (resp. η) by omitting occurrences of variables of H. Then both θ^- and η^- are in MU(1) and have distinct variables.

The following is an example of such formulas

Let θ^+ is the clause obtained from θ by iteratively applying sDP reduction on variables in θ^-. Likewise for η^+. Then we have

$$\{\theta^+\} + H, \ \{\eta^+\} + H \text{ and } \{\theta^+ \vee \eta^+\} + H \text{ are all in } \mathrm{MU}(1)$$

We consider the first case. That F contains only MU(1) subformula.