

STRING MANIPULATION, GUESS-and-CHECK, APPROXIMATIONS, BISECTION

(download slides and .py files ••• follow along!)

6.0001 LECTURE 3

LAST TIME

- strings
- branching – if/elif/else
- while loops
- for loops

TODAY

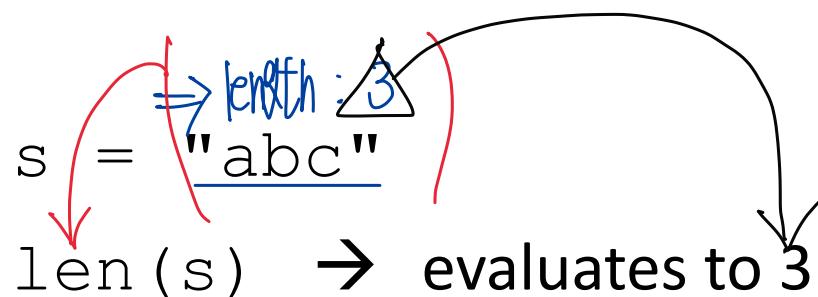
- string manipulation
- guess and check algorithms
- approximate solutions
- bisection method

STRINGS

- think of as a **sequence** of case sensitive characters
- can compare strings with ==, >, < etc.
- `len()` is a function used to retrieve the **length** of the string in the parentheses

`s = "abc"`

`len(s)` → evaluates to 3



STRINGS

- square brackets used to perform **indexing** into a string to get the value at a certain index/position

`s = "abc"`

index: 0 1 2 ← indexing always starts at 0

index: -3 -2 -1 ← last element always at index -1

`s[0]` → evaluates to "a"

`s[1]` → evaluates to "b"

`s[2]` → evaluates to "c"

`s[-4]` = `s[3]` → trying to index out of bounds, error ~~***~~

`s[-1]` → evaluates to "c"

`s[-2]` → evaluates to "b"

`s[-3]` → evaluates to "a"

	P(t)	n(-)
a	0	-3
b	1	-2
c	2	-1

MAX Position : 2,

STRINGS

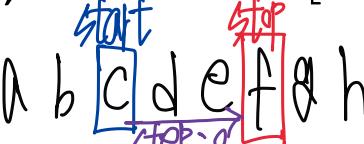
- can **slice** strings using `[start:stop:step]`
- if give two numbers, `[start:stop]`, `step=1` by default
- you can also omit numbers and leave just colons

`s = "abcdefg"`

0 1 2 3 4 5 6 7

`s[3:6]` → evaluates to "def", same as `s[3:6:1]`

start *stop* *step*
If unsure what some command does, try it out in your console!

`s[3:6:2]` → evaluates to "df" 

`s[::]` → evaluates to "abcdefg", same as `s[0:len(s):1]`

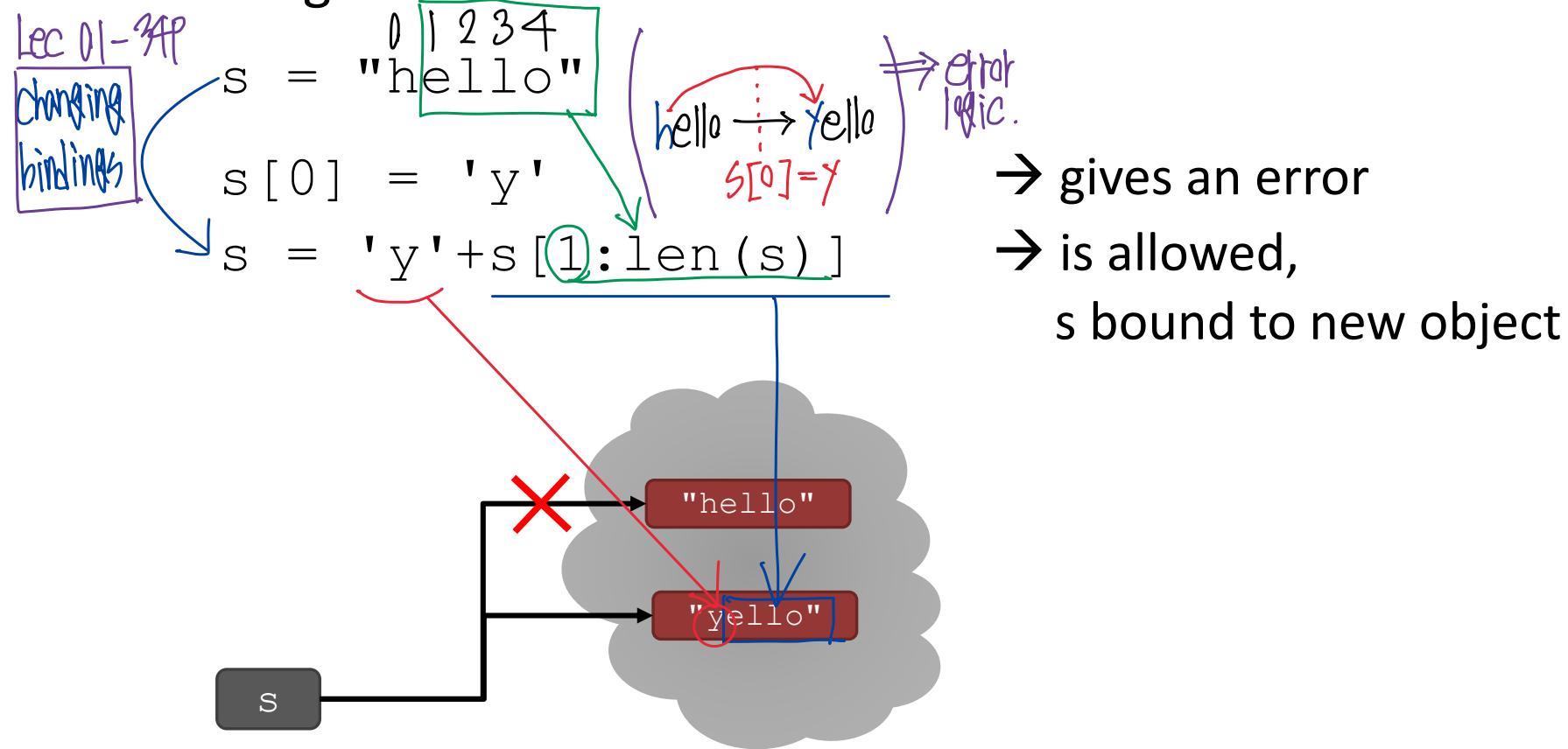
start *stop* *stop*
⇒ *abcdefg*

`s[:::-1]` → evaluates to "hgfedcba", same as `s[-1:-(len(s)+1):-1]`

`s[4:1:-2]` → evaluates to "ec" 

STRINGS

- strings are “**immutable**” – cannot be modified



for LOOPS RECAP

- for loops have a **loop variable** that iterates over a set of values

$\Rightarrow 0, 1, 2, 3$

```
for var in range(4) :      → var iterates over values 0,1,2,3
    <expressions>          → expressions inside loop executed
                            with each value for var
```

```
for var in range(4, 6) : → var iterates over values 4,5
    <expressions>
```

- range is a way to iterate over numbers, but a for loop variable can **iterate over any set of values**, not just numbers!

STRINGS AND LOOPS

- these two code snippets do the same thing
- bottom one is more “pythonic” 

```
s = "abcdefg"  
  
for index in range(len(s)):  
    if s[index] == 'i' or s[index] == 'u':  
        print("There is an i or u")  
  
  
for char in s:  
    if char == 'i' or char == 'u':  
        print("There is an i or u")
```

CODE EXAMPLE: ROBOT CHEERLEADERS

```
an_letters = "aefhilmnorsxAEFHILMNORSX"  
  
word = input("I will cheer for you! Enter a word: ")  
times = int(input("Enthusiasm level (1-10): "))
```

```
i = 0  
while i < len(word):  
    char = word[i]  
    if char in an_letters:  
        print("Give me an " + char + "! " + char)  
    else:  
        print("Give me a " + char + "! " + char)  
i += 1  
print("What does that spell?")  
for i in range(times):  
    print(word, "!!!!")
```

for char in word: 

EXERCISE

```
s1 = "mit u rock"  
s2 = "i rule mit"  
if len(s1) == len(s2):  
    for char1 in s1:  
        for char2 in s2:  
            if char1 == char2:  
                print("common letter")  
                break
```

GUESS-AND-CHECK

- the process below also called **exhaustive enumeration**
- given a problem...
- you are able to **guess a value** for solution
- you are able to **check if the solution is correct**
- keep guessing until find solution or guessed all values

GUESS-AND-CHECK

– cube root

```
cube = 8

for guess in range(cube+1):
    if guess**3 == cube:
        print("Cube root of", cube, "is", guess)
```

GUESS-AND-CHECK

– cube root

```
cube = 8

for guess in range(abs(cube)+1):
    if guess**3 >= abs(cube):
        break

if guess**3 != abs(cube):
    print(cube, 'is not a perfect cube')
else:
    if cube < 0:
        guess = -guess
    print('Cube root of ' + str(cube) + ' is ' + str(guess))
```

for math.

Computer Can't understand.

Computer Can't understand.

APPROXIMATE SOLUTIONS

- **good enough** solution
- start with a guess and increment by some **small value**
- keep guessing if $|\text{guess}^3 - \text{cube}| \geq \text{epsilon}$ for some **small epsilon**
 - in Python `abs()`
 - if `abs(Luciano) : |Luciano|`
- decreasing increment size → slower program
- increasing epsilon → less accurate answer

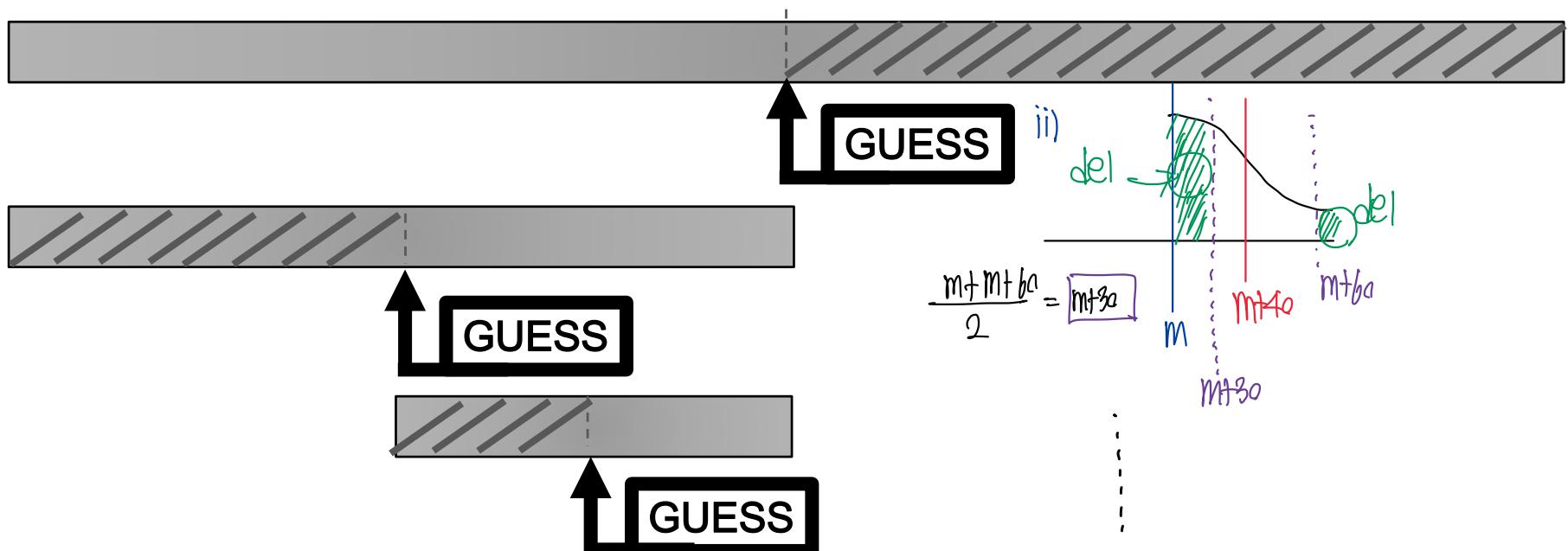
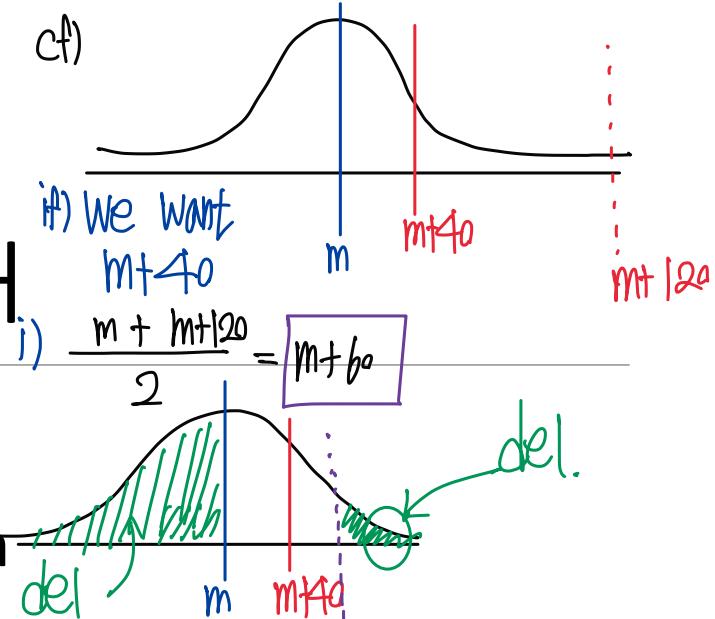
APPROXIMATE SOLUTION

– cube root

```
cube = 27
epsilon = 0.01
guess = 0.0
increment = 0.0001
num_guesses = 0
while abs(guess**3 - cube) >= epsilon and guess <= cube :
    guess += increment
    num_guesses += 1
print('num_guesses =', num_guesses)
if abs(guess**3 - cube) >= epsilon:
    print('Failed on cube root of', cube)
else:
    print(guess, 'is close to the cube root of', cube)
```

BISECTION SEARCH

- half interval each iteration
- new guess is halfway in between
- to illustrate, let's play a game!



BISECTION SEARCH

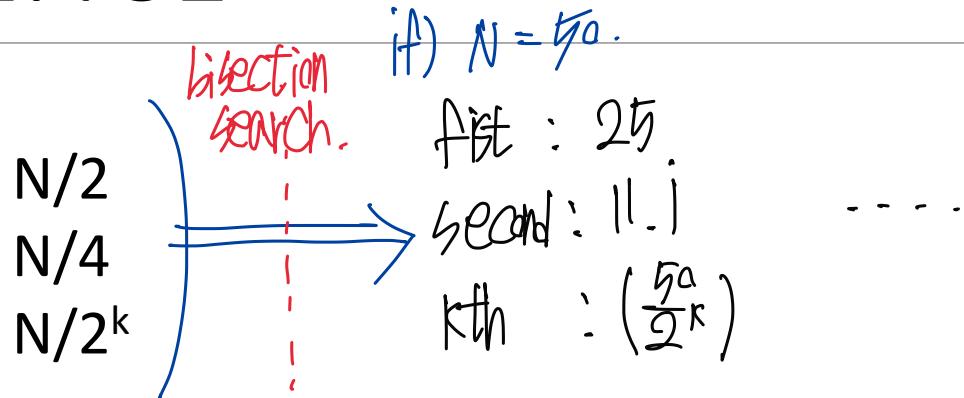
– cube root

```
cube = 27
epsilon = 0.01
num_guesses = 0
low = 0
high = cube
guess = (high + low)/2.0
while abs(guess**3 - cube) >= epsilon:
    if guess**3 < cube :
        low = guess
    else:
        high = guess
    guess = (high + low)/2.0
    num_guesses += 1
print 'num_guesses =', num_guesses
print guess, 'is close to the cube root of', cube
```

Start Point

BISECTION SEARCH CONVERGENCE

- search space
 - first guess: $N/2$
 - second guess: $N/4$
 - kth guess: $N/2^k$



- guess converges on the order of $\log_2 N$ steps
- bisection search works when value of function varies monotonically with input
 - T: 1 2 3 4 ... *Always increase*
 - F: 1 2 4 3 -1 ... *Always*
- code as shown only works for positive cubes > 1 – why?
- challenges → modify to work with negative cubes!
→ modify to work with $x < 1$!

$x < 1$

- if $x < 1$, search space is 0 to x but cube root is greater than x and less than 1
- modify the code to choose the search space depending on value of x

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6.0001 Introduction to Computer Science and Programming in Python

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