

$$y' = 2 \cos(2x+6) \cdot (-\sin(2x+6)) \cdot 2 = -4 \cos(2x+6) \sin(2x+6) = -2 \sin(4x+12)$$

$$y'(-4) = -2 \sin(-4) = 2 \sin 4$$

$$y'' = -2 \cos(4x+12) \cdot 4 = -8 \cos(4x+12)$$

$$y''(-4) = -8 \cos 4$$

$$y''' = -8 \cdot 4 \cdot (-\sin(4x+12)) = 32 \sin(4x+12)$$

$$y'''(-4) = 32 \sin 4$$

$$y^{(4)} = 32 \cdot 4 \cos(4x+12) = 128 \cos(4x+12)$$

$$y^{(4)}(-4) = 128 \cos 4$$

$$y^{(5)} = 128 \cdot 4 \cdot (-\sin(4x+12)) = -512 \sin(4x+12)$$

$$y^{(5)}(-4) = -512 \sin 4$$

$$\Rightarrow y \sim \cos^2 2 + 2 \sin 4 \cdot (x+4) - 4 \cos 4 \cdot (x+4)^2 - \frac{32}{6} \cdot (x+4)^3 + \frac{128}{24} \cos 4 \cdot (x+4)^4 + \frac{512}{120} \cdot \sin 4 \cdot (x+4)^5 + o((x+4)^5)$$

$$a) \lim_{x \rightarrow 0} \frac{2 - \cos x - e^{x^2}}{x - \ln(1+x)} \stackrel{N19}{=} \lim_{x \rightarrow 0} \frac{F(x)}{G(x)} = L$$

$$\cos x \sim 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$e^{x^2} \sim 1 + x^2 + \frac{x^4}{2!} + \dots \Rightarrow F(x) \sim -\frac{1}{2}x^2 + o(x^2)$$

$$\ln(1+x) \sim x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \Rightarrow G(x) \sim \frac{1}{2}x^2 + o(x^2)$$

$$\Rightarrow L = \lim_{x \rightarrow 0} \frac{-\frac{1}{2}x^2 + o(x^2)}{\frac{1}{2}x^2 + o(x^2)} = -1$$

$$b) \lim_{x \rightarrow 0} \frac{e^x + e^{2x} - 2 \cos x - 3 \sin x}{\sqrt{1+2x} + \sqrt[3]{1-3x} - 2 \cos x} = \lim_{x \rightarrow 0} \frac{F(x)}{G(x)} = L$$

$$e^x \sim 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{2x} \sim 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$$

$$\cos x \sim 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\sin x \sim x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\Rightarrow F(x) \sim (1+1-2) + (1+2-3)x + \left(\frac{1}{2}+2-1\right)x^2 + \dots = \frac{3}{2}x^2 + o(x^2)$$

