1 General Problems

Configuration and Specification (Sets)

Equivalence Problem

Input: $C = \{\alpha_1, \cdots, \alpha_n\}$
Query: Whether $\exists K \subseteq C : K \equiv \beta$

Retricted Equivalence(R-equivalence

Input: $C = \{\alpha_1, \cdots, \alpha_n\}$, $beta$, and a set V of variables
Query: Whether $\exists K \subseteq C : K \equiv^V \beta$

Implication Problem

Input: $C = \{\alpha_1, \cdots, \alpha_n\}$, $beta$
Query: Whether $\exists K \subseteq C : K \models \beta$

Note: In all problems K is demanded satisfiable.

$$\alpha \equiv_V \beta \Longleftrightarrow \forall \gamma \text{ over } V, (\alpha \models \gamma \Leftrightarrow \beta \models \gamma)$$

$$\alpha \models_V \beta \Longleftrightarrow \forall \gamma \text{ over } V, (\beta \models \gamma \Longrightarrow \alpha \models \gamma)$$

Euivalence Problem is in $\mathrm{P}^{\mathrm{NP[log]}}$
R-Equivalence Problem same as above
Implication Σ_2^P?

2 Resticted to DHORN

C: set of DHORN, β: DHORN

Equivalence problem is in PTIME

Idea: Let

$$\mathrm{IMP}(C, \beta) := \{\alpha \mid \alpha \in C \text{ and } \beta \models \alpha\}$$

Then

$$\exists K \subseteq C (K \equiv \beta) \Longleftrightarrow \mathrm{IMP}(C, \beta) \models \beta$$

Implication problem is trivial

The existence of K is equivalent to $C \models \beta$

The V-equivalence problem

$\mathrm{IMP}(C, \beta, V) := \{\alpha \in C \mid \beta \models_V \alpha\}$

The existence of $K \equiv_V \beta$ is equivalent to $\mathrm{IMP}(C, \beta, V) \equiv_V \beta$.

we guess it is co-NP complete.
See Hans's book page 251

3 Restricted to HORN

Equivalence Problem: same as the DHORN case (PTIME).

iff $\mathrm{IMP}(C, \beta)$ is satisfiable and $\mathrm{IMP}(C, \beta) \models \beta$

V-Equivalence Problem: idea Same as DHORN but Σ_2^P-complete

$$\Phi := \exists y_1, \cdots, y_k \forall x_1, \cdots, x_m \, (c_1 \vee \cdots \vee c_n)$$

Where c_i is a conjunction of literals.

for each variable z, introduce a new variable $\pi(\neg z)$. Let $\pi(z) = z$.

introduce U.

for a clause $c = L_1 \wedge \cdots \wedge L_s$, we write $\pi(c) := \pi(L_1) \wedge \cdots \wedge \pi(L_s)$

$$\begin{aligned} C := \quad & \{\pi(c_i) \to U \mid i = 1, \cdots, n\} \cup \\ & \{\neg\pi(\neg x_j) \vee \neg x_j \mid j = 1, \cdots, m\} \cup \\ & \{\neg\pi(\neg y_i) \vee \neg y_i \mid i = 1, \cdots, k\} \cup \\ & \{y_i, \pi(\neg y_i) \mid i = 1, \cdots, k\} \end{aligned}$$

$$\beta := U \vee \left(\bigvee_{j=1}^{k} (\neg\pi(\neg x_j) \wedge \neg x_j) \right)$$

(Remark: using Tsting algorithm)

Let

$$V := \{U\} \cup \{x_1, \cdots, x_m\}$$

($\exists K \subseteq C$ such that $K \equiv_V \beta$) if and only if Φ is true.

Implication Problem
NP-complete
Given a 3CNF F

$$\bigwedge_{i=1}^{m} (L_{i,1} \vee L_{i,2} \vee L_{i,3}) \quad \text{over } x_1, \cdots, x_n$$

For each $i = 1, \cdots, m$, pick a new variable z_i. For each $j = 1, \cdots, n$ we pick a new variable $\pi(\neg x_j)$. For convenience, we also write x_j as $\pi(x_j)$.

Define C

$$
\begin{aligned}
C := \quad & \bigcup_{i=1}^{m} \{\pi(L_{i,1}) \to z_i, \pi(L_{i,2}) \to z_i, \pi(L_{i,3}) \to z_i\} \cup \\
& \bigcup_{j=1}^{n} \{\to x_j, \to \pi(\neg x_j\} \cup \\
& \{z_1 \wedge \cdots \wedge z_m \to z\}
\end{aligned}
$$

Define

$$\beta := z \wedge \bigwedge (\neg \pi(\neg x_j) \vee \neg x_j)$$

Impliation problem iff F is satisfiable

Specification Problem

Given a partial configuration K, demand β, a set of variables V, W
Looking for σ over W such that

1. $K \wedge \sigma(W) \equiv \beta$
2. $K \wedge \sigma(W) \equiv_V \beta$
3. $K \wedge \sigma(W) \models \beta$

Query Learning
black box α
equivalence query. Guess a β ask whether $\alpha \equiv \beta$. If the answer is no, output a truth assignment satisfying α and $\neg\beta$ or satisfying $\neg\alpha$ and β.

membership query
guess a truth assignment v answer $v(\alpha) = 0$ or 1.