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### Abstract

Given  $f, g$ , and  $H$  such that  $f + H \in \text{MU}(1)$ ,  $g + H \in \text{MU}(1)$ . Then

$\forall h \in H, (H - h) + \{f, g\}$  is satisfiable

Proof. For  $n = 1$ , it is clearly true.

Then by induction and the disjoint splitting property of  $\text{MU}(1)$  formulas

Suppose  $\Phi := Q\varphi$  with  $\varphi = H + \{f, g\}$  with the above property. Then (conjecture)  $\Phi$  is true iff both  $Q(H + f)$  and  $Q(H + g)$  are true

Need the following property.

$F \in \text{MU}(1)$ .  $f, g \in F$ , a path from  $f$  to  $g$

$f_1 = f, \dots, f_n = g$  and  $L_1, \dots, L_n$  such that

$L_1 \in f_1, \neg L_i, L_{i+1}$  are in  $f_{i+1}$  for  $i = 1, \dots, n - 1, \neg L_n \in f_n$ .

Claim: Let  $f, g \in F$ ,  $\pi_1, \pi_2$  are two paths from  $f$  to  $g$ . Then the two paths are compatible, that is, they do not contain complementary literals.

Proof. For  $n = 1$  clearly. For  $n > 1$  by using disjoint splitting.

**So, in this case, QMU(2) is solvable in polynomial time.**

Suppose  $F$  is lean  $d(F) = 2$ .

Case 1.  $F - \{f\} \in \text{MU}(1)$ , and for all  $h \in F - \{f\}$ ,  $F - \{h\}$  is satisfiable.

Case 2.  $F - \{f\} \in \text{MU}(1)$ , and there is  $g \in F - f$ ,  $F - g$  is in  $\text{MU}(1)$  (then this case)

Case 3.  $F' \subseteq F \in \text{MU}(1)$  such that  $F - F'$  has more than one clauses.

Case 4.  $F \in \text{MU}(2)$

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