

Calculus Assignment #2

Date: _____

Q1 = Find $\frac{dy}{dx}$.

$$(1) \quad 8 = 3x^2 + y^4$$

$$(3) \quad x^2 + 2y^5 = 10xy$$

Differentiating w.r.t x

$$0 = 6x + 4y^3 \frac{dy}{dx}$$

$$4y^3 \frac{dy}{dx} = -6x$$

$$\frac{dy}{dx} = \frac{-6x}{4y^3} \Rightarrow \frac{-3x}{2y^3}$$

$$(2) \quad \sin(2x-y) = 4x$$

Differentiating w.r.t x

$$\cos(2x-y) (\cancel{2x} - \frac{dy}{dx}) = 4$$

$$2 \cos(2x-y) - \frac{dy}{dx} \cos(2x-y) = 4$$

$$\frac{dy}{dx} \cos(2x-y) = 2 \cos(2x-y) - 4$$

$$\frac{dy}{dx} = \frac{2 \cos(2x-y) - 4}{\cos(2x-y)}$$

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$$(3) \quad x^2 + 2y^5 = 10xy$$

Differentiating wrt x

$$2x + 10y^4 \frac{dy}{dx} = 10 \left[x \frac{dy}{dx} + y(1) \right]$$

$$2x + 10y^4 \frac{dy}{dx} = 10x \frac{dy}{dx} + 10y$$

$$10y^4 \frac{dy}{dx} - 10x \frac{dy}{dx} = 10y - 2x$$

$$10 \frac{dy}{dx} (y^4 - x) = 10y - 2x$$

$$\frac{dy}{dx} = \frac{10y - 2x}{10(y^4 - x)}$$

(4) Implicit Differentiation to find $\frac{d^2y}{dx^2}$

$$7xy = 8$$

Differentiating wrt x

$$7 \left[x \frac{dy}{dx} + y(1) \right] = 0$$

$$7x \frac{dy}{dx} + 7y = 0$$

∴

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$$x \frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Again differentiating w.r.t x

$$\frac{d^2y}{dx^2} = - \left[x \left[+ \frac{dy}{dx} \right] - y(1) \right] \div x^2$$

$$\frac{d^2y}{dx^2} = x \frac{dy}{dx} - y \div x^2$$

Putting the value of $\frac{dy}{dx}$

$$\frac{d^2y}{dx^2} = -x \left(-\frac{y}{x} \right) + y \div x^2$$

$$\frac{d^2y}{dx^2} = \frac{y + y}{x^2} \quad *$$

$$\frac{d^2y}{dx^2} = \frac{2y}{x^2}$$

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$$(5) e^x + y^2 = 4$$

Differentiating w.r.t x :-

$$e^x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{e^x}{2y}$$

Again differentiating w.r.t x :-

$$\frac{d^2y}{dx^2} = -\left[\frac{2y e^x - e^x 2 \frac{dy}{dx}}{4y^2} \right]$$

$$\frac{d^2y}{dx^2} = -\frac{2ye^x + 2e^x \frac{dy}{dx}}{4y^2}$$

Putting value of $\frac{dy}{dx}$:-

$$\frac{d^2y}{dx^2} = -2ye^x + 2e^x \left(-\frac{e^x}{2y} \right)$$

$$\frac{d^2y}{dx^2} = -2ye^x + \frac{4y^2}{e^{2x}/y} = \frac{2ye^{2x} + e^{2x}}{4y^2}$$

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$$\frac{d^2y}{dx^2} = -2ye^x - e^{2x}$$
$$4y^3$$

$$Q6 = x^2 y + y^2 + 4 = 0 , \frac{dy}{dx} \Big|_{x=2}$$

For y at $x=2$

$$(2)^2 y + y^2 + 4 = 0$$

$$4y + y^2 + 4 = 0$$

$$(y)^2 + 2(y)(2) + (2)^2 = 0$$

$$(y+2)^2 = 0$$

$$\boxed{y = -2} \quad (x, y) = (2, -2)$$

Differentiating w.r.t x :-

$$\cancel{2xy} + x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx} (x^2 + 2y) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + 2y}$$

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Putting x & y value.

$$\frac{dy}{dx} = \frac{-2(2)(-2)}{(2)^2 + 2(-2)} \Rightarrow \frac{8}{4-4}$$

$$\frac{dy}{dx} = \frac{8}{0}$$

(E) Non-Existent

$$Q7) = \text{If } x^2 - y^2 = 5, \frac{d^2y}{dx^2} \Big|_{(3,2)} = ?$$

$$2x = 2y$$

Differentiating w.r.t. x

$$2x - 2y \frac{dy}{dx} = 5$$

$$2y \frac{dy}{dx} = 2x + 5$$

$$\frac{dy}{dx} = \frac{2x+5}{2y} \quad \text{--- (1)}$$

After putting values :-

~~$$\frac{dy}{dx} = \frac{2(3)+5}{2(2)} \rightarrow \frac{11}{4}$$~~

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Q7 = $x^2 - y^2 = 5$, $\frac{d^2y}{dx^2}$ at $x, y = (3, 2)$

$$2x - 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = +\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = y(1) + x \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = y + \frac{x^2}{y^2} \Rightarrow \frac{y^2 + x^2}{y^3}$$

$$\frac{d^2y}{dx^2} = \frac{(2)^2 - (3)^2}{(2)^3} = -\frac{5}{8}$$

$$(D) -\frac{5}{8}$$

Q8 =

$$\frac{dx}{dt} \Big|_{x=1} = -30 \text{ mil/h}$$

$$\frac{dy}{dt} \Big|_{y=0.4} = 15 \text{ mil/h}$$

Let D be the distance b/w them

$$D = \sqrt{x^2 + y^2}$$

$$D = \sqrt{1^2 + 0.4^2}$$

$$D = \sqrt{29}$$

$$D^2 = x^2 + y^2$$

$$2D \frac{dD}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\cancel{2} \left[\frac{\sqrt{29}}{5} \right] \frac{dD}{dt} = \cancel{2}(1) [30] + \cancel{2}(0.4)(15)$$

$$\frac{dD}{dt} = \frac{5}{\sqrt{29}} (-30 + 6)$$

$$\frac{dD}{dt} = \frac{-120}{\sqrt{29}} \Rightarrow \cancel{2} - 22.283 \text{ mil/h}$$

Q9 =

Let the Surface Area = S

$$\frac{dS}{dt} = 0.2$$

$$V = S^3$$

$$\frac{dV}{dt} = 3S^2 \frac{dS}{dt}$$

$$\frac{dV}{dt} = 3 \left(\frac{S}{6}\right) (0.2)$$

$$\frac{dV}{dt} = 0.1 S \text{ cm/s}$$

$$g = 6S^2$$

$$S^2 = \frac{5}{6}$$

(A) 0.1

Q10 = Inc / Dec, Concavity, Inflection
, Critical and relative extrema

$$(a) f(x) = x^5 - 5x^4$$

$$f'(x) = 5x^4 - 20x^3$$

$$f''(x) = 20x^3 - 60x^2$$

① C.P :-

$$f'(x) = 0 \quad \& \quad f'(x) = \text{undefined}$$

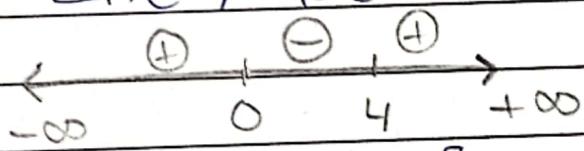
Not possible for this question

$$5x^4 - 20x^3 = 0$$

$$5x^3(x - 4) = 0$$

$$\boxed{x=0} \quad \& \quad \boxed{x=4}$$

② Inc / Dec :-



Inc: $(-\infty, 0] \cup [4, +\infty)$

Dec: $[0, 4]$

③ Inflection Points

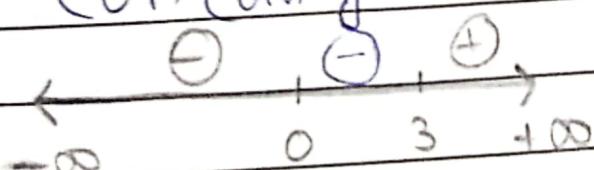
$$f''(x) = 20x^3 - 60x^2 = 0 \quad \& \quad f''(x) = \text{undefined}$$

Not Possible

$$20x^2(x - 3) = 0$$

$$\boxed{x=0} \quad \& \quad \boxed{x=3}$$

④ Concavity :-



Concave up: $(-\infty, 3)$

Concave down: $(3, +\infty)$

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(5) Relative Extrema :-

At $x=0$:-

$$f''(0) = 20(0)^3 - 60(0)^2$$

$$f''(0) = 0$$

we cannot find the sign of $f''(x)$
so, it is an inflection point

$$\text{inflection point} = f(0) = (0)^5 - 5(0)^4 \\ = 0$$

inflection(0, 0)

At $x = 4$:-

$$f''(4) = 20(4)^3 - 60(4)^2 \\ = 320 > 0 \text{ minima}$$

So $f''(x)$ has minimum value at $x=4$.

$$\text{minimum value} = f(4) = (4)^5 - 5(4)^4 \\ = -256$$

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(b)

$$\textcircled{2} \quad Q10: f(x) = x^{1/5}$$

$$f'(x) = x^{-4/5} \Rightarrow 1$$

$$f''(x) = \frac{5}{25} x^{-9/5}$$

① Critical Points

② Incl/Dec

$$f'(x) = 0$$

$$1 = 0$$

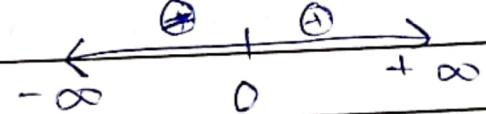
$$5x^{4/5}$$

$$1 = 0 \times$$

$$f'(x) = \text{undefined}$$

$$x^{4/5} = 0$$

$$\boxed{x = 0}$$



Inc: $[0, +\infty)$

Dec: $(-\infty, 0]$

③ Inflection Points:

$$f''(x) = \frac{-4x^{-9/5}}{25} = 0$$

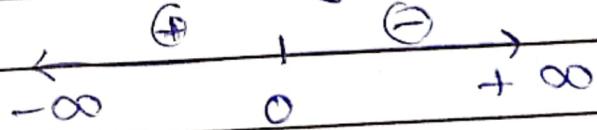
$$\frac{-4}{25x^{9/5}} = 0$$

$$-4 = 0 \times$$

$$x^{9/5} = 0$$

$$\boxed{x = 0}$$

④ Concavity



Concave Up: $(-\infty, 0]$

Concave Down: $(0, +\infty)$

✓

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⑤ Relative Extrema:-

At $x = 0$:-

$$f''(0) = -4 \quad \#'$$

$$f''(0) < 0$$

D.N.E

C) $f(x) = (x+1)(x-1)^2$

$$f'(x) = (x+1)2(x-1)(1) + (x-1)^2(1)$$

$$f'(x) = (x-1)[2x+2 + x-1]$$

$$f'(x) = (x-1)(3x+1)$$

$$f''(x) = (x-1)(3) + (3x+1)(1)$$

$$f''(x) = 3x-3 + 3x+1$$

$$f''(x) = 6x - 2$$

① Critical Point:-

$$f'(x) = 0$$

$$\& f'(x) = \text{undefined}$$

$$(x-1)(3x+1) = 0$$

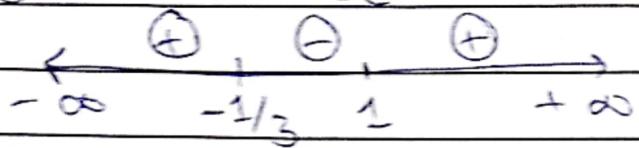
D.N.E

$$x = 1$$

$$x = -\frac{1}{3}$$

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② Inc / Dec :-



Inc: $(-\infty, -\frac{1}{3}) \cup (1, +\infty)$

Dec: $(-\frac{1}{3}, 1)$

③ Inflection Points:-

$$f''(x) = 0$$

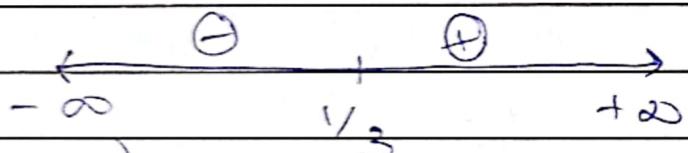
$$6x - 2 = 0$$

$$\begin{cases} x = 1 \\ 3 \end{cases}$$

\notin $f''(x)$ = undefined

D.N.E

④ Concavity:-



Concave Up: $(\frac{1}{3}, +\infty)$

Concave down: $(-\infty, \frac{1}{3})$

⑤ Relative Extrema:-

At $x = 1$:-

$$\begin{aligned} f''(1) &= 6(1) - 2 \\ &= 4 > 0 \text{ Minima} \end{aligned}$$

At $x = \underline{\underline{-\frac{1}{3}}}$:-

$$\begin{aligned} f''(-\frac{1}{3}) &= 6(-\frac{1}{3}) - 2 \\ &= -2 - 2 = -4 \\ &= -4 < 0 \text{ Maxima} \end{aligned}$$

Minimum Value = $f(1)$

$$\begin{aligned} f(1) &= (1+1)(1-1)^2 \\ &= 0 \end{aligned}$$

Max Value = $f(-\frac{1}{3})$:-

$$\begin{aligned} f(-\frac{1}{3}) &= (-\frac{1}{3}+1)(-\frac{1}{3}-1)^2 \\ &= \frac{32}{27} \end{aligned}$$

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$$(d) f(x) = \frac{x^2 - 3}{x - 2}$$

$$\begin{aligned}f'(x) &= \frac{(x-2)(2x) - (x^2 - 3)}{(x-2)^2} \\&= \frac{2x^2 - 4x - x^2 + 3}{(x-2)^2}\end{aligned}$$

$$f'(x) = \frac{x^2 - 4x + 3}{(x-2)^2} \Rightarrow \frac{(x-1)(x-3)}{(x-2)^2}$$

$$\begin{aligned}f''(x) &= \frac{(x-2)^2(2x-4) - (x^2 - 4x + 3)^2}{(x-2)^4} \\&= \frac{(x-2)\{(x-2)(2x-4) - 2(x^2 - 4x + 3)\}}{(x-2)^3} \\&= \frac{(x-2)(2x-4) - 2x^2 + 8x - 6}{(x-2)^3} \\&= \frac{2x^2 - 4x - 4x + 8 + 2x^2 + 8x - 6}{(x-2)^3}\end{aligned}$$

$$f''(x) = \frac{2}{(x-2)^3}$$

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① Critical Point :-

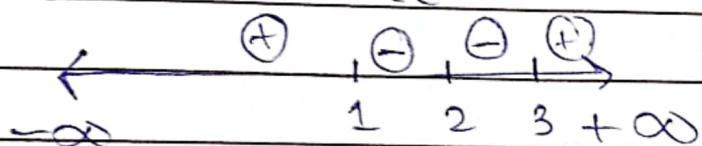
$$(x-1)(x-3) = 0$$

$$\boxed{x=1}, \boxed{x=3}$$

$$; (x-2)^2 = 0$$

$$\boxed{x=2}$$

② Inc / Dec :-



$$\text{Inc: } (-\infty, 1] \cup [3, +\infty)$$

$$\text{Dec: } [1, 2] \cup [2, 3]$$

$$\text{or } (1, 3)$$

③ Inflection Points :-

$$f''(x) = 0$$

$$2 = 0$$

$$(x-2)^3$$

$$2 = 0 \quad \times$$

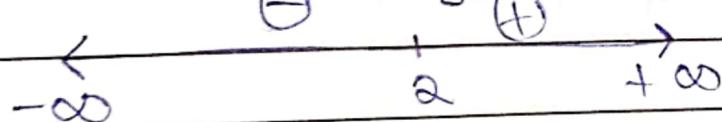
$$\& f''(x) = \text{undefined}$$

$$(x-2)^3 = 0$$

$$x-2 = 0$$

$$\boxed{x=2}$$

④ Concavity :-



$$\text{Concave up: } (-\infty, 2]$$

$$\text{Concave down: } [2, +\infty)$$

⑤ Relative Extrema

$$\text{At } x = 1$$

$$f''(1) = -2 < 0$$

Maxima

~~Minimum~~

$$\text{Maximum Value} = f(-2) = -1$$

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At $x = 3$:-

$$f''(3) = 2 > 0 \text{ Minimum}$$

$$\text{Minimum value} = f(3) = 6$$

At $x = 2$:-

$$f''(2) = \text{D.N.E}$$

$$(e) f(x) = x \sqrt{8-x^2}$$

$$f'(x) = \frac{1}{2\sqrt{8-x^2}} (-2x)(x) + \sqrt{8-x^2}$$

$$f'(x) = -\frac{x^2}{\sqrt{8-x^2}} + \sqrt{8-x^2}$$

$$f'(x) = \frac{x^2 + 8 - x^2}{\sqrt{8-x^2}} \Rightarrow \frac{8-2x^2}{\sqrt{8-x^2}}$$

$$f''(x) = \frac{8-2x^2(-4x) - (8-2x^2)}{(8-x^2)x} \left(\frac{1}{2\sqrt{8-x^2}} \right)$$

$$f''(x) = \frac{-4x(8-x^2) + x(8-2x^2)}{(8-x^2)^{3/2}}$$

$$= -24x + 2x^3$$

$$(8-x^2)^{3/2}$$

$$= \frac{2x(-12+x^2)}{(8-x^2)^{3/2}}$$

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① Critical Points:-

$$8 - 2x^2 = 0$$

$$\sqrt{8-x^2}$$

$$8 - 2x^2 = 0$$

$$x^2 = 4$$

$$x = 2, x = -2$$

$$\sqrt{8-x^2} = 0$$

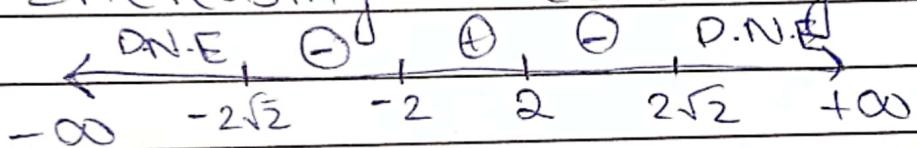
$$8 - x^2 = 0$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

$$x = 2\sqrt{2}, -2\sqrt{2}$$

② Increasing / Decreasing :-



$$\text{Inc: } [-2, 2]$$

$$\text{Dec: } [-2\sqrt{2}, -2] \cup [2, 2\sqrt{2}]$$

③ Inflection Points :-

$$f''(x) = 0$$

$$f''(x) = \text{undefined}$$

$$2x(-12+x^2) = 0$$

$$(8-x^2)^{3/2} = 0$$

$$(8-x^2)^{3/2}$$

$$8 - x^2 = 0$$

$$2x = 0$$

$$x^2 = 12$$

$$x^2 = 8$$

$$x = 0$$

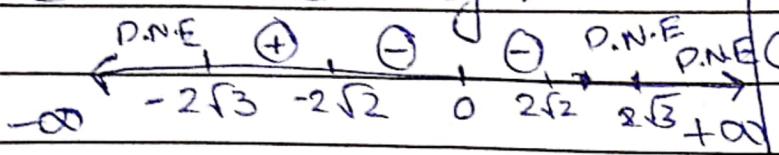
$$x = \pm 2\sqrt{3}$$

$$x = \pm 2\sqrt{2}$$

$$x = 2\sqrt{3}, -2\sqrt{3}$$

$$x = 2\sqrt{2}, -2\sqrt{2}$$

(4) Concavity :-



Concave up: $[-2\sqrt{3}, -2\sqrt{2}]$

Concave down: $[2\sqrt{2}, 0] \cup [0, 2\sqrt{2}]$
or $(-2\sqrt{2}, 2\sqrt{2})$

(5) Relative Extrema :-

At $x = 2$:-

$$f''(2) = -4 < 0$$

Relative Maxima

At $x = -2$:-

$$f''(-2) = 4 > 0$$

Relative minima

Maximum Value :-

$$f(2) = 4$$

Minimum Value :-

$$f(-2) = -4$$

$$(f) f(x) = \frac{12}{x^2 + 12}$$

$$f'(x) = \frac{(x^2 + 12)(0) - 12(2x)}{(x^2 + 12)^2} \quad f''(x) = \frac{(x^2 + 12)^2(-24)}{-24x(x^2 + 12)(2x)}$$

$$f'(x) = \frac{-24x}{(x^2 + 12)^2} \quad f''(x) = \frac{(x^2 + 12)[-24(x^2 + 12) + 96x^2]}{(x^2 + 12)^4}$$

$$f''(x) = \frac{-24[x^2 + 12 - 4x^2]}{(x^2 + 12)^3}$$

$$f''(x) = \frac{-24(-3x^2 + 12)}{(x^2 + 12)^3}$$

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① Critical Point :-

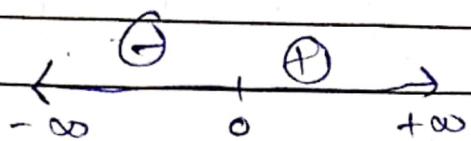
$$-24x = 0 \quad (x^2 + 12) \geq 0$$

$$(x^2 + 12)^2 \quad x^2 + 12 = 0$$

$$-24x = 0 \quad x^2 = -12$$

$x = 0$ Complex Roots

② Inc / Dec :-



Inc: $(-\infty, 0)$

Dec: $(0, +\infty)$

③ Inflection Point :-

$$-24(-3x^2 + 12) = 0 \quad (x^2 + 12)^{\frac{3}{2}} \geq 0$$

$$(x^2 + 12)^3 \quad x^2 + 12 = 0$$

$$-3x^2 + 12 = 0$$

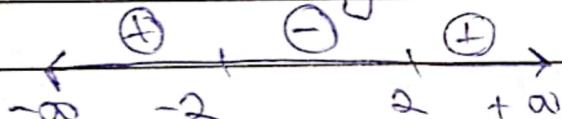
$$x^2 = 4$$

$$x = 2, -2$$

$$x = \pm\sqrt{12}$$

Complex roots

④ Concavity :-



Concave up: $(-\infty, 2) \cup (2, +\infty)$

Concave down: $(-2, 2)$

⑤ Relative Extrema :-

At $x = 0$:-

$$f''(0) = -24 \frac{(-3(0)^2 + 12)}{(0^2 + 12)^3} \Rightarrow -1 < 0 \text{ Maxima}$$

$$f(0) = \frac{12}{(0)^2 + 12} \Rightarrow 1 \Rightarrow \text{Maximum Value}$$

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$$(9) = f(x) = x^2 \ln x$$

$$f'(x) = 2x \ln x + x$$

$$f''(x) = 2x + 2\ln x + 1 \Rightarrow 2\ln x + 3$$

① Critical Points:-

$$f'(x) = 0$$

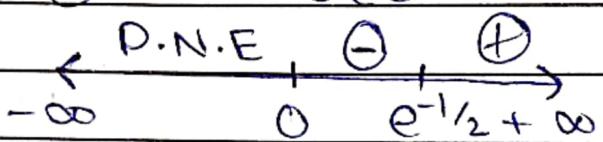
$$2x \ln x + x = 0$$

$$\frac{d}{dx}(2\ln x + 1) = 0$$

$$x = 0 \quad \ln x = -\frac{1}{2}$$

$$x = e^{-1/2}$$

② Inc / Dec:-



Inc : $(e^{-1/2}, +\infty)$

Dec : $(0, e^{-1/2})$

③ Inflection Points

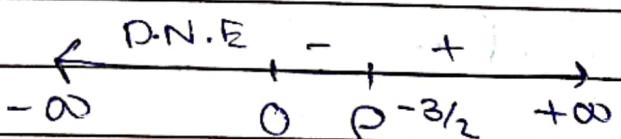
④ Concavity

$$f''(x) = 0$$

$$2\ln x + 3 = 0$$

$$\ln x = -\frac{3}{2}$$

$$x = e^{-\frac{3}{2}}$$



Concave Up: $(e^{-3/2}, +\infty)$

Concave down: $(0, e^{-3/2})$

and line is undefined

at $x=0$ so $x=0$ is also inflection point

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⑤ Relative Extrema :-

At $x=0$:-

$$f''(0) = 2\ln(0) + 3$$

At $x=e^{-1/2}$:-

$$\begin{aligned} f''(e^{-1/2}) &= 2\ln(e^{-1/2}) + 3 \\ f''(e^{-1/2}) &= 2 > 0 \end{aligned}$$

Minima

Minimum Value :-

$$f(e^{-1/2}) = (e^{-1/2})^2 \ln(e^{-1/2})$$

$$f(e^{-1/2}) = -0.183$$

and find f_{\min}

(h) $\cos x - 9x = f(x); [0, 4\pi]$

$$f'(x) = -\sin x - 9$$

$$f''(x) = -\cos x$$

① Critical Points :-

$$f'(x) = 0 \quad \& \quad f'(x) = \text{undefined}$$

$$-\sin x - 9 = 0 \quad \text{D.N.E}$$

$$\sin x = -9$$

$$x \in \emptyset$$

② Increasing / Decreasing:-

$$\leftarrow \rightarrow_{+\infty}$$

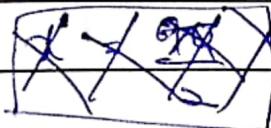
Inc: \emptyset or {} Dec: ~~[0, 4\pi]~~ $[0, 4\pi]$

③ Inflection Points :-

$$f''(x) = 0 \quad \& \quad f''(x) = \text{undefined}$$

$$-\cos x = 0$$

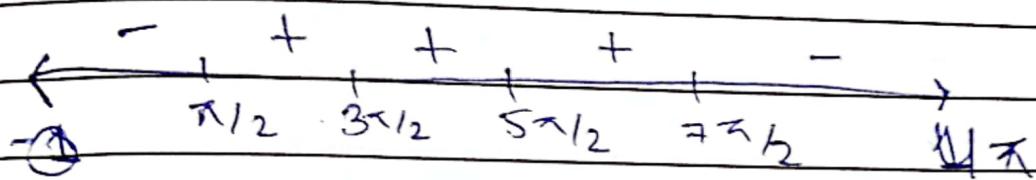
$$\cos x = 0$$



$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2},$$

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(iv) Concavity :-



Concave Up: $(\pi/2, 7\pi/2)$

Concave Down: $(0, \pi/2) \cup (7\pi/2, 4\pi)$

(v) Relative Extrema:-

Since, No critical points exist

So, Relative Extrema does not exists.

$$(i) f(x) = \ln \sqrt{x^2+4}$$

$$f'(x) = \frac{1}{\sqrt{x^2+4}} \left(\frac{1}{x^2+4} \right) (2x)$$

$$\therefore = \frac{2x}{x^2+4}$$

$$\begin{aligned} f''(x) &= \frac{(x^2+4)(1) - x(2x)}{(x^2+4)^2} \\ &= \frac{(x^2+4-2x^2)}{(x^2+4)^2} \\ &= \frac{4-x^2}{(x^2+4)^2} \end{aligned}$$

(i) Critical Points :-

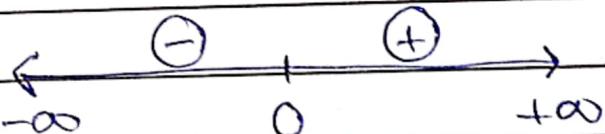
$$x = 0, x^2+4=0$$

$$x = 0$$

$$x^2 = -4$$

complex

(ii) Inc / Dec :-



inc: $(0, +\infty)$

dec: $(-\infty, 0)$

(iii) Inflection Point:

$$4-x^2=0$$

$$x = \pm 2$$



$$x^2+4=0$$

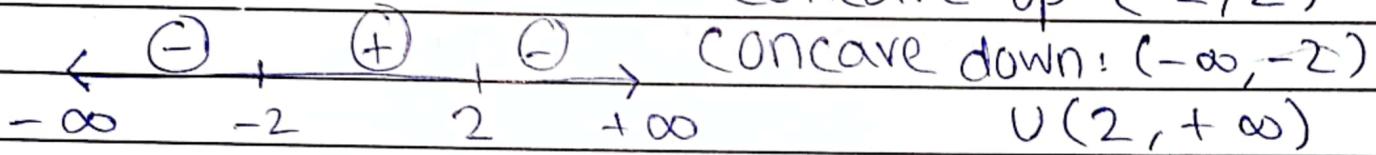
complex roots

Date: _____

(i) $\ln \sqrt{x^2 + 4} = f(x)$

$$f'(x) = \frac{1}{\sqrt{x^2 + 4}} \left(\frac{1}{2\sqrt{x^2 + 4}} \right) (2x)$$

(4) Concavity:-



(5) Relative Extrema:-

At $x = 0$:-

$$f''(0) = \frac{4 - (0)^2}{[(0)^2 + 4]^2} \Rightarrow \frac{1}{4} > 0 \text{ Minima}$$

$$\begin{aligned}\text{Minimum Value} &= f(0) = \ln(0^2 + 4) \\ &= \ln(4) \\ &= 1.386\end{aligned}$$

Date: _____

(j) $f(x) = xe^{-7x}$

$$f'(x) = e^{-7x}(1) + xe^{-7x}(-7)$$
$$= e^{-7x}(1 - 7x)$$

$$f''(x) :-$$

$$= e^{-7x}(-7) + (1 - 7x)e^{-7x}(-7)$$
$$= e^{-7x}(-7 + 49x - 7)$$
$$= e^{-7x}(49x - 14)$$

① Critical Points:-

$$e^{-7x}(1 - 7x) = 0$$

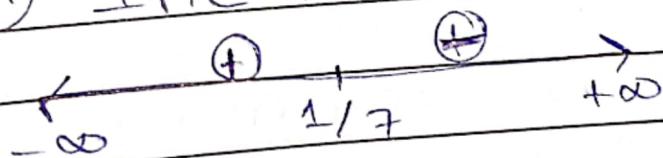
$$e^{-7x} = 0$$

D.N.E

$$1 - 7x = 0$$

$$x = \frac{1}{7}$$

② Inc / Dec :-



Inc: $(-\infty, 1/7)$

Dec: $(1/7, +\infty)$

③ Inflection Points:-

$$e^{-7x}(49x - 14) = 0$$

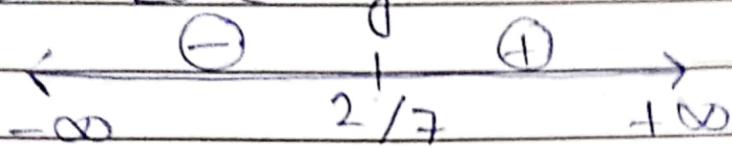
$$e^{-7x} = 0$$

D.N.E

$$49x - 14 = 0$$

$$x = \frac{2}{7}$$

(iv) Concavity:-

Concave Up: $(2/7, +\infty)$ Concave Down: $(-\infty, 2/7)$.

(v) Relative Extrema:-

At $x = \frac{1}{7}$,

$$\begin{aligned}f''\left(\frac{1}{7}\right) &= e^{-7\left(\frac{1}{7}\right)} \left[49\left(\frac{1}{7}\right) - 14 \right] \\&= e^{-1}(-7) \\&= -\frac{7}{e} < 0 \text{ Maxima}\end{aligned}$$

$$\begin{aligned}\text{Maximum Value} &= f\left(\frac{1}{7}\right) e^{-7\left(\frac{1}{7}\right)} \\&= \left(\frac{1}{7}\right) e^{-1} \\&= \frac{1}{7e}\end{aligned}$$