Вар 16

№1

$$\left(\lim_{n\to\infty} \frac{2n}{1+n^2}=0\right) \Leftrightarrow (\forall \varepsilon>0)(\exists n_\varepsilon \in \mathbb{N})\left(n>n_\varepsilon \to \left|\frac{2n}{1+n^2}\right|<\varepsilon\right)$$

Зафиксируем $\varepsilon>0$.

$$\left|\frac{2n}{1+n^2}\right| = \frac{2n}{1+n^2} < \varepsilon$$

Т.к. $1+n^2>0$, то

$$2n < \varepsilon + \varepsilon n^2$$

$$\varepsilon n^2 - 2n + \varepsilon > 0$$

$$n^2 - 2\cdot\frac{1}{\varepsilon} n + 1 > 0$$

$$n_{1,2} = \frac{1}{\varepsilon} \pm \sqrt{\frac{1}{\varepsilon^2}-1}$$

$$\left[\begin{array}{l} n > \frac{1}{\varepsilon}+\sqrt{\frac{1}{\varepsilon^2}-1} \\ n < \frac{1}{\varepsilon}-\sqrt{\frac{1}{\varepsilon^2}-1} \end{array}\right.$$

$$\Rightarrow n > \frac{1}{\varepsilon}+\sqrt{\frac{1}{\varepsilon^2}-1}, \quad \underline{\underline{n_\varepsilon = \left[\frac{1}{\varepsilon}+\sqrt{\frac{1}{\varepsilon^2}-1}\right]+1}}$$

$$\varepsilon = 10^{-1} \Rightarrow n_{10^{-1}} = \left[10+\sqrt{100-1}\right]+1 = \underline{20}$$

Т.о., по определению $\lim\limits_{n\to\infty}\frac{2n}{1+n^2}=0$.

№2

$$\lim_{n\to\infty}\left(\sqrt{n^2-2n-1}-\sqrt{n^2-7n+3}\right) = \lim_{n\to\infty}\frac{n^2-2n-1-(n^2-7n+3)}{\sqrt{n^2-2n-1}+\sqrt{n^2-7n+3}} =$$

$$= \lim_{n\to\infty}\frac{5n-4}{\sqrt{n^2-2n-1}+\sqrt{n^2-7n+3}} = \frac{5}{2}.$$

№3

$$\left(\lim_{x\to+\infty} f(x) = +\infty\right) \Leftrightarrow (\forall \varepsilon>0)(\exists \delta_\varepsilon>0)(x>\delta_\varepsilon \to f(x)>\varepsilon).$$

Геометрически: