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Brief Paper

Parameter Identification for a Traffic Flow Model*

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Key Words—Parameter identification; modelling; traffic flow; model validation; nonlinear programming.

Abstract—In this paper, a macroscopic model is presented which describes the traffic flow on a freeway by a set of nonlinear, deterministic difference equations. The model is deduced from simple physical and empirical considerations and contains a set of free parameters which have to be estimated using real traffic data. This identification procedure is formulated here as a parameter optimization problem which is solved by nonlinear programming. In addition, the sensitivity of the model with respect to parameter changes and structural changes is investigated. Although stochastic events play a role in traffic dynamics, the results demonstrate that the validated model copes surprisingly well with real traffic behaviour.

Introduction

FROM the viewpoint of an individual driver, traffic phenomena may seem to be mainly stochastic in nature. However, in a macroscopic treatment where traffic dynamics are represented by aggregate variables, traffic flow turns out to be a surprisingly reproducible, deterministic process. This makes it possible and also meaningful to develop mathematical models for the description of the dynamics of traffic flow. Overloaded and congested roads in most industrialized nations give a well-founded motivation to use these models for numerous purposes such as system analysis, traffic simulation and prediction, data processing and the development and judgement of control strategies (Nahi and Trivedi, 1973; Isaksen and Payne, 1973; Gosh and Knapp, 1978; Cremer 1978; Looze and colleagues, 1978).

This paper proposes a model for traffic flow on freeways which is an improved version of a model originally developed by Payne (1971). The model is formulated as a set of nonlinear difference equations and is expected to describe traffic dynamics in the whole range of possible traffic densities. The equations contain a number of parameters which are used to adapt the model to real traffic data. This is done in a thorough, extensive identification process, where the model calibration is formulated as a parameter optimization problem which is solved by means of nonlinear programming. The data sets used cover a large variety of different traffic situations to make sure that the results are reliable and representative. In order to investigate the quality of the model a performance index is formulated, which quantifies the coincidence between the model's behaviour and the real traffic flow process. In addition this index makes it possible to measure the degradation of the model if certain simplifications are made or if the sampling time is increased.

The paper is organized as follows. In the next section the aggregate variables of traffic flow are introduced and the traffic flow model is established from physical and empirical considerations. The third section starts with some general reflections concerning the causality of the process of traffic flow. Then the identification of the model's parameters is formulated as a least squares optimization problem which is solved by a nonlinear programming routine using iterative comparison of model behaviour with real traffic data collected from a German autobahn. The results are presented in the fourth section. Using the parameter optimization procedure the model is then validated with respect to its parameter sensitivity and to its sensitivity to the choice of the applied data set. Furthermore, the influence of the sampling interval and of the width of the spatial discretization is discussed as well as the model's degradation due to several simplifications of the model equations. Finally a summary with conclusions is given in the last section.

Mathematical model of traffic flow

Before we can start developing the model equations, we must clarify what we mean by the state of traffic flow on a longer road section and by what variables it could be represented. Consider a section of a two-lane freeway having a length of several kilometres. This section is thought to be subdivided into segments of length 400–800 m (Fig. 1).

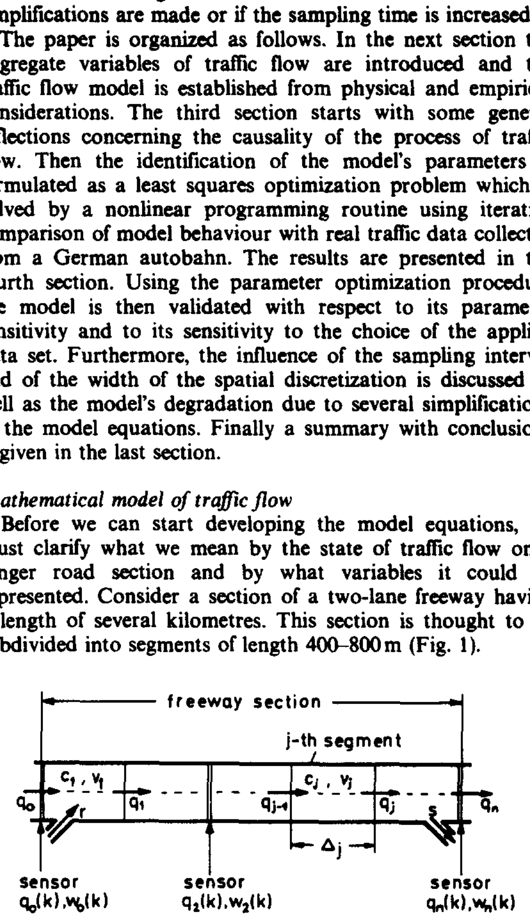

FIG. 1. Section of a two-lane freeway.

If there are any on- and off-ramps, they are supposed to be at the ends of the section only. A whole freeway is considered to be built up as a tandem connection of such sections.

With respect to this configuration, the following variables are defined for each segment

$c_j(k)$ — *traffic density* in segment j at time $k \cdot T$ (veh/km)

$v_j(k)$ — *mean speed* of the vehicles within segment j at time $k \cdot T$ (km/h)

$q_j(k)$ — volume from the jth segment to the $(j+1)$th segment, that is the number of vehicles which pass from the jth segment to the $(j+1)$th segment during the time interval $kT \leq t < (k+1)T$ divided by T(veh/h)

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$r(k), s(k)$ entering or leaving *ramp volumes* related to the time interval $kT \leq t < (k+1)T$ (veh/h)

$w_j(k)$ *time mean speed*, harmonic mean of the individual vehicle velocities which are measured at cross-section j during $kT \leq t < (k+1)T$.

Here T is a fixed time interval and k is a running integer time index. The state of the traffic flow process will then be defined by the set of density and mean speed values in the segments $c_j(k)$, $v_j(k)$, while the volumes $q_j(k)$ can be expressed by these variables as will be shown below.

Having defined the process variables we try to find relations which describe the transition from state at time $k \cdot T$ to the state at time $(k+1) \cdot T$. From a simple balance of vehicles we obtain

$$c_j(k+1) = c_j(k) + \frac{T}{\Delta_j}[q_{j-1}(k) - q_j(k)] \tag{1}$$

where Δ_j is the length of the jth segment. (On- or off-ramp volumes have to be added eventually for $j=1$ or $j=n$ within the brackets.) It should be pointed out, that in this form equation (1) holds exactly.

Following the ideas of Payne (1971) the mean speed v_j is influenced by three terms: a relaxation term, a convection term and a density gradient term. This is expressed by the following difference equation

$$\begin{aligned} v_j(k+1) = v_j(k) &+ \frac{T}{\tau}[V(c_j) - v_j]_{(k)} \\ &+ \frac{T}{\Delta_j}\left[v_j(v_{j-1} - v_j)\,\mathrm{sat}\left(\frac{c_{j-1}}{c_j}\right)\right]_{(k)} \\ &+ \frac{\nu}{\Delta_j}\frac{T}{\tau}\left[\frac{c_j - c_{j+1}}{c_j + \kappa}\right]_{(k)} \end{aligned} \tag{2}$$

The relaxation term (first bracket term) effects according to the yet unknown time constant τ that the actual mean speed v_j approaches asymptotically the steady state characteristic $V(c_j)$ which is a monotonically decreasing function of the density c. The convection term (second bracket term) accounts for the propagation of a speed difference $(v_{j-1} - v_j)$ into the jth segment. However, vehicles leaving segment $j-1$ contribute to the mean speed in segment j according to the ratio of vehicle numbers, that is, to the ratio of densities. (Without the factor sat (c_{j-1}/c_j) which was not contained in Payne's model, difficulties may arise in cases of congestion.) The density gradient term reflects the driver's anticipation to a foreseen relative density change which is weighted by a sensitivity factor ν. The density parameter κ takes into account that this effect becomes negligible for low density values. (Without this parameter, which was not contained in Payne's equation, the model has shown abnormal behaviour in the low density range.)

While the volumes $q_0(k)$ and $q_n(k)$ are considered to be known as external inputs (see the argument in the next section), the volumes $q_j(k)$ are regarded as internal variables and are expressed in analogy to hydromechanics as a weighted local mean of products of density and velocity

$$q_j(k) = \alpha[c_j v_j]_{(k)} + (1-\alpha)[c_{j+1} v_{j+1}]_{(k)}. \tag{3}$$

Here the weighting factor α is again a free parameter. For the steady-state characteristic $V(c)$ in equation (2) May and Keller (1967) proposed a rather general relationship

$$V(c) = V_f[1 - (c/c_{\max})^l]^m \tag{4}$$

where V_f is the free velocity at zero density, $c_{\max}$ is the jam density and l, m are positive real parameters.

For the identification of the model as described in the next section we will use measured time sequences of volumes $q(k)$ and time mean speed values $w(k)$. While the volumes are related to the state variables by equation (3), for the values $w_j(k)$ we write similarly

$$w_j(k) = \alpha v_{j-1}(k) + (1-\alpha) v_j(k). \tag{5}$$

The model equations (1)–(5) contain the free parameters V_f, $c_{\max}$, l, m, α, κ, ν and τ which have to be identified by the calibration of the model. Besides, we will consider the length of a segment Δ_j and the sampling time T as parameters which are to be selected appropriately. It is the subject of the identification procedure to determine the parameter values which give the best coincidence between the model and the real process and to mark the admissible range for the values of Δ_j and T. In addition we will investigate to what extent simplifications of the model equations will affect the performance of the model.

Model identification

Traffic flow on a freeway section is now considered as a causal process on which certain stimuli act as inputs while any combination of internal variables may be chosen as the system's reaction or output. When defining the input variables we have to take into account different traffic situations: in the case of low density, traffic flow dynamics are mainly determined by the entering vehicles, that is by the conditions at the section entry. At high density, however, growing congestion originates from the section exit and propagates in upstream direction. Depending on the state of the traffic, either the conditions at the section entry or those at the section exit have more influence on the system's behaviour. Since the model is to describe traffic dynamics in the whole density range $0 \leq c \leq c_{\max}$, the identification should be carried out on the basis of traffic data from low to high density. To have in every case a correct assignment of stimulus and reaction the measured data at both ends of the section are treated as system inputs. The reaction or output of the system is then to be taken from a third location inside the section.

Consequently a configuration with three sites of data collection must be chosen (cf. Fig. 1). The data from both ends of the section, i.e. $q_0(k)$, $w_0(k)$ and $q_n(k)$, $w_n(k)$, are treated as input sequences while the data from an internal location $q_J(k)$, $w_J(k)$ are taken as output sequences.

Let us introduce at this point

a state vector $\mathbf{x}^T = [c_1 v_1 \ldots c_n v_n]$

an input vector $\mathbf{u}^T = [q_0 w_0 q_n w_n]$

and

an output vector $\mathbf{y}^T = [q_J w_J]$

for the whole freeway section, and the vector of unknown parameters $\boldsymbol{\beta} = [V_f,\ c_{\max},\ l,\ m,\ \alpha,\ \kappa,\ \nu,\ \tau]$. Furthermore, let $\hat{\mathbf{u}}(k)$, $\hat{\mathbf{y}}(k)$, $k = 1, \ldots K$ be the time sequences of measured data collected from a real traffic flow which contains transitions through the whole spectrum of possibly density values in a representative manner.

By substituting equation (3) into (1) and equation (4) into (2) for all segments, a nonlinear, dynamic state vector equation can be derived having the general form

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), \boldsymbol{\beta}]. \tag{6}$$

The output vector equation is built up by equations (3) and (5) taken for $j=2$ and has the form

$$\mathbf{y}(k) = \mathbf{g}[\mathbf{x}(k), \boldsymbol{\beta}]. \tag{7}$$

The estimation of the unknown parameters is not a conventional problem in our case, since the model equations are nonlinear in both the state variables and the parameters. The most common approach for the identification of nonlinear systems is the least squares output error method which minimizes the discrepancy between the model and the real process with respect to some quadratic output error functional. This approach has the additional advantage that it needs no further a priori information about the probabilistic properties of the parameter values (Åström and Eykhoff, 1971) which are not available in our case.

The parameter identification problem may now be formulated as the following least squares output error problem.

Given the time sequences of measured data

$$\hat{\mathbf{u}}(k), \hat{\mathbf{y}}(k), k=1, \ldots K$$

and the initial state of the traffic flow process $x(0)$. Find the set of parameters $\boldsymbol{\beta}$ which minimizes the criterion

$$I(\boldsymbol{\beta}) = \sum_{k=1}^{K} [\mathbf{y}(k)-\hat{\mathbf{y}}(k)]^T \mathbf{Q} [\mathbf{y}(k)-\hat{\mathbf{y}}(k)] \quad (8)$$

where $\boldsymbol{\beta}$ has to be taken from a closed admissible region in the eight-dimensional parameter space which has to be defined on the basis of physical considerations. $y(k)$ is the output vector as generated by the model equations (6) and (7).

$\mathbf{Q}$ is a positive definite 2×2 matrix which was chosen to be

$$Q = \begin{pmatrix} \gamma & 0 \\ 0 & 1 \end{pmatrix}$$

with a weighting factor γ which has to be chosen appropriately, for example

$$\gamma = \sigma_w^2 / \sigma_q^2 \quad (9)$$

where σ_q^2, σ_w^2 are the variances of the stochastic components in the measured variables which cannot be modelled by the deterministic equations of the model. Here we have chosen

$$\gamma = 0.001 \text{ km}^2/\text{veh}^2.$$

There is obviously no direct functional but only a procedural relationship between the parameter vector $\boldsymbol{\beta}$ and the performance criterion $I(\boldsymbol{\beta})$. This rules out optimization techniques which use derivatives of the criterion with respect to the parameters β_i like standard variational techniques. Consequently, the determination of the optimal parameter set must be performed by means of a nonlinear programming search routine where for each choice of a new parameter vector $\boldsymbol{\beta}$ the value of the performance criterion (8) is computed by a simulation run of the model equations driven by the measured inputs according to Fig. 2.

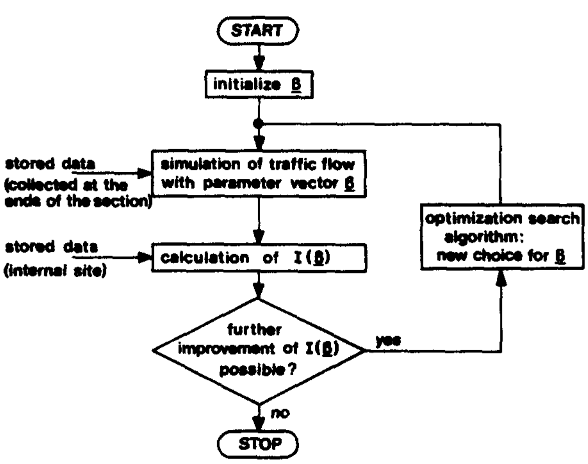

FIG. 2. Two stage structure of optimization.

By the same reason it is not easy to decide whether there is only one global optimum or a number of local optima. As a consequence, gradient methods or steepest descent methods are less suitable to solve this optimization problem, since these methods tend to find only a local optimum point and, additionally, require a numerical computation of the gradient of $I(\boldsymbol{\beta})$ which involves additional simulation runs for each iteration in our case. Therefore, the Complex algorithm of Box (1965) was applied here which does not require any derivatives of the performance index.

This algorithm starts with an initial set of points $\boldsymbol{\beta}^i$ which are randomly scattered throughout the admissible region in the parameter space. Then in each iteration step the parameter set with the worst value of the performance index is replaced by a new parameter set which is chosen appropriately. In this way the algorithm has a greater chance of finding the global (or at least a 'good') optimum. The procedure is terminated when the points $\boldsymbol{\beta}^i$ reach a sufficiently small region around the optimum so that no further improvement of the performance criterion $I(\boldsymbol{\beta})$ can be achieved by further iterations. Even with this algorithm, however, it is not easy to decide whether the global optimum is actually reached. For this reason it is useful to repeat the procedure with different sets of starting points.

Results and model validation

(a) *Optimal parameter set.* For the execution of the outlined identification procedure a number of different data sets were available which were collected from a section of the autobahn from Frankfurt to Basel. The section has a total length of 2650 m which was subdivided into $n=5$ segments of 500 m ($j=1,2$) and 550 m ($j=3,4,5$) length. Sensors for volumes and velocities are installed at both ends and within the section at a distance of 1000 m from the section's entry, i.e. behind the second segment with $J=2$. This distance is passed by a vehicle with free velocity within half a minute, that is, within the time constant as will be apparent from the results below. Thus, the distance between the input sensors at both ends and the internal output sensor is short enough to ensure a clear causal dependency. On the other hand, it is long enough for the dynamic laws of traffic flow to noticeably affect the outputs.

The available data sets contained a number of different traffic situations including free as well as congested traffic flow. From these a representative set of a $3\frac{1}{2}$ hours observation period was selected where in the last hour traffic became more and more crowded and finally collapsed. By the identification procedure described in the preceding section the following set of optimal parameter values was obtained (Table 1).

The optimization with the complex algorithm of Box was performed on a Cyber 175 computer. Convergence was achieved after 148 iterations which took about 5 min computation time. The sampling time T was chosen to be 10 s which means that each simulation run over the $3\frac{1}{2}$ h real time period contained $K=1260$ sampling time intervals. The optimization was carried out several times with different sets of starting points whereat in most cases the optimal parameter set of Table 1 was found. As was mentioned above, this makes it rather likely that the absolute optimum was achieved.

In Figs 3 and 4 the time response of local mean speed $w_2(k)$ and volume $q_2(k)$ as generated by the calibrated model are presented together with the measured sequences $\hat{w}_2(k)$ and $\hat{q}_2(k)$ of the real traffic process.

(b) *Transferability of the model.* To validate the model it was investigated to what extent the performance of the identified model was sensitive with respect to the chosen data set. For this the model with the nominal parameters of Table 1 was applied to a number of different data sets. First, in each case the performance criterion was evaluated for the nominal parameter values. Then the identification procedure was carried out again to find out what improvement of the model's quality could be obtained by an individual parameter optimization.

These investigations have shown that the value of the performance criterion was improved generally by less than 20% when the nominal parameter set was replaced by the individually optimized set. This demonstrates the flexibility and the transferability of the identified model.

In Fig. 5 the results are depicted for a critical data set where traffic collapses only for a short time of 10 min and then returns to normal flow. The curves show that the breakdown is modelled very effectively by the nominal model

TABLE 1. OPTIMAL PARAMETER SET β^0

V_f	c_{max}	l	m	α	κ	ν	τ
123 km/h	200 veh/km	4,0	1,4	0,8	20 veh/km	21,6 km^2/h	0,01 h

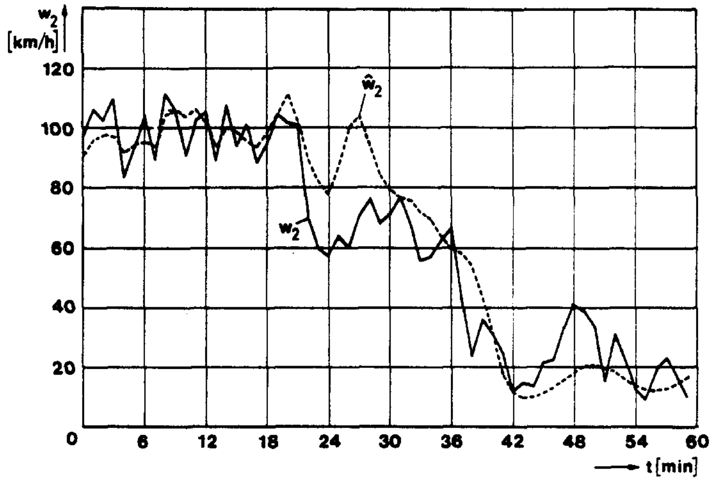

FIG 3. Local mean speed $w_2(k)$; measurements (——) and model (-----).

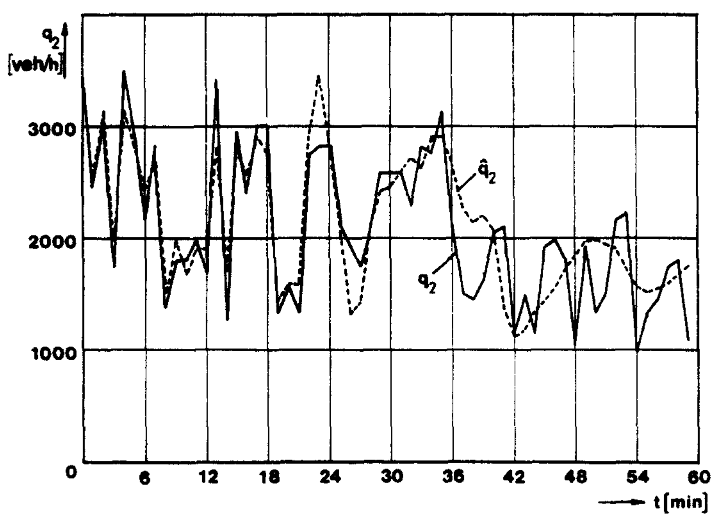

FIG. 4. Volume $q_2(k)$; measurements (——) and model (-----).

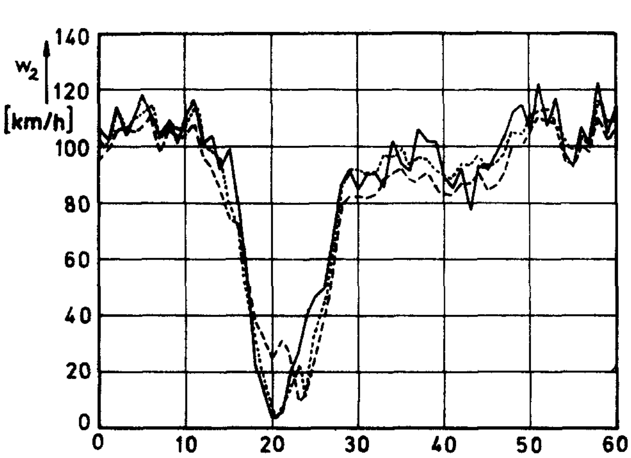

FIG. 5. Local mean speed $w_2(k)$; measurements (——), model with nominal parameters (-----) and model with individually adapted parameters (.....).

while the special shape of the phenomenon is fitted even better by an individual parameter adaptation.

In addition it was inspected to what extent the values of the optimal parameters differ from the nominal values of Table 1 when the optimization procedure was applied individually to the different data sets. It turned out that the changes were less than 3% for most of the parameter values. Only the free velocity V_f may vary in different situations up to 5%. In an on-line application of the model e.g. for traffic surveillance by filtering (Nahi and Trivedi, 1973; Cremer, 1976) this parameter might be adapted from time to time [for example according to Grewal and Payne (1976)].

(c) *Sensitivity with respect to parameter changes.* It was further investigated to what extent the performance of the model is degraded when small parameter changes are considered. For this purpose the performance criterion was evaluated with the nominal data for a perturbed parameter set $\beta=\beta^0+\delta\beta$. The sensitivity with respect to a certain parameter change $\delta\beta$ was quantified by the index

$$\sigma(\delta\beta)=\frac{I(\beta^0+\delta\beta)-I(\beta^0)}{I(\beta^0)}100\%. \qquad (10)$$

In Table 2 the results are listed when each parameter is changed by $\pm 5\%$ successively.

It is seen that the model is not very sensitive to small parameter changes. Only deviations in the free velocity V_f may cause a noteworthy degradation which might require an adaptation of this parameter in an on-line application as already mentioned above.

(d) *Influence of sampling time and segment length.* In the above optimization procedure the length Δ_j of a segment and the sampling time T were kept fixed. We will now investigate whether and to what extent another choice of both values will lead to an improvement or a deterioration of the model's performance. Intuitively, the introduced aggregate variables c and v become meaningless if Δ_j is too small while too large a value of Δ_j will inhibit the modelling of local inhomogeneities. Too small a value of sampling time T makes the variables q and w (which relate to ensembles of vehicles over a time interval) meaningless, while if T is chosen too long it becomes difficult for the model to follow the faster dynamics of traffic flow. The introduced cost functional (8) and the instrument of the optimization routine offers the possibility to investigate the influence of Δ_j and T on the model's performance.

For this reason, the freeway section from which the data were collected was in the first case formally subdivided into three segments with length of 1000 m, 825 m (internal sensor behind the first segment) and in the second case into eight segments with length of 330 m (internal sensor behind the third segment).

The value of the cost functional for different sampling times T and different segment lengths Δ_j was determined repeating the parameter optimization procedure for each case. The results are summarized in Table 3.

From Table 3 it becomes apparent that there exists a critical lower bound for the ratio Δ_j/T at a value of 25 m/s. If the ratio has a smaller value, the quality of the model is degraded drastically. The main reason for this is that for too small a ratio Δ_j/T, vehicles may pass a whole segment during T thus already influencing the state of the second downstream segment. The convection term in equation (2) would then become erroneous and lead to a worse performance of the model.

For a ratio Δ_j/T greater than 30 m/s Table 3 reveals that increasing the segment length beyond 500 m gives slight degradation. This might still be acceptable in a practical case, especially since for a given freeway configuration the number of segments, and thus the order of the system, is reduced when longer segments are chosen. Increasing the value of Δ_j/T beyond 50 m/s by shortening the sampling interval gives no further improvement while the computational effort grows and may inhibit real time applications. A convenient compromise might be obtained by the values $T=10$ s, $\Delta_j=500$ m which were chosen for the identification procedure.

TABLE 2. SENSITIVITY INDEX $\sigma(\delta\beta)$ FOR SMALL PARAMETER CHANGES

Parameter	V_f	c_{max}	l	m	α	κ	ν	τ
$\sigma(\delta\beta_i=+5\%)$	9 %	4 %	6 %	0 %	1,0 %	1,0 %	0,6 %	0,2 %
$\sigma(\delta\beta_i=-5\%)$	3 %	1 %	1 %	2,5 %	0 %	0,5 %	0 %	0,3 %

TABLE 3. PERFORMANCE OF THE MODEL FOR DIFFERENT VALUES OF SAMPLING TIME T AND SEGMENT LENGTH Δ_j

Sampling-time T	segment length Δ_j 330 m	500-550 m	825-1000 m
5s	119	120	188
10s	141	120	184
15s	3494	127	162
20s	5339	578	156
30s	5769	4036	249
60s	6444	5833	6044

TABLE 4. PERCENTAGE OF LOSS OF PERFORMANCE DUE TO VARIOUS SIMPLIFICATIONS OF THE MODEL

simplification	loss of performance ρ mixed traffic	free traffic	congestion
Payne's model $\alpha = 1; \varkappa = 0$ simple convection term	21 %	33 %	3 %
density gradient term dropped ($\nu = 0$)	142 %	0 %	425 %
convection term dropped	93 %	130 %	20 %
no dynamic delay for v_j $v_j = V(c_j)$	228 %	80 %	524 %

(e) *Simplifications of the model.* Finally the elaborated procedure offers the possibility of quantifying the loss of performance if some shortcuts are made in the model equations for reasons of simplification and reduction of numerical effort. To investigate the consequences of some simplification the whole parameter optimization procedure was repeated yielding a new optimal value $I^0_{\rm red}$ of the criterion, thus allowing the remaining parameters to compensate the shortcut by altered values. The loss of performance can then be quantified by the following index

$$\rho = \frac{I^0_{\rm red} - I^0}{I^0} 100\% \tag{11}$$

where I^0 is the optimal value of the performance criterion as obtained for the nominal model. In Table 4 the value of ρ is presented for various simplifications.

The results indicate that the modifications by which the model of Payne was refined here bring considerable improvement while only few additional numerical operations are necessary. If the density gradient term is dropped, congestion is modelled rather poorly. This is a consequence of the fact that propagation of congestion in upstream direction (which is a characteristic phenomenon of traffic flow) can only be modelled in this case by setting the volume entering a segment to zero when jam density is reached in the segment. The convection term on the other hand has its main effect in the low density range. Consequently, degradation of the model's behaviour is confined mainly to the free traffic flow if this term is dropped. Neglecting the dynamic inertia for the mean speed and replacing equation (2) simply by the static relation $v_j = V(c_j)$ results in a drastic deterioration of the model's quality.

Conclusions

The paper presented a dynamic traffic flow model in the form of a set of cascaded, nonlinear difference equations. The model was designed to represent traffic dynamics in any density range and its time discrete form is especially suitable for use on a digital computer. A number of free parameters were used to adapt the model to observations of the real traffic process.

For this purpose, data from a freeway section with three measurement sites were used, where the measurements at both ends acted as the inputs while the data from an inner location were taken to be the output of the system thus giving a unique assignment of causality.

The identification process was formulated as a parameter optimization problem and was performed on the basis of extensive data material. The result is a rather general, highly realistic traffic flow model which is capable of describing traffic behaviour in the whole density range. Moreover, the introduced performance index made it possible to investigate the sensitivity of the model with respect to different data sets and to parameter changes as well as to quantify its loss of performance when some simplifications of the model's equations are made. As might be expected, it has been shown that the quality of the model is the more degraded the more its equations are simplified.

The model identification was performed using data collected under no speed limitation. The identified parameter values can be considered to be representative of traffic flow on freeways in most European countries where no or only light restriction is imposed on the maximum speed. When applied to traffic flow in the United States slightly different parameter values are to be expected due to the considerable limitation of maximum speed and to the different size of cars.

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