COMS10014 Worksheet 3: Logic 2

1. Truth Tables and Terminology

a. (★★) For the following terms, draw a truth table, then classify them as one of: tautology, contingency, or contradiction.

1. $p \to (q \to p)$
2. $\neg p \vee (\neg p \to q)$
3. $(p \vee q) \to (p \wedge q)$
4. $(p \wedge q) \to (p \vee q)$
5. $(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
6. $p \vee (q \to p)$
7. $(p \to q) \vee (q \to p)$
8. $\neg(p \to q) \wedge p$

b. (★) Which of the above terms are satisfiable?

2. Implications (★★)

State the converse, the inverse, and the contrapositive of each of the following propositions:

1. If we have frost tonight, then I won't cycle in tomorrow.
2. My cat comes in whenever it is hungry.
3. When you hear the fire alarm, you need to vacate the building.
4. If it is hot tomorrow, then we will go swimming.
5. People who don't pay their tax by the deadline will be fined.

3. NAND

1. (★) Draw a truth table for the NAND ($\uparrow$) operation.
2. (★) Is $\uparrow$ commutative?
3. (★★) Is $\uparrow$ associative?

Express the following propositions using only the NAND operation (and brackets).

4. (★★) $p \to \neg q$
5. (★★) $p \wedge (q \wedge r)$
6. (★★★) $\neg(p \oplus q)$

4. Normal Forms

1. (★★) What is the big difference in the proofs of the following two claims:
    a. The set $\{\wedge,\ \vee,\ \neg\}$ is functionally complete.
    b. The set $\{\wedge,\ \neg\}$ is functionally complete.

2. (★★★) Give a procedure to turn a formula, presented as a truth table, into a term in CNF. (The ideas are similar to the DNF algorithm discussed in the lectures and notes, but there is an extra step.) Note, this exercise has three parts:
    a. Find a procedure.
    b. Show that your procedure always produces a formula in CNF.
    c. Show that your procedure always produces a formula that is equivalent to the original formula.

3. (★★) Aisha has written in her notebook: "$a \vee b \vee c$ is a formula in CNF". Brenda leans over and says this formula is in DNF as it has $\vee$ operators, not $\wedge$ ones.
    I propose a compromise: Aisha and Brenda are both correct! Explain this.

4. (★) Based on your insights from 3., which of the following are in CNF, DNF, or both?
    1. $\neg A \wedge B$
    2. $\neg A$
    3. $\neg A \wedge \neg B$
    4. $\neg A \vee B$
    5. $\neg(A \wedge B)$
    6. $\neg(A \vee B)$
    7. $\neg(A \wedge \neg B)$
    8. $(A \wedge B) \vee (C \wedge D)$

5. (★★★) Darren has read the part of the lecture notes about how turning a CNF formula into DNF is hard to do efficiently – at least, as far as we know. He has also noticed that we have procedures for turning truth tables into both DNF (in the lectures) and CNF (part 2 of this exercise). He wonders why, starting with a formula in CNF, we cannot simply turn it back into a truth table efficiently as each clause "encodes" exactly one of the rows, then make the DNF formula from that.
    a. Find the mistake in Darren's argument, that is not related to efficiency.
    b. Find an example where a CNF formula with n variables, for which Darren's procedure does work, produces a 'much bigger' DNF formula. What is the worst possible case?