National University of Computer & Emerging Sciences, Karachi

Fall-2024 FAST School of Computing

MT-1003

Calculus and Analytical Geometry

- Marks:100 points
- Submission date start from 12th October 2024
- 1th November will be last day of submission
- In any circumstances no submission will be accepted after 1th November, 24

6.1 Implicit Differentiation

Step 1: Take the derivative normally. Each time a "y" is involved, include a $\frac{dy}{dx}$.

Step 2: Gather all terms with $\frac{dy}{dx}$ on the left side, everything else on the right.

Step 3: If necessary, factor out the $\frac{dy}{dx}$ to create only one $\frac{dy}{dx}$ term.

Step 4. Solve for $\frac{dy}{dx}$.

Find $\frac{dy}{dx}$.

1. $8 = 3x^2 + y^4$	2. $\sin(2x - y) = 4x$	3. $x^2 + 2y^5 = 10xy$

Use implicit differentiation to find $\frac{d^2y}{dx^2}$.

4. $7xy = 8$	5. $e^x + y^2 = 4$

$\frac{dy}{dx} =$

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$\frac{d^2y}{dx^2} =$

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6. If $x^2y + y^2 + 4 = 0$, then when $x = 2$, the value of $\frac{dy}{dx}$ is

(A) -2 (B) -1 (C) 0 (D) 2 (E) nonexistent

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7. If $x^2 - y^2 = 5$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 2)$?

(A) $-\frac{13}{8}$ (B) $-\frac{11}{8}$ (C) $-\frac{7}{8}$ (D) $-\frac{5}{8}$ (E) $-\frac{1}{4}$

6.2 Related Rates

Guidelines to solving related rate problems
1. Draw a picture.
2. Make a list of all known and unknown rates and quantities.
3. Relate the variables in an equation.
4. Differentiate with respect to time.
5. Substitute the known quantities and rates and solve.

8. Brust is riding his bicycle north away from an intersection at a rate of 15 miles per hour. Sully is driving his car towards the intersection from the west at a rate of 30 miles per hour. If Brust is 0.4 miles from the intersection, and Sully is 1 mile from the intersection, at what rate is the distance between the two of them increasing or decreasing?

9. The side of a cube is increasing at a constant rate of 0.2 centimeters per second. In terms of the surface area S, what is the rate of change of the volume of the cube, in cubic centimeters per second?

(A) $0.1S$ (B) $0.2S$ (C) $0.6S$

(D) $0.04S$ (E) $0.008S$

Q:10 Find the intervals of increasing/decreasing, concavity, inflection points and relative extrema for each of functions:

a. $f(x) = x^5 - 5x^4$

b. $f(x) = x^{\frac{1}{5}}$

c. $f(x) = (x+1)(x-1)^2$

d. $f(x) = \frac{x^2-3}{x-2}$

e. $f(x) = x\sqrt{8-x^2}$

f. $f(x) = \frac{12}{x^2+12}$

g. $f(x) = x^2 \ln x$

h. $f(x) = \cos x - 9x$, $(0,4\pi)$

i. $f(x) = \ln\sqrt{x^2+4}$

j. $f(x) = xe^{-7x}$