COMS10014 Class Test 2 Solutions

Question 1

The original statement is an implication with the antecedent “Ash likes one Pokémon” and the consequent “Ash likes more than one”. For an implication to be false, the antecedent must be true and the consequent false, so we can deduce that Ash likes exactly one Pokémon.

Therefore:

a. Satisfiable (namely if this is the only one he likes)
b. Unsatisfiable (he cannot like more than one)
c. Unsatisfiable (if the antecedent is false, the original implication is true)
d. Satisfiable (in fact, a tautology if the original statement is false)
e. Unsatisfiable (he cannot like more than one)
f. Satisfiable (in the three cases where he likes exactly one of the others)

Question 2

Line 2 is $\wedge$ elimination. We conclude $\neg a$ from $\neg a \wedge \neg b$.

Line 5 is $\neg$ elimination. We conclude F since both a and $\neg a$ are in scope.

Line 11 is $\vee$ elimination on the $a \vee b$ on line 10, as both $a \vdash F$ and $b \vdash F$ are in scope.

Line 13 is $\neg$ introduction. From $(a \vee b) \vdash F$ on line 12, we conclude $\neg(a \vee b)$.

Question 3

A statement of the form $\exists x \forall y. Z(x, y)$ always implies the matching $\forall y\, \exists x. Z(x, y)$ one (namely, one can always pick the single x value that satisfied the original formula). This makes c) and d) tautologies; the and/or distinction is just a distraction here.

The other way round is false. If we set both $P(x, y)$ and $Q(x, y)$ to $x = y$ then we have a counter-example to both a) and b), so they are not tautologies.

Question 4

The induction hypothesis is on Line 8 where we substitute $L(n) = R(n)$.

The base case is on Line 4 where we check $L(1) = R(1)$.

Question 5

a. We have $\mathcal{P}(A) = \{\emptyset, \{1\}, \{3\}, \{1,3\}\}$ and $\mathcal{P}(B) = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$ so the intersection is $\{\emptyset, \{3\}\}$.
b. The complement of A is $\{2\}$ with powerset $\{\emptyset, \{2\}\}$.
c. This is $\{\emptyset, 1, 3, \{2\}, \{3\}, \{2,3\}\}$.
d. $A \setminus B = \{1\}$ so $\{\emptyset, \{1\}\}$.

Question 6

1. r = a (option g.)

2. In the end, we want $a_0 = qb_0 + r$ (we need a_0 here because we have modified a, but $a_0 = qb + r$ would do just as well as b does not change). The invariant is $a_0 = qb_0 + a$, so $X = a_0$, $Y = q$, $Z = a$ and $\triangle$ stands for $+$.

- At the start, $a = a_0$ and $q = 0$ so the invariant holds.
- Suppose that before a pass through the loop, $a_0 = Qb_0 + A$. Then we set $q = Q + 1$ and $a = A - b_0$ (b does not change so $B = b = b_0$ always). Therefore,
$$a_0 = (q-1)b_0 + (a + b_0) = qb_0 + a$$
so the invariant is preserved.
- At the end of the loop, $a_0 = qb_0 + a$ and $a < b$ (the termination condition) and $a \geq 0$ (the loop variant, not part of the question here) therefore $r = a$ meets the conditions of Euclid's theorem.