MATHEMATICS 2019

Time: 2 Hours 40 Minutes   Marks: 80

SECTION 'B' (SHORT-ANSWER QUESTIONS)(50)

NOTE: Answer any 10 part questions from this section, selecting at least thee part questions from each question.

COMPLEX NUMBER, ALGEBRA & MATRICES

2.(i) Solve the compledx equation $(x,y).(2,3) = (-4,7)$.

OR   Find the real and imaginary parts of $\dfrac{2-i}{3i}$

(ii) Solve the equation $4.2^{2x+1} - 9.2^x + 1 = 0$.

(iii) By using he properties of determinants, show that

$$\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix} = 0$$

(iv) Form an equation whose roots are $\dfrac{1}{2}$ and $\dfrac{1}{6}$.

(v) Determine the nature of roots of thefollowing equation:

$$2x^2 + 9 = 9x$$

OR   Solve the equation: $\sqrt{2x+7} + \sqrt{x+3} = 1$

GROUPS, SEQUENCES & SERIES AND COUNTING PROBLEMS

3.(i) Let * be defined in Z by $m*n = m + n + 2$.
   (a) Show that * is associative and commutative.
   (b) Identity w.r.t. * exists in Z.
   (c) Every element of Z has an inverse under *.

(ii) Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may becomes the G.M. between a & b.

OR   *Express* $0.3\overline{48}$ as a vulgar fraction.

(iii) Prove by Mathematical induction, following preposition:
$2 + 6 + 12 + \cdots + n(n + 1) = 1/3\, n(n+1)(n+2)$.

(iv) If in a G.P., the fifth term is 9 times the third term and its second term is 6, find the G.P.

OR   Insert 4 A.M.s between 18 & 3.

(v) If there are 3 children in a family, what is probability that: (a) the third child is a girl?
  (b) the two children are boys and one child is a girl?

TRIGONOMETRY

4.(i) By using definition of Radian function, find the remaining trigonometric functions if $\tan\theta = 1/3$ and $p(\theta)$ is in 4th quadrant.

(ii) Draw the graph of $y = \sin 2x$, where $0 \leq x \leq \pi$.

OR   Without using calculator, prove that:
$\sin 19° \cos 11° + \sin 11° = \frac{1}{2}$

(iii) Find the area of the triangle when:
A = 9.1cm, b = 8.2cm, c = 7.3cm

(iv) Solve $\cos\theta - 2\sin\theta = 0$.

(v) Show that $\tan^{-1}\theta = \sin^{-1}\dfrac{\theta}{\sqrt{1+\theta^2}}$

SECTION 'C' (DETAILED- ANSWER QUESTIONS)

NOTE: Answer any Two questions from this section, including Question 5 which is compulsory.

5.(a) The sums of the first n terms of two A.P's are in the ratio $3n + 31 : 5n - 3$. Show that ther 9th terms are equal.

(b) Prove the law of tangent $\dfrac{\tan\left(\dfrac{\alpha-\beta}{2}\right)}{\tan\left(\dfrac{\alpha+\beta}{2}\right)} = \dfrac{a-b}{a+b}$

OR   In $\triangle ABC$, prove that the area of the triangle $\Delta = \frac{1}{2}\, ab \sin\gamma$

6.(a) Find the coefficient of x^6 in expansion of $(a^3 + 3bx^2)^{-6}$

(b) Apply Cramer's rule tosolve the following system of equation:
$$x + y = 5$$
$$Y + z = 7$$
$$Z + x = 6$$

7.(a) An aeroplane is flying at a height of 9000 metres. If the angle of depression to a field marker measures 23°, find he aerial distance.

(b) Prove any two of the following:

(i) $\dfrac{\tan\theta+\sin\theta}{\csc\theta+\cot\theta} = \tan\theta \sin\theta$, $(\cos\theta \neq 0, -1)$

(ii) $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2$ (iii) $\dfrac{1+\tan^2\frac{\pi}{6}}{\tan^2\frac{\pi}{6}} = \csc^2\frac{\pi}{6}$

(c) Solve and check: $4x^2 + y^2 = 25$
$$Y^2 - 2x = 5$$