POST-PROCESS GPS NAVIGATION SOFTWARE DEVELOPMENT

The laboratory work is to develop the necessary software to solve receiver coordinates for each observation epoch stored in the corresponding RINEX file using the navigation RINEX file.

P2 pseudoranges to the GPS satellites are used as observations and the results are saved to a .txt file, the corresponding graphs should be generated and the solution will be plotted directly to google maps.

The process has three distinct steps that must be linked to solve the system equations.

1 Observation RINEX file reading

89090590.11o file (a car kinematic observation).

The software should read:

a) Header:
   - APPROX POSITION XYZ
   - ANTENNA: DELTA H/E/N
   - # / TYPES OF OBSERV (a variable should be generated to store the P2 position)
   - TIME OF FIRST OBS

b) Observations:
   - Beggining of the epoch line/s (if more tan 12 satellites have been observed there will be more tan one line)
     - Read Epoch
     - Read OK flag
     - Number satellites per epoch
     - Satellite identificator (letter and number)
   - Observations line/s for every satellite (if there is more tan 5 observations, two lines per satellite will be used). RINEX format:

     ftp://igscb.jpl.nasa.gov/pub/data/format/

     http://gage14.upc.es/gLAB/HTML/LaunchHTML.html

As an idea, an output file with all the observations can be generatted with the format below:

Epoch hour min. Sec. Num_sats OK_flag PRN observations

(If there is no a specific observation in the RINEX file for a satellite a “none” should be introduced in the corresponding cell).

2 Navigation RINEX file Reading

Coordinates of the satellites (X_{Sat}, Y_{Sat}, Z_{Sat}) in the emission time should be computed (for every epoch and satellite) from brdc0590.11n or 89090590.11n file.

The inputs of this part of the software are the observation epoch, the satellite identification and the P2 observation.

The software sould read:

a) LEAP SECONDS and Alpha and Beta coeffients for the inospheric delay correction from the header.

b) Navigation RINEX files enter the orbital parameters for each satellite every two hours and, sometimes, in a random way (this is no the usual if a brdc file is used). The first step is to find the appropiate satellite parameters epoch for the corresponding observation. After that, the reading should be done properly (*D* should be replaced by *e* in the numerical formats) :

   •ftp://igscb.jpl.nasa.gov/pub/data/format/
   •http://gage14.upc.es/gLAB/HTML/LaunchHTML.html

The computational procedure can be followed from:

- http://www.navipedia.net/index.php/GPS_and_Galileo_Satellite_Coordinates_Computation.
- http://ccar.colorado.edu/asen5050/projects/projects_2008/xiaofanli/

Special attention in the calculation of the emission epoch is required, the procedure is:

$$T_{Emis1} = \left(T_{Rec} - \frac{P2}{c}\right) - Toe$$

T_{Rec} is the reception time in GPS week seconds and c is the speed of light.

If $T_{Emis1}>302400$, $T_{Emis1}= T_{Emis1} - 604800$; If $T_{Emis1}<302400$, $T_{Emis1}= T_{Emis1} + 604800$

In order to correct possible jumps in the GPS week due to the interpolation.

After that, the correction to the satellite clock with respect to the GPS time should be computed using a0, a1, and a2 parameters form the ephemeris file:

$$T_{Corr1} = a_0 + a_1(T_{Emis1}) + a_2(T_{Emis1})^2$$

Where *Toe* is the reference time (GPS week seconds) for the orbital ephemeris parameters.

A second computation of the emission time will be performed using this correction:

$$T_{Emis2} = \left(T_{Rec} - \frac{P2}{c} - T_{corr1}\right) - Toe$$

If $T_{Emis2}>302400$, $T_{Emis2}= T_{Emis2} - 604800$; If $T_{Emis2}<302400$, $T_{Emis2}= T_{Emis2} + 604800$

A second calculation of the satellite clock correction is needed:

$$T_{Corr2} = a_0 + a_1(T_{Emis2}) + a_2(T_{Emis2})^2$$

Finally, the emission time is:

$$T_{Emis} = \left(T_{Rec} - \frac{P2}{c} - T_{corr2}\right) - Toe$$

If $T_{Emis}>302400$, $T_{Emis}= T_{Emis} - 604800$; If $T_{Emis}<302400$, $T_{Emis}= T_{Emis} + 604800$

This time (T_{Emis}) is the one used to compute the satellite coordinates.

The sequence of computation is:

Semi-Major axis of the satellite orbit (Ephemeris RINEX file gives only the square root):

$$a = \left(\sqrt{a}\right)^2$$

The mean motion is computed as:

$$n = \sqrt{\frac{\mu}{a^3}} + \Delta n$$

Where μ is the geocentric gravitational constant (3.986005e14).

Δn is the mean motion difference from computed value, and can be obtained from the navigation RINEX file.

The mean anomaly is found as:

$$M = M_0 + n * T_{Emis}$$

M_0 is the Mean anomaly at a reference time, and can be obtained from the navigation RINEX file.

Eccentric anomaly can be found throught iterative calculation as:

$$E = M + e * sinE$$

Where E=M in the first iteration. *e* is the eccentricity of the satellite orbit, and can be obtained from the navigation RINEX file.

The true anomaly is calculated as:

$$\nu = tan^{-1}\left[\frac{\left(\sqrt{1-e^2} * sinE\right)}{(cosE - e)}\right]$$

The argument of latitude is found as:

$$\phi_0 = \nu + \omega$$

Where ω is the argument of the perigee and can be obtaided from the RINEX navigation file.

The orbital correction terms are:

$$\delta u = C_{us} * sin2\phi + C_{uc} * cos2\phi$$
$$\delta r = C_{rs} * sin2\phi + C_{rc} * cos2\phi$$
$$\delta i = C_{is} * sin2\phi + C_{ic} * cos2\phi$$

All *C* terms can be obtained from the RINEX navigation file.

The argument of latitude, radius and inclination are corrected as:

$$\phi = \phi_0 + du$$
$$r = a(1 - e * \cos E) + dr$$
$$i = i_0 + di + \dot{i} * T_{Emis}$$

Where i_0 is the inclination angle at reference time and $\dot{i}$ is the rate of inclination angle, both can be obtained from the RINEX navigation file.

The position in the orbital plane is:

$$x_{op} = r * \cos \phi$$
$$y_{op} = r * \sin \phi$$

The corrected longitude of ascending node is:

$$\Omega = \Omega_0 + \left(\dot{\Omega} - \Omega_e\right) * T_{Emis} - \Omega_e * Toe$$

Where Ω_0 is the longitude of ascending node of the orbit plane at weekly epoch, $\dot{\Omega}$ is the rate of right ascensión (from the navigation RINEX file) and Ω_e is the value of the Earth's rotation rate (7.2921151467e-5).

Finally, the satellite coordinates are:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_{op} * \cos \Omega - y_{op} * \cos i * \sin \Omega \\ x_{op} * \sin \Omega + y_{op} * \cos i * \cos \Omega \\ y_{op} * \sin i \end{bmatrix}$$

Two final corrections should be applied to the computed T_{Corr2} in order to obtain the final correction to the satellite clock (T_{Corr}) to be used in the navigation equations. The first one is a relativistic effect due to the orbit eccentricity:

$$\Delta t_{rel} = -2 \frac{\sqrt{\mu a}}{c} e \sin E$$

Where μ is the geocentric gravitational constant (3.986005e14), a is the semimajor axis, e is the eccentricity and E the eccentric anomaly of the satellite orbit.

The second one is the Total Group Delay (TGD), or satellite instrumental bias, the satellites broadcast in their navigation messages this number in seconds for the L1 frequency, so, in order to use it for the L2 frequency the following expression should be used:

$$\Delta t_{TGD}^{L2} = \left(\frac{f_1}{f_2}\right)^2 \Delta t_{TGD}^{L1} = 1.65\ \Delta t_{TGD}^{L1}$$

Finally, the clock correction to be used in the equations is:

$$T_{Corr} = T_{Corr2} + \Delta t_{rel} - \Delta t_{TGD}^{L2}$$

As an idea, ECEF satellite coordinades can be added to the previous output file, including T_{Corr} and Sec_of_Week in each line ("none" in the satellite coordinates if a GLONASS satellite is observed).

3 Main module: solving the system equations for every epoch

Only epochs with OK_flag=0 should be solved.

The process consist of 6 iterations to solve the coordinates of the navigation point for every epoch. The approximate unkowns for the first iteration will be the coordinates and receiver clock correction obtained in the previous epoch and, for the first epoch, the APPROX POSITION XYZ from the observation file and zero for the receiver clock.

The first step, in the iteration process, is to transform the satellite coordinates from the system tied to the earth at "emission time" to the system tied to the earth at "reception time" (which is common for all measurements). In order to do so, one must consider the earth rotation during the time interval that the signal takes to propagate from the satellite to the receiver:

$$\omega\tau = \omega * travel\_time$$

$$travel\_time = \frac{\sqrt{\left(X_{Sat\_rot} - X_{Est.}\right)^2 + \left(Y_{Sat\_rot} - Y_{Est.}\right)^2 + \left(Z_{Sat\_rot} - Z_{Est.}\right)^2}}{c}$$

ω = 7.2921151467e-5 (Earth rotation speed)

$$R3 = \begin{pmatrix} cos(\omega\tau) & sen(\omega\tau) & 0 \\ -sen(\omega\tau) & cos(\omega\tau) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X_{Sat\_rot} \\ Y_{Sat\_rot} \\ Z_{Sat\_rot} \end{pmatrix} = R3 * \begin{pmatrix} X_{Sat} \\ Y_{Sat} \\ Z_{Sat} \end{pmatrix}$$

The elevation and azimut for every satellite at every epoch should be computed (coordinate transformation from X, Y, Z to Latitude, Longitude and elliposoidal height, and transformation to horizontal coordinates: dN, dE, dh, elevation and azimut), only satellites with an elevation higher than 10 degrees will form an equation.

Tropospheric delay should be computed using the Saastamoinen method ((Hoffmann, Lichtenegger, Wasle (2008): GNSS global navigation satellite systems, pg. 135):

$$d_{Tropo} = \frac{0.002277}{cos(z)} \left[p + \left(\frac{1255}{T} + 0.05 \right) e - B\ tan^2(z) \right] +\ dR$$

Where *z* is the satellite zenit angle, and the theoretical values for pressure, *p* (in Milibars), temperature, *T* (in Kelvin degrees), and *e* the partial pressure of water vapor (in Millibars) can be obtained from:

Pres = 1013.25*(1-0.000065*h_ECEF)**(5.225)

Temp = 291.15-0.0065*h_ECEF

H = 50*numpy.exp(-0.0006396*h_ECEF)

e_wp=(H*0.01)*numpy.exp(-37.2465+0.213166*Temp(0.000256908*Temp^2))

H is the humidity in % and the orthometric height is assumed to be the elipsoidal height.

dR correction will not be implemented and *B* parameter (a correction which depends on the station height), can be computed from the table below:

High (Km)	B (mb)
0.0	1.156
0.5	1.079
1.0	1.006
1.5	0.938
2.0	0.874
2.5	0.813
3.0	0.757
4.0	0.654
5.0	0.563

Finally, the ionospheric delay (d_{Ion}), should be taken into account using the Klobuchar model:

- http://www.navipedia.net/index.php/Klobuchar_Ionospheric_Model

The sequence for the calculation is:

1.- Calculate the Earth-center angle (elevation E in semicircles: E(in degrees/180)

$$\psi = \frac{0.0137}{E + 0.11} - 0.022 \quad (semicircles)$$

2.- Compute the latitude of the ionospheric Pierce Point (IPP). A is the azimuth of the satellite (in radians) and ϕ_u is the geodetic latitude (in semicircles: latitude in degrees / 180) of the user (rover receiver).

$$\phi_i = \phi_u + \psi \cos A \quad (semicircles)$$

$$if\ \phi_1 > 0.416 \rightarrow \phi_1 = 0.416$$
$$if\ \phi_1 < -0.416 \rightarrow \phi_1 = -0.416$$

3.- Compute the longitude of the IPP. λ_u is the geodetic longitude (in semicircles: longitude in degrees / 180) of the user (rover receiver). A and ϕ_i in radians.

$$\lambda_i = \lambda_u + \frac{\psi \sin A}{\cos \phi_i} \quad (semicircles)$$

4.- Find the geomagnetic latitude of the IPP. (λ_i-1.617) should be multiplied by π in order to transform to radians for the calculation of the cosinus.

$$\phi_m = \phi_i + 0.064\ \cos(\lambda_i - 1.617) \quad (semicircles)$$

5.- Find the local time at the IPP.

$$t = 43200 * \lambda_i + t_{\sec of\ week} \quad (seconds)$$

where ≤ *t*< 86400,

therefore: if $t \geq 86400$, substract 86400. If $t < 0$, add 86400

6.- Compute the amplitude of ionospheric delay.

$$A_i = \sum_{n=0}^{3} \alpha_n \phi_m^n \quad (seconds)$$

If $A_I < 0$, thence $A_I = 0$

7.- Compute the period of ionospheric delay.

$$P_i = \sum_{n=0}^{3} \beta_n \phi_m^n \quad (seconds)$$

If $P_I < 72000$, thence $P_I = 72000$

8.- Compute the phase of ionospheric delay.

$$X_i = \frac{2\pi(t - 50400)}{P_i} \quad (radians)$$

9.- Compute the slant factor (elevation E in semicircles)

$$F = 1.0 + 16.0(0.53 - E)^3$$

10.- Compute the ionospheric time delay.

$$I_{L1} = \begin{cases} \left[5.10^{-9} + A_i\left(1 - \frac{X_I^2}{2} + \frac{X_I^4}{24}\right)\right] * F & ; \ |X_i| \leq 1.57 \\ 5.10^{-9} & ; \ |X_i| \geq 1.57 \end{cases}$$

11.- Compute the ionospheric time delay for L2 (seconds).

$$I_{L2} = \left(\frac{f_{L1}}{f_{L2}}\right)^2 * I_{L1} = 1.65 * I_{L1}$$

12.- Transform to meters (c is the speed of light):

$$I_{L2}(m) = I_{L2}(s) * c$$

In the system equations for every epoch, each satellite generates the following equation:

$$\left[-\frac{(X_{Sat\_rot}-X_{Sta})}{Dist} \quad -\frac{(Y_{Sat\_rot}-Y_{Sta})}{Dist} \quad -\frac{(Z_{Sat\_rot}-Z_{Sta})}{Dist} \quad 1\right] = \left[P2 - Dist - dT^{Rec.}_{Pre.Ite.} + c * T_{corr} - d_{Trop} - d_{Ion}\right]$$

Where $Dist. = \sqrt{(X_{Sat\_rot}-X_{Sta.})^2 + (Y_{Sat\_rot}-Y_{Sta.})^2 + (Z_{Sat\_rot}-Z_{Sta.})^2}$

And the weight is:

$$Pi = \frac{sen(el_i)^2}{e^2_{P2}}$$

Where e_{P2} is equal to 1.5*(0.30) meters (Hofmann et al. 2008).

The parameters to be solved are the coordinates of the navigation point, and the delay of the receiver clock (the solution will be obtained in meters, for a time units the solution should be divided by the speed of light).

The final solution should will be saved in a file with the following columns:

Hour Min. Sec. X Y Z Xerror Yerror Zerror Lat. Lon. h

Only the solutions with an error less than 1.5 multiplied by the mean error of all the navigation points (máximum error value again), will be plotted using matplotlib library. The plots to be generated are: (X, Y, X), (Lat., Lon.) and (h).

Finally, the .html file will be generated using Pygmaps library, placing a circle for every navigation epoch on google maps. The web navigator should be open directly from the software using webbrowser library.

4 Partial Results

Orbital Parameters

epoch 1

satellite= 2.0
hour_ref (navigation file hour-epoch hour)= 0.469444444444
jde= 2455621.0637731482275
sec_of_week= 135110.000007
T_{Emis1}= 135109.924863
Toe= 136800.0
dt_eph0(T_{Emis1}-Toe)= -1690.07513741
T_{Corr1}= 0.000322746571865

dt_eph1((T_{Emis1}-T_{Corr1})-Toe)= -1690.07546015
T_coor2= 0.000322746571864
dt_eph2((T_{Emis1}-T_{Corr2})-Toe)= -1690.07546015
mean motion= 0.000145856177504
mean anomaly= -1.49519531165
ano_ecc= -1.50522458968
true_ano= -1.51525734066
phi= -4.58635680549
T_{rel}= 2.29639815455e-08
T_{Corr}= 0.000322797964467
du= -2.69086694427e-06
dr= -238.072496744
di= 1.25158610262e-09
u_eph= -4.58635949635
r_eph= 26543000.1928
i_eph= 0.939351632001
omega= -9.23134314432
x1= -3336352.10533
y1=1 26332482.1061
sat_x= 6262234.04949
sat_y= -14613110.9779
sat_z= 21254935.2267

satellite= 4.0
hour ref= (navigation file hour-epoch hour)=(14:00:00-13:31:50)=0.469444444444
jde= 2455621. 0637731482275
sec_of_week= 135110.000007
T_{Emis1}= 135109.931313
toe= 136800.0
dt_eph0(T_{Emis1}-Toe)= -1690.06868701
T_{Corr1} = 4.78864889998e-05
dt_eph1 ((T_{Emis1}-T_{Corr1})-Toe)= -1690.06873489
T_{Corr2}= 4.78864889998e-05
dt_eph2 ((T_{Emis1}-T_{Corr2})-Toe)= -1690.06873489
T_{rel}= -2.19797297574e-08
T_{Corr}= 4.78744977042e-05
mean_motion= 0.00014586579188
mean_anomaly= 1.52824192856
ano_ecc= 1.5378415543
true_ano= 1.54744284698
phi= 2.25971282235
du= -7.11183316323e-06
dr= -67.0188662344
di= 2.45406585183e-08
u_eph= 2.25970571052
r_eph= 26551099.9286
i_eph= 0.93843559198
omega= -9.21384712706

x1= -16878419.2681
y1= 20495850.0782
sat_x= 19040662.0127
sat_y= -8311756.52015
sat_z= 16532658.6212
…
…
…
satellite= 23.0
hour ref= (navigation file hour-epoch hour)=(14:00:00-13:31:50)=0.469444444444
jde= 2455621. 0637731482275
sec_of_week= 135110.000007
T_{Emis1}= 135109.927325
toe= 136800.0
dt_eph0 (T_{Emis1}-Toe)= -1690.07267501
T_{Corr1}= 0.000323132455779
dt_eph1((T_{Emis1}-T_{Corr1})-Toe)= -1690.07299814
T_{Corr2}= 0.000323132455779
dt_eph2 ((T_{Emis1}-T_{Corr2})-Toe)= -1690.07299814
T_{rel}=1.59123485071e-08
T_{corr}=0.000323181406797
mean_motion= 0.000145865413454
mean_anomaly= -1.44951718329
ano_ecc= -1.45646690052
true_ano= -1.46341944901
phi= 1.66630620623
du= -5.12121056468e-06
dr= -204.234152066
di= 5.67837878201e-08
u_eph= 1.66630108502
r_eph= 26538062.5133
i_eph= 0.965516493005
omega= -7.10926601189
x1= -2530660.05432
y1= 26417125.5373
sat_x= 9336918.79375
sat_y= 12048291.2834
sat_z= 21723935.7979

Parameters for the system equations

Epoch 1

itera = 1

Satellite values:
sat_id=G02
omegatau= 5.49875083649e-06
sat_x_rot= 6262153.69554
sat_y_rot= -14613145.4121
sat_z_rot= 21254935.2267
el_sat= 32.1926877409
aci_sat= 310.365695268

troposphere:
relative_humidity= 9.88283763309
corr_trop= 4.43538019603

ionosphere:
psi= 0.0254297465115
lat_i= 0.235810850931
lon_i= -0.0281147580174
lat_gm= 0.263987676582
l_t= 47495.4424605
sf= 1.69279450999
A_i= 1.50664711795e-08
per= 87300.3907406
dIon= 10.0170140051
dIon*1.65= 16.528073108

sat_id=G04

Satellite Values:
omegatau= 5.00833545146e-06
sat_x_rot= 19040620.3844
sat_y_rot= -8311851.88207
sat_z_rot= 16532658.6212
el_sat= 66.4528487462
aci_sat= 274.491856591

Troposphere:
Relative_humidity= 9.88283763309
corr_trop= 2.57901336887

ionosphere:
psi= 0.0065903600385
lat_i= 0.21985706383
lon_i= -0.0103812917082
lat_gm= 0.244790455634
l_t= 48261.5282051
sf= 1.06654569398

A_i= 1.48442380443e-08
per= 89756.3764061
dIon= 6.29196077549
dIon*1.65= 10.381735279
…
…
…
sat_id=G23

satellite values:
omegatau= 5.31922601949e-06
sat_x_rot= 9336982.8812
sat_y_rot= 12048241.6181
sat_z_rot= 21723935.7979
el_sat= 41.8680881185
aci_sat= 48.0022627314

Troposphere:
Relative_humidity= 9.88283763309
corr_trop= 3.54108651306

ionosphere:
psi= 0.0179882674368
lat_i= 0.231376896304
lon_i= 0.0160327171571
lat_gm= 0.251338857388
l_t= 49402.613388
sf= 1.42086298444
A_i= 1.49162920008e-08
per= 88927.9419704
dIon= 8.46785344086
dIon*1.65= 13.971958177
…
…
…
itera = 6

sat_id=G02

Satellite values:
omegatau= 5.4987512563e-06
sat_x_rot= 6262153.69553
sat_y_rot= -14613145.4122
sat_z_rot= 21254935.2267
el_sat= 32.1926980946
aci_sat= 310.365692316

Troposphere:
Relative_humidity= 9.92074206773

corr_trop= 4.44133822047

ionosphere:
psi= 0.0254297370664
lat_i= 0.235810867169
lon_i= -0.0281147459783
lat_gm= 0.263987690647
l_t= 47495.4429806
sf= 1.69279416954
A_i= 1.09129771063e-08
per= 87300.3889115
dIon= 7.95505738179
dIon*1.65= 13.1258446799

Sat_id=G04

Satellite values:
omegatau= 5.00833626485e-06
sat_x_rot= 19040620.3844
sat_y_rot= -8311851.88208
sat_z_rot= 16532658.6212
el_sat= 66.4528523773
aci_sat= 274.491844111

Troposphere:
Relative_humidity = 9.92074206773
corr_trop= 2.58247726226

Ionosphere:
psi= 0.00659035883489
lat_i= 0.219857085659
lon_i= -0.0103812861186
lat_gm= 0.244790476427
l_t= 48261.5284465
sf= 1.06654566894
A_i= 1.12728645771e-08
per= 89756.3737914
dIon= 5.16281244185
dIon*1.65= 8.51864052906
…
…
…
Sat_id=G23

Satellite values:
omegatau= 5.31922654288e-06
sat_x_rot= 9336982.8812
sat_y_rot= 12048241.6181
sat_z_rot= 21723935.7979

el_sat= 41.8680991394
aci_sat= 48.0022667268

Troposphere:
Relative_humidity = 9.92074206773
corr_trop= 3.54584303856

ionosphere:
psi= 0.0179882602904
lat_i= 0.231376913944
lon_i= 0.0160327167164
lat_gm= 0.251338875112
l_t= 49402.613369
sf= 1.42086272451
A_i= 1.11512868845e-08
per= 88927.9397148
dIon= 6.86807603209
dIon*1.65= 11.332325453

Final coordinates

Hour	minute	second	X	Y	Z	Satellite number
13	31	50.0	4929474.94651	-28766.5668131	4033738.32092	9

eX	eY	eZ	Latitude	Longitude	h
0.7737	0.3694	0.5796	39.4813705133	-0.334352883046	48.7464

Hour	minute	second	X	Y	Z	Satellite number
13	31	55.0	4929475.16583	-28766.257238	4033738.30884	9

eX	eY	eZ	Latitude	Longitude	h
1.0261	0.4900	0.7689	39.4813691837	-0.334349270071	48.9066

…
…

Hour	minute	second	X	Y	Z	Satellite number
14	4	20.0	4929472.01463	-28762.8753547	4033740.09682	9

eX	eY	eZ	Latitude	Longitude	h
1.2659	0.5950	1.0507	39.4813997726	-0.33431017715	47.5961

Hour	minute	second	X	Y	Z	Satellite number
14	4	25.0	4929471.37283	-28762.8722439	4033739.80181	9

eX	eY	eZ	Latitude	Longitude	h
1.3494	0.6341	1.1202	39.4814013973	-0.334310184518	46.9131