GPS satellite position computation from broadcast ephemeris

Example solution sheet

Problem: determine the position of the PRN 11 GPS satellite in the orbital plane and spatial WGS84 coordinates from the broadcast ephemeris given below. Determine also the range to the satellite using the known observer position.

Input:
No.= 0

Epoch: August 21, 2005. (1^{st} day of GPS Week 1337.) 4 hours (No.+1)*5 minutes

Parameters of the WGS84 reference frame: $GM = \mu = 3.986005 \cdot 10^{14}$ m3/s2
$\omega_e = 7.2921157 \cdot 10^{-5}$ rad/s

<u>GPS broadcast ephemeris:</u>

ephemeris reference time: *GPSWeek* = 1337.	$T_{0e} = 14400$ (seconds of the GPS Week)
square root of semimajor axis:	$\sqrt{a} = 5153.688\,850\,40$ m$^{1/2}$
eccentricity of orbit:	$e = 4.392\,384\,667\,880 \cdot 10^{-3}$
mean motion difference from computed:	$\Delta n = 6.677\,063\,840\,800 \cdot 10^{-9}$ rad/s
mean anomaly at reference time:	$M_0 = 1.947\,876\,00$ rad
argument of perigee:	$\omega = 0.233\,996\,741\,3720$ rad
corrections to argument of latitude:	$C_{uc} = -1.553\,446\,054\,460 \cdot 10^{-6}$ rad
	$C_{us} = 3.330\,409\,526\,820 \cdot 10^{-6}$ rad
corrections to orbit radius:	$C_{rc} = 283.218\,75$ m
	$C_{rs} = -31.968\,75$ m
corrections to inclination angle:	$C_{ic} = -8.754\,432\,201\,390 \cdot 10^{-8}$ rad
	$C_{is} = 1.434\,236\,764\,910 \cdot 10^{-7}$ rad
rate of inclination angle:	$\dot{i} = -3.314\,423\,773\,340 \cdot 10^{-10}$ rad/s
inclination angle at reference time:	$i_0 = 0.900\,298\,2524$ rad
longitude of ascending node at ref. epoch:	$\Omega_0 = -1.092\,228\,18$ rad
rate of right ascension:	$\dot{\Omega} = -9.302\,887\,502\,600 \cdot 10^{-9}$ rad/s

Solution
1) coordinates of the GPS satellite in the orbital plane

Seconds elapsed from the reference epoch:

t_{obs} = (GPS Week -1) · 168 · 3600 + (GPS day – 1) · 24 · 3600 + hour · 3600 + min · 60 + sec =
(GPS Week – 1) · 168 · 3600 + 4 · 24 · 3600 + 12 · 3600 + 10 · 60 =
(GPS Week -1) · 168 · 3600 + 389400 s

t_{ref} = (GPS Week -1) · 168 · 3600 + T_{oe} = (GPS Week -1) · 168 · 3600 + 388800 s

$\Delta t = t_{\text{obs}} - t_{\text{ref}} = 300$ s

Mean motion:

$$n_0 = \sqrt{\frac{GM}{a^3}} = \sqrt{\frac{3{,}986005 \cdot 10^{14}}{5153.68885040^6}} = 1{,}458\ 526\ 536 \cdot 10^{-4} \text{ rad/s}$$

$$n = n_0 + \Delta n = 1{,}458\ 526\ 536 \cdot 10^{-4} + 0{,}667\ 706\ 384\ 08 \cdot 10^{-8} = 1{,}458\ 593\ 307 \cdot 10^{-4} \text{ rad/s}$$

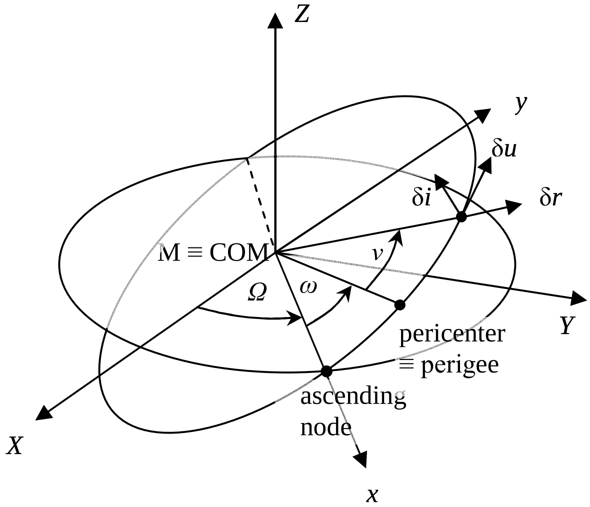
Mean anomaly:

$$M = M_0 + n\Delta t = 1.947\ 876\ 00 + 1{,}458\ 593\ 307 \cdot 10^{-4} \cdot 300 = 1.991\ 633\ 804 \text{ rad}$$

Excentric anomaly by iteration:

Kepler's equation: $M = E - e \sin E$
iteration: $E_0 = M$
$E_i = M + e \sin E_{i-1}$
stop condition: $E_i = E_{i-1} = E$
results:

$E_1 =$ 1.995 642 9405
$E_2 =$ 1.995 635 7143
$E_3 =$ 1.995 635 7274
$E_4 =$ 1.995 635 7274
$E =$ 1.995 635 7274 rad



True anomaly:

$$\cos v = \frac{\cos E - e}{1 - e \cos E}$$

$$\sin v = \frac{\sqrt{1-e^2} \sin E}{1 - e \cos E}$$

v = 1.999 634 0454 rad

Argument of longitude:

$$\delta u = C_{uc} \cos 2(v + \omega) + C_{us} \sin 2(v + \omega) = -0.000\ 002\ 8539 \text{ rad}$$

$$u = v + \omega + \delta u = 2.233\ 627\ 9329 \text{ rad}$$

Radius:

$\delta r = C_{rc}\cos 2(\nu+\omega) + C_{rs}\sin 2(\nu+\omega)$ = –37.718 m
$r = a(1-e\cos E) + \delta r$ = 26 608 556.958 m

coordinates of the GPS satellite in the orbital plane:

$x = r\cos u$ = –16 373 611.121 m
$y = r\sin u$ = 20 974 273.819 m

Check: $\sqrt{x^2+y^2} = r$

2) coordinates of the GPS satellite in the spatial WGS84 system

Orbit inclination:

$\delta i = C_{ic}\cos 2(\nu+\omega) + C_{is}\sin 2(\nu+\omega)$ = –0.000 000 1179 rad
$i = i_0 + i\Delta t + \delta i$ = 0.900 298 0351 rad

Longitude of ascending node:

$\Omega = \Omega_0 + (\dot{\Omega} - \omega_e)\Delta t - \omega_e T_{oe}$ = –2.164 171 9761 rad

WGS84 coordinates:

$X = x\cos\Omega - y\sin\Omega\cos i$ = 19 960 559.708 m
$Y = x\sin\Omega + y\cos\Omega\cos i$ = 6 287 146.678 m
$Z = y\sin i$ = 16 433 598.090 m

Check: $\sqrt{X^2+Y^2+Z^2} = r$

Range to the satellite from the observer

$d = \sqrt{(X-X_P)^2 + (Y-Y_P)^2 + (Z-Z_P)^2}$

WGS84 coordinates of the observer (BUTE):

X: 4 081 882.424 m
Y: 1 410 011.130 m
Z: 4 678 199.424 m

d = 20 349 649.659 m