Given the general rendering equation

$$L_o(x_o, \vec{\omega}_o) = L_e(x_o, \vec{\omega}_o) + L_r(x_o, \vec{\omega}_o)$$

$$L_o(x_o, \vec{\omega}_o) = L_e(x_o, \vec{\omega}_o) + \int_A \int_{2\pi} S(x_i, \vec{\omega}_i, x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i)(\vec{\omega}_i \cdot \vec{n}_i) d\vec{\omega}_i dA$$

The inner integarl can be eliminated if we consider only one directional light source. That leads to the following equation:

$$L_o(x_o, \vec{\omega}_o) = \sum_{i=0}^{N} S(x_i, \vec{\omega}_d, x_o, \vec{\omega}_o)\ (\vec{\omega}_d \cdot \vec{n}_i)\ V(n_i, \vec{\omega}_d)\ L_d\ A_i$$

Where A_i is the barycentric area (sum of vertex to the neighboring triangles to closest barycenter), N is the number of vertices of the mesh, and $\vec{\omega}_d$, L_d are the direction towards and the radiance of a directional light. $V(n_i, \vec{\omega}_d)$ is a visibility function defined as follows:

$$V(n_i, \vec{\omega}_d) = \begin{cases} 1 & \text{if } \vec{\omega}_d \cdot \vec{n}_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

That holds for the simple models used.
As we can see from the following results, there it seems to be a slight overshoot on the directional dipole model: needs investigation.

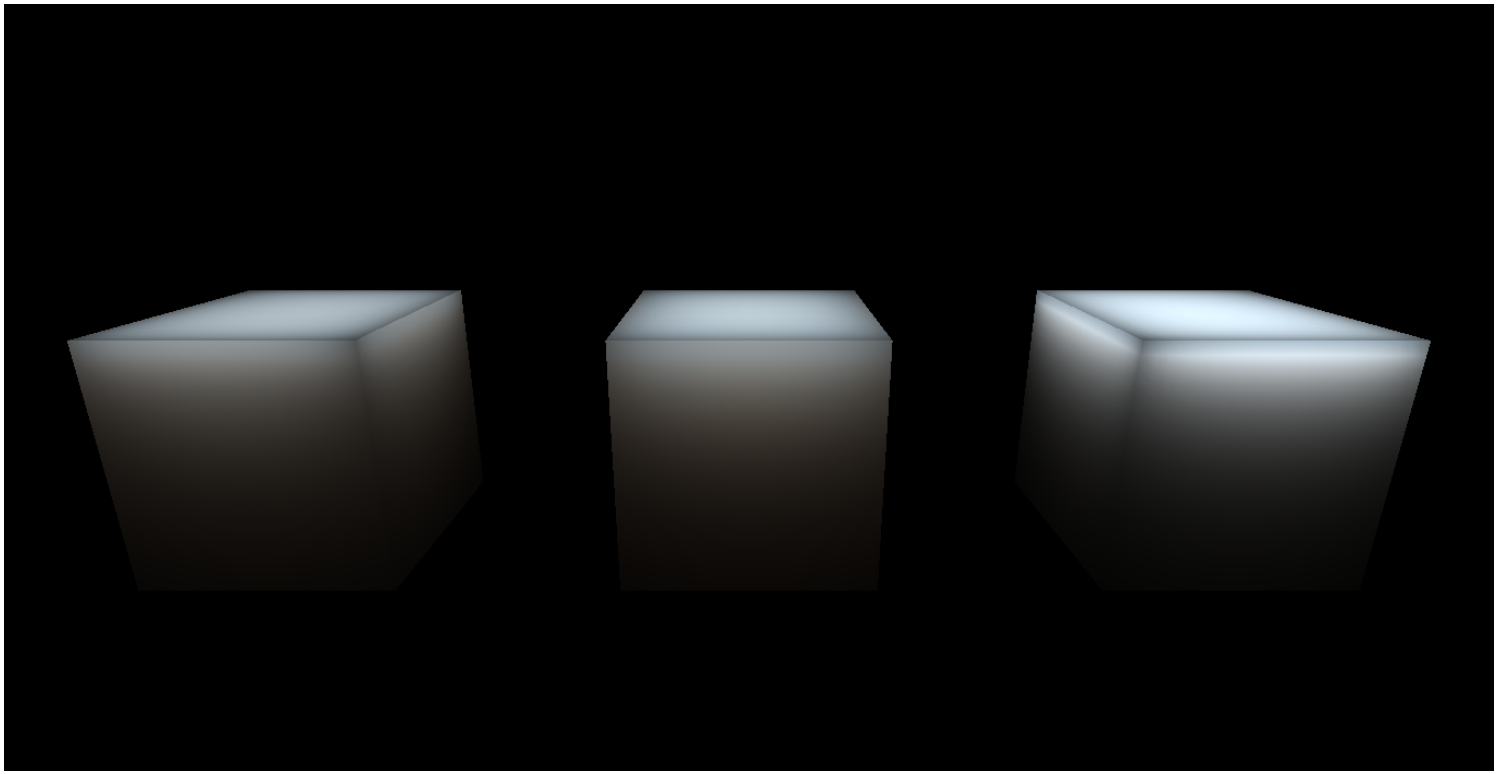

Figure 1: Marble set of cubes. Left to right: Jensen et al. dipole, D'Eon et al. better dipole and Frisvad et al. directional dipole. Material properties for marble.

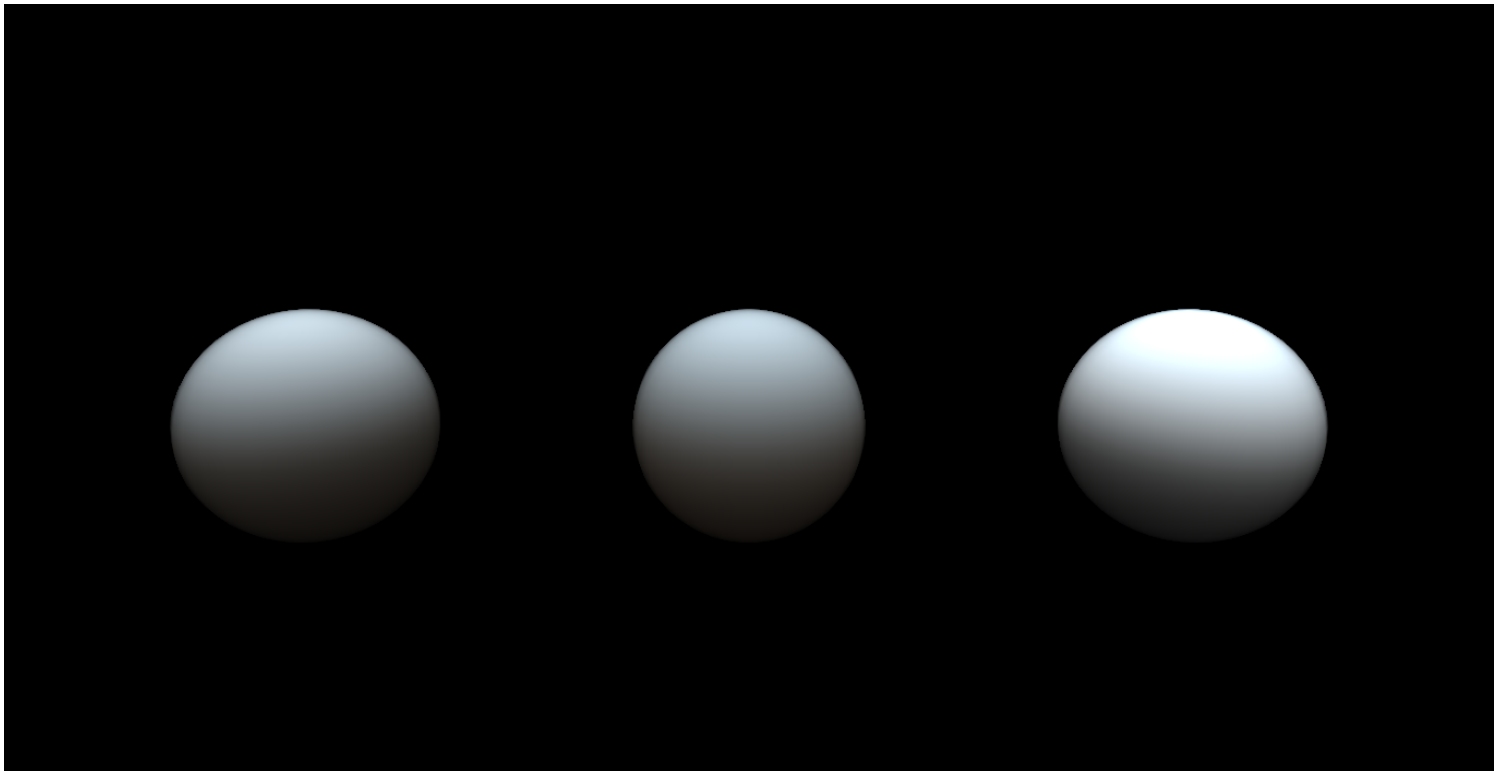

Figure 2: Marble set of spheres. Left to right: Jensen et al. dipole, D'Eon et al. better dipole and Frisvad et al. directional dipole. Material properties for marble.

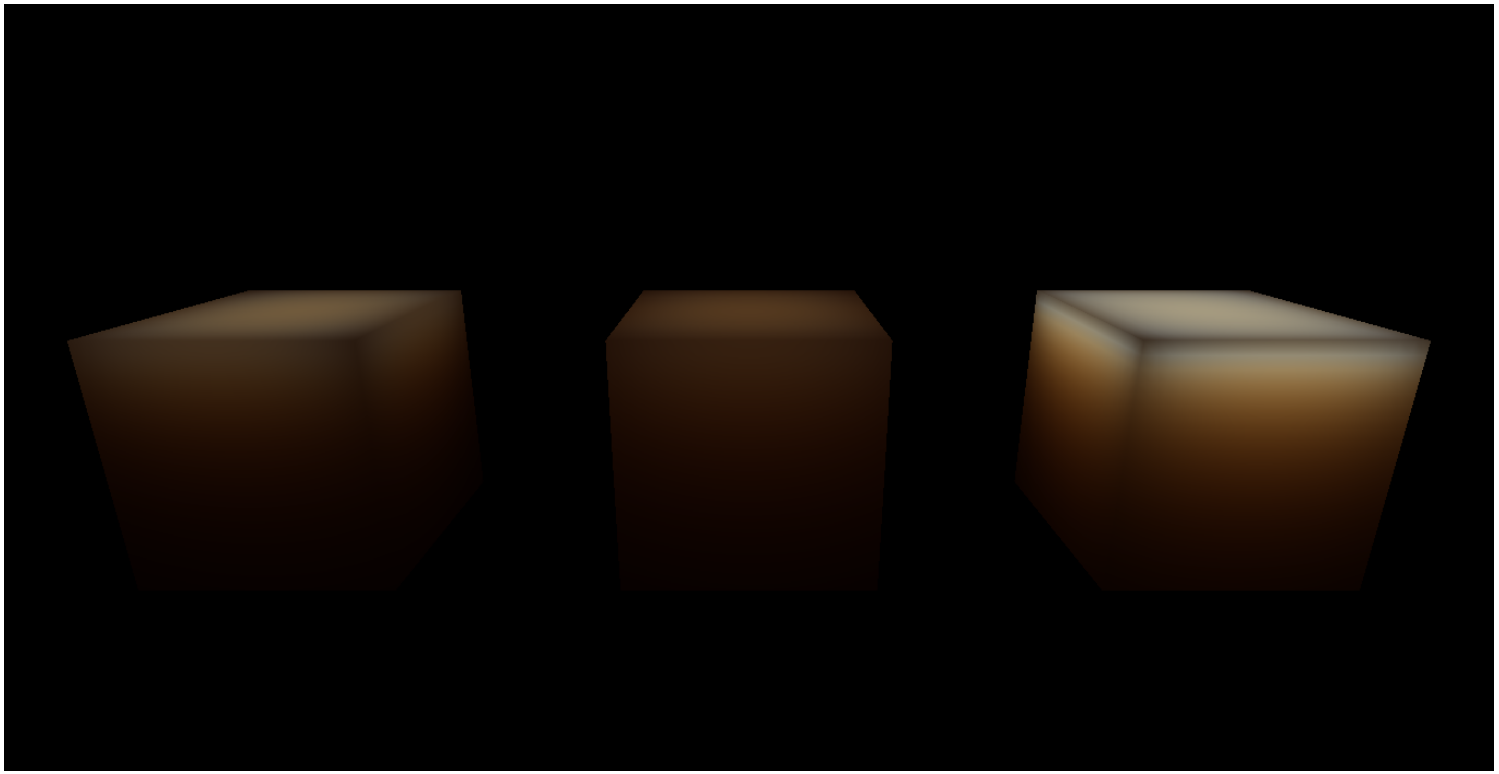

Figure 3: Regular chocolate milk set of cubes. Left to right: Jensen et al. dipole, D'Eon et al. better dipole and Frisvad et al. directional dipole. Material properties for regular chocolate milk.

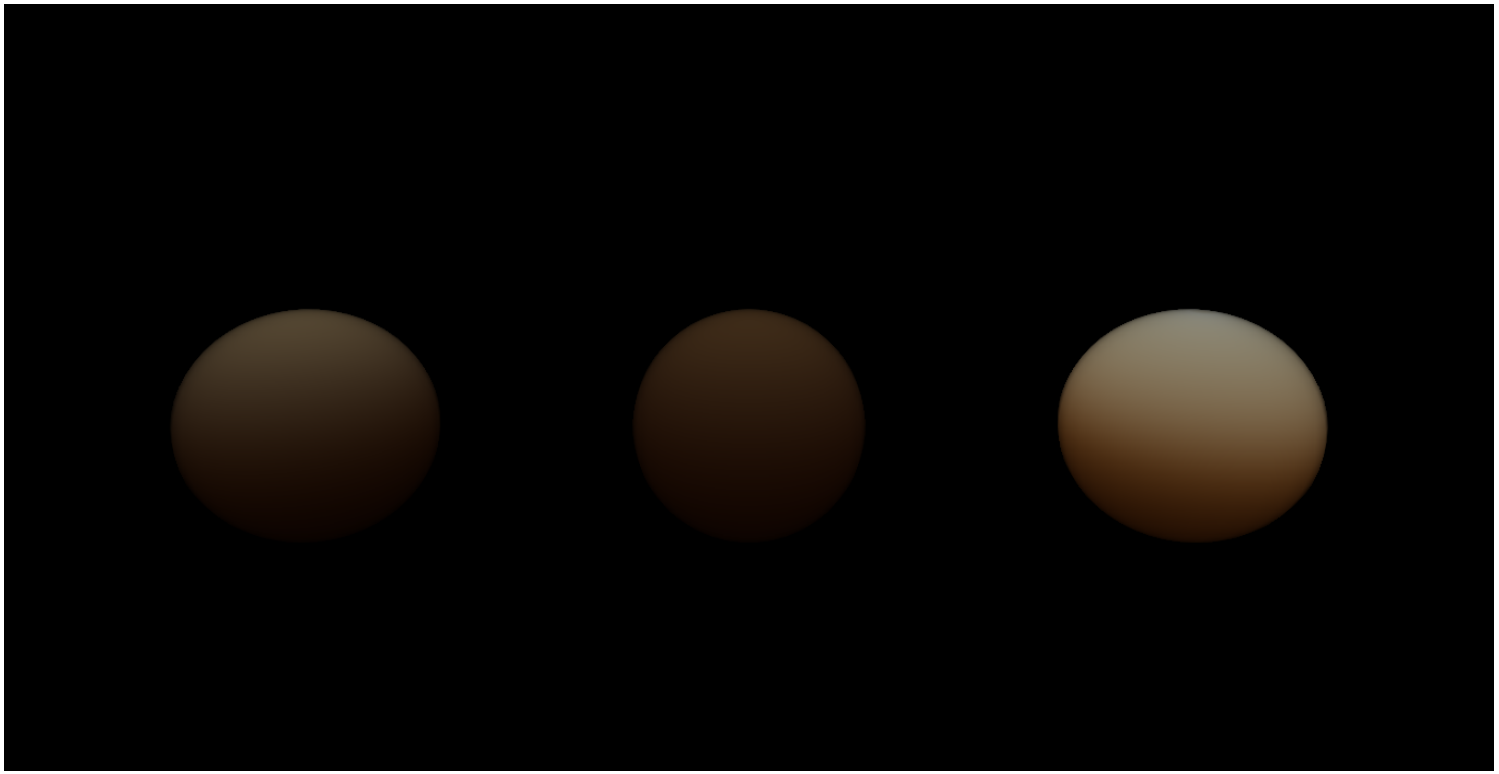

Figure 4: Regular chocolate milk set of spheres. Left to right: Jensen et al. dipole, D'Eon et al. better dipole and Frisvad et al. directional dipole. Material properties for regular chocolate milk.

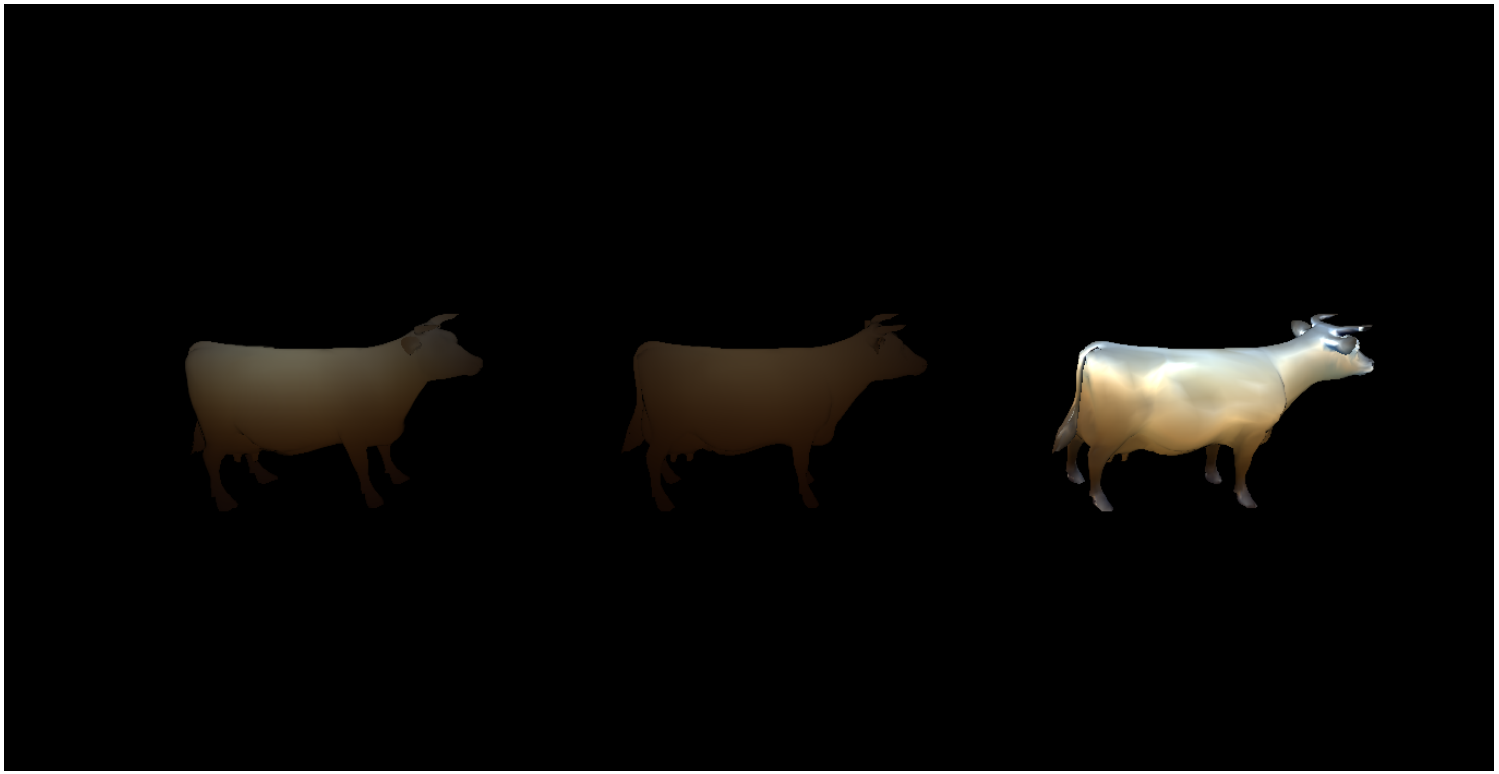

Figure 5: Regular chocolate milk set of cow. Left to right: Jensen et al. dipole, D'Eon et al. better dipole and Frisvad et al. directional dipole. Material properties for regular chocolate milk.