MATHEMATICS 2015

Time: 20 Minutes Max. Marks: 20

SECTION "A" (MULTIPLE CHOICE QUESTIONS)

1. **Choose the correct answer for each from the given options:**

(i) $\sum n =$
- $\frac{n(n+1)}{2}$ • $\frac{n+1}{2}$ • $\frac{n^2(n+1)^2}{2}$ • $\frac{n(n+2)}{2}$

(ii) The angle 135^o in radians is:
- $\frac{5\pi}{4}$ • $\frac{3\pi}{4}$ • $\frac{2\pi}{3}$ • 135π

(iii) If $\sin\theta < 0$ and $\cos\theta > 0$ then $P(\theta)$ is in:
- 1^{st} Quadrant • 2^{nd} Quadrant
- 3^{rd} Quadrant • 4^{th} Quadrant

(iv) The distance between (a, 0) and (0, b) is:
- $a + b$ • $a^2 + b^2$ • $\sqrt{a+b}$ • $\sqrt{a^2+b^2}$

(v) The period of sin x is:
- $\pi/2$ • π • $-\pi$ • 2π

(vi) If the sides of a triangle are a, b and c then $\frac{a-b+c}{2} =$
- s • s – a • s – b • s – c

(vii) The matrix $[1 \quad 2 \quad 3]^t$ is a:
- row matrix • column matrix
- singular matrix • non-singular matrix

(viii) If a, b, c are the sides of a triangle ABC then R =:
- $\frac{abc}{4}$ • $\frac{4\Delta}{abc}$ • $\frac{\Delta}{s}$ • $\frac{abc}{4\Delta}$

(ix) $\sin(180o + \theta) =$
- $\cos\theta$ • $-\cos\theta$ • $\sin\theta$ • $-\sin\theta$

(x) If roots of the equation $ax^2 + bx + c = 0$ are real then $b^2 - 4ac$ is:
- Positive • negative • zero • perfect square

(xi) If angle a in ΔABC is in standard position, the law of cosine is:
- $a^2 = b^2 + c^2 + 2bc\cos\alpha$ • $a^2 = b^2 + c^2 + bc\cos\alpha$
- $a^2 = b^2 + c^2 - 2bc\cos\alpha$ • $a^2 = b^2 + c^2 - bc\cos\alpha$

(xii) $\sum_{n=3}^{20} n^o =$ • 17 • 18 • 19 • 20

(xiii) If ω is a complex cube root of unity then $\omega^3 + \omega^4 + \omega^5 =$
- 1 • ω • ω^2 • 0

(xiv) A square matrix A is said to be singular if:
- $|A| = 1$ • $A = 0$ • $|A| = 0$ • $A = 1$

(xv) The real and imaginary parts of I (3 – 2i) are respectively:
- –2 and 3 • 2 and –3 • 2 and 3 • –2 and –3

(xvi) If $z = -4 + 3i$ then $\bar{z}$ is equal to:
- $4 + 3i$ • -4 • $4 - 3i$ • $-4 + 3i$

(xvii) The product of the roots of the equation $2x^2 - 6x - 15 = 0$ is:
- –15 • 15 • –15/2 • 15/2

(xviii) If $I = \sqrt{-1}$ then value of $(-i^3)^2$ is:
- 1 • i • –i • –1

(xix) The G.M. between 2 and 8 is:
- 5 • 16 • ±8 • ±4

(xx) ${}^nP_r$ is equal to:
- $\frac{n!}{r!(n-r)!}$ • $\frac{n!}{r!}$ • $\frac{n!}{n!-r!}$ • $\frac{n!}{(n-r)!}$

MATHEMATICS 2015

Time: 2 Hours 40 Minutes Marks: 80

SECTION 'B' (SHORT-ANSWER QUESTIONS)

ALGEBRA (35 MARKS)

NOTE: Answer 7 questions from this section.

2.(i) Prove that the roots of the equation $y^2 - 2\left(m + \frac{1}{m}\right)y + 3 = 0$ are real; $\forall m \in \Re$.

OR Solve the complex equation (x, y) (2, 3) = (–4, 7)

(ii) Prove that the cube roots of –125 are $-5, -5\omega, -5\omega^2$ & their sum is zero (where ω is the complex cube root of unity).

OR Solve the equation $\left(t + \frac{1}{t}\right)^2 = 4\left(t + \frac{1}{t}\right)$

(iii) Solve the system of equations: $4x^2 + y^2 = 25$, $y^2 - 2x = 5$

(iv) Find the values of x, y, z and v so that:

$$\begin{bmatrix} 4 & x+y \\ z+v & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & v \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2v \end{bmatrix}$$

(v) Using the properties of determinants, evaluate:

$$\begin{vmatrix} a+x & a & a \\ a & a+x & a \\ a & a & a+x \end{vmatrix}$$

(vi) Prove by mathematical induction:

$$2^3 + 4^3 + 6^3 + \text{--------} + (2n)^3 = 2[n(n+1)]^2$$

(vii) Find the simplified form the term independent of x in the binary expansion of $\left(\frac{4x^2}{3} - \frac{3}{2x}\right)^9$.

(viii) The P^{th} term of an A.P. is q and q^{th} term is P. Find the $(P + Q)^{th}$ term.

(ix) A = {1, –1, i, –i}, construct the multiplication table for complex numbers multiplication (*) in A, also show that (*) is commutative in A.

(x) Find n, if ${}^nP_4 = 24\,{}^nC_5$.

OR In how many ways can 3 English, 2 Urdu and 2 Sindhi books be arranged on a shelf so as to keep all the books in a language together?

TRIGONOMETRY (15)

Note: Attempt 3 questions from this Section.

3.(i) Using the definition of radian function, find the remaining trigonometric functions of $\cos\theta = ½$ and $\tan\theta$ is positive.

OR if $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = \frac{1}{\sqrt{2}}$, both $P(\alpha)$ and $P(\beta)$ lie in the first quadrant, find the value of $\tan(\alpha + \beta)$.

(ii) How far does a boy on a bicycle travel in 10 revolutions if the diameter of the wheel of his bicycle each equal to 56 cm?

(iii) Prove any two of the following:

(i) $\frac{\cot\theta + \operatorname{cosec}\theta}{\sin\theta + \tan\theta} = \operatorname{cosec}\theta\cot\theta$

(ii) $\frac{\sin(\theta+\phi)}{\cos\theta\cos\phi} = \tan\theta + \tan\phi$ (iii) $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

(iv) Prove that $\Delta = ½\, ab\sin\gamma$, where Δ denotes the area of Δ ABC.

(v) Draw the graph of $\sin\theta$, where $0 \le \theta \le 2\pi$.

OR Solve the triangle ABC when a = 10cm, $\alpha = 30^o$, $\beta = 40^o$.

SECTION C (DETAILED- ANSWER QUESTIONS)(30)

NOTE: Answer 2 questions from this section.

4.(a) Solve the following system of equations using matrix method:

$$x + y = 5$$
$$y + z = 7$$
$$z + x = 6$$

(b) If α, β are roots of the equation $px^2 + qx + r = 0$, $p \neq 0$, then find the equation whose roots are $\frac{-1}{\alpha^3}, \frac{-1}{\beta^3}$

5.(a) If c be a quantity so small that c^3 may be neglected in comparison with l^3, prove that:

$$\sqrt{\frac{\ell}{\ell+c}} + \sqrt{\frac{\ell}{\ell-c}} = 2 + \frac{3c^2}{4\ell^2}$$

(b)(i) Which term of the sequence 18, 12, 8 ----- is $\frac{512}{729}$?

(ii) Find the 17^{th} term of an H.P. whose first two terms are 6 and 8.

6.(a) Without using the calculator, prove that

$$\tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{4} = \tan^{-1}\frac{1}{3}.$$

(b) Prove that $4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} = r_1$

(c) A man observes that the angle of elevation of the top of a mountain measures 45^o from a point on the ground. On walking 100 meters away from the point, the angle of elevation measures 43.45^o. Find the height of the mountain.

OR

Solve $4\sin^2\theta\tan\theta + 4\sin^2\theta - 3\tan\theta - 3 = 0$.