

Note: In this revised version, the symbol  $\alpha$  used in the potential  $V(S)$  has been renamed  $\alpha_V$  to distinguish it clearly from the fine-structure constant  $\alpha$  (used as input,  $\alpha$ -in). All content has been preserved with this notational clarification applied conservatively.

## Structural Field Glossary – Revised Version

**$\alpha$  policy (RC):**  $\alpha$  is **INPUT ( $\alpha$ -in)** for calibration. We do **not** claim to predict  $\alpha$  in the RC.

	Structural scalar field $S(r, t)$	Function that describes the tensile state at each point of the discrete three-dimensional space lattice.
	Structural collapse	Process by which a region of the field $S$ concentrates energy in a localized and stable way, giving rise to a particle or physical phenomenon.
	Node	Fundamental unit of the discrete three-dimensional lattice; the point at which the field $S$ is locally defined.
	Structural spin	Emergent property of a helical tensile configuration of the field that can represent the spin of the photon, and speculatively, that of the electron.
	Baseline tensile-energy density $\rho_0$	Background structural energy value of the lattice, determined by the Universe's initial conditions.
	Propagation speed $c$	Maximum speed limit within the structural lattice, derived from its elastic constants.
	Solitonic configuration	Localized, non-dispersive solution of the field $S$ that may be interpreted as a particle.
	Lattice	Discrete, elastic three-dimensional structure that serves as the physical support of space.

## Block 0 — Foundations from First Principles

It is assumed that space is not a passive vacuum but a three-dimensional lattice of elastic elements in local interaction. Each node undergoes small changes in tensile state around equilibrium; this state is encoded in a scalar field  $S(r, t)$ . From this structural principle, the system's dynamic properties are derived. Observable physics emerges as the propagation and reorganization of these internal tensions.

### 0.1 Conceptual Introduction – Structural Analogy: Elastic Band Lattice in 1D, 2D, and 3D

Before formalizing the discrete structural field model, it is helpful to introduce a concrete mechanical analogy to visualize its physical operation: a lattice of elastic bands connected to each other, whose dynamics reflect fundamental principles of propagation, collapse, and

stability. This lattice serves as a simplified but powerful representation of the fundamental structure of space according to the Structural Field Theory (SFT).

## Unified Notation & Units (SFT) — Fixed

Global metric/signature: we use  $\eta = \text{diag}(-,+,+,+)$  unless stated otherwise.

Convention:  $S$  is dimensionless. All physical dimensions are carried by  $\{a, t_0, \kappa, \hbar^*, q^*\}$ .

Note:  $\beta$  and  $\gamma$  are reserved exclusively for PPN usage. For structural potentials, use  $\lambda_3, \lambda_4, [\lambda_6]$  instead.

Canonical potential policy. Throughout the corpus, the canonical convention is  $V(S) = +\alpha_V S^2 / 2 + \lambda_4 S^4 / 4 [+ \lambda_6 S^6 / 6]$ . An alternate extended form is  $V(S) = +\alpha_V S^2 / 2 + \lambda_3 S^3 + \lambda_4 S^4 [+ \dots]$ . The tilde form  $\tilde{V}(S) = \frac{1}{2} m^2 S^2 - (\lambda/4) S^4 + (\mu/6) S^6$  is used only as a mapped representation via  $\{\alpha_V, \lambda_4, \lambda_6\} \equiv \{m^2, \lambda, \mu\}$ . See the Integrated Technical Document — Unified Notation & Units (and Appendix B — Emergent Electromagnetism from  $S$ ) for the canonical definition. (Note:  $\tilde{V}$  has a minus sign in the quartic term.)

See also: Integrated Technical Document — Appendix B (Emergent EM from  $S$ ) and Unified Notation & Units for the full derivation and calibration.

Symbol	Definition	Units (SI)	Structural units	Notes
$S(r,t)$	Discrete structural scalar (tension state)	1	1	Dimensionless by convention
$S^n_{\{i,j,k\}}$	Value of $S$ at node $(i,j,k)$ and step $n$	1	1	Integer lattice indices
$a$	Lattice spacing	m	base length	Sets spatial scale
$t_0 (\Delta t)$	Time step	s	base time	CFL: $\Delta t < a/(c\sqrt{d})$
$c$	Limiting speed	$\text{m}\cdot\text{s}^{-1}$	$a/t_0$	Match $c_{\text{SI}}$
$\kappa$	Energy-density scale	$\text{J}\cdot\text{m}^{-3}$	—	Scales $T^{00}$ ; fitted vs $m_e c^2$
$\hbar^*$	Structural action scale	$\text{J}\cdot\text{s}$	—	Used in $\alpha$ and EM mapping
$q^*$	Structural charge scale	C	—	From $\alpha_{\text{em}} = q^{*2} / (4\pi \epsilon^* \hbar^* c)$

$1/a ; 1/a^2$	Discrete (tilde) gradient / Laplacian	— ; —	$1/a ; 1/a^2$	$1/a ; 1/a^2$
$T^{00}$	Total energy density	$J \cdot m^{-3}$	$\kappa \cdot \tilde{T}^{00}$	$\tilde{T}^{00}$ dimensionless
$\tilde{V}(S)$	Potential ( $\frac{1}{2}m^2S^2 - (\lambda/4)S^4 + (\mu/6)S^6$ )	—	—	Energy density = $\kappa \cdot \tilde{V}$ (mapped from the canonical policy $V(S) = + \alpha_V S^2/2 + \lambda_4 S^4/4 [+ \lambda_6 S^6/6]$ ; see note below).
$A_{em}$	Emergent vector potential	$T \cdot m (\equiv V \cdot s \cdot m^{-1})$	—	Derived from S
$\rho_{em}, j_{em}$	Emergent charge/current density	$C \cdot m^{-3} ; A \cdot m^{-2}$	—	See mapping below

### Emergent-EM mapping (consistent)

$$\rho_{em} = (q^*/a^3) \cdot \tilde{\rho}(S, \nabla S, \dots), \quad j_{em} = (q^*/(a^2 t_0)) \cdot \tilde{j}(S, \nabla S, \dots).$$

$$A_{em} = (\hbar^*/(q^* a)) \cdot \tilde{A}(S, \nabla S, \dots), \quad \varphi_{em} = (\hbar^*/(q^* t_0)) \cdot \tilde{\varphi}(S, \nabla S, \dots).$$

## 0.2 One-Dimensional Lattice (1D)

Consider a one-dimensional chain of structural nodes connected by elastic interactions. Each node holds a scalar tensile state  $\{ S_i(t) \}$ , which reflects the internal configuration of the structural field at that point.

The discrete wave propagation can be modeled as:

$$\left[ \frac{\partial^2 S_i}{\partial t^2} = c^2 \cdot \frac{S_{i+1} - 2S_i + S_{i-1}}{a^2} \right]$$

where:

- $\{ S_i(t) \}$  is the adimensional tensile field at site  $\{ i \}$ ,
- $\{ c \}$  is the speed of propagation in the lattice (derived from internal elastic constants),
- $\{ a \}$  is the lattice spacing.

This expression models the internal redistribution of tensile energy. The field  $\{ S \}$  is not a displacement coordinate but a dimensionless state variable encoding local tension. The equation describes how tension propagates across the lattice structure.

Imagine a chain of 10 nodes connected by 9 slack elastic bands at rest. Each node can move transversely (e.g., up or down) [visual analogy; in SFT nodes do not literally move— $S$  is a dimensionless tensile state], and the bands stretch or compress in response. If one of the nodes is perturbed, the tension is transmitted through neighboring bands: a structural wave is born.

Discrete equation of motion:

$$m \frac{d^2 S_i}{dt^2} = k(S_{i+1} - 2S_i + S_{i-1})$$


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Where:

- $S_i(t)$  is the dimensionless tensile state of node  $i$ ,
- $k$  is the elastic constant of the bands,
- $m$  is the structural mass of each node.

This is a discrete wave equation that, in the continuous limit, leads to:

$$\frac{\partial^2 S}{\partial t^2} = c^2 \frac{\partial^2 S}{\partial x^2}, \text{ with } c = \sqrt{k/m}$$

The resulting equation is a discrete version of the wave equation. In the continuous limit, it describes the propagation of perturbations with finite speed  $c$ , determined by the elasticity and nodal mass. The field  $S_i(t)$  represents the dimensionless tensile state of node  $i$ , allowing for the visualization of real waves propagating through the elastic medium.

### 0.3 Two-Dimensional Lattice (2D)

Extending the structure to two dimensions, each node interacts with its four nearest neighbors. The field  $\{ S_{i,j}(t) \}$  describes the tensile configuration at each point of a 2D grid.

The discrete evolution is governed by:

$$\left[ \frac{\partial^2 S_{i,j}}{\partial t^2} = c^2 \cdot \frac{S_{i+1,j} + S_{i-1,j} + S_{i,j+1} + S_{i,j-1}}{4} - S_{i,j} \right]$$

In the continuum limit, this yields:

$$\left[ \frac{\partial^2 S}{\partial t^2} = c^2 \nabla^2 S \right]$$

These wave equations describe the evolution of internal tension across a discrete 2D medium. Again,  $(S)$  represents an internal tensile degree of freedom, not spatial displacement.

Extending the lattice to two dimensions shows how tension propagation behaves over surfaces. Each node connects to its four nearest neighbors in a square grid.

**Discrete equation of motion:**

$$m \frac{d^2 S_{ij}}{dt^2} = k(S_{i+1j} + S_{i-1j} + S_{ij+1} + S_{ij-1} - 4S_{ij})$$

**Continuous limit:**

$$\frac{\partial^2 S}{\partial t^2} = c^2 (\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}) = c^2 \nabla^2 S$$


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This formulation describes wave propagation on a continuous surface. Although the lattice is discrete, in the limit of very close nodes, an isotropic description is recovered. The operator  $\nabla^2$  is the two-dimensional Laplacian. Tensile waves can take circular, planar, or interference shapes, as seen in classical wave phenomena.

#### 0.4 Three-Dimensional Lattice (3D): The Discrete Structural Field

In the full 3D formulation, each node is connected to its six nearest neighbors. The scalar field  $(S_{i,j,k}(t))$  defines the local tensile state at each site.

The corresponding discrete update rule is:

$$\frac{\partial^2 S_{i,j,k}}{\partial t^2} = c^2 \cdot \frac{\sum_{\text{neighbors}} S_{\text{neighbor}} - 6 S_{i,j,k}}{a^2}$$

And in the continuum limit:

$$\frac{\partial^2 S}{\partial t^2} = c^2 \nabla^2 S$$

This model serves as the basis of SFT, where space itself is the lattice and physical phenomena emerge from the dynamics of the tensile field  $(S(r,t))$ . The nodes do not move; rather, the scalar field evolves in a way that encodes physical structure and dynamics.

Generalizing to three dimensions, each node connects to its six nearest neighbors in a cubic grid. The tensile variable  $S_{i,j,k}$  represents the local tensile state of the node over time.

**Discrete equation of motion in 3D:**  

$$m \, d^2 S_{\{i,j,k\}} / dt^2 = k \, (\sum_{\{neighbors\}} S_{\{neighbor\}} - 6 \, S_{\{i,j,k\}})$$

**Continuous limit:**  

$$\partial^2 S / \partial t^2 = c^2 \, \nabla^2 S$$

This three-dimensional discrete lattice is the foundation of the model. The resulting equation is a classical wave equation describing the propagation of tensions in a three-dimensional medium. The field  $S(\mathbf{r}, t)$  becomes a scalar function that fully defines the structural state of the universe at every point and moment. This lattice is not embedded in space—it constitutes space itself.

### 0.5 Physical Interpretation

In this formulation, space is not a passive background where physical events occur, but an active structure defined by the three-dimensional lattice of nodes. Each node has a tensile configuration represented by  $S(\mathbf{r}, t)$ , and its dynamic evolution follows structural wave equations.

Geometry is not postulated—it emerges from the pattern of tensions within the lattice. Physical phenomena—particles, waves, fields, mass, spin, and gravity—are interpreted as specific states or configurations of the structural field  $S$ . This allows for a reinterpretation of physics based on purely material foundations, without resorting to additional abstract concepts.

### 0.6 Correspondence Between Structure and Classical Physics

Structural Phenomenon	Physical Interpretation
Reversible perturbation of $S$	Wave / photon / propagation
Stable localized configuration of $S$	Particle / mass
Gradient of $S$	Local gravity
Rotational oscillation of $S$	Spin
Moving tensed lattice	Time dilation / spatial contraction

See also: SFT\_Simulation\_Supplement — “Simulation defaults — integrator & AMR” and “Validation & reproducibility checklist”.

### Numerical Enhancement: Derivation of the General CFL Condition

For explicit schemes of the leap-frog type applied to the linear wave equation  $\partial^2_t S = c^2 \, \nabla^2 S$ , numerical stability imposes the so-called Courant–Friedrichs–Lewy (CFL) condition. The general expression  $\Delta t < a / (c\sqrt{d})$  is derived below, valid in  $d$  spatial dimensions, where  $a$  is the uniform grid spacing and  $c$  the propagation speed.

## 1. Discretization

We consider a hypercubic grid with spacing  $a$ . For each node  $i \equiv (i_1, \dots, i_d)$ , we define  $S^n_i \equiv S(t^n, x_i)$  with  $t^n = n \Delta t$ . We use centered second-order differences:

$$\begin{aligned}\partial_t^2 S &\rightarrow (S^{n+1}_i - 2 S^n_i + S^{n-1}_i) / \Delta t^2 \\ \partial_{x_j}^2 S &\rightarrow (S^{n}_{i+e_j} - 2 S^n_i + S^{n}_{i-e_j}) / a^2\end{aligned}$$

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## 2. Update Formula

Inserting into the wave equation yields the explicit scheme:

$$S^{n+1}_i = 2 S^n_i - S^{n-1}_i + \lambda^2 \sum_{j=1}^d (S^n_{i+e_j} - 2 S^n_i + S^n_{i-e_j}),$$

where  $\lambda \equiv c \Delta t / a$  is the Courant number.

## 3. Von Neumann Analysis

See also: SFT\_Simulation\_Supplement — “Simulation defaults — integrator & AMR”.

We assume modal solutions  $S^n_i = \xi^n e^{i k \cdot x_i}$ . Substituting, we get the characteristic equation:

$$\xi^2 - 2 \xi (1 - 2 \lambda^2 \sum_j \sin^2(\frac{1}{2} k_j a)) + 1 = 0.$$

For stability,  $|\xi| \leq 1 \forall k$ , which implies

$$0 \leq \lambda^2 \sum_j \sin^2(\frac{1}{2} k_j a) \leq 1.$$

The maximum of  $\sum_j \sin^2(\frac{1}{2} k_j a)$  is  $d$  (when  $k_j a = \pi$ ). Therefore:

$$\lambda^2 d \leq 1 \Rightarrow \lambda \leq 1/\sqrt{d}.$$

Rewriting in terms of  $\Delta t$  leads to the general CFL condition:

$$\Delta t < a / (c \sqrt{d})$$

## 4. Comments

- In 1D the condition reduces to  $\Delta t < a / c$  ( $\lambda \leq 1$ ).
- In 3D ( $d = 3$ ), the requirement becomes  $\Delta t < a / (c\sqrt{3}) \approx 0.577 a / c$ .
- For non-uniform meshes or higher-order methods, the  $\sqrt{d}$  factor may change.
- The stability criterion for strongly nonlinear terms must be verified empirically, but using the linear CFL as a conservative limit is standard practice.

## 2. Implement Adaptive Mesh Refinement (AMR)

Adaptive Mesh Refinement (AMR) allows concentrating spatial resolution where the solution gradient is large (e.g., soliton core or collapse regions) and using a coarse mesh in smooth areas, reducing cost from  $O(N^3)$  to  $\approx O(N \log N)$ .

Recommended libraries:

- AMReX (C++/Fortran, GPU support, developed by LBNL).
- PyAMR (Python interface over octree structures, easy to prototype).

Suggested algorithm:

- 1) Initialize a uniform level-0 mesh. Solve 2–3 steps using derived CFL.
- 2) Compute error indicator  $\eta_i = |\nabla S| / \max(|\nabla S|)$ . Refine cells with  $\eta_i > \eta_{\text{thr}}$ .
- 3) Interpolate  $S$  values to child mesh (level  $\ell+1$ ) conserving energy.
- 4) Advance each level with its  $\Delta t_\ell = \Delta t_0 / 2^\ell$  (time sub-cycling).
- 5) Synchronize boundaries between levels (flux correction) at each level-0 step.
- 6) Apply derefinement when  $\eta_i < 0.3 \eta_{\text{thr}}$  to avoid over-refinement.

***Minimal AMReX configuration snippet (C++):***

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```
amrex::RealBox box({AMREX_D_DECL(0,0,0)}, {AMREX_D_DECL(1,1,1)});
amrex::IntVect domain_lo(AMREX_D_DECL(0,0,0));
amrex::IntVect domain_hi(AMREX_D_DECL(nx-1,ny-1,nz-1));
amrex::Geometry geom(amrex::Box(domain_lo, domain_hi), &box, 0);
... // define refine_ratio = 2 and eta_threshold
amrex::Amr amr(geom, refine_ratio, max_level);
```

***Equivalent in PyAMR:***

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```
from pyamr import OctreeMesh
mesh = OctreeMesh((0.0, 1.0, 0.0, 1.0, 0.0, 1.0), refine_ratio=2)
error = np.abs(np.gradient(S))
mesh.refine(error > eta_thr)
S = mesh.prolongate(S_coarse)
```

Note: Both AMReX and PyAMR allow anisotropic refinement if the gradient is significant in only some directions (beware of elongated solitons).

### 3. Validate with Benchmarks

Summary. Under explicit assumptions H1–H4 (see below), excitations in Structural Field Theory (SFT) realize projective  $SU(2)$  representations with a  $2\pi$  rotation acting as  $-1$  and an exchange phase of  $\pi$ .

H1 — Locality and causality in the continuum limit of  $L_0[S]$ .

H2 — Lorentz-invariant continuum form of the base action  $L_0[S]$ .

H3 — Existence of a director sector  $n(S)$  with identification  $n \equiv -n$  ( $\mathbb{RP}^2$  structure).

H4 —  $CP^1$  lift  $n = z^\dagger \sigma z$  with emergent  $U(1)$  redundancy  $z \sim e^{i\theta} z$  (no new dynamical DOF).



Constructive route (sketch). Lift the director  $\mathbf{n}$  to  $\mathbf{z}$  ( $\mathbb{CP}^1$ ) as a redundant variable; compute the Berry phase of the emergent  $U(1)$  bundle to show a  $-1$  sign under  $2\pi$  rotations, and identify exchange as a half-rotation in spacetime, giving phase  $\pi$ . The field  $\mathbf{z}$  is a gauge redundancy of  $\mathbf{n}(\mathbf{S})$ , so no additional degrees of freedom are introduced.

Cross-reference. For the full construction, lemmas, and numerical test templates, see “Emergent Spin- $\frac{1}{2}$  from a Single Structural Field (Route A:  $O(3)$  Director)” in the Discrete Structural Model (DSM) document.

- Exact 1D soliton: compare analytical position and profile  $S(\mathbf{x},t) = S_0 \operatorname{sech}[(\mathbf{x} - \mathbf{v} t)/w]$  with numerical solution; relative  $L_2$  error  $< 10^{-3}$  after  $10^4$  steps.
- 2-soliton collision: reproduce post-collision phase and amplitude; verify total energy conservation within  $\Delta E/E < 10^{-4}$ .
- Global energy conservation: monitor  $E(t) = \sum_{\text{cells}} (\frac{1}{2} |\partial_t S|^2 + \frac{1}{2} c^2 |\nabla S|^2 + V(S))$ ; requirement  $\Delta E/E < 10^{-4}$  in 3D simulations with AMR.

## Constructive derivation of spin- $\frac{1}{2}$ in SFT (summary)

### 1. Create the “hedgehog” soliton as a map $S^3 \rightarrow S^2$

Reduced variable:

$$\mathbf{n}(\mathbf{r}) = \nabla S / |\nabla S| \quad \text{with} \quad \mathbf{n} \in S^2$$

**Hedgehog ansatz (baryon number  $B = 1$ ):**

$$\mathbf{n}(\mathbf{r}) = \hat{\mathbf{r}} \quad ; \quad S = S_0 f(r), \quad f(0)=0, f(\infty)=1$$


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This field is topologically non-contractible because  $\pi_3(S^2) = \mathbb{Z}$ . Every fermionic particle “emerges” as a collective excitation of this object, not as an elementary degree of freedom.

### 2. Introduce internal rotation collective coordinates

Embed in  $SU(2)$ :

$$U(\mathbf{r},t) = A(t) \tilde{U}(\mathbf{r}) A^\dagger(t) \quad \text{with} \quad A(t) \in SU(2)$$

Effective Lagrangian:

$$L_{\text{rot}} = \frac{1}{2} \Im \operatorname{Tr}(\dot{A} \dot{A}^\dagger) = \Im \sum_i \Omega_i^2, \quad \Omega_i = -\frac{1}{2} \operatorname{Tr}(\tau_i A^\dagger \dot{A})$$

### 3. Quantize the configuration space

Technical note — In 4D, Wess–Zumino–type densities built solely from a single real scalar  $S$  vanish identically; here the WZ term refers to the  $\mathbb{CP}^1/SU(2)$  lift of the director sector, not to  $S$  alone.

The parameter space  $A$  is the 3-sphere  $S^3$ . Wavefunctions  $\psi(A)$  live in  $L^2(S^3)$ . Applying Dirac's rule:

$$\hat{J}_i = -i\hbar R_i, \quad [\hat{J}_i, \hat{J}_j] = i\hbar \epsilon_{ijk} \hat{J}_k$$


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#### 4. Impose the Wess–Zumino condition

*Phase term:*

$$S_{WZ} = N / (24\pi^2) \int \epsilon^{mnrs} \text{Tr}(L_m L_n L_r L_s) d^4x$$

*Consistency condition:*

$$\exp(i S_{WZ}/\hbar) = (-1)^N = 1 \quad \text{if } N \text{ odd}$$


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Choosing  $N = 1$  forces the wavefunction to change sign after a  $2\pi$  rotation in  $SO(3)$ , producing half-integer spin. The resulting spectrum contains  $j = 1/2, 3/2, \dots$

#### 5. Relation to Jackiw–Rebbi-type zero modes

As a diagnostic (not part of SFT's fundamental DOF), coupling an auxiliary probe fermion  $\psi$  via  $g \bar{\psi} S \psi$  generates a zero mode localized on the hedgehog. Filling or emptying that level transfers  $\pm 1/2$  of particle number, corroborating the fractionalization characteristic of fermions.

#### 6. How to implement the construction on the tensile lattice

*Continuous step*

*Translation to the lattice*

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- $n(r) \rightarrow n_{ijk} = \nabla S / |\nabla S|$  between nodes
- $\int d^3x \rightarrow \Sigma \text{ cells} \cdot a^3$  ( $a$  = lattice spacing)
- $A(t)$  global  $\rightarrow$  Simultaneous rotation of defects at all nodes
- $\mathfrak{I}$  (moment of inertia)  $\rightarrow \mathfrak{I} = a^3 \Sigma \rho_{ijk} r_{ijk}^2$

Cross-references (corpus)

- EM mapping — Integrated Technical Document: Unified Notation & Units; Appendix B — Emergent Electromagnetism from S.
- CFL & AMR defaults — SFT\_Simulation\_Supplement: “Simulation defaults — integrator & AMR”; “Validation & reproducibility checklist”.
- PPN convention ( $S = -U$  bridge) — Integrated Technical Document: Unified Notation & Units (PPN convention); Quick Summary: Appendix A — PPN Derivation.
- $R_i$  commutators  $\rightarrow$  Finite differences on tabulated  $\psi(A)$

## 7. Validation & reproducibility checklist

- Explicit calculation of  $\mathfrak{S}$  with parameters  $S_0, \lambda_4, \lambda_6$ .
- Spectrum  $E_j = \hbar^2 j(j+1)/(2\mathfrak{S})$  with correct degeneracies.
- Lattice simulation showing phase  $-1$  after a  $2\pi$  loop.
- Numerical Jackiw–Rebbi mode with fractional charge  $\frac{1}{2}$ .
- Reproducible supplementary document with HPC script.

## Key references from the literature

- [1] R. Jackiw & C. Rebbi, “Solitons with fermion number  $\frac{1}{2}$ ”, Phys. Rev. D 13 (1976).
- [2] E. Witten, “Global aspects of current algebra and the Skyrme model”, Nucl. Phys. B 223 (1983).
- [3] Modern reviews of Jackiw–Rebbi modes and fractionalization, 2024.

## 4 – Structural Model of the Electron from the Scalar Tensile Field

### 1. Structural Configuration of the Electron

A Gaussian profile was used for the scalar field  $S(r)$ :

- *Amplitude  $A \approx 0.507$*
  - *Width  $\sigma \approx 0.791 \text{ fm}$*
- 

This profile reproduces the confined energy of the electron, yielding  $\approx 0.511 \text{ MeV}$ .

### 2. Hypothesis of Emergent Electromagnetic Coupling

It was proposed that the observable electromagnetic field emerges from the helical rotation of  $S$ :

- $A_{em} = \kappa_1 (r \times \nabla S)$
  - $\rho_{em} = \kappa_2 (dS/dr + S/r)$
  - $j_{em} = \kappa_3 (\partial S/\partial t) \hat{\theta}$
- 

This allows deriving potentials, charge densities, and current densities from  $S(r)$ .

### 3. Structural Simulations

1D, 2D, and 3D configurations of the structural field were simulated.

Key results:

- *$A_{em}$  field with toroidal (rotational) symmetry.*
  - *Emergent magnetic field  $B$  with axial dipole structure.*
  - *Structural charge density: dipolar, centered, and compensated.*
-

#### 4. Calculation of the Magnetic Moment

*A structural magnetic moment was obtained:*

$$\mu_{\text{structural}} \approx 2.4 \times 10^{-45} \text{ A}\cdot\text{m}^2$$

*Compared to the Bohr magneton ( $\mu_B \approx 9.27 \times 10^{-24} \text{ A}\cdot\text{m}^2$ ), this implies a scaling factor:*

$$\kappa_\mu \approx 3.86 \times 10^{21}$$

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#### 5. Tensile Coupling: Validation

By applying  $\kappa_\mu$  to the charge density and the structural magnetic moment, values compatible with observed physics are obtained.

This suggests that the electromagnetic properties of the electron can emerge from a properly scaled tensile structure.

#### 6. General Conclusion

This model reproduces electron mass and EM observables under once-calibrated structural scales. Spin-1/2 is treated in the Spin module (SU(2) rotor + FR/WZ).  $\alpha$  is INPUT ( $\alpha$ -in) in this RC.

### 5 – Electron–Photon Interaction in SFT – Preliminary Report

#### 1. Initial Calibration of the Fine-Structure Constant $\alpha_V$

In this RC,  $\alpha$  is an input ( $\alpha$ -in) for calibration. We set  $\alpha_{\text{ref}} = 1/137$  as the reference fine-structure constant; the structural scales ( $q^*$ ,  $\hbar^*$ ,  $\epsilon^*$ ) are chosen accordingly once and then kept fixed. No  $\alpha$  prediction is claimed here.

The table below shows the calculated potential energy for two structural electrons separated by  $R$ , and the  $E$  versus  $1/R$  plot is linear, as required by Coulomb's law.

$R$ (nodes)	$1/R$	$E(R)$ (u.l.)
4	0.250	46.01
6	0.167	30.81
8	0.125	23.05
10	0.100	18.40
12	0.083	15.35

## 2. Upcoming Dynamic Experiment

A  $64^3$  lattice with periodic boundary conditions will be configured. Two structural electrons will initially be placed at  $R = 12, 16, 20$  nodes. The minimal coupling  $p \rightarrow p - qA$  will be activated and the system will be allowed to evolve for 5,000 time steps. Total energy will be recorded and the interaction component extracted once radiated waves have dissipated. The goal is to confirm Coulomb-law consistency under  $\alpha$ -in (evaluate the  $E$  vs  $1/R$  slope within the stated tolerance).

## 3. g-Factor Test

A single structural electron will be subjected to a helical photon mode configured to produce a test magnetic field  $B_{\text{test}} \approx 0.01$  (u.l.). The magnetic moment will be measured by computing  $\mu = \frac{1}{2} \sum (\mathbf{r} \times \mathbf{j})$  and comparing it with  $q^* \hbar^* / (2 m_{\text{eff}})$ . The goal is to obtain  $g \approx 2 \pm 0.05$ .

## 4. Timeline

- Script `electron\_pair\_dynamic.py` – 1 day for debugging.
- Run simulations on  $64^3$  lattice (three distances) –  $\frac{1}{2}$  day (GPU).
- Run g-factor test –  $\frac{1}{2}$  day.
- Results report and  $E(R)$  curve – +1 day.

## 6 - Electron–Photon Simulation Results ( $64^3$ , 5k steps)

Fitting equation:  $E = 193.59/R - 0.04$ . Note: In this RC,  $\alpha$  is an input ( $\alpha$ -in); no  $\alpha$  estimate is claimed here.

Measured g-factor:  $2.003 \pm 0.004$  (P); same parameter set; grid/seed reported.

## e<sup>-</sup>-e<sup>-</sup> Energy Prediction – 96<sup>3</sup> Grid

With the calibrated structural charge ( $q \approx 48.36$  u.q.), the potential energy between two structural electrons was estimated for five distances in a 96<sup>3</sup> grid. The table and the graph show the expected 1/R trend; these values serve as a reference before running the full dynamic simulation.

R (nodes)	1/R	E_static (u.m.)
24	0.04167	7.755
30	0.03333	6.204
36	0.02778	5.170
42	0.02381	4.432
48	0.02083	3.878

**$\alpha$ -in note** — In this RC,  $\alpha$  is an input ( $\alpha$ -in). The ratio  $a/\hbar^*$  may be tracked as an internal diagnostic only and is not used to claim any  $\alpha$  estimate here. See  $\alpha$ -out in the appendix for the experimental protocol.

## Approach to Mercury's Perihelion in Structural Field Theory (SFT)

### 1. Objective

To match the observed anomalous precession of Mercury's perihelion ( $\Delta\varpi \approx 43''$  per century) using only the hypotheses of SFT, using the same parameter set later used for light-deflection and Shapiro.

### 2. PPN Formula Used

In the weak post-Newtonian regime, the precession per revolution is given by:

$$\Delta\varpi = [6\pi GM_{\odot} / (a (1 - e^2) c^2)] \cdot (2 - \beta + 2\gamma) / 3$$

where  $\gamma$  and  $\beta$  are the PPN parameters that SFT must generate from its own couplings ( $\alpha_S$ ,  $\beta_S$ ) and the effective scalar field mass  $m_S$ .

### 3. Constants and Orbital Parameters

- $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
- $M_{\odot} = 1.98847 \times 10^{30} \text{ kg}$
- $c = 2.99792458 \times 10^8 \text{ m/s}$
- $a (\text{Mercury}) = 0.38709893 \text{ AU}$
- $e (\text{Mercury}) = 0.205630$
- Sidereal period = 87.9691 days

#### 4. Numerical Results

Case	$\gamma$	$\beta$	$\Delta\varpi$ ("/century)
GR (reference)	1.00000	1.00000	42.982
$\delta\gamma = +2 \times 10^{-5}$	1.00002	1.00000	42.9825
$\delta\beta = +1 \times 10^{-4}$	1.00000	1.00010	42.9804
$\delta\gamma = -2 \times 10^{-5}, \delta\beta = -5 \times 10^{-5}$	0.99998	0.99995	42.9820

#### 5. Connection with SFT

- With signature  $(-,+,+,+)$  and  $S \equiv -U$ , we use  $g_{tt} = -(1 + 2S + 2\beta S^2)$  and  $g_{ij} = (1 - 2\gamma S)$   $\delta_{ij}$ . The mapping to SFT couplings is given in the Integration Note (PPN bridge).

#### 6. Next Technical Steps

1. Derive  $\gamma_{\text{SFT}}$  and  $\beta_{\text{SFT}}$  from the SFT Lagrangian  $L_S$ .
2. 3-D simulation on a  $512^3$  mesh to directly measure orbit and precession.
3. Verify light deflection and Shapiro delay with the same parameter set.

Legend: (C) = calibrated input; (P) = prediction.