Given the general rendering equation

$$L_o(x_o, \vec{\omega}_o) = L_e(x_o, \vec{\omega}_o) + L_r(x_o, \vec{\omega}_o)$$

$$L_o(x_o, \vec{\omega}_o) = L_e(x_o, \vec{\omega}_o) + \int_A \int_{2\pi} S(x_i, \vec{\omega}_i, x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i)(\vec{\omega}_i \cdot \vec{n}_i) d\vec{\omega}_i dA$$

I discretize it this way (not considering emitted radiance):

$$L_o(x_o, \vec{\omega}_o) = \sum_{i=0}^{N} S(x_i, \vec{\omega}_i, x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i) A_i$$

Where A_i is the barycentric area (sum of vertex to the neighboring triangles to closest barycenter), and N is the number of vertices of the mesh.

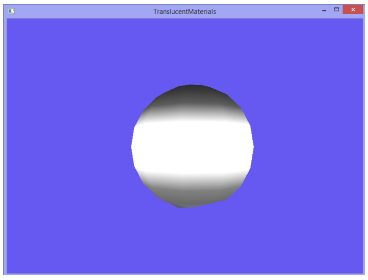
The result we obtain looks correct, apart from a singularity band that happens when the refracted light direction and the normal are perpendicular, that is:

$$\vec{\omega}_{12} \cdot \vec{n}_o \approx 0$$

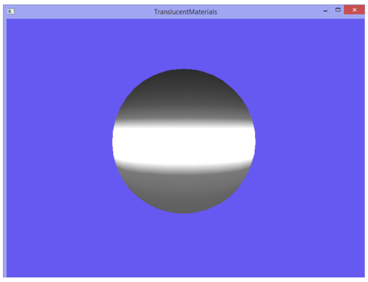
In this case, the frontlit/backlit adjustment gives a value for d_r close to zero, that actually causes the singularity, since

$$S_d \propto \frac{1}{{d_r}^3}$$

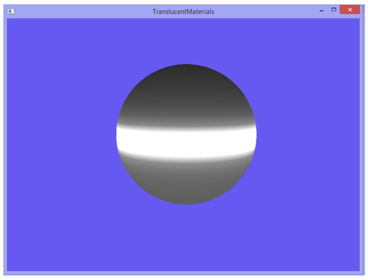
Because of the coarse approximation used with the vertices, we are missing the points that can give us a better estimation at the point of interest. In fact, if we use a better tessellation for the mesh, we can reduce the band ideally to zero, as we can see in 1. However, for the tests I was able to prepare this shrinking seems slow to converge.



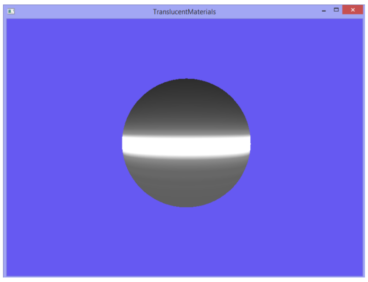
(a) 100 vertices



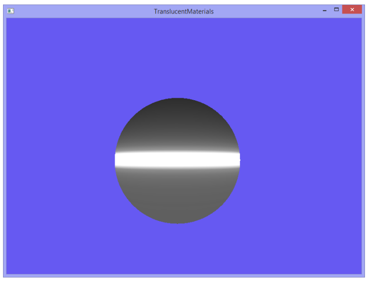
(b) 400 vertices



(c) 900 vertices



(d) 1600 vertices



(e) 2500 vertices

Figure 1: Shrinking band for growing number of vertices.

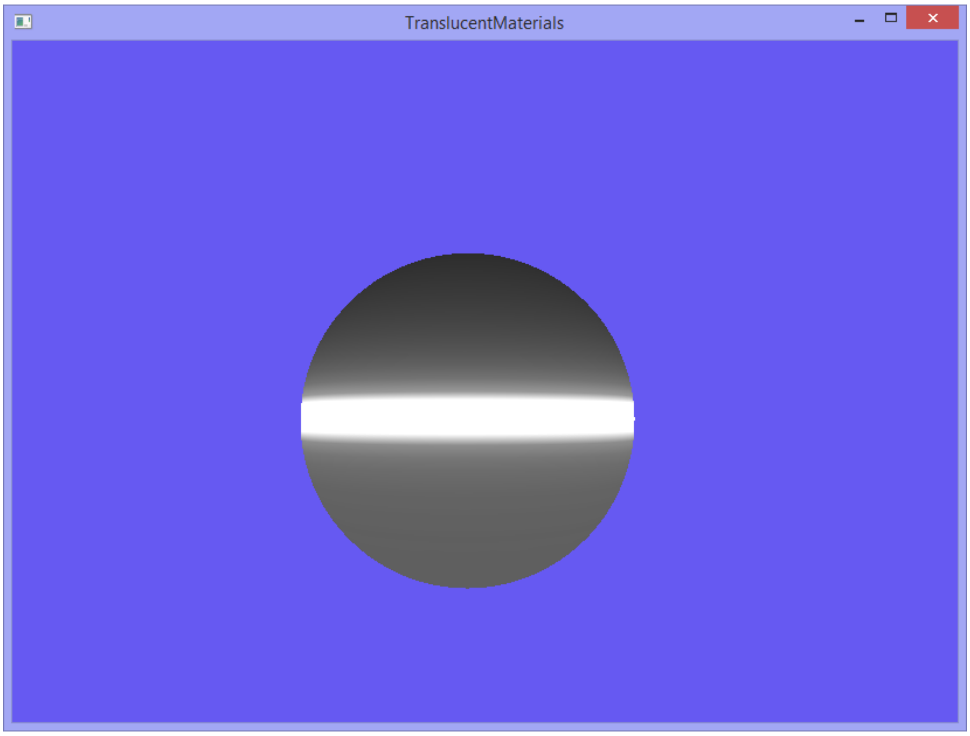
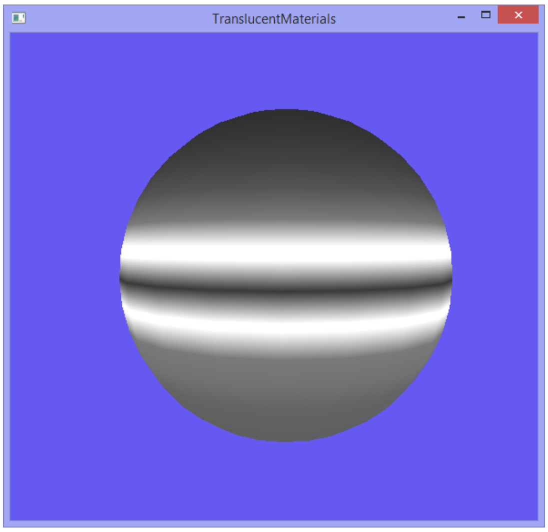
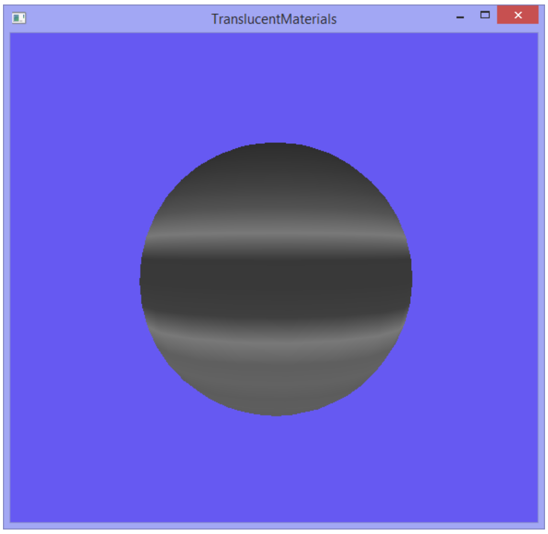


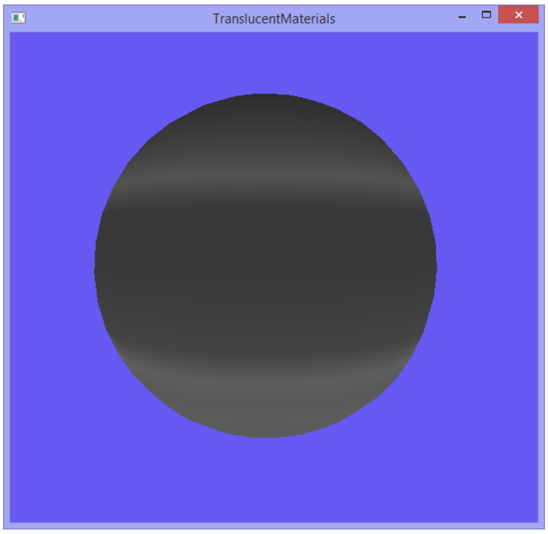
Figure 2: Full image.



(a) 0.05



(b) 0.2



(c) 0.4

Figure 3: Different Thresolds for μ_0.