COMS10014 Worksheet 7: Predicates

1. Introduction (★)

a. Let $P(x)$ denote the predicate $x > 42$. What are the truth values of
   1. $P(1)$
   2. $P(42)$
   3. $P(128)$

b. Let $Q(x)$ be the predicate "the sentence x contains the word **Gruffalo**". (No-one said that variables always had to stand for numbers.) What are the truth values of
   1. *P("The truth is out there.")*
   2. *P("Oh no! Oh my! It's a Gruffalo!")*
   3. *P("There is no Gruffalo in this sentence.")*

c. Let $R(x, y)$ be the predicate "x is planet number y of our solar system", where the planet closest to the centre is number 1. Feel free to look up the order online if you need to. What are the truth values of
   1. *R(Earth, 3)*
   2. *R(Jupiter, 4)*
   3. *R(Pluto, 9)*

2. Quantifiers (★)

Consider the domain of all humans and the predicates

$P(x)$: x is perfect
$F(x)$: x is your friend.

Write the following sentences as formulas:

a. No one is perfect.
b. Not everyone is perfect.
c. All your friends are perfect.
d. At least one of your friends is perfect.
e. Everyone is your friend and is perfect.
f. Not everybody is your friend or someone is not perfect.

3. Negation of Quantifiers (★★)

For each of the following statements, first write the statement as a formula, clearly defining your predicates. Next, negate the statement, and simplify the negated statement so that negation operators only appear directly before predicates. Finally, write the negation of the statement in English. Do not simply put "It is not true that" or words to that effect in front of the statement.

a. Some old dogs can learn new tricks.
b. No rabbit knows calculus.
c. Every bird can fly. Assume the domain is all birds.
d. Every bird can fly. Assume the domain is all *animals*.
e. There is no dog that can talk.
f. No-one in this class knows both French and Spanish.

4. Predicates in English (★★)

Translate the following formulas a. – d. into English, using the predicates

$F(p)$	Printer p is out of service.
$B(p)$	Printer p is busy.
$L(j)$	Print job j is lost.
$Q(j)$	Print job j is queued.

a. $\exists p. \big(F(p) \wedge B(p)\big) \rightarrow \exists j\,.\,L(j)$
b. $\forall p. B(p) \rightarrow \exists j\,.\,Q(j)$
c. $\exists j. \big(Q(j) \wedge L(j)\big) \rightarrow \exists p. F(p)$
d. $(\forall p. B(p) \wedge \exists j. Q(j)) \rightarrow \exists j. L(j)$

5. Quantifiers and Integers

1. (★) Let $Q(x)$ be the statement $x + 1 > 2x$, in the domain of all integers. Determine the truth values of the following statements.

a. $Q(0)$
b. $Q(-1)$
c. $Q(1)$
d. $\exists x. Q(x)$
e. $\forall x. Q(x)$
f. $\exists x. \neg Q(x)$
g. $\forall x. \neg Q(x)$

2. (★★) Translate the following statements into English, over the domain of "numbers". Then decide whether they are true for the integers.

a. $\forall x \exists y. (x > y)$
b. $\forall x \forall y. \big((x = 0) \wedge (y = 0) \rightarrow (xy = 0)\big)$
c. $\exists z \forall x \forall y. (xy = z)$
d. $\forall x \forall y \exists z. (xy = z)$

6. Oliver!

1. (★) Let $L(x, y)$ be the statement "x loves y", over the domain of people in the world. Write the following statements with quantifiers.
    a. Everybody loves Oliver.
    b. Everybody loves somebody.
    c. There is somebody whom everybody loves.
    d. Nobody loves everybody.
    e. There is somebody whom Oliver does not love.
    f. There is somebody whom nobody loves.

2. (★★) Negate the statements a-f above, rewrite the negations so that negation operators only appear directly before predicates, and translate the results to English.

3. (★★) Some slightly harder statements – write these as formulas:

    a. There is exactly one person whom everybody loves.
    b. There are exactly two people whom Oliver loves.
    c. There is someone who loves no one besides themselves.