Dated: 21-10-24

ASSIGNMENT # 2

ROLL NO. 24K-3034

Q1.

Solution:

we have to prove that:

$$c = \frac{b+a}{2}$$

Now;

In mean value theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

• Consider e.g:

$$f(x) = Ax^2+Bx+C$$

$$\Rightarrow f'(x) = 2Ax+B$$

$$\therefore f'(c) = 2Ac+B$$

$$\therefore f(b) = Ab^2+Bb+C$$

$$\& \; f(a) = Aa^2+Ba+C$$

Now;

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\therefore 2Ac+B = \frac{Ab^2+Bb+\cancel{C}-Aa^2-Ba-\cancel{C}}{b-a}$$

$$2Ac+B = \frac{A(b^2-a^2)+B(b-a)}{b-a}$$

$$2Ac+B = \frac{A(b-a)(b+a)+B(b-a)}{b-a}$$

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$$2AC + B = \frac{\cancel{(b-a)}\{A(b+a)+B\}}{\cancel{(b-a)}}$$

$$2AC + \cancel{B} = A(b+a) + \cancel{B}$$

$$2\cancel{A}C = \cancel{A}(b+a)$$

$$\therefore C = \frac{b+a}{2} \rightarrow \text{proved}$$

Q2.

Solution

Given e.q is:

$$f(t) = 2t + e^{-2t}$$

$$\therefore f'(t) = 2 - 2e^{-2t} \text{ ......} \rightarrow \text{derivative existis}$$

$\because f(t)$ is continious at $(-2,3)$:

$$f'(c) = 2 - 2e^{-2c}$$

$$\& \quad f(3) = 6 + e^{-6}$$

$$f(-2) = -4 + e^{4}$$

Now;

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \left\} \begin{array}{l} \text{Here} \\ a=-2 \\ b=3 \end{array} \right.$$

$$\therefore 2 - 2e^{-2c} = \frac{6+e^{-6}+4-e^{4}}{3+2}$$

$$\cancel{10} - 10e^{-2c} = \cancel{10} + e^{-6} - e^{4}$$

$$\cancel{-}10e^{-2c} = \cancel{-}54.59$$

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$$e^{-2c} = \frac{54.59}{10}$$

$$e^{-2c} = +5.45$$

$$-2c = \ln(5.45)$$

$$c = \frac{\ln(5.45)}{-2}$$

$$c = -0.84$$

Q3.

$$\therefore P(x) = -x^3 + 9x^2 + 120x - 400$$

where $x \geq 5$

$$\therefore P'(x) = -3x^2 + 18x + 120$$

Put $P'(x) = 0$:

$$0 = -3x^2 + 18x + 120$$

$$0 = x^2 - 6x - 40$$

$$0 = (x-10)(x+4)$$

Either	Or
$x = 10$	$x = -4$ ↳ neglected

Now;

$$P''(x) = -6x + 18$$

At $x = 10$

$$P''(x=10) = -6(10) + 18 = -42$$

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$\because P''(x=10) < 0$ then function is maxima at points $(x=10)$

* for maximum profit:

At $(x=10)$:

$$P(x) = -(10)^3 + 9(10)^2 + 120(10) - 400$$
$$P(x) = 700$$

RESULT:

$\therefore$ 10 hundred thousands of tires must be sold to get maximize profit i.e 700 thousand dollars

Q4.

Solution

$$f(x) = 3x^4 - 16x^3 + 18x^2$$

$$\therefore f'(x) = 12x^3 - 48x^2 + 36x$$

put $f'(x) = 0$:

$$0 = x(12x^2 - 48x + 36)$$
$$0 = x(x-3)(x-1)$$

$\therefore$ $x = 0$, $x = 3$, $x = 1$ } Critical points

decreasing, increasing, increasing, decreasing

$-\infty$ 0 1 3 $+\infty$

Consider:

* $f'(-1) < 0$ ......→ decreasing
* $f'(0.5) > 0$ ......→ increasing

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* $f'(2) < 0$ ⟶ increasing
* $f'(4) > 0$ ⟶ decreasing

increasing: $(0, 3)$

decreasing: $(-\infty, 0) \cup (3, +\infty)$

* for concavity:

$$\therefore f''(x) = 36x^2 - 96x + 36$$

let $f''(x) = 0$:

$$0 = 36x^2 - 96x + 36$$

Either $x = \frac{4+\sqrt{7}}{3}$ Or $x = \frac{4-\sqrt{7}}{3}$

inflection points

$x = 2.21$ $x = 0.45$

concave up, concave down, concave up

$-\infty$ 0.45 2.21 $+\infty$

* $f''(0) = 36$, $f''(0) > 0$ ⟶ concave up
* $f''(2) = -12$, $f''(2) < 0$ ⟶ concave down
* $f''(3) = 72$, $f''(3) > 0$ ⟶ concave up

concave down: $(0.45, 2.21)$

concave up: $(-\infty, 0.45) \cup (2.21, +\infty)$

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* for Absolute Extrema :

it is given that:
* End points ⇒ $[1,4]$
* critical points ⇒ $\{0, 1, 3\}$

⇒ $f(1) = 5$
⇒ $f(0) = 0$
⇒ $f(3) = -27$
⇒ $f(4) = 32$

3 is the absolute minima point whereas -27 is the absolute minima.

4 is the absolute maxima point whereas 32 is the absolute maxima.

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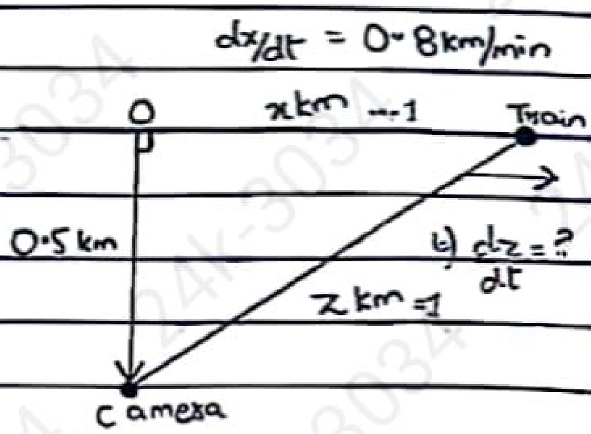
Q5.



Data:

$$\frac{dx}{dt} = 0.8 \text{ km/min}$$

$$y = 0.5\text{km}$$

Solution:

* for (a):

using pythagoras theorem:

$$H^2 = P^2 + B^2$$

Here;

$$\left.\begin{aligned} H &= z \\ P &= y \\ B &= x \end{aligned}\right\} \text{put in above}$$

$$z = \sqrt{(0.5)^2 + x^2}$$

$$z = \sqrt{x^2 + \frac{1}{4}}$$

* for (b):

given that:

$$z = 1\text{km}$$

Consider:

$$z = \sqrt{x^2 + (0.5)^2}$$

derivate w.r.t 't'

$$\frac{dz}{dt} = \frac{dx}{dt} \cdot \frac{1}{2}\left\{x^2 + (0.5)^2\right\}^{-1/2} \cdot 2x \rightarrow \text{eq } i$$

* for value of 'x':

$$z = \sqrt{x^2 + (0.5)^2}$$

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$$(1)^2 = x^2 + (0.5)^2$$

$$\frac{3}{4} = x^2$$

$$x = 0.866 \text{ km} \quad \underline{\text{OR}} \quad \sqrt{3/4} \text{ km}$$

e.q i =>

$$\frac{dz}{dt} = \frac{1}{2} \cdot \left[\frac{1}{\sqrt{x^2+(0.5)^2}}\right]^{1} \cdot 2(x) \cdot \frac{dx}{dt}$$

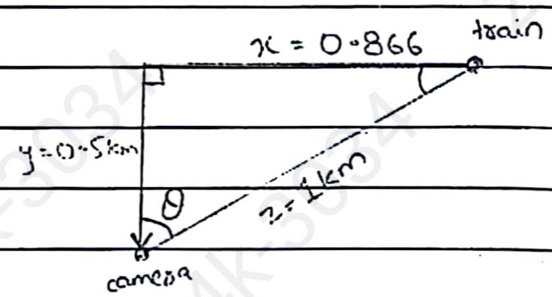
put value of x:

$$\frac{dz}{dt} = \frac{1}{2} \cdot \left[\frac{1}{\sqrt{(0.866)^2+(0.5)^2}}\right] \cdot 2 \times \sqrt{\frac{3}{4}} \cdot (0.8)$$

$$\frac{dz}{dt} = \frac{\sqrt{3}}{2} \cdot (0.8) \text{ km/min} \Rightarrow \frac{2\sqrt{3}}{5} \text{ km/min}$$

$$\frac{dz}{dt} = (0.8) \times 0.866 \text{ km/min} \Rightarrow 0.69 \text{ km/min}$$

* for (c):



$$\because \tan\theta = \frac{P}{B} =$$

* from part (b) $x = 0.866$ km

$$\tan\theta = 0.866/0.5$$

$$\tan\theta = 1.732$$

$$\theta = \tan^{-1}(1.732)$$

$$\theta = 60^\circ \quad \underline{\text{OR}} \quad \pi/3 \text{ rad}$$

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Consider:

$$\tan\theta = \frac{x}{y}$$

$$\tan\theta = x/0.5$$

derivate:

$$\frac{d\theta}{dt} \cdot \sec^2\theta = 1/0.5 \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = 1/0.5 \cdot \frac{dx}{dt} \cdot \frac{1}{\sec^2\theta} \rightarrow \text{eq ii}$$

for $\sec^2\theta$:

$$\sec^2\theta = \tan^2\theta + 1$$

$$= (1.732)^2 + 1$$

$$\sec^2\theta = 3.99 \approx 4$$

eq ii =>

$$\frac{d\theta}{dt} = 1/0.5 \cdot \frac{dx}{dt} \cdot \frac{1}{4}$$

Now;

$$\left.\begin{aligned} x &= 0.866 \\ dx/dt &= 0.8 \end{aligned}\right\} \text{put in above}$$

$$\frac{d\theta}{dt} = 1/0.5 \cdot 0.8 \cdot \frac{1}{4}$$

$$\frac{d\theta}{dt} = 0.4 \text{ rad/min}$$

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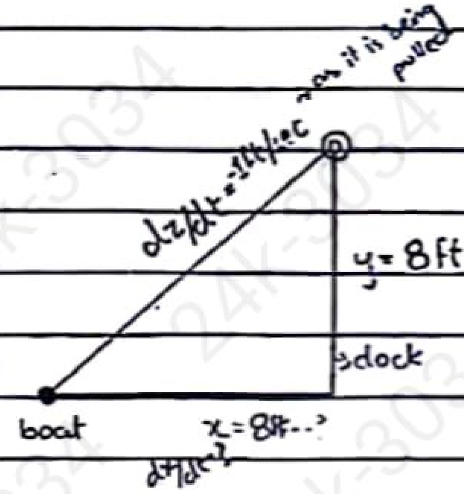
Q6.



Given:

* $y = 8ft$

According to given condition:

* $x = 8ft$

* $\frac{dz}{dt} = \boxed{-1}$ ft/sec   rope length decreasing

Solution:

According to pythagoras theorem

$$z^2 = x^2 + \underbrace{\boxed{y^2}}_{\text{constant}}$$

derivate w.r.t 't':

$$\cancel{2}z \cdot \frac{dz}{dt} = \cancel{2}x \cdot \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{z}{x} \cdot \frac{dz}{dt} \rightarrow eq\,i$$

* for value of z:

$$z = \sqrt{x^2+y^2} = \sqrt{8^2+8^2} = 8\sqrt{2}$$

$$z = 11.31\ ft$$

* e.q. i ⇒

$$\frac{dx}{dt} = \frac{11.31}{8} \cdot -1$$

$$\frac{dx}{dt} = -\sqrt{2} = -1.41 \text{ ft/sec}$$

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Q7.

a) $\int \frac{e^{x^2} - 2x}{e^{x^2}} \, dx$

Solution:

let;

$$u = x^2 \qquad \rightarrow \qquad x = \sqrt{u}$$

$$du = 2x \; dx$$

$$dx = \frac{du}{2x}$$

$$\Rightarrow \int \frac{e^u - 2x}{e^u} \cdot \frac{du}{2x} \qquad 2$$

$$\Rightarrow \frac{1}{2}\int \frac{du}{x} - \frac{1}{\cancel{2}}\int \frac{\cancel{2}\cancel{x}}{e^u \cdot \cancel{x}} \, du$$

$$\Rightarrow \frac{1}{2} \cdot \frac{u^{-1/2+1}}{-1/2+1} + e^{-u} + C$$

$$\Rightarrow \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \sqrt{u} + e^{-u} + C$$

$$\Rightarrow \sqrt{x^2} + e^{-x^2} + C$$

$$\Rightarrow x + \frac{1}{e^{x^2}} + C$$

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b) $\displaystyle\int \frac{x^2+2x}{\sqrt{x^3+3x^2+4}}\,dx$

Solution:

$$\Rightarrow \int (x^3+3x^2+4)^{-1/3} \cdot (x^2+2x)\,dx$$

$$\Rightarrow \int (x^3+3x^2+4)^{-1/3} \cdot \frac{1}{3}(3x^2+6x)\,dx$$

$$\Rightarrow \frac{1}{3}\int (x^3+3x^2+4)^{-1/3}\,(3x^2+6x)\,dx$$

$$\int (f(x))^n \cdot f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + C$$

$$\Rightarrow \frac{1}{\cancel{3}} \cdot (x^3+3x^2+4)^{2/3} \cdot \frac{\cancel{3}}{2} + C$$

$$\Rightarrow \frac{1}{2}(x^3+3x^2+4)^{2/3} + C$$

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c) $\displaystyle\int \frac{\cos(2x)}{1+\sin(2x)}\, dx$

Solution:

$\Rightarrow \displaystyle\int (1+\sin(2x))^{-1} \cdot \cos(2x)\, dx$

$\Rightarrow \displaystyle\frac{1}{2}\int (1+\sin(2x))^{-1}\,(2\cos(2x))\, dx$

$\therefore \displaystyle\int (f(x))^{-1}\cdot f'(x)\, dx = \ln[f(x)] + C$

$\Rightarrow \displaystyle\frac{1}{2}\cdot \ln(1+\sin(2x)) + C$

$\Rightarrow \ln\sqrt{1+\sin(2x)} + C$

d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\, dx$

Solution

let,

$x = 3\sec\theta$

$\therefore\ dx = 3(\sec\theta\tan\theta)\, d\theta$

$\Rightarrow \displaystyle\int \frac{3(\sec\theta\tan\theta)}{(3\sec\theta)^2\sqrt{(3\sec\theta)^2-9}}\, d\theta$

$\Rightarrow \displaystyle\int \frac{\cancel{3}\,\sec\theta\tan\theta}{9\sec^2\theta\cdot\cancel{3}\sqrt{1(\sec^2\theta-1)}}\, d\theta$

$\Rightarrow \displaystyle\frac{1}{9}\int \frac{\cancel{\tan\theta}}{\sec\theta\cdot\cancel{\tan\theta}}\, d\theta \qquad \because \sec^2\theta - 1 = \tan^2\theta$

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$$\Rightarrow \frac{1}{9}\int \cos\theta \, d\theta$$

$$\Rightarrow \frac{1}{9} \cdot \sin\theta + C \rightarrow e.q\, i$$

* for θ:

Consider

$$x = 3\sec\theta$$

$$\sec\theta = \frac{x}{3} \rightarrow \frac{H}{B}$$

$$P = \sqrt{H^2 - B^2}$$

$$= \sqrt{x^2 - 9}$$

$$\therefore \sin\theta = \frac{P}{H} = \frac{\sqrt{x^2-9}}{x}$$

e.q i $\Rightarrow$

$$\Rightarrow \frac{1}{9} \cdot \frac{\sqrt{x^2-9}}{x} + C$$

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e) $\int \frac{x^3}{\sqrt{x^2-9}}\,dx$

let,

$x = 3\sec\theta \quad \dashrightarrow \quad \sec\theta = \frac{H}{B} = \frac{x}{3}$

$dx = 3\sec\theta\tan\theta\, d\theta$

$\Rightarrow \int \frac{(3\sec\theta)^3 \cdot 3\sec\theta\tan\theta}{\sqrt{(3\sec\theta)^2-9}}\, d\theta$

$\Rightarrow \int \frac{27 \cdot \cancel{3} \cdot \sec^4\theta \tan\theta}{\cancel{3}\sqrt{\sec^2\theta - 1}}\, d\theta$

$\Rightarrow 27 \int \frac{\sec^4\theta \cancel{\tan\theta}}{\cancel{\tan\theta}}\, d\theta.$

$\Rightarrow 27 \int \sec^2\theta \cdot \sec^2\theta \, d\theta$

$\Rightarrow 27 \int \sec^2\theta\,(1+\tan^2\theta)\, d\theta$

$\Rightarrow 27 \int \sec^2\theta\, d\theta + 27 \int \sec^2\theta \tan^2\theta\, d\theta$

$\Rightarrow 27\tan\theta + \overset{9}{\cancel{27}}\,\frac{\tan^3\theta}{\cancel{3}} + c$

$\Rightarrow 9\left\{3\tan\theta + \tan^3\theta\right\} + c$

$\therefore \tan\theta = \sqrt{x^2-9}/\cancel{x}3$

$\Rightarrow 9\left\{\sqrt{x^2-9} + \frac{1}{(3)^3}(x^2-9)^{3/2}\right\} + c$

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$$\Rightarrow 9\sqrt{x^2-9} + \frac{1}{3}\sqrt{(x^2-9)^3} + C$$

OR

$$\Rightarrow 9\sqrt{x^2-9} + \frac{1}{3}(x^2-9)^{3/2} + C$$

9) $\int x^2 \cos(4x)\, dx \dashrightarrow y = \int x^2 \cos(4x)\, dx$

Solution

Let;

$$u = x^2 \;,\; v = \cos 4x$$

$$\Rightarrow u\int v\,dx - \int\left\{du \cdot \int v\,dx\right\}dx$$

$$\Rightarrow x^2 \int \cos(4x) - \int\left[2x \cdot \int \cos(4x)\right]dx$$

$$\Rightarrow x^2 \cdot \frac{\sin 4x}{4} - \int\left[2x \cdot \frac{1 \sin 4x}{4_2}\right]dx$$

$$\Rightarrow \frac{1}{4}x^2\sin 4x - \frac{1}{2}\int x \cdot \sin 4x\, dx$$

$$\Rightarrow \frac{1}{4}x^2\sin 4x - \frac{1}{2}\left[x \cdot -\frac{\cos 4x}{4} - \frac{1}{4}\int\left\{-\cos 4x\right\}dx\right]$$

$$\Rightarrow \frac{1}{4}x^2\sin 4x + \frac{1}{8}x\cos 4x - \frac{1}{32}\sin 4x + C$$

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h) $\int e^{-x}(4x^3-9x^2+7x+3)\,dx$

Solution:

let $u = 4x^3-9x^2+7x+3$

$v = e^{-x}$

Using tabular method

U	V
1. $\Rightarrow 4x^3-9x^2+7x+3$	$\Rightarrow e^{-x}$
2. $12x^2-18x+7$	$+$ 1. $-e^{-x}$
3. $24x-18$	$-$ 2. e^{-x}
4. 24	$+$ 3. $-e^{-x}$
5. 0	$-$ 4. e^{-x}
	$+$ 5. $-e^{-x}$

$\therefore \int e^{-x}(4x^3-9x^2+7x+3)\,dx =$

$(-e^{-x}(4x^3-9x^2+7x+3)+e^{-x}(12x^2-18x+7)-e^{-x}(24x-18)+(24e^{-x}))$

$\Rightarrow -e^{-x}\{+(4x^3-9x^2+7x+3)+(12x^2-18x+7)+(24x-18)+24\}+c$

$\Rightarrow -e^{-x}\{+4x^3-9x^2+7x+3+12x^2-18x+7+24x-18+24\}+c$

$\Rightarrow -e^{-x}\{4x^3+3x^2+13x+16\}+c$

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i) $\int e^{2z} \cos\left(\frac{z}{4}\right) dz$ ......> $I = \int e^{2z} \cos\left[\frac{z}{4}\right] dz$

Solution

let;

$u = e^{2z}$

$v = \cos(z/4)$

$$I \Rightarrow e^{2z} \int \cos\left\{\frac{z}{4}\right\} dz - \int \left[2e^{2z} \cdot \int \cos\left\{\frac{z}{4}\right\} dz\right] dz$$

$$I \Rightarrow e^{2z} \cdot 4 \sin\left\{\frac{z}{4}\right\} - \int \left\{2 \cdot 4\, e^{2z} \sin\left\{\frac{z}{4}\right\}\right\} dz$$

$$I = 4e^{2z} \cdot \sin\left\{\frac{z}{4}\right\} - 8 \left[e^{2z} \cdot 4\left(-\cos\left\{\frac{z}{4}\right\}\right) + 8 \int \left[e^{2z} \cdot \cos\left\{\frac{z}{4}\right\}\right] dz\right.$$

$$I = 4e^{2z} \cdot \sin\left\{\frac{z}{4}\right\} + 32\, e^{2z} \cos\left\{\frac{z}{4}\right\} - 64\, I + C$$

$$65 I = 4e^{2z}\left[\sin\left\{\frac{z}{4}\right\} + 8 \cos\left\{\frac{z}{4}\right\}\right] + C$$

$$I = \frac{4e^{2z}}{65}\left\{\sin\left(\frac{z}{4}\right) + 8 \cos\left(\frac{z}{4}\right)\right\} + C$$

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f) $\displaystyle\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx$

Solution

$$\Rightarrow \int \frac{x^2+1}{(x^2-2x+1+1)^2}\,dx$$

$$\Rightarrow \int \frac{x^2+1}{((x-1)^2+1)^2}\,dx$$

let;

$$x-1 = \tan\theta \quad \cdots\cdots \tan = \frac{P}{B} = \frac{x-1}{1}$$

$$x = 1+\tan\theta$$

$$\therefore dx = \sec^2\theta\, d\theta \qquad H = \sqrt{(x-1)^2+1}$$

$$\Rightarrow \int \left[\frac{(1+\tan\theta)^2+1}{(\tan^2\theta+1)^2}\cdot\sec^2\theta\right] d\theta$$

$$\Rightarrow \int \left[\frac{1+2\tan\theta+\tan^2\theta+1}{\sec^{\cancel{4}}\theta}\cdot\cancel{\sec^2\theta}\right] d\theta$$

$$\Rightarrow \int \left[\frac{\tan^2\theta+2\tan\theta+2}{\sec^2\theta}\right] d\theta$$

$$\Rightarrow \int \tan^2\theta\cos^2\theta\, d\theta + 2\int \tan\theta\cos^2\theta\, d\theta + 2\int \cos^2\theta\, d\theta$$

$$\Rightarrow \int \sin^2\theta\, d\theta + \int 2\sin\theta\cos\theta\, d\theta + 2\int \cos^2\theta\, d\theta$$

$$\Rightarrow \int \left[\frac{1-\cos2\theta}{2}\right] d\theta + \int \sin2\theta\, d\theta + \cancel{2}\int \frac{1+\cos2\theta}{\cancel{2}}\, d\theta$$

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$$\Rightarrow \frac{1}{2}\int d\theta - \frac{1}{2}\int \cos 2\theta\, d\theta + \int \sin 2\theta\, d\theta + \int d\theta + \int \cos 2\theta\, d\theta$$

$$\Rightarrow \frac{3}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta\, d\theta + \int \sin 2\theta\, d\theta$$

$$\Rightarrow \frac{3}{2}\theta + \frac{1}{4}\cdot \sin 2\theta - \frac{1}{2}\cos 2\theta + C$$

$\because \theta = \tan^{-1}(x-1)$

$$\sin\theta = \frac{P}{H} = \frac{x-1}{\sqrt{(x-1)^2+1}}, \quad \cos\theta = \frac{B}{H} = \frac{1}{\sqrt{(x-1)^2+1}}$$

$\because \sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\cdot \frac{x-1}{\sqrt{(x-1)^2+1}}\cdot \frac{1}{\sqrt{(x-1)^2+1}} \Rightarrow \frac{2(x-1)}{(x-1)^2+1}$$

$\because \cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \frac{1}{(x-1)^2+1} - \frac{(x-1)^2}{(x-1)^2+1} \Rightarrow \frac{1-(x-1)^2}{(x-1)^2+1}$$

$$\Rightarrow \frac{3}{2}\cdot \tan^{-1}(x-1) + \frac{1}{2\cancel{4}}\cdot \frac{\cancel{2}(x-1)}{(x-1)^2+1} - \frac{1}{2}\cdot \frac{1-(x-1)^2}{(x-1)^2+1} + c$$

$$\Rightarrow \frac{3}{2}\tan^{-1}(x-1) + \frac{1}{2}\cdot \frac{x-1}{(x-1)^2+1} - \frac{1}{2}\cdot \frac{1-(x-1)^2}{(x-1)^2+1} + C$$

OR

$$\Rightarrow \frac{3}{2}\tan^{-1}(x-1) + \frac{x-1}{2(x^2-2x+2)} - \frac{1-(x-1)^2}{2(x^2-2x+2)} + C$$

$$\Rightarrow \frac{3}{2}\tan^{-1}(x-1) - \frac{x^2-3x+3}{2(x^2-2x+2)} + C$$

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Q8.

Solution

it is given that:

$$f'(x) = \frac{1}{2}x^2 + \frac{3}{4}x$$

& $f(1) = 2$

* for integral of $f'(x)$

$$\int f'(x)\,dx = \frac{1}{2}\int x^2\,dx + \frac{3}{4}\int x\,dx$$

$$f(x) = \frac{1}{2}\cdot\frac{x^3}{3} + \frac{3}{4}\cdot\frac{x^2}{2} + C$$

$$f(x) = \frac{1}{6}x^3 + \frac{3}{8}x^2 + C$$

* for 'c':

put $f(1)$ in $f(x)$:

$$f(1) = \frac{1}{6} + \frac{3}{8} + C$$

$$2 = \frac{13}{24} + C$$

$$\frac{35}{24} = C$$

$$\therefore f(x) = \frac{1}{6}x^3 + \frac{3}{8}x^2 + \frac{35}{24}$$

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Q9

a) $\int \sec^n x \, dx$

Derivation

let;

$$I = \int \sec^n x \, dx$$

$$\therefore I = \int \left[\sec^{n-2} x \cdot \sec^2 x\right] dx$$

$I =$ By parts:

$$I = \sec^{n-2} x \int \sec^2 x \, dx - \int \left[\left[(n-2)\sec^{n-3}(\sec x \tan x)\right] \int \sec^2 x \, dx\right] dx$$

$$I = \sec^{n-2} x \cdot \tan x - (n-2) \int \left[\sec^{n-2} \tan^2 x\right] dx$$

$$I = \sec^{n-2} x \cdot \tan x - (n-2) \int \left[\sec^{n-2} (\sec^2 x - 1)\right] dx$$

$$I = \sec^{n-2} x \cdot \tan x - (n-2) \int \left[\sec^n x - \sec^{n-2} x\right] dx$$

$$I = \sec^{n-2} x \cdot \tan x - (n-2) \int \left[\sec^n x\right] dx + (n-2) \int \sec^{n-2} x \, dx$$

$$I = \sec^{n-2} x \cdot \tan x - (n-2) \cdot I + (n-2) \int \sec^{n-2} x \, dx$$

$$(n-2)I + I = \sec^{n-2} x \cdot \tan x + (n-2) \int \sec^{n-2} x \, dx$$

$$I = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{(n-2)}{(n-1)} \int \sec^{n-2} x \, dx$$

derived!

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b) $\int \tan^n x \, dx$

Solution to derives

let;

$$I = \int \tan^n x \, dx$$

$$= \int [\tan^{n-2} x \ \tan^2 x] \, dx$$

$$= \int \tan^{n-2} x \,(\sec^2 x - 1)\, dx$$

$$= \int \tan^{n-2} x \sec^2 x \, dx - \int \tan^{n-2} x \, dx$$

$$\because \int (f(x))^n \cdot f'(x) = \frac{(f(x))^{n+1}}{n+1}$$

$$= \frac{\tan^{n-1}}{n-1} - \int \tan^{n-2} x \, dx$$

derived

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Q10

a) $\int \sec^4 x \, dx$

Solution

* from reduction formula (derived before)

$$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$\because n = 4$

$$\therefore \int \sec^4 x \, dx = \frac{1}{4-1} \cdot \sec^{4-2} x \tan x + \frac{4-2}{4-1} \int \sec^{4-2} x \, dx$$

$$= \frac{1}{3} \sec^2 x \tan x + \frac{2}{3} \int \sec^2 x \, dx$$

$$= \frac{1}{3} \sec^2 x \tan x + \frac{2}{3} \tan x + c$$

b) $\int \tan^4 x \, dx$

Solution

* from reduction formula (derived before)

$$\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$$

$\because n = 4$

$$\int \tan^4 x \, dx = \frac{1}{4-1} \tan^{4-1} x - \int \tan^{4-2} x \, dx$$

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$$= \frac{\tan^3 x}{3} - \int \{\sec^2 x - 1\} dx$$

$$= \frac{1}{3}\tan^3 x - \int \sec^2 x \, dx + \int dx$$

$$= \frac{1}{3}\tan^3 x - \tan x + x + C$$