MATHEMATICS 2018

Time: 2 Hours 40 Minutes Marks: 80

SECTION 'B' (SHORT-ANSWER QUESTIONS)(50)

NOTE: Answer any 10 part questions from this section, selecting at least thee part questions from each question.

COMPLEX NUMBER, ALGEBRA & MATRICES

2.(i) Solve the complex equation for x and y $(x + 2yi)^2 = xi$:

OR Solve the complex equation for x and y:

$$X(1 + 2i) + y(3 + 5i) = -3i \text{ (where } i = \sqrt{-1})$$

(ii) Solve the equation: $\left(x+\frac{1}{x}\right)^2 = 4\left(x-\frac{1}{x}\right)$.

(iii) Determine the value of m in the equation that will make the roots equal: $(m + 1)y^2 + 2(m + 3)y + (2m + 3) = 0$

(iv) If $A=\begin{bmatrix}\sin\theta & -\cos\theta \\ \cos\theta & \sin\theta\end{bmatrix}$ and $B=\begin{bmatrix}\sin\theta & \cos\theta \\ -\cos\theta & \sin\theta\end{bmatrix}$, then verify that $AB = BA = I_2$

(v) Using properties of determinants, show that:

$$\begin{vmatrix}1 & x & yz \\ 1 & y & zx \\ 1 & z & xy\end{vmatrix} = \begin{vmatrix}1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2\end{vmatrix}$$

GROUPS, SEQUENCES & SERIES AND COUNTING PROBLEMS

3.(i) Let $G = \{1,\omega,\omega^2\}$, ω being a complex cube root of unity. Show that $(G, \bullet)$ is an abelian group, where '$\bullet$' is an ordinary multiplication.

(ii) If three books are picked at random from a shelf containg 3 novels, 4 books of poems and a dictionary, what is the probability that:

(i) the dictionary is selected

(ii) one novel and 2 books of poems are selected.

OR In how many ways can a party of 5 students and 2 teachers be formed out of 15 students and 5 teachers?

(iii) Prove by mathematical induction that $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$, $\forall$ natural numbers n.

OR Without using the calculator, find the sum of $21^2 + 22^2 + 23^2 + \dots\dots + 50^2$.

(iv) Find the sum of an A.P. of nineteen terms whose middle term is 10.

(v) Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may becomes the H.M. between a and b.

OR Find the first term of a G.P. whose second term is 3 and sum to infinity is 12.

TRIGONOMETRY

4.(i) A belt, 24.75 metres long, passes around a 3.5 cm diameter pulley. The belt makes three complete revolution in a minute. How many radians does the wheel turn in two seconds?

(ii) Draw the graph of $y = \cos x$, where $0 \le x \le \pi$.

OR Show that $\tan\theta$ is a periodic function of period π.

(iii) In ΔABC, if $a = b = c$, then prove that : $r : R : r_1 = 1:2:3$

(iv) Solve the equation: $\tan 2\theta \cot\theta = 3$.

(v) Prove that $\text{Tan}^{-1}\frac{1}{5} + \text{Tan}^{-1}\frac{1}{4} = \text{Tan}^{-1}\frac{9}{19}$

OR Prove that: $\sin^{-1}A+\sin^{-1}B=\sin^{-1}\left(A\sqrt{1-B^2}+B\sqrt{1-A^2}\right)$

SECTION'C' (DETAILED- ANSWER QUESTIONS)

NOTE: Answer any Two questions from this section, including Question 5 which is compulsory.

5.(a) Divide 600 rupees among 5 boys, so that their shares are in A.P., and the two smallest shares together make one-seventh of what the other three boys get.

OR In an H.P., the 10^{th} term is 35 and the 35^{th} term is 25. If the last term is 2, find the number of terms.

(b) Prove the Law of cosine $a^2 = b^2 + c^2 - 2bc\cos\alpha$

OR Prove the fundamental law, $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

6.(a) Show that: $\sqrt{2}=1+\frac{1}{2^2}+\frac{1.3}{2!.2^4}+\frac{1.3.5}{3!.2^6}+\dots\dots\dots$

(b) Apply Cramer's rule to solve the system of equations:

$$X + y + z = d$$
$$X + (1 + d)y + z = 2d$$
$$X + y + (1 + d)z = 0 \quad (d \ne 0)$$

7.(a) By using the definition of radian function, if $\sin\theta = 0.6$ and $\tan\theta$ is negative, find the remaining trigonometric functions.

(b) Prove any two of the following:

(i) $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$ (ii) $\frac{\sin 2\theta}{\sin\theta}-\frac{\cos 2\theta}{\cos\theta}=\sec\theta$

(iii) $\frac{\sin 7\theta-\sin 5\theta}{\cos 7\theta+\cos 5\theta}=\tan\theta$ OR $\tan\frac{\theta}{2}=\frac{\sin\theta}{1+\cos\theta}$

(c) Solve and check:

$$Yz + 15 = 0$$
$$Y^2 + z^2 - 34 = 0$$