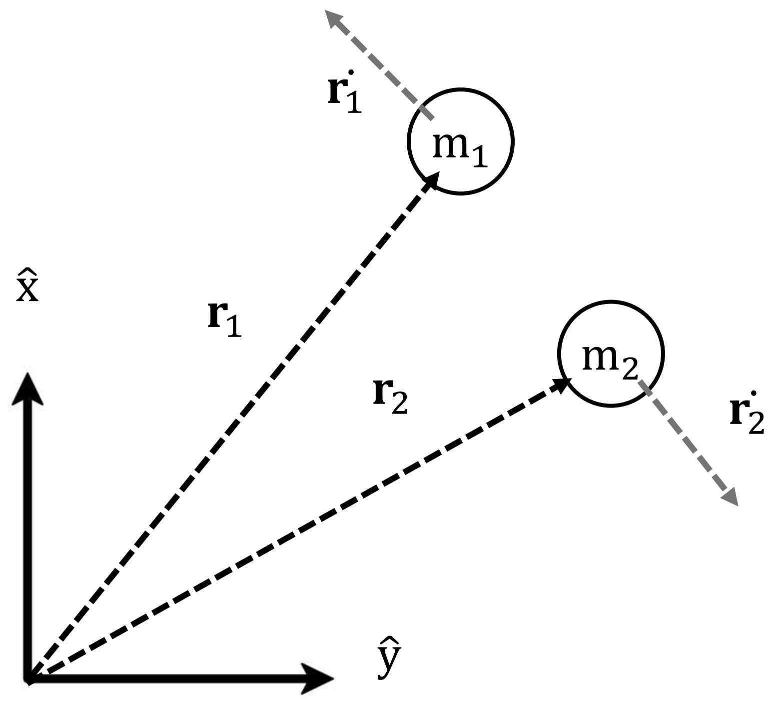
The N-Body Problem

AU25, Physics 5300, Xander Carroll

The Coordinates



The Lagrangian

The Lagrangian

$$T = \sum_{i=1}^{N} \frac{1}{2} m_i |\dot{\mathbf{r}}_{\mathrm{i}}|^2$$

The Lagrangian

$$U = -G \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{m_i m_j}{\left|\mathbf{r_i} - \mathbf{r_j}\right|}$$

The Lagrangian

$$\mathcal{L} = T - U = \sum_{i=1}^{N} \frac{1}{2} m_i |\dot{\mathbf{r}}_{\mathrm{i}}|^2 + G \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{m_i m_j}{|\mathbf{r_i} - \mathbf{r_j}|}$$

The Equations of Motion

$$\frac{\partial \mathcal{L}}{\partial \mathbf{r_i}} = -G\sum_{j \neq i} \frac{m_i m_j(\mathbf{r_i} - \mathbf{r_j})}{\left|\mathbf{r_i} - \mathbf{r_j}\right|^3}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{r}}_{\mathrm{i}}} = \frac{\mathrm{d}}{\mathrm{dt}}[m_i \dot{\mathbf{r}}_{\mathrm{i}}] = m_i \ddot{\mathbf{r}}_{\mathrm{i}}$$

The Equations of Motion

$$\frac{\mathrm{d}}{\mathrm{dt}}\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{r}}_i} = \frac{\partial \mathcal{L}}{\partial \mathbf{r}_\mathbf{i}}$$

$$m_i \ddot{\mathbf{r}}_\mathrm{i} = -G \sum_{j \neq i} \frac{m_i m_j \left(\mathbf{r}_\mathbf{i} - \mathbf{r}_\mathbf{j}\right)}{\left|\mathbf{r}_\mathbf{i} - \mathbf{r}_\mathbf{j}\right|^3}$$

Leapfrog Integration

$$\mathbf{v}_{\mathbf{i+1/2}} = \mathbf{v}_{\mathbf{i}} + \mathbf{a}(\mathbf{r}_{\mathbf{i}})\Delta t/2$$
$$\mathbf{r}_{\mathbf{i+1}} = \mathbf{r}_{\mathbf{i}} + \mathbf{v}_{\mathbf{i+1/2}}\Delta t$$
$$\mathbf{v}_{\mathbf{i+1}} = \mathbf{v}_{\mathbf{i+1/2}} + \mathbf{a}(\mathbf{r}_{\mathbf{i+1}})\Delta t/2$$

3-Body Animation

Solar System (N=9) Animation

Solar System (N=109) Animation

Expanding the Simulation

1. We would like to run the simulation for $n \rightarrow \infty$ bodies.

Expanding the Simulation

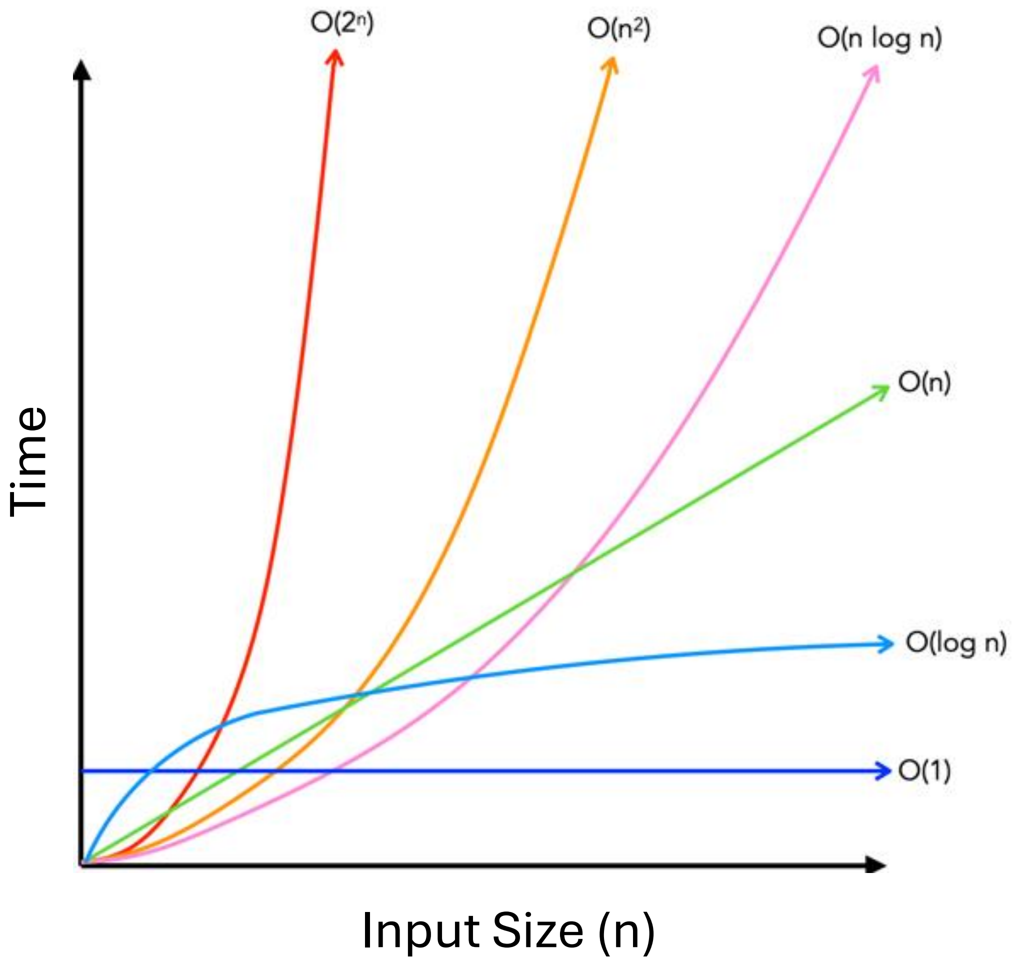
1. We would like to run the simulation for n → ∞ bodies.

2. We would like to do this without emptying Dr. Brandenburg’s OSC budget.

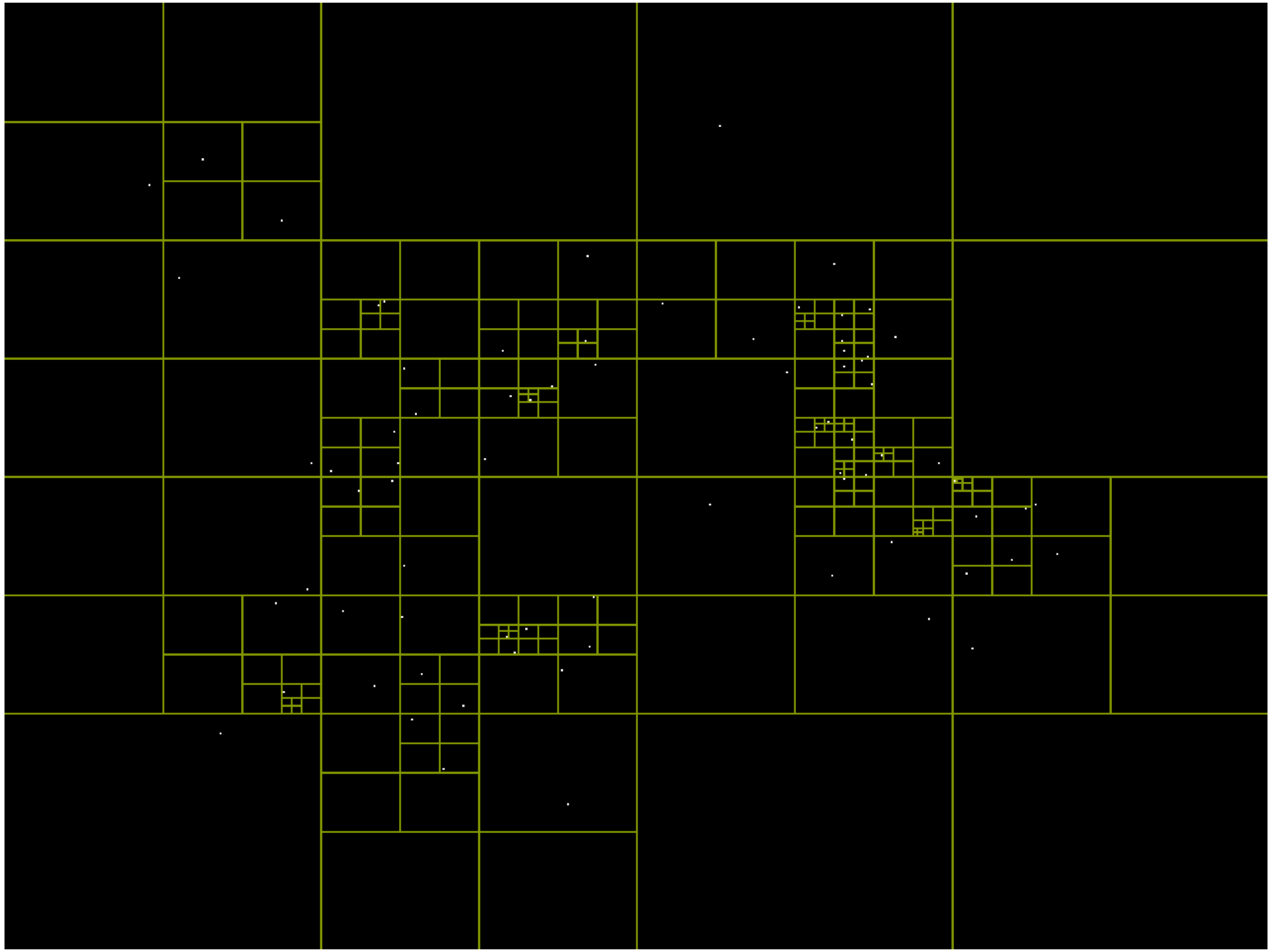
Running Time

- Making every comparison takes (n^2) "running time".

- The "Barnes-Hut" approximation takes $(n \log n)$ "running time".



Barnes-Hut



What I Learned

1. How to use the Manim Library

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1. How to use the Manim library.

2. What a Hamiltonian is.