COMS10014 Worksheet 2: Logic 1

1. Truth Tables (★)

1. Use truth tables to determine whether logical implication $a \to b$ is commutative or not.
2. Show, using truth tables, that logical or $(a \vee b)$ is associative and commutative.
3. Prove DeMorgan's Laws using truth tables.
4. Show, using truth tables, that $a \to b$ is semantically equivalent to $\neg a \vee b$.
5. Show, using truth tables, that $a \oplus b$ is semantically equivalent to $(a \vee b) \wedge \neg(a \wedge b)$.
6. Show, using truth tables, that $a \oplus b$ is semantically equivalent to $(a \wedge \neg b) \vee (\neg a \wedge b)$.

Note: the method to use in this question is purely mechanical – if you had a computer program to parse the expressions involved, then you could program a computer to do this exercise, with no thinking involved. In fact, there are online tools that will do exactly this for you. This question is one-star because once you have practiced the method enough, it should be almost automatic for you, even if longer expressions take more time.

The equivalences that you have shown in 4.–6. are worth remembering.

2. Propositions

Using

p	It is raining.
q	It is snowing.
r	It is freezing.

express the following statements in propositional logic:

(★) statements:

1. It is not snowing.
2. It is raining, but it is not snowing.
3. If it is freezing, then it is snowing.
4. If it is freezing or snowing, then it is not raining.

(★★) statements:

5. Either it is not raining or, if it is not raining, then it is snowing.
6. Either it is not raining or, if it is not raining but freezing, then it is snowing.
7. It is raining if and only if it is not snowing.
8. If it is both raining and freezing, then it is snowing.

(★★) Next, express the following in clear English:

9. $p \to r$
10. $q \leftrightarrow r$
11. $q \to (p \wedge r)$
12. $(p \vee r) \leftrightarrow q$

3. Brackets (★★)

Eliminate as many brackets as possible in the following expressions. You can eliminate brackets that lead to a different term-as-a-tree, as long as it is semantically equivalent to the original one.

1. $(p \to q) \to r$
2. $(a \lor b) \land (c \lor d)$
3. $(a \land b) \lor (c \land d)$
4. $\neg(a \lor b)$
5. $(a \land b) \land c$
6. $((p \land q) \to (p \to q))$
7. $\left(\left((p \lor (\neg q)) \lor s\right) \leftrightarrow \left(\left(\neg(s \land (\neg r))\right)\right)\right)$
8. $\left(\left(((\neg p) \lor s) \land q\right) \lor (\neg \left(\neg(\neg(s))\right)\right)$

4. Theory Questions

1. (★) For two variables *p* and *q*, how many assignments of the truth values T and F exist?
2. (★★) Let $V = \{T, F\}$ be the set of truth values. How many binary operations on truth values exist, that is functions that take two truth values as input and output one truth value? Two functions are considered the same, if they have the same truth tables.
3. (★★) Give a truth table that lists all functions of degree 2 in some natural order, and explain your approach. Identify the functions whose names you already know, such as ∧ (conjunction, logical AND) and so on.
4. (★★) In general, how many assignments of truth values exist for a compound proposition with *n* distinct variables – that is, how many functions exist that take *n* truth values as input, and produce one truth value as output?