Traffic as a compressible liquid

Non Linear eq. of 2nd-order model

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i \cdot \lambda_i}\left[q_{i-1}(k) - q_i(k) + r_i(k)\right]$$

$$V_i(\rho_i) = v_{free} \cdot \exp\left(-\frac{1}{a}\left(\frac{\rho_i}{\rho_{cr}}\right)^a\right)$$

$$v_i(k+1) = v_i(k) + \frac{T}{\tau}\left[V\left[\rho_i(k)\right] - v_i(k)\right]$$
$$+ \frac{T}{L} \cdot v_i(k) \cdot \left[v_{i-1}(k) - v_i(k)\right]$$
$$- \frac{T \cdot \eta}{\tau \cdot L} \cdot \frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \kappa} - \frac{\delta \cdot T}{L \cdot \lambda_i} \cdot \frac{r_i(k) v_i(k)}{\rho_i(k) + \kappa}$$

q = Traffic Flow

ρ = Traffic Density

v = Space Mean Speed of Traffic

λ_i = # of Lanes

r_i = Flow of on-ramp

a, β, ρ_{cr}, κ, τ, δ, v_{free}, & η = constants

State Space Form of the Centered System

$$\Delta x(k) = x(k) - x^*(k)$$

$$\Delta x(k+1) = A\Delta x(k) + B\Delta u(k) + H\Delta d(k)$$

x^* = Setpoints

$x \in \mathbb{R}^m$ denotes System States

$u \in \mathbb{R}$ denotes control inputs

$d \in \mathbb{R}$ denotes disturbances of the System

$A \in \mathbb{B}$