

Exercici 1: Especificació i OCm

1. Precondició: $(x = \bar{x} \wedge m = \bar{n} \wedge x \geq 0 \wedge m > 0)$

Postcondició: $(P = \bar{x}^{\bar{m}})$

Invariant: $(x > 0 \wedge m \geq 0 \wedge \bar{x}^{\bar{m}} = P * x^m)$
 $T: m$

2. $O(\log_2 m)$ és $\log_2(m)$ ja que a cada iteració del bucle m es redueix a la meitat.

Exercici 1: Verificació formal

```
private int p;  
public void power(int x, int m) {
```

```
    if (B  $x \neq 0$ ) {
```

```
        p = 1
```

```
        while (C  $m \neq 0$ ) {
```

```
            if (D  $m \% 2 \neq 0$ )  $p = x;$ 
```

```
             $m /= 2; x = x^2;$ 
```

```
        } else p = 0; }
```

Verificació

$$1.1) P \Rightarrow \text{dom}(B) \text{ int}$$

$$1.2) P \wedge B \Rightarrow \text{wp}(S_1, R)$$

$$P \wedge x! = \emptyset \Rightarrow \text{wp}(S_1, R)$$

$$1.2.1) Q \Rightarrow P$$

$$\text{wp}(P=1, P) = \{x \mid x \neq \emptyset \wedge m \geq 0 \wedge \bar{x}^{\bar{N}} = 1 \cdot x^m\}$$

$$1.2.2) P \wedge C \Rightarrow \text{wp}(S_2, P)$$

$$1.2.2.1) U \equiv \text{wp}(m/2; x^2 = \bar{x}, P) \wedge \bar{x}^{\bar{N}} = P \cdot (x^2)^{\frac{m}{2}} \wedge x^2 > 0 \wedge \frac{m}{2} \geq 0$$

$$1.2.2.2) P \wedge m! = \emptyset \Rightarrow \text{wp}(I\{U\})$$

$$1.2.2.2.1) U \Rightarrow \text{dom}(c) \text{ int}$$

$$1.2.2.2.2) P \wedge C \wedge D \Rightarrow \text{wp}(P^0 = x, U)$$

$$\text{wp}(P^0 = x, \bar{x}^{\bar{N}} = P \cdot (x^2)^{\frac{m}{2}} \wedge x^2 > 0 \wedge \frac{m}{2} \geq 0)$$

$$\bar{x}^{\bar{N}} = (P \cdot x^m \wedge x^2 > 0 \wedge \frac{m}{2} < 0 \wedge x^2! = 0 \wedge \frac{m}{2} \% 2 = 0) \Rightarrow$$

$$(\bar{x}^{\bar{N}} = P \cdot x^2 \cdot \left(\frac{m-1}{2}\right) \wedge x^2 > 0 \wedge \frac{m-1}{2} \geq 0 \wedge x^2! = 0 \wedge \frac{m}{2} / 2 = 0) \Rightarrow$$

$$\bar{x}^{\bar{N}} = P \cdot x^m \wedge x^2 > 0 \wedge m \geq 0$$

$$1.2.2.2.3) P \wedge C \wedge \neg D \Rightarrow \text{wp}(\text{null}, U)$$

$$\text{wp}(\text{null}, U) = \bar{x}^{\bar{N}} = P \cdot x^m \wedge x^2 > 0 \wedge \frac{m}{2} \geq 0$$

$$1.2.2.3) P \wedge \neg (\neg \exists R = P = \bar{X}^{\bar{N}})$$

$$\bar{X}^{\bar{N}} = P \cdot X^m \wedge X > \emptyset \wedge m \geq \emptyset \wedge m \leq \emptyset \Rightarrow$$

$$\bar{X}^{\bar{N}} = P$$

$$1.2.2.4) P \wedge C \rightarrow \tau > \emptyset \quad (\tau = m)$$

$$\bar{X}^{\bar{N}} = P \cdot X^m \wedge X > \emptyset \wedge m \geq \emptyset \wedge m \leq m \neq \emptyset$$

$$m > \emptyset \Rightarrow \tau > \emptyset$$

$$1.2.2.5) P \wedge C \wedge m \leq \tau + 1 \Rightarrow WP(S_3, m \leq \tau)$$

$$\bar{X}^{\bar{N}} = P \cdot X^m \wedge X > \emptyset \wedge m \geq \emptyset \wedge m \leq \tau + 1 \Rightarrow$$

$$m > \emptyset \wedge m \leq \tau + 1$$

$$\Rightarrow \frac{m}{2} < m \wedge m \leq \tau + 1$$

$$\Rightarrow \frac{m}{2} < \tau + 1 \Rightarrow \frac{m}{2} \leq \tau$$

$$1.3) P \wedge \neg B$$

$$P \wedge X = \emptyset \Rightarrow WP(S_3, R)$$

$$P = \bar{X}^{\bar{N}} = \emptyset \rightarrow \emptyset^m = 0$$