Threshold Measurements in the Bosonic backend

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• States in the Bosonic backend

An M-mode state in the bosonic backend is specified by a collection of coefficients $\{c_\ell\}_{\ell=1}^{K}$, means $\{\vec{r}_\ell\}_{\ell=1}^{K}$ & covariances $\{\sigma_\ell\}_{\ell=1}^{K}$

note that each $\vec{r}_\ell$ is a vector of dimension 2M

& each covariance is a matrix of dimension $2M \times 2M$

• Threshold Measurements:

Threshold measurement are <u>single</u> mode measurements with two outcomes:

- No click "0"
- click "1"

To do a threshold measurement in the Bosonic backend we first need to calculate the probability of clicking. Let's assume without loss of generality that we measure the last mode. We write each covariance matrix as

$$\sigma^{(e)} = \begin{pmatrix} \sigma_A^{(e)} & \sigma_{AB}^{(e)} \\ \left(\sigma_{AB}^{(e)}\right)^T & \sigma_B \end{pmatrix}$$

$\sigma_A^{(e)} : (M-2)\times(M-2)$

$\sigma_{AB}^{(e)} : (M-2)\times 2$

$\sigma_B^{(e)} : 2\times 2$

Similarly we split the vectors of means

$$\vec{r}_e = \begin{pmatrix} \vec{r}_A^{(e)} \\ \vec{r}_B^{(e)} \end{pmatrix}$$

$\vec{r}_A^{(e)} : M-2$

$\vec{r}_B^{(e)} : 2$

With this notation the probability of no click is simply

$$p_0 = \sum c_\ell \frac{\exp\left(-\frac{1}{2}(r_\ell^{(B)})^T\left(\sigma_{(\ell)}^{(B)}+\frac{\hbar}{2}\mathbb{I}_2\right)(r_\ell^{(B)})\right)}{\sqrt{\det\left(\frac{1}{\hbar}\sigma_{(\ell)}^{(B)}+\frac{\mathbb{I}_2}{2}\right)}}$$

with $\mathbb{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

After deciding whether the detector click or not we need to decide how to update the first $M-1$ modes.

(A) No click

In this case number of covariance matrices does not change i.e. we still have K of them.

They are now of dimension $(M-2)\times(M-2)$ and given by

$$\sigma_A^{(\ell)} \longrightarrow \underbrace{\left(\sigma_A^{(\ell')}\right)}_{\text{the new ones}} = \sigma_A^{(\ell)} - \sigma_{AB}^{(\ell)}\left(\frac{1}{\sigma_B^{(\ell)}+\sigma_M}\right)\left(\sigma_{AB}^{(\ell)}\right)^T$$

$$\vec{r}_A^{(e)} \longrightarrow \underbrace{\left(r_A^{(e)}\right)'}_{\text{the new ones}} = r_A^{(e)} + \sigma_{AB}^{(e)} \left(\frac{1}{\sigma_B^{(e)} + \sigma_M}\right) \left(-r_B^{e)}\right)$$

with $\sigma_M = \frac{\hbar}{2} \mathbb{I}_2$

and $$C^{(e)} \longrightarrow \underbrace{\left(C^{e}\right)'}_{\text{the new ones}} = \frac{C^{(e)}}{p_0}$$

These are generalizations of Eq. 5.142 & 5.143 of Serafinis' quantum continuous variables

Now consider the case of

- A click event

In this case the number of covariance variance matrices, vector of means & coefficients double)

- the first K vectors of means & covariances matrices are precisely the old ones $\sigma_A^{(l)}$, $r_A^{(l)}$ with coefficents

$$\frac{C^{(l)}}{1-p_0}$$

- Then the second set of K cov mats, means and coeffs are $(\sigma_A^{(l)})'$, $(r_A^{(l)})'$ with coefficients

$$-\frac{p_0 C^{(l)}}{1-p_0}$$