1 General Problems

Configuration and Specification (Sets)

Equivalence Problem

Input: $C = \{\alpha_1, \cdots, \alpha_n\}$
Query: Whether $\exists K \subseteq C : K \equiv \beta$

Retricted Equivalence(R-equivalence

Input: $C = \{\alpha_1, \cdots, \alpha_n\}$, $beta$, and a set V of variables
Query: Whether $\exists K \subseteq C : K \equiv^V \beta$

Implication Problem

Input: $C = \{\alpha_1, \cdots, \alpha_n\}$, $beta$
Query: Whether $\exists K \subseteq C : K \models \beta$

Note: In all problems K is demanded satisfiable.

$$\alpha \equiv_V \beta \iff \forall \gamma \text{ over } V, (\alpha \models \gamma \Leftrightarrow \beta \models \gamma)$$

$$\alpha \models_V \beta \iff \forall \gamma \text{ over } V, (\beta \models \gamma \Longrightarrow \alpha \models \gamma)$$

Euivalence Problem is in $\mathrm{P}^{\mathrm{NP}[\log]}$
R-Equivalence Problem same as above
Implication Σ_2^P?

2 Resticted to DHORN

C: set of DHORN, β: DHORN

Equivalence problem is in PTIME

Idea: Let

$$\mathrm{IMP}(C,\beta) := \{\alpha \mid \alpha \in C \text{ and } \beta \models \alpha\}$$

Then

$$\exists K \subseteq C(K \equiv \beta) \Longleftrightarrow \mathrm{IMP}(C,\beta) \models \beta$$

Implication problem is trivial

The existence of K is equivalent to $C \models \beta$

The V-equivalence problem

$\mathrm{IMP}(C,\beta,V) := \{\alpha \in C \mid \beta \models_V \alpha\}$

The existence of $K \equiv_V \beta$ is equivalent to $\mathrm{IMP}(C,\beta,V) \equiv_V \beta$.

we guess it is co-NP complete.
See Hans's book page 251

3 Restricted to HORN

Equivalence Problem: same as the DHORN case.

iff $\mathrm{IMP}(C,\beta)$ is satisfiable and $\mathrm{IMP}(C,\beta) \models \beta$

V-Equivalence Problem: idea Same as DHORN but coNP-complete iff $\mathrm{IMP}(C, \beta, V)$ is sat and $\mathrm{IMP}(C, \beta, V) \models_V \beta$

Implication Problem
NP-complete