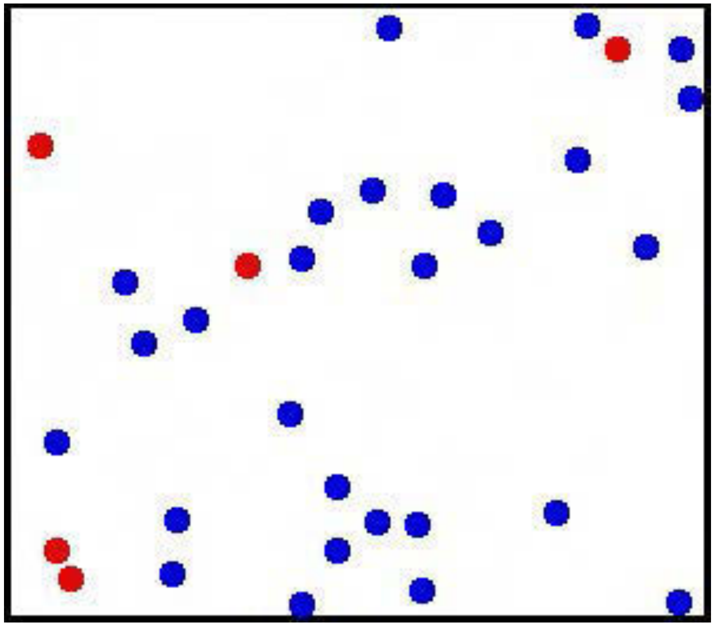
Lecture 5: Random Walks

Relevant Reading

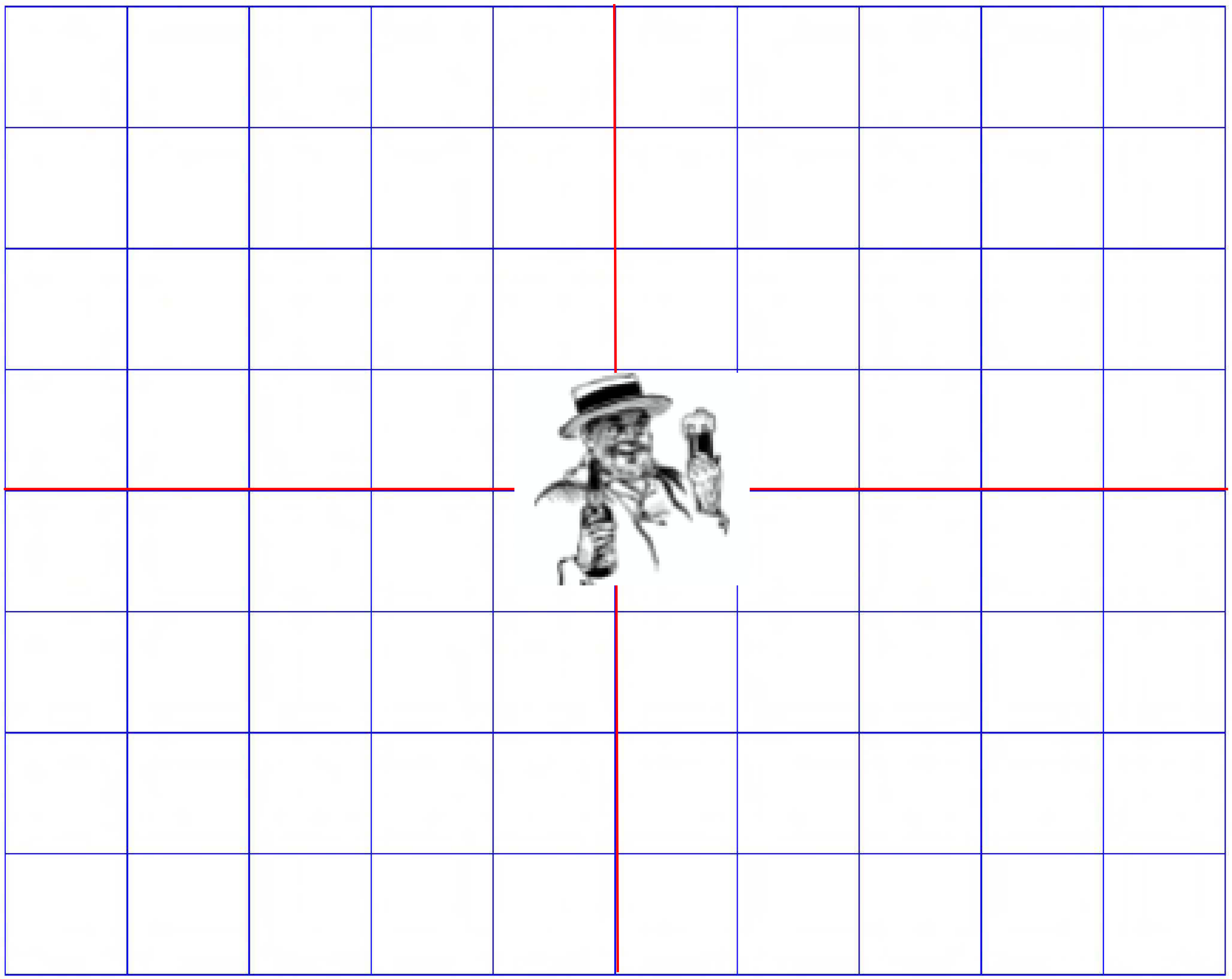
- Chapter 11
- Chapter 14

Why Random Walks?

- Random walks are important in many domains
  - Understanding the stock market (maybe)
  - Modeling diffusion processes
  - Etc.
- Good illustration of how to use simulations to understand things
- Excuse to cover some important programming topics
  - Practice with classes
  - Practice with plotting

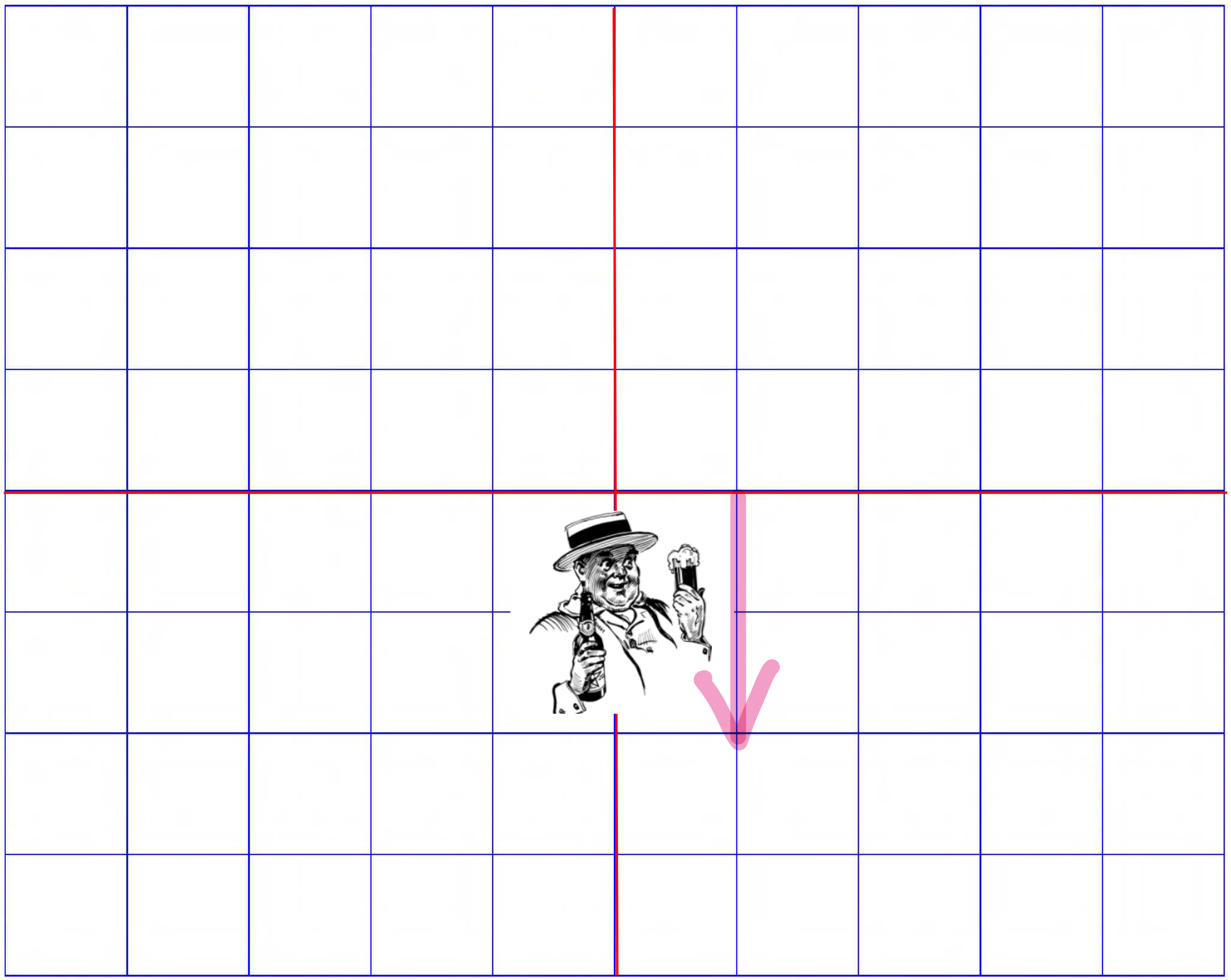


Drunkard's Walk



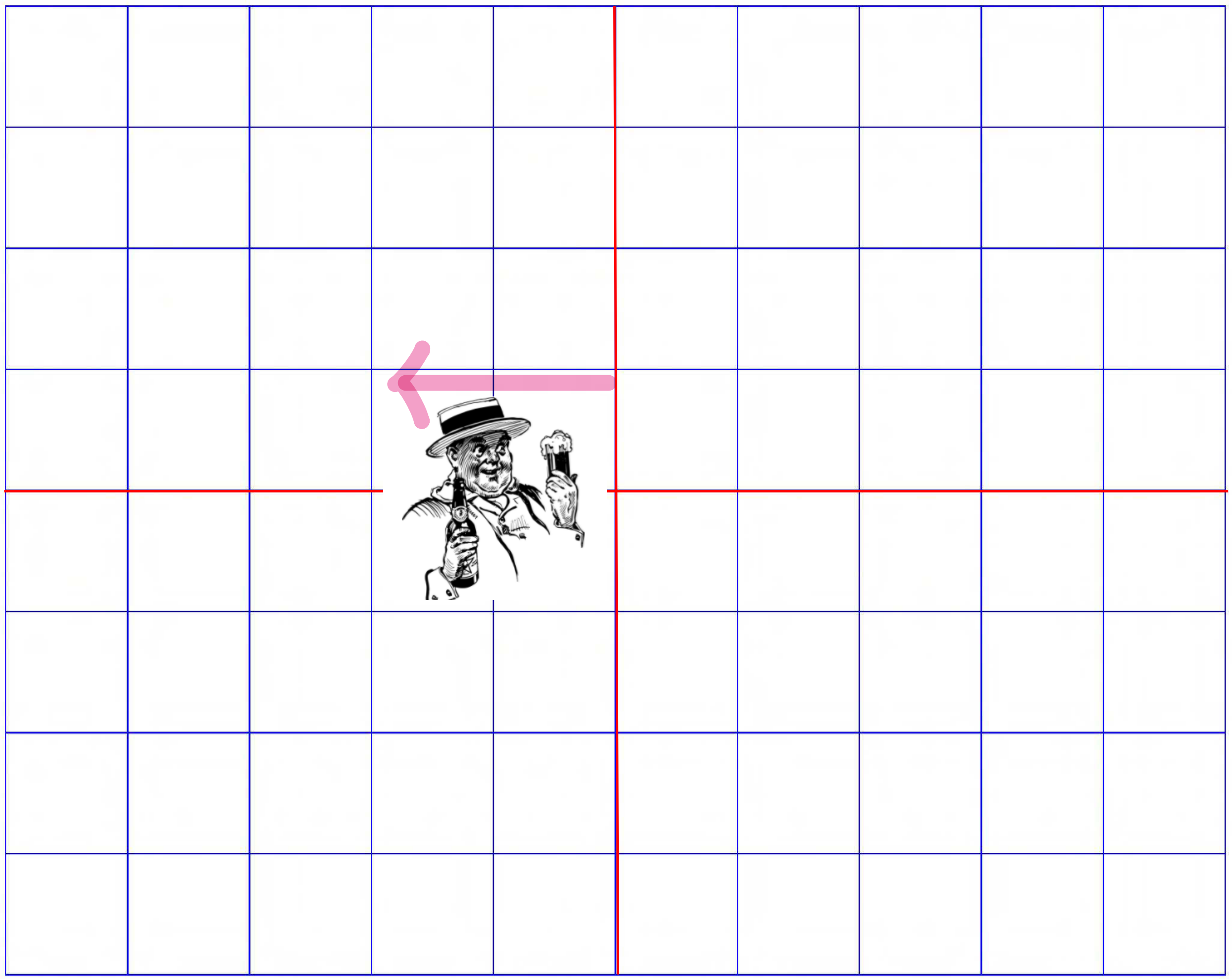
Case i) 1st step : down.

One Possible First Step



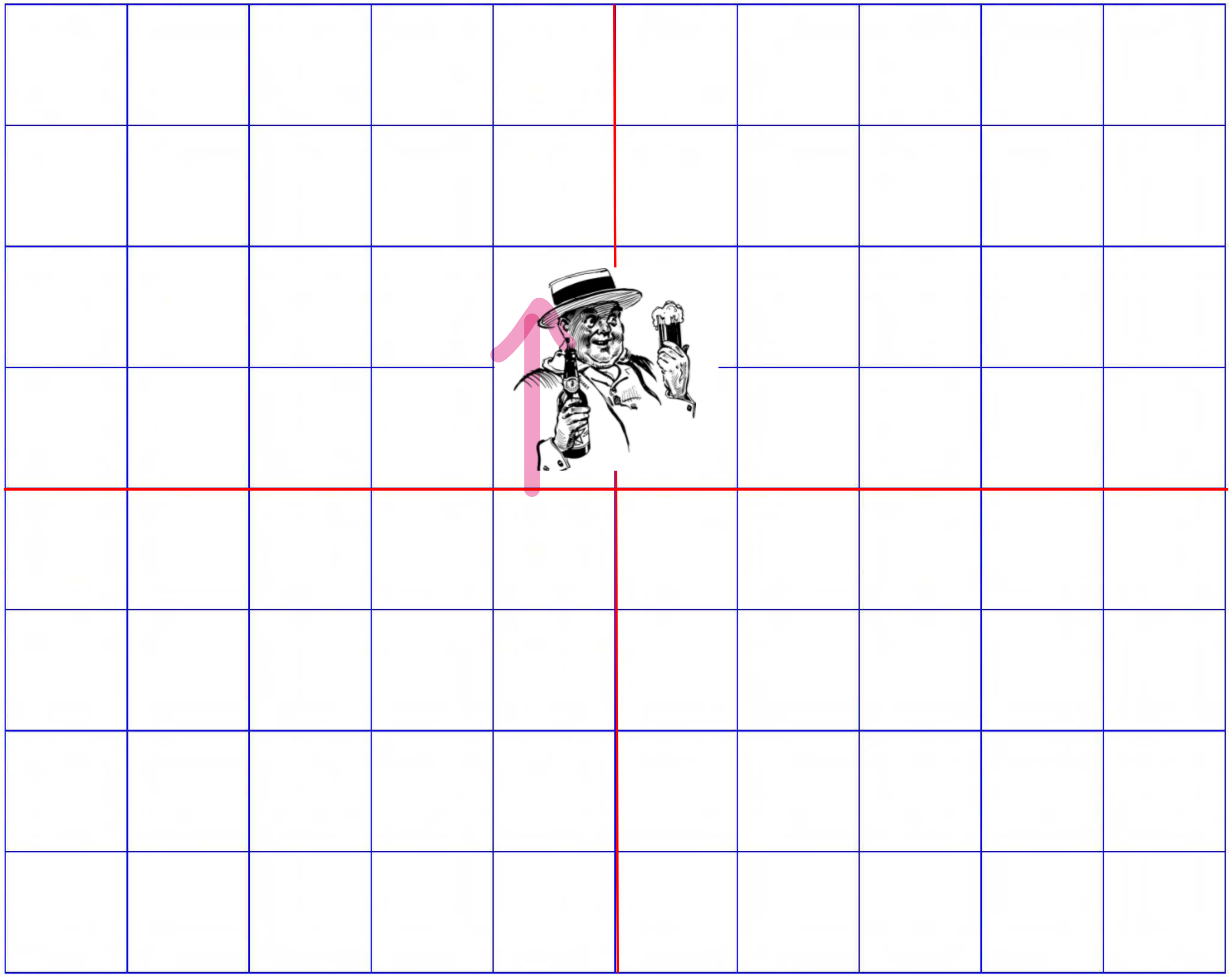
Another Possible First Step

case ii) 1st step : left.



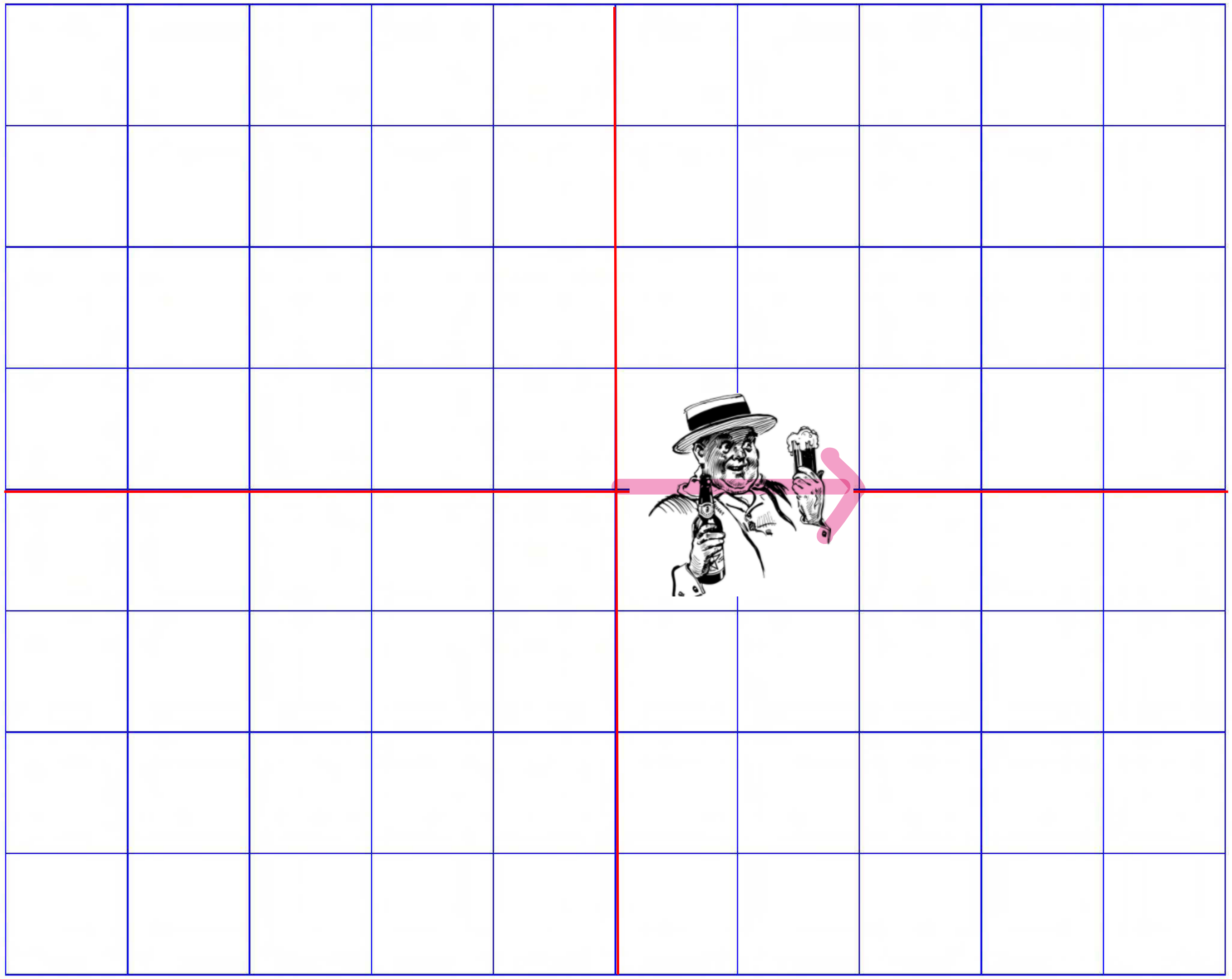
Yet Another Possible First Step

Case iii) 1st step : up

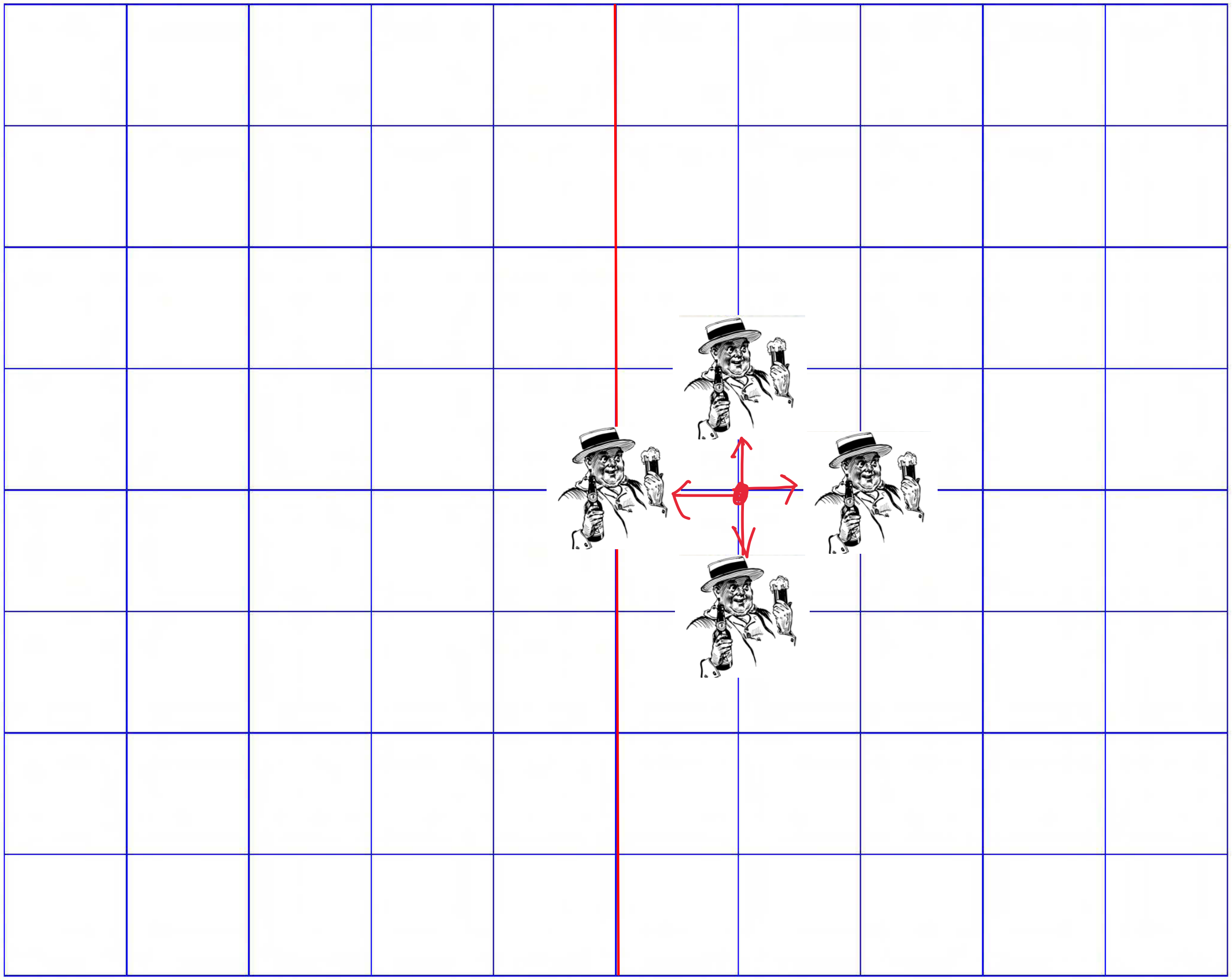


Last Possible First Step

Case iv) 1st step : right.



Possible Distances After Two Steps



Case

- up
- down
- left
- right

Expected Distance After 100,000 Steps?

- Need a different approach to problem
- Will use simulation

Structure of Simulation

- Simulate one walks of k steps
- Simulate n such walks
- Report average distance from origin

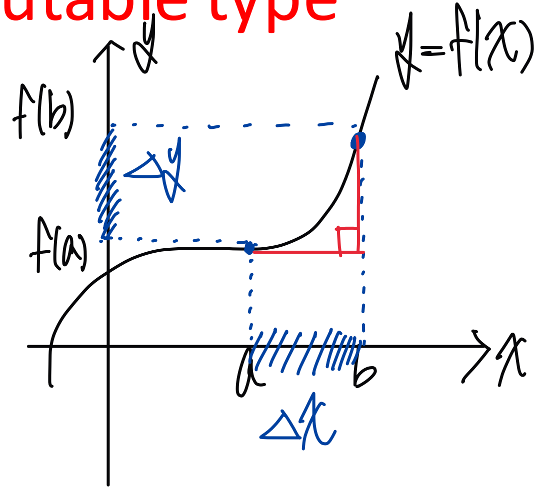
First, Some Useful Abstractions

- Location—a place
- Field—a collection of places and drunks
- Drunk—somebody who wanders from place to place in a field

Class Location, part 1

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Immutable type




```
class Location(object):
    def __init__(self, x, y):
        """x and y are floats"""
        self.x = x
        self.y = y

    def move(self, deltaX, deltaY):
        """deltaX and deltaY are floats"""
        return Location(self.x + deltaX,
                        self.y + deltaY)

    def getX(self):
        return self.x

    def getY(self):
        return self.y
```

distance of x = Δx

distance of y = Δy

return new location.

$\Rightarrow x + \Delta x + y + \Delta y$

Class Location, continued

≒ x

⇒ return self.x

```
def distFrom(self, other):
        xDist = self.x - other.getX()
        yDist = self.y - other.getY()
        return (xDist**2 + yDist**2)**0.5
```

⇒ return self.y

≒ y

≒ x

```
def __str__(self):
        return '<' + str(self.x) + ', '\
                    + str(self.y) + '>'
```

≒ y

Class Drunk

```
class Drunk(object):
    def __init__(self, name = None):
        """Assumes name is a str"""
        self.name = name

    def __str__(self):
        if self != None:
            return self.name
        return 'Anonymous'
```

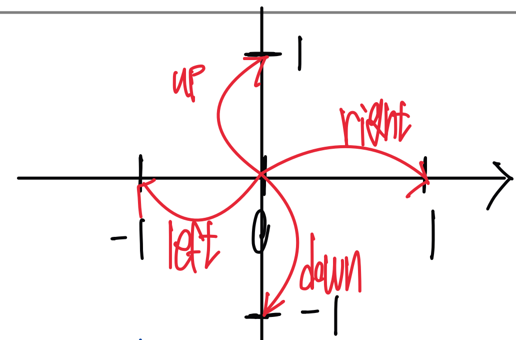
Not intended to be useful on its own

A base class to be inherited

Two Subclasses of Drunk

- The "usual" drunk, who wanders around at random
- The "masochistic" drunk, who tries to move northward

Two Kinds of Drunks




```
import random

class UsualDrunk(Drunk):
    def takeStep(self):
```

up down right left

```
        stepChoices = [(0,1), (0,-1), (1, 0), (-1, 0)]
        return random.choice(stepChoices)

class MasochistDrunk(Drunk):
    def takeStep(self):
        stepChoices = [(0.0,1.1), (0.0,-0.9),
                       (1.0, 0.0), (-1.0, 0.0)]
        return random.choice(stepChoices)
```

Immutable or not?

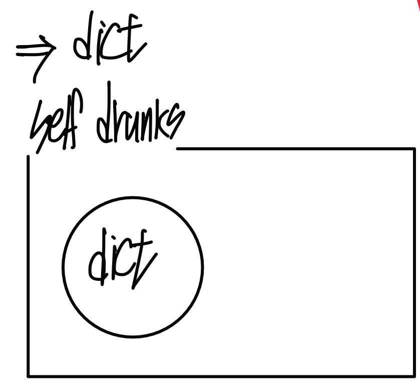
Class Field, part 1

```
class Field(object):
    def __init__(self):
        self.drunks = {}

    def addDrunk(self, drunk, loc):
        if drunk in self.drunks:
            raise ValueError('Duplicate drunk')
        else:
            self.drunks[drunk] = loc

    def getLoc(self, drunk):
        if drunk not in self.drunks:
            raise ValueError('Drunk not in field')
        return self.drunks[drunk]
```

dict



→ raise ValueError.

Class Field, continued

```
def moveDrunk(self, drunk):
    if drunk not in self.drunks:
        raise ValueError('Drunk not in field')
    xDist, yDist = drunk.takeStep()
    #use move method of Location to get new location
    self.drunks[drunk] =\
        self.drunks[drunk].move(xDist, yDist)
```

Immutable or not?

Simulating a Single Walk

Field
Drunk.

f : Field
d : Drunk in f.

```
def walk(f, d, numSteps):
    """Assumes: f a Field, d a Drunk in f, and
       numSteps an int >= 0.
       Moves d numSteps times; returns the distance
       between the final location and the location
       at the start of the  walk."""
    start = f.getLoc(d)
    for s in range(numSteps):
        f.moveDrunk(d)
    return start.distFrom(f.getLoc(d))
```

min.

Simulating Multiple Walks

```
def simWalks(numSteps, numTrials, dClass):
    """Assumes numSteps an int >= 0, numTrials an
         int > 0, dClass a subclass of Drunk
       Simulates numTrials walks of numSteps steps
         each. Returns a list of the final distances
         for each trial"""
    Homer = dClass()
    origin = Location(0, 0)
    distances = []
    for t in range(numTrials):
        f = Field()
        f.addDrunk(Homer, origin)
        distances.append(round(walk(f, Homer,
                                     numTrials), 1))
    return distances
```

Putting It All Together

```
def drunkTest(walkLengths, numTrials, dClass):
    """Assumes walkLengths a sequence of ints >= 0
         numTrials an int > 0,
         dClass a subclass of Drunk
       For each number of steps in walkLengths,
         runs simWalks with numTrials walks and
         prints results"""
    for numSteps in walkLengths:
        distances = simWalks(numSteps, numTrials,
                             dClass)
        print(dClass.__name__, 'random walk of',
              numSteps, 'steps')
        print(' Mean =',
              round(sum(distances)/len(distances), 4))
        print(' Max =', max(distances),
              'Min =', min(distances))
```

Let's Try It

```
drunkTest((10, 100, 1000, 10000), 100,
          UsualDrunk)

UsualDrunk random walk of 10 steps
 Mean = 8.634
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 100 steps
 Mean = 8.57
 Max = 22.0 Min = 0.0
UsualDrunk random walk of 1000 steps
 Mean = 9.206
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 10000 steps
 Mean = 8.727
 Max = 23.5 Min = 1.4
```

Plausible?

Let's Try a Sanity Check

- Try on cases where we think we know the answer
  - A very important precaution!

Sanity Check

```
drunkTest((0, 1, 2) 100, UsualDrunk)

UsualDrunk random walk of 0 steps
 Mean = 8.634
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 1 steps
 Mean = 8.57
 Max = 22.0 Min = 0.0
UsualDrunk random walk of 2 steps
 Mean = 9.206
 Max = 21.6 Min = 1.4
```

```
distances.append(round(walk(f, Homer,
                            numTrials), 1))
```

Let's Try It

```
drunkTest((10, 100, 1000, 10000), 100,
          UsualDrunk)

UsualDrunk random walk of 10 steps
 Mean = 2.863
 Max = 7.2 Min = 0.0
UsualDrunk random walk of 100 steps
 Mean = 8.296
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 1000 steps
 Mean = 27.297
 Max = 66.3 Min = 4.2
UsualDrunk random walk of 10000 steps
 Mean = 89.241
 Max = 226.5 Min = 10.0
```

And the Masochistic Drunk?

```
random.seed(0)
simAll((UsualDrunk, MasochistDrunk),
       (1000, 10000), 100)
```

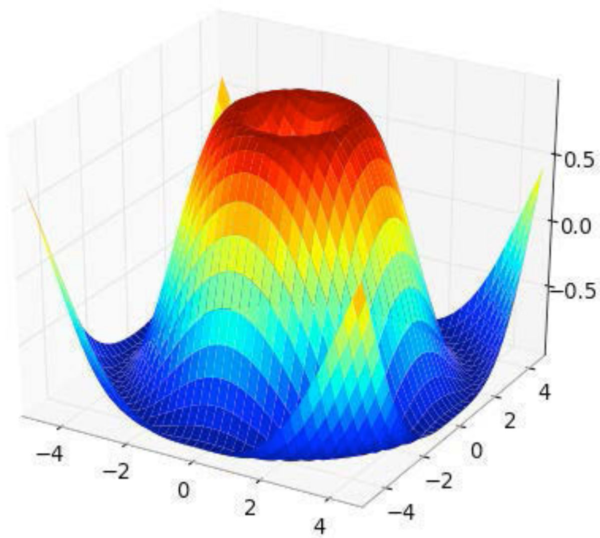
```
UsualDrunk random walk of 1000 steps
 Mean = 26.828
 Max = 66.3 Min = 4.2
UsualDrunk random walk of 10000 steps
 Mean = 90.073
 Max = 210.6 Min = 7.2
MasochistDrunk random walk of 1000 steps
 Mean = 58.425
 Max = 133.3 Min = 6.7
MasochistDrunk random walk of 10000 steps
 Mean = 515.575
 Max = 694.6 Min = 377.7
```

Visualizing the Trend

- Simulate walks of multiple lengths for each kind of drunk
- Plot distance at end of each length walk for each kind of drunk

Pylab

- NumPy adds vectors, matrices, and many high-level mathematical functions
- SciPy adds mathematical classes and functions useful to scientists
- MatPlotLib adds an object-oriented API for plotting
- PyLab combines the other libraries to provide a MATLAB®- like interface



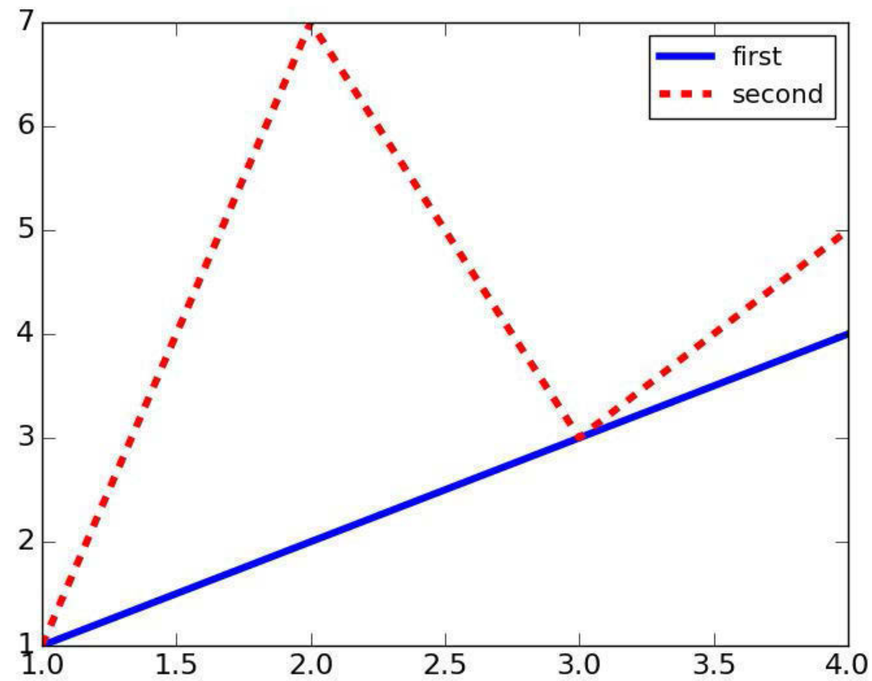
plot

- The first two arguments to `pylab.plot` must be sequences of the same length.
- First argument gives x-coordinates.
- Second argument gives y-coordinates.
- Many optional arguments
- Points plotted in order. In default style, as each point is plotted, a line is drawn connecting it to the previous point.

Example

```
import pylab

xVals = [1, 2, 3, 4]
yVals1 = [1, 2, 3, 4]
pylab.plot(xVals, yVals1, 'b-', label = 'first')
yVals2 = [1, 7, 3, 5]
pylab.plot(xVals, yVals2, 'r--', label = 'second')
pylab.legend()
```

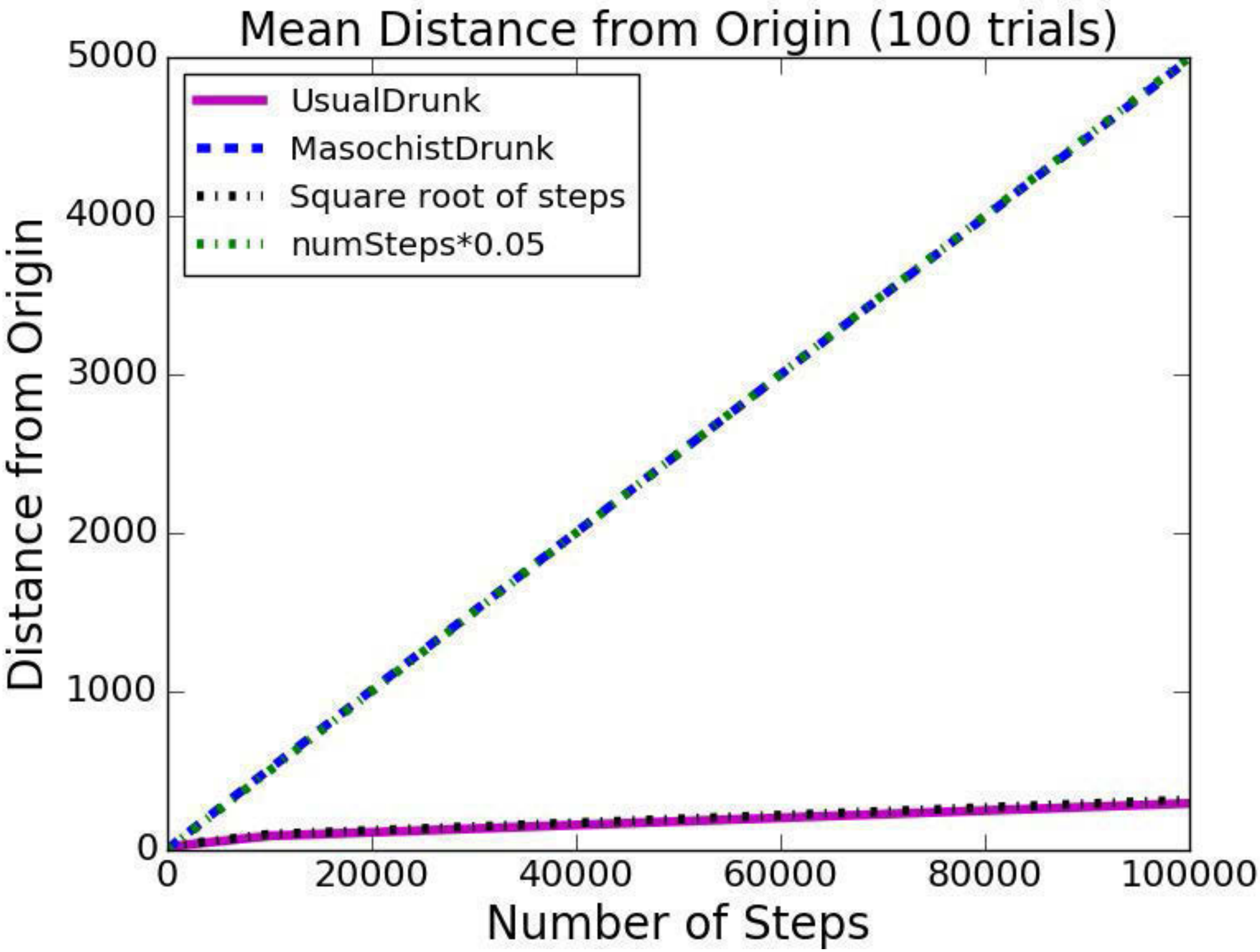


Details and Many More Examples

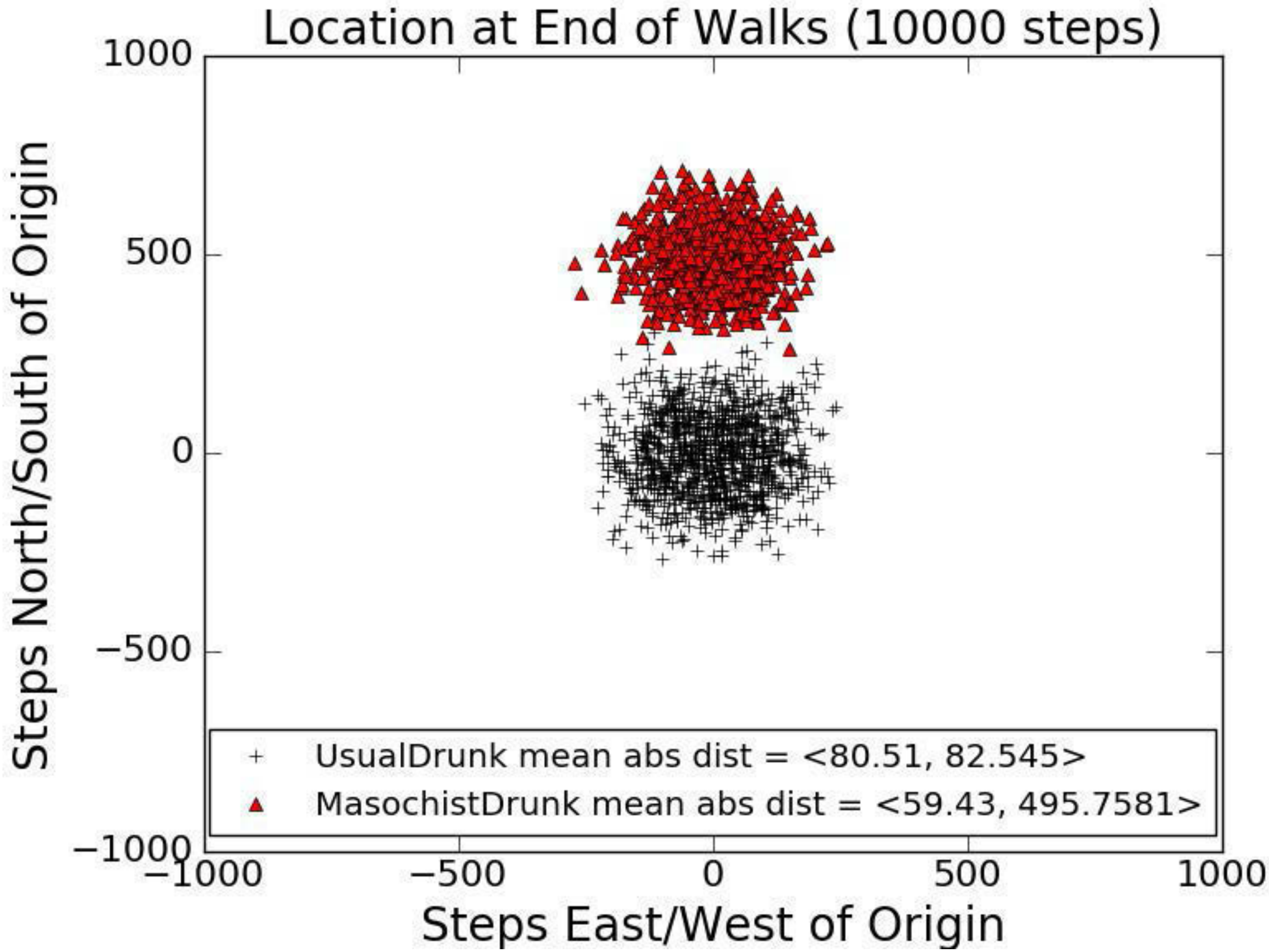
- Assigned reading
- Video of Prof. Grimson's lecture from 6.00x.1
- Code for this lecture
- matplotlib.org/api/pyplot_summary.html
- www.scipy.org/Plotting_Tutorial

You should learn how to produce the plots that I will show you

Distance Trends



Ending Locations



Fields with Wormholes

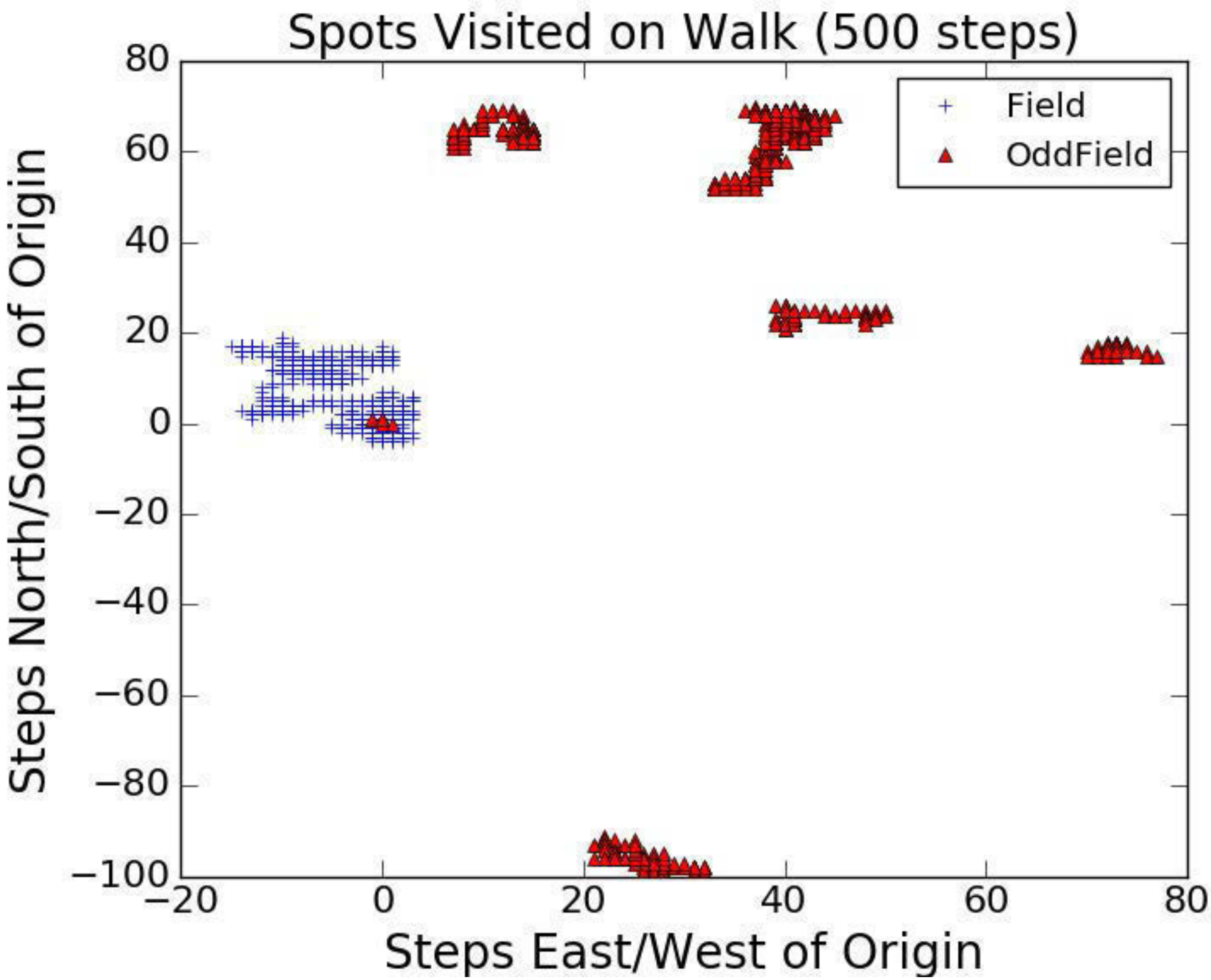
A Subclass of Field, part 1

```
class OddField(Field):
    def __init__(self, numHoles = 1000,
                 xRange = 100, yRange = 100):
        Field.__init__(self)
        self.wormholes = {} ···dict
        for w in range(numHoles):  → 1000
            x = random.randint(-xRange, xRange)
            y = random.randint(-yRange, yRange)
            newX = random.randint(-xRange, xRange)
            newY = random.randint(-yRange, yRange)
            newLoc = Location(newX, newY)
            self.wormholes[(x, y)] = newLoc
```

A Subclass of Field, part 2

```
def moveDrunk(self, drunk):
    Field.moveDrunk(self, drunk)
    x = self.drunks[drunk].getX()
    y = self.drunks[drunk].getY()
    if (x, y) in self.wormholes:
        self.drunks[drunk] = self.wormholes[(x, y)]
```

Spots Reached During One Walk



Summary

- Point is not the simulations themselves, but how we built them
- Started by defining classes
- Built functions corresponding to
  - One trial, multiple trials, result reporting
- Made series of incremental changes to simulation so that we could investigate different questions
  - Get simple version working first
  - Did a sanity check!
  - Elaborate a step at a time
- Showed how to use plots to get insights