Study of gamma ray absorption in different materials using a NaI detector.

Xabier García Andrade

UPPSALA UNIVERSITET

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Abstract

The task we were commissioned was to find out which material would be the best option to build a radiation shield. In order to find out, we designed an experiment where we measured the intensity of the radiation with a material for different thicknesses. We used a Cs-137 source to emit the radiation and an NaI detector to detect the gammas. After that, the signal is amplified and displayed as a spectrum in a computer. After using this values, we can plot the results and perform a linear fit to obtain the relevant quantity here, the linear attenuation coefficient. We checked the results for each material and resolved that the one which would make a better shield would be Pb (Lead).

1 Introduction

The scope of the task was to study how γ-rays are absorbed in different materials in order to build a radiation shield. We need to determine the dimensions of the shield after reflecting on how radiaton is absorbed for each component.

The theoretical background behind gamma ray absorption is given by the following equation:

$$dI(x) = -I(x) \cdot n \cdot \sigma \cdot dx \tag{1}$$

Where n ($\frac{atoms}{cm^3}$) represents the number of atoms per unit of volume and σ (cm^2) is the cross section. x (cm) would represent the thickness of the material. This represents the change of intensity for a narrow beam of mono-energetic photons as a function of the thickness of a material. We can integrate the equation to get:

$$I(x) = I_0 e^{-n\sigma x} \tag{2}$$

Where I_0 is the initial intensity of the incoming radiation. This can also be expressed in terms of the linear attenuation coefficient μ (cm^{-1}):

$$I(x) = I_0 e^{-\mu x} \tag{3}$$

Interaction of electromagnetic radiation with matter can occur by means of fundamentally three proccesses:

Photoelectric effect:

Photoelectric absorption occurs when a photon approaches a material and its energy is transferred into releasing an electron, usually one from the inner electron shells. The photoelectron is released with kinetic energy:

$$T = E_\gamma - B_e$$

In this case, T is the kinetic energy of the released electron, E_γ is the energy of the incoming radiation and B_e is the binding energy of the electron.

Compton Scattering:

Compton effect is the pure collision of a photon with an electron, which leads to part of the photon´s energy being converted into kinetic energy for the electron. The photon is scattered and its wavelength is shifted. The photon´s final energy is given by:

$$E_\gamma' = \frac{E_\gamma}{1 + (E_\gamma/mc^2)(1 - cos\theta)} \tag{4}$$

In this formula E_γ is again the energy of the incoming radiation, while the primed energy refers to the energy of the photon after being scattered. θ is the angle of scattering.

Pair production:

If a photon has the sufficient energy, its entire energy is converted into the creation of an electron-positron pair with total kinetic energy given by:

$$T_{-} + T_{+} = E_{\gamma} - 2mc^2 \tag{5}$$

T_{-} and T_{+} are respectively the kinetic energy of the electron and the positron. E_{γ} is the energy of the photon and $2mc^2$ refers to the rest mass of both created particles.

Now we can discuss how we treated the error of the physical quantities that we measured. We followed the criteria of writing the error up to two significant figures, writing the value of the measurement in a agreement with this. With this measurements we also calculated other values, whose error has to be propagated according to the following formula:

$$\sigma_{(y)} = \sqrt{\sum_{i}^{n} \left[\left(\frac{\partial y}{\partial x_i} \right)^2 \cdot \sigma_i^2 \right]} \tag{6}$$

2 Experimental Method and setup

The key component of the experiment is the ^{137}Cs collimated source, which emits gamma radiation. Encased in plastic, it is pointed towards pieces of different materials with variable thickness. The setup is shown in the next figure:

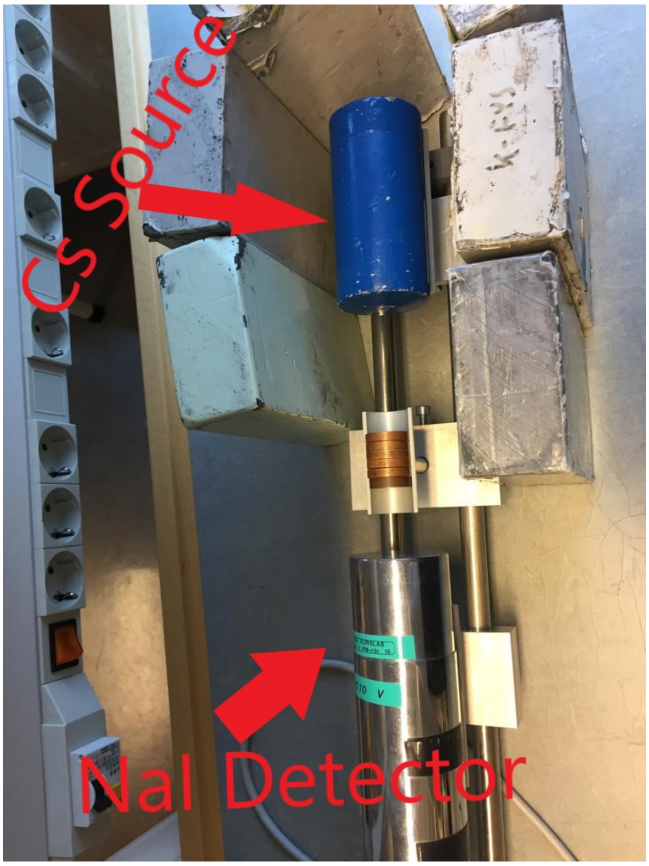


Figure 1: Source and NaI detector.

After going through these slices, the radiation is attenuated and it is captured by a NaI detector. The signal must be amplified using a spectroscopy amplifier. In order to visualize the final signal and confirm its validity, we also attached an oscilloscope (figure (3)). The important features for the amplifier can be seen in the figure as well:

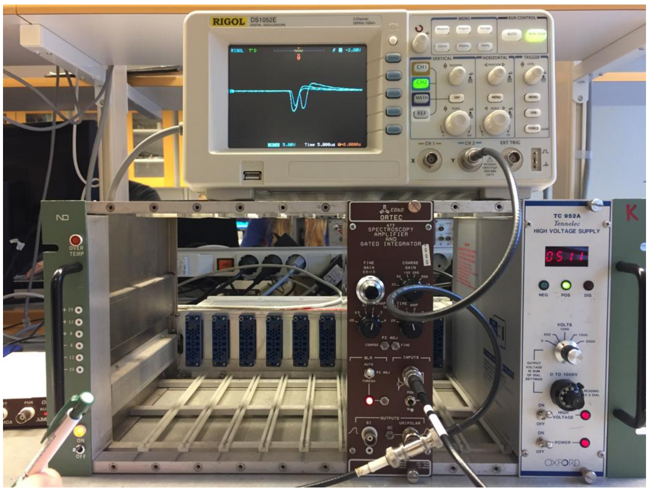


Figure 2: Oscilloscope and amplifier.

After going through these slices, the radiation is attenuated and it is captured by a NaI detector.

The result of the measurement is shown in the computer by means of the software MAESTRO. We will show an example of how the spectrum appears in MAESTRO [2]:

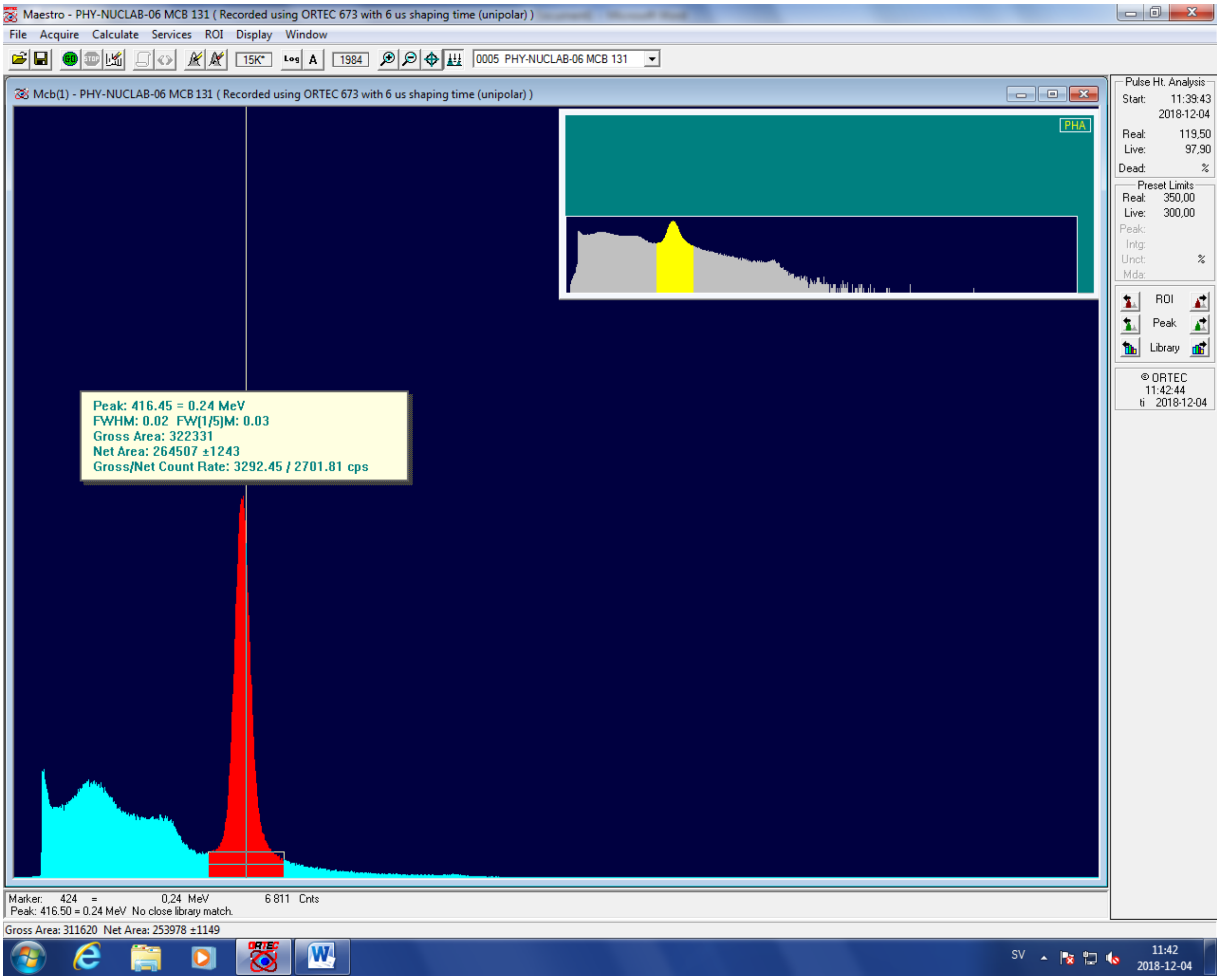


Figure 3: Sample of spectrum as displayed by MAESTRO.

For each measurement, we wrote down the net area, live time and net count rate. Apart from that, as we increased the thickness, we measured each piece of material with a sliding gauge.

3 Method and Error estimation

The valuable quantity that will give us the intensity is the net area, which is the result of substracting the background noise to the gross area. In this case, the net area relates to the number of counts. We have to take into consideration that the number of counts is Poisson distributed. Knowing this, we can determine its standard deviation.

$$\sigma_{(N)} = \sqrt{N} \tag{7}$$

Then we can easily calculate the intensity with the following formula:

$$I = \frac{N}{t} \qquad \sigma_{(I)} = \sqrt{\left(\frac{\partial I}{\partial N}\right)^2 \cdot \sigma_{(N)}} = \sqrt{\frac{1}{t^2} \cdot N} = \frac{\sqrt{N}}{t} \tag{8}$$

Where t is the live time. Then we will need to do a linear fit, so it will be interesting to show the error propagation for the logarithm of the intensity:

$$\sigma_{(ln(I))} = \sqrt{\left(\frac{\partial ln(I)}{\partial I}\right)^2 \cdot \sigma_{(I)}} = \sqrt{\frac{1}{I^2}\frac{N}{t^2}} = \frac{1}{\sqrt{N}} \tag{9}$$

We can also consider the error that we assign to the measurement of the thickness of each material. We decided that the resolution of the sliding gauge would be a correct quantity:

$$\sigma_{(x)} = 0.10 \;\; cm \tag{10}$$

In order to increase the thickness of our sample, we stacked several pieces of the same material. Since each piece was measured separaterly, the total error will be given by:

$$\sigma_{(t)} = \sqrt{\left(\sum_{i=1}^{n} \sigma_i\right)^2} \tag{11}$$

Where n is the total number of pieces stacked in each measurement.

With all this quantities, we can calculate μ for each material. In order to do that, we will take logarithms in equation (3). We end up having:

$$ln(I) = -\mu \cdot x + ln(I_0) \tag{12}$$

If we plot $ln(I)$ as a function of thickness, we can verify whether this follows a linear behaviour. Then, we can try a linear regression $y = a \cdot x + b$ and retrieve the linear attenuation coefficient as $\mu = -a$ with $\sigma_{(\mu)} = \sigma_{(a)}$.

After finding this value, we can also calculate the half-thickness of an element, which is defined as the thickness which leads to a 50% attenuation. Then:

$$0.5 I_0 = I_0 e^{-\mu x}$$

$$x_{1/2} = \frac{0.693}{\mu} \qquad \sigma_{(x_{1/2})} = \frac{\sigma_\mu \cdot 0.693}{\mu^2} \tag{13}$$

In order to perform a linear regression $y = b \cdot x + a$, we need to calculate the coefficients that reduce the mean squared error. Each coefficient is calculated with the following formula:

$$a = \frac{\sum_i w_i y_i \sum_i w_i x_i^2 - \sum_i w_i x_i \sum_i w_i x_i y_i}{\Delta} \qquad i = 0, 1, \ldots, N-1 \tag{14}$$

$$b = \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\Delta} \quad i = 0, 1, \ldots, N-1 \tag{15}$$

Where $w_i = \frac{1}{\sigma(y_i)^2}$ is the statistical weight and σ_i is the error of each measurement. Δ is the determinant of the following matrix:

$$\begin{pmatrix} \sum_i w_i & \sum_i w_i x_i \\ \sum_i w_i x_i & \sum_i w_i x_i^2 \end{pmatrix} \tag{16}$$

The error of both a and b can be calculated as:

$$s(a) = \sqrt{\frac{\sum_i w_i x_i^2}{\Delta}} \tag{17}$$

$$s(b) = \sqrt{\frac{\sum_i w_i}{\Delta}} \tag{18}$$

And finally the Pearson coefficient will be:

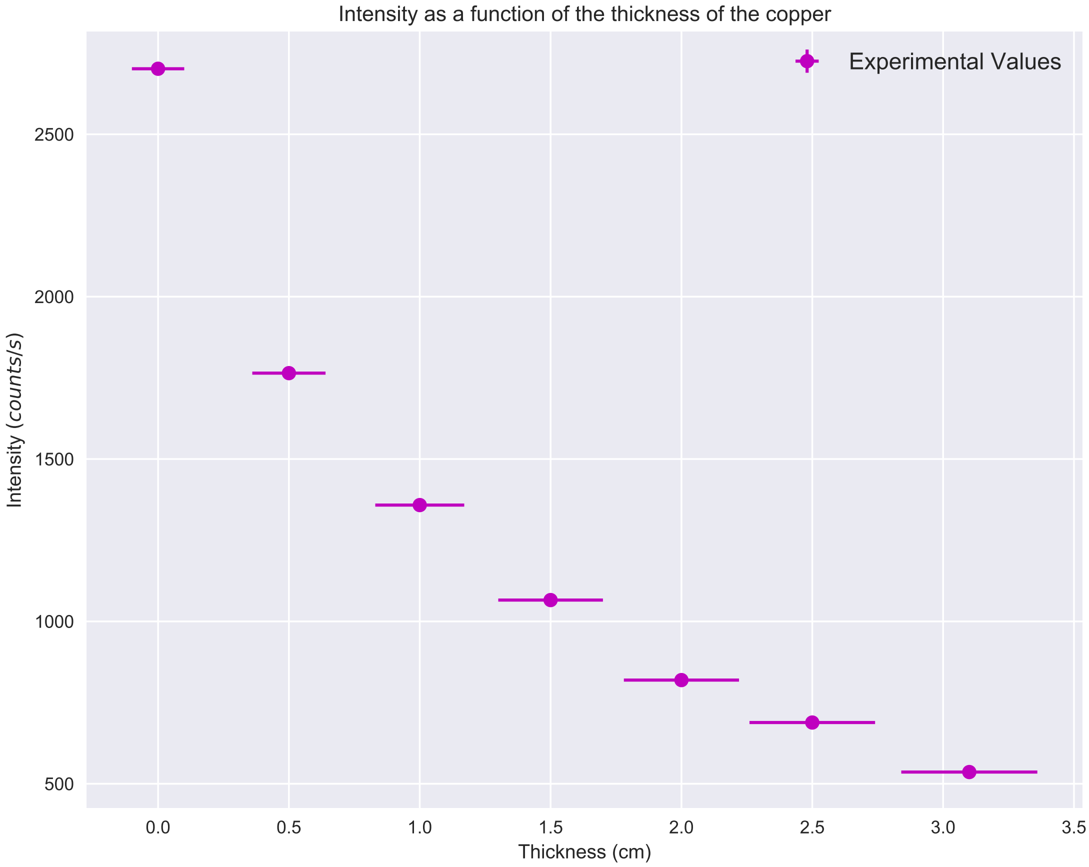
$$r = \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\sqrt{\left(\sum_i w_i \sum_i w_i x_i^2 - (\sum_i w_i x_i)^2\right)\left(\sum_i w_i \sum_i w_i y_i^2 - (\sum_i w_i y_i)^2\right)}} i = 0, 1, \ldots, N-1 \tag{19}$$

4 Measurements and results

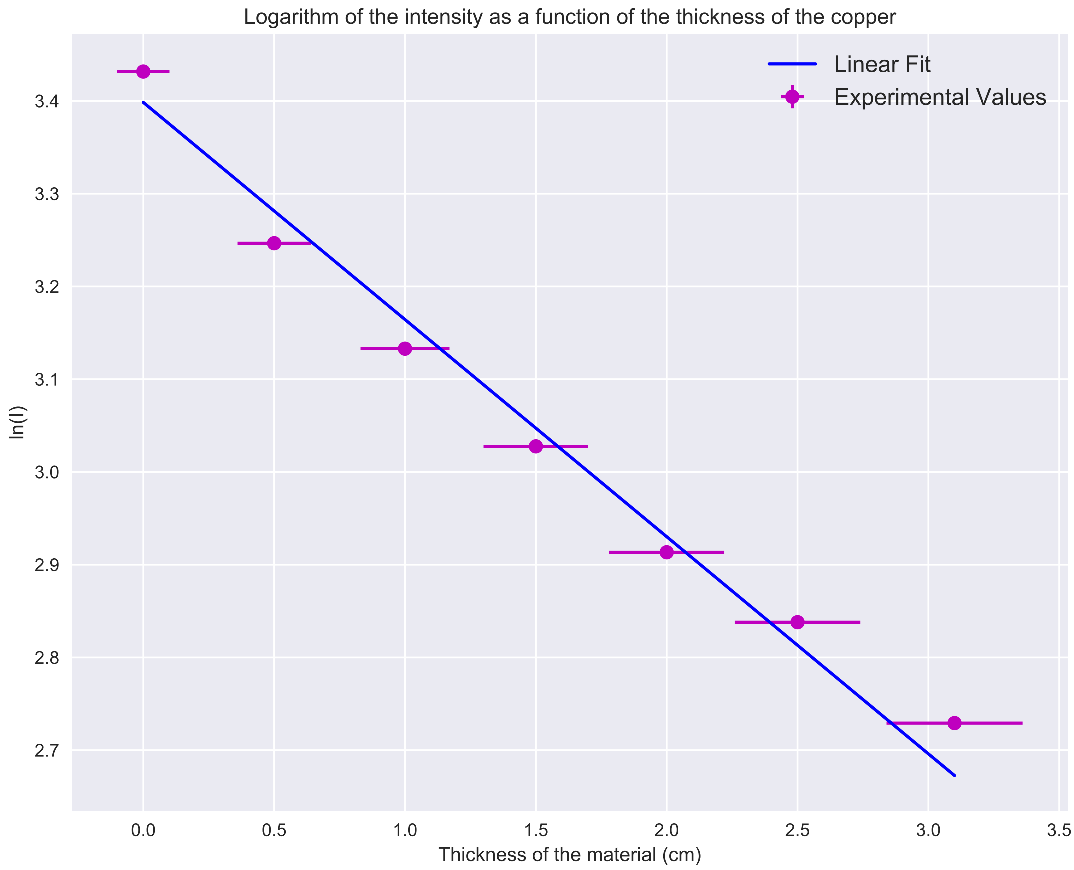
4.1 Copper

x (cm)	$\sigma_{(x)}$ (cm)	N	$\sigma_{(N)}$	I $counts/s$	$\sigma_{(I)}$ counts/s	$ln(I)$	$\sigma_{(ln(I))}$
0.00	0.00	2.6E+05	5.1E+02	2701.8	5.3	3.4317	0.0019
0.50	0.10	1.7E+05	4.2E+02	1764.5	4.2	3.2466	0.0024
1.00	0.14	1.3E+05	3.6E+02	1358.2	3.7	3.1329	0.0027
1.50	0.17	1.0E+05	3.2E+02	1065.7	3.3	3.0276	0.0031
2.00	0.20	8.0E+04	2.8E+02	819.3	2.9	2.9134	0.0035
2.50	0.22	6.7E+04	2.6E+02	688.6	2.7	2.8380	0.0039
3.10	0.24	5.2E+04	2.3E+02	536.1	2.3	2.7292	0.0044

This values can be better visualized with a plot:



We can certify that this follows the expected exponential behaviour. In order to obtain the linear attenuation coefficient we take logarithms in both sides and perform a linear regression. The result is the following:



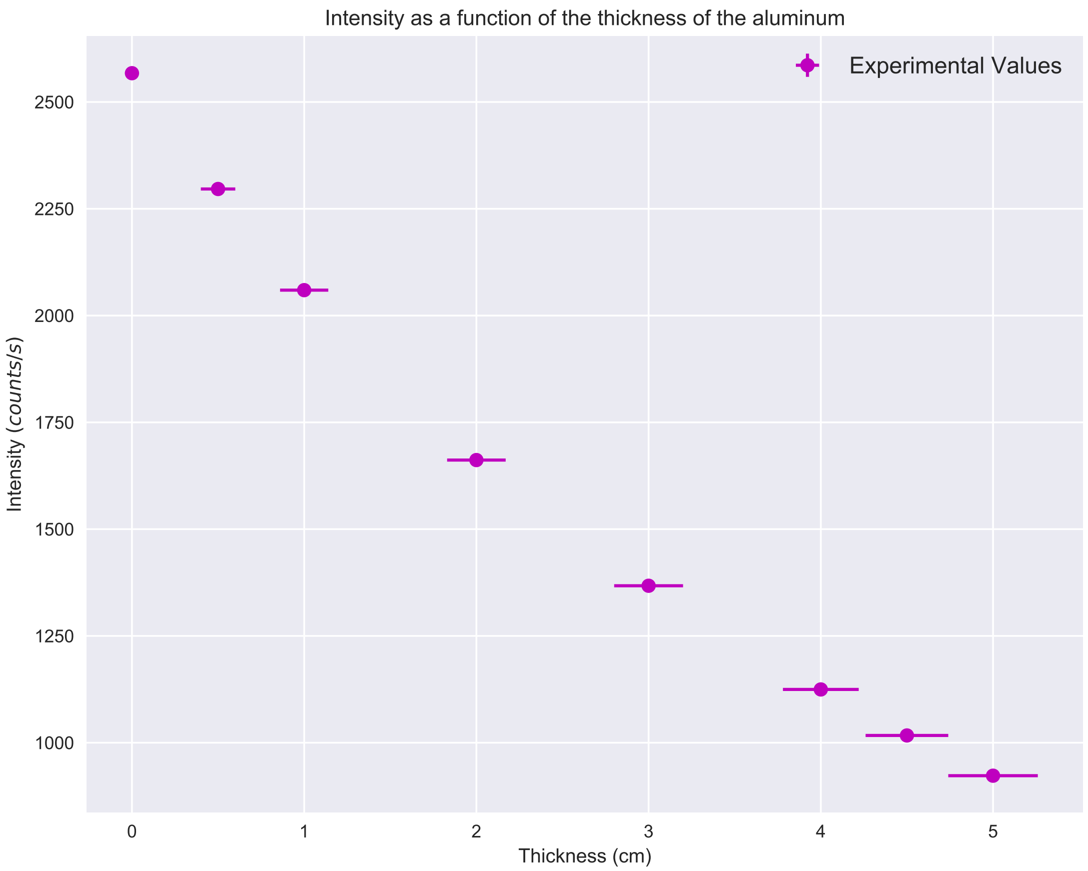
Obtained values after the fit:

$$a = (-0.234 \pm 0.015)\ \ cm^{-1} \qquad\qquad b = (3.398 \pm 0.021)$$

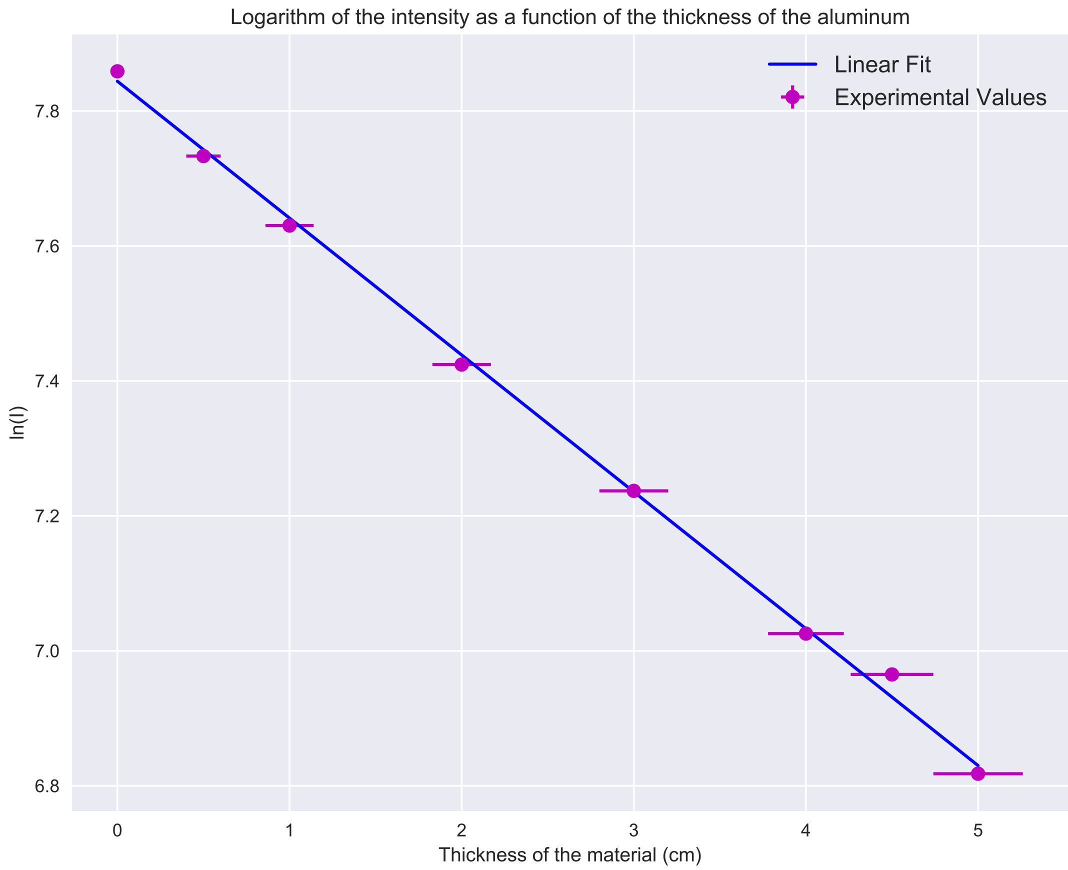
4.2 Aluminum

This values can be better visualized with a plot:

x (cm)	$\sigma_{(x)}$ (cm)	N	$\sigma_{(N)}$	I $counts/s$	$\sigma_{(I)}$ counts/s	$ln(I)$	$\sigma_{(ln(I))}$
0.00	0.00	2.9E+05	5.4E+02	2567.4	4.8	7.7095	0.0019
0.50	0.10	1.1E+05	3.3E+02	2296.3	6.9	7.6610	0.0030
1.00	0.14	2.3E+05	4.8E+02	2059.5	4.3	7.5138	0.0021
2.00	0.17	1.7E+05	4.2E+02	1661.7	4.0	7.2206	0.0024
3.00	0.20	8.1E+04	2.8E+02	1367.5	4.8	7.1359	0.0035
4.00	0.22	1.3E+05	3.6E+02	1124.7	3.1	7.0511	0.0028
4.50	0.24	1.2E+05	3.4E+02	1017.0	3.0	6.9073	0.0029
5.00	0.26	1.1E+05	3.3E+02	922.8	2.8	6.8651	0.0031



We can certify that this follows the expected exponential behaviour, even though that if we only considered the intensity for low values of thickness, we were not able to observe this trend. Then, we took logarithms as in the previous case and carried out a linear regression. The result was the following:



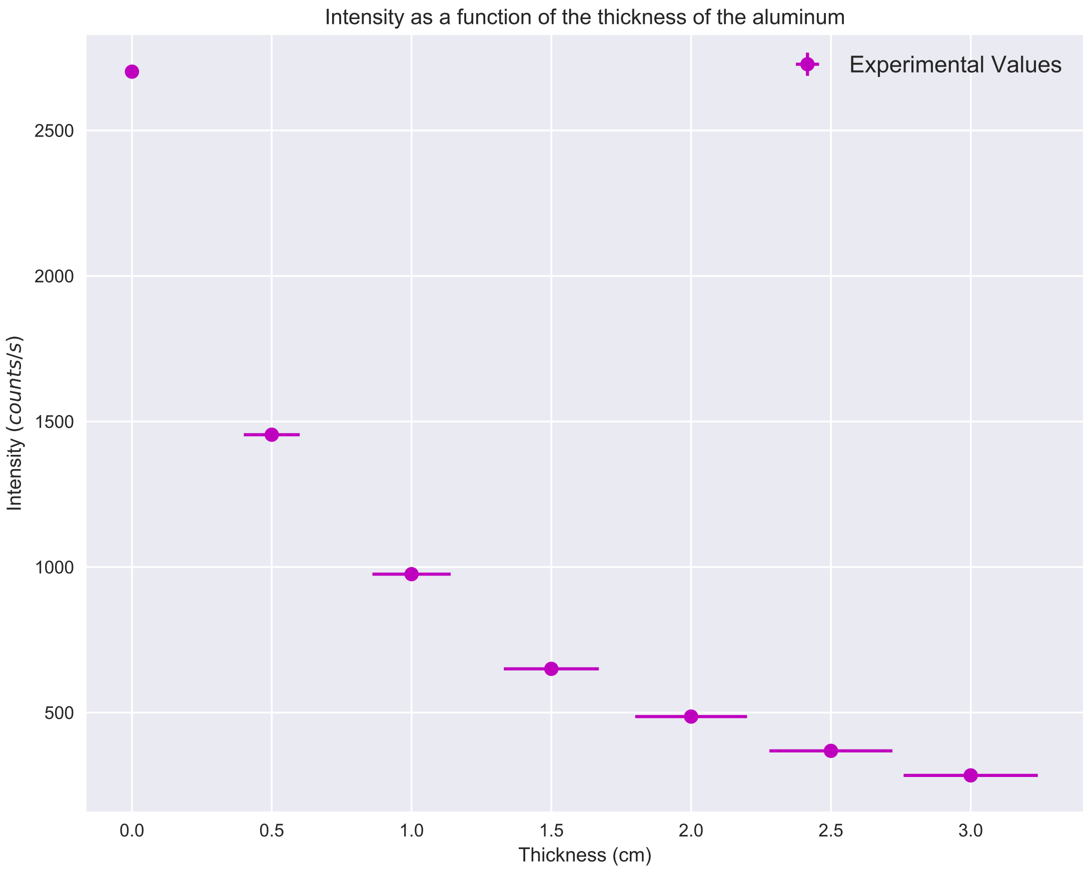
Obtained values after the fit:

$$a = (-0.202 \pm 0.035) \;\; cm^{-1} \qquad\qquad b = (7.844 \pm 0.023)$$

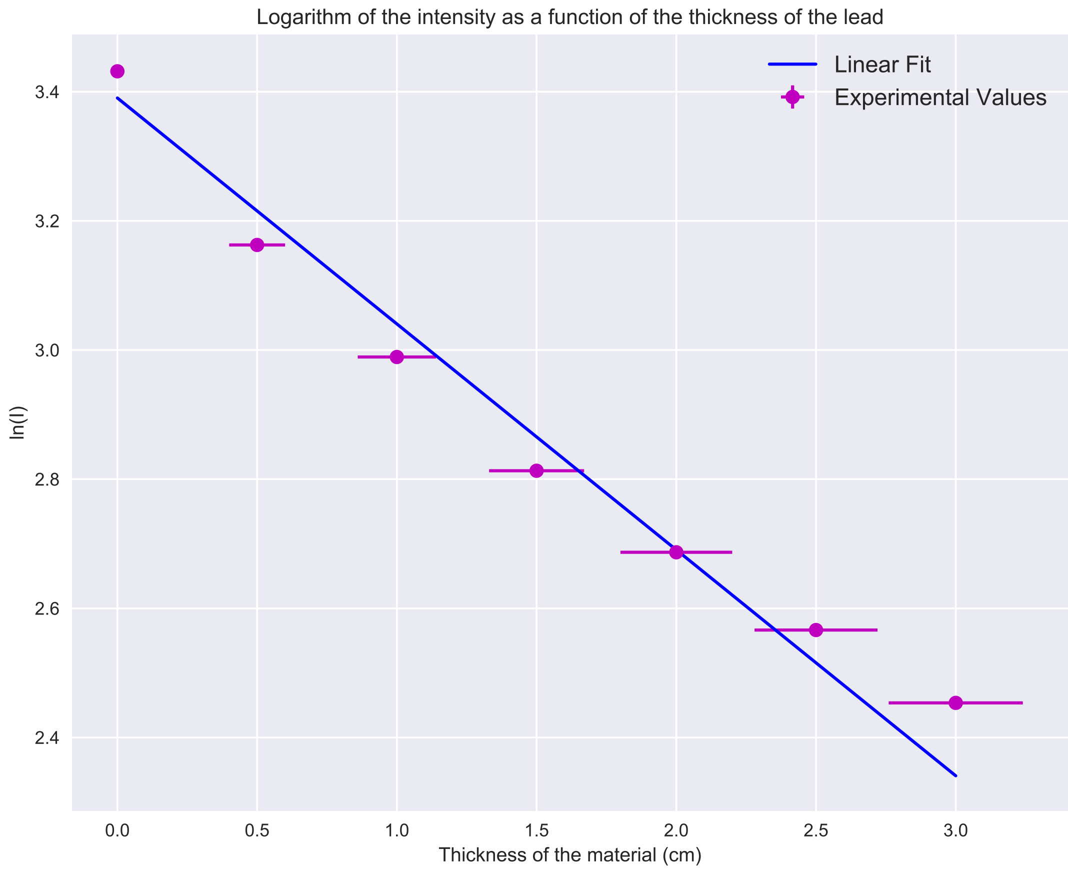
4.3 Lead

This values can be better visualized with a plot:

x (cm)	$\sigma_{(x)}$ (cm)	N	$\sigma_{(N)}$	I $counts/s$	$\sigma_{(I)}$ counts/s	$ln(I)$	$\sigma_{(ln(I))}$
0.00	0.00	2.6E+05	5.1E+02	2701.8	5.3	3.4317	0.0019
0.50	0.10	1.4E+05	3.8E+02	1454.5	3.9	3.1627	0.0026
1.00	0.14	9.6E+04	3.1E+02	975.5	3.2	2.9892	0.0032
1.50	0.17	6.4E+04	2.5E+02	650.2	2.6	2.8131	0.0040
2.00	0.20	4.8E+04	2.2E+02	486.2	2.2	2.6868	0.0046
2.50	0.22	3.6E+04	1.9E+02	368.5	1.9	2.5664	0.0053
3.00	0.24	2.8E+04	1.7E+02	284.2	1.7	2.4536	0.0060



We can certify that this follows the expected exponential behaviour. In order to obtain the linear attenuation coefficient we take logarithms in both sides and perform a linear regression. The result is the following:



Obtained values after the fit:

$$a = (-0.350 \pm 0.026)\ \ cm^{-1} \qquad\qquad b = (3.390 \pm 0.030)$$

5 Discussion

A reasonable estimation of the quality of our results can be done after comparing them with the tabulated ones. We can look up the vales in [1]. The database does not have the linear attenuation coefficient for the exact value of our peak (0.66 MeV), so we must extrapolate according to:

$$y = y_a + (y_b - y_a)\,\frac{x - x_a}{x_b - x_a} \text{ at the point } (x, y) \tag{20}$$

Apart from that, the result given is the linear attenuation coefficient divided by the density of the material. In the following table we will show the tabulated values opposed to the ones we obtained, as well as the relative error:

	Cu	Al	Pb
Theoretical μ (cm^{-1})	0.633	0.211	1.283
Experimental μ (cm^{-1})	0.234	0.202	0.350
Relative Error (%) ϵ	63.0	4.1	72.7

We can also calculate the half-thickness as explained in the theoretical introduction. We obtained the following results:

Cu $x_{(1/2)}$ (cm)	Al $x_{(1/2)}$ (cm)	Pb $x_{(1/2)}$ (cm)
2.96 ± 0.19	3.43 ± 0.59	1.98 ± 0.15

From the relative errors we can tell that our results cannot be compared to the theoretical values. In the next section, we will discuss about the factors that lead to this difference.

6 Conclusions

Our results did not match the tabulated ones for the Copper and the Lead. We had some issues with the experimental equipment since the beginning of the experiment, but did not notice anything malfunctioning for any material except for the aluminum. After spotting this error, we repeated several times the measurements with a different device. With this change, we obtained the right answer for the aluminum. The reason for this is still unclear to the date.

An option to improve the accuracy of our method is having more data for more extreme cases of the thickness. Sometimes the proper relation between intensity and thickness can only be seen in a certain range of thicknesses. Then again, we could use different radioactive sources and then obtain the linear coefficient by averaging the result that we obtained.

With these findings, we can give a rational answer to which material should be used for shielding purpouses. The one with the higher attenuation coefficient will be the best one at protecting everything from the radiation. Both our experiment and the tabulated values state that lead should be the right option for shielding.

References

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