chapter #09.

Ex 9.1

Q write out first five terms and check series converges or diverges.

(Q7) $\left\{\frac{n}{n+2}\right\}_{n=1}^{+\infty}$

$\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}$

series is converging.

so, finding limit.

$\lim_{n\to\infty} \frac{n}{n+2} \cdot \left(\frac{\infty}{\infty}\right)$

$\lim_{n\to\infty} \frac{n(1)}{n(1+\frac{2}{n})}$

$= \boxed{1}$.

(Q8) $\left\{\frac{n^2}{2n+1}\right\}_{n=1}^{+\infty}$

$\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11}$

series converge so.

$\lim_{n\to\infty} \frac{n^2}{2n+1} \cdot \left(\frac{\infty}{\infty}\right)$

$\frac{2n}{2} = \boxed{\infty}$

(Q9) $\{2\}_{n=1}^{+\infty}$

2 converges

$\lim_{n\to\infty} 2 = \boxed{2}$

(Q10) $\left\{\ln\left(\frac{1}{n}\right)\right\}_{n=1}^{+\infty}$

$\ln 1, \ln\frac{1}{2}, \ln\frac{1}{3}, \ln\frac{1}{4}, \ln\frac{1}{5}$

$0, -0.69, -1.09, -1.38, -1.60$

diverges $\lim_{n\to\infty} \ln\left(\frac{1}{n}\right)$

$= \boxed{-\infty}$

(Q11) $\left\{\frac{\ln n}{n}\right\}_{n=1}^{+\infty}$

$\ln 1, \frac{\ln 2}{2}, \frac{\ln 3}{3}, \frac{\ln 4}{4}, \frac{\ln 5}{5}$

converges

$\lim_{n\to\infty} \frac{\ln n}{n}$

apply L-hopital

$\frac{\frac{1}{n}}{1} = \boxed{0}$

$\frac{d}{dx} a^x = a^x \ln(a)$

Q12) $\left\{ n \sin\frac{\pi}{n} \right\}_{n=1}^{+\infty}$

$\sin\pi$, $2\sin\pi/2$, $3\sin\pi/3$, $4\sin\pi/4$, $5\sin\pi/5$.

converges.

$\lim_{n\to\infty} n\sin\pi/n$

$\frac{\sin\pi/n}{1/n}$

applying L-hopitol

$\frac{\cos\pi/n \; (-\pi/\cancel{n^2})}{-1/\cancel{n^2}}$

$\lim_{n\to\infty} \pi \left(\cos\frac{\pi}{n}\right)$

applying limits.

$= \boxed{\pi}$

Q13) $\left\{ 1+(-1)^n \right\}_{n=1}^{+\infty}$

0, 2, 0, 2, 0

diverges.

Q14) $\left\{ \frac{(-1)^{n+1}}{n^2} \right\}_{n=1}^{+\infty}$

$1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}$

$= \boxed{\text{diverges}}$

$\lim_{n\to\infty} \frac{(-1)^{n+1}}{n^2}$

applying limit

$= \boxed{0}$

Q15) $\left\{ (-1)^n \frac{2n^3}{n^3+1} \right\}_{n=1}^{+\infty}$

$\frac{-2}{2}, \frac{16}{9}, \frac{-54}{28}, \frac{128}{65}, \frac{-250}{126}$

$= \boxed{\text{diverges}}$

Q16) $\left\{ \frac{n}{2^n} \right\}_{n=1}^{+\infty}$

$\frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}, \frac{5}{32}$

converges

$\lim_{n\to\infty} \frac{n}{2^n} \quad = \left(\frac{\infty}{\infty}\right)$

applying L-hopitol

$\lim_{n\to\infty} \frac{1}{2^n \ln(2)}$

applying limit

$\boxed{= 0}$

(Q17) $\left\{\dfrac{(n+1)(n+2)}{2n^2}\right\}_{n=1}^{+\infty}$

$\dfrac{6}{2}, \dfrac{12}{8}, \dfrac{20}{18}, \dfrac{30}{32}, \dfrac{42}{50}$

converging.

$\lim\limits_{n\to\infty} \dfrac{(n+1)(n+2)}{2n^2}$.

$\lim\limits_{n\to\infty} \dfrac{\cancel{n^2}(1+1/n)(1+2/n)}{2\cancel{n^2}}$

$= \boxed{\dfrac{1}{2}}$

(Q18) $\left\{\dfrac{\pi^n}{4^n}\right\}_{n=1}^{+\infty}$

$\dfrac{\pi}{4}, \dfrac{\pi^2}{16}, \dfrac{\pi^3}{64}, \dfrac{\pi^4}{256}, \dfrac{\pi^5}{1028}$

$\lim\limits_{n\to\infty} \dfrac{\pi^n}{4^n} = \left(\dfrac{\pi}{4}\right)^n$ $\boxed{\text{as } \dfrac{\pi}{4} < 1}$

$\cancel{\dfrac{\pi^n \ln \pi}{4^n \ln 4}}$ $\cancel{\log y = n \ln(\pi/4)}$

~~logy = taking~~

so

$\left(\dfrac{\pi}{4}\right)^{\infty} = \boxed{0}$

(Q19) $\left\{n^2 e^{-n}\right\}_{-1}^{+\infty}$

$1/e, 4/e^2, 9/e^3, 16/e^4, 25/e^5$

converges

$\lim\limits_{n\to\infty} n^2 e^{-n}$

$\lim\limits_{n\to\infty} \dfrac{n^2}{e^n}$

$\lim\limits_{n\to\infty} \dfrac{2n}{e^n} \Rightarrow \dfrac{2}{e^n}$

$= \boxed{0}$

Exercise 9.5

Q5 - 10) use limit comparision test to determine whether the series converges.

5) $\sum_{k=1}^{\infty} \frac{4k^2-2k+6}{8k^7+k-8}$

$a_n = \frac{4k^2-2k+6}{8k^7+k-8}$

$b_n = \frac{\cancel{1}4k^{\cancel{2}}}{\cancel{2}8k^{\cancel{7}5}} = \frac{1}{2k^5}$

~~$a_n > b_n$~~

$\cancel{\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\left(\frac{4k^2-2k+6}{8k^7+k-8}\right)}{\left(\frac{4k^2}{8k^7}\right)}}$

$\cancel{= \frac{32k^9-16k^8+48k^7}{32k^9+8k^2-32k^2}}$

~~$= 1$~~

$\lim_{k\to\infty} \frac{a_n}{b_n} = \frac{\left(\frac{4k^2-2k+6}{8k^7+k-8}\right)}{\left(\frac{1}{2k^5}\right)}$

$= \frac{8k^7-4k+6}{8k^7+k-8}$

$= 1$ limit exist

Since limit exists and $\sum_{n=1}^{\infty} \frac{1}{2k^5}$ converges so the original also converges.

6) $\sum_{k=1}^{\infty} \frac{1}{9k+6}$

$a_n = \frac{1}{9k+6}$

$b_n = \frac{1}{9k}$

$\lim_{k\to\infty} \frac{a_n}{b_n} = \frac{\left(\frac{1}{9k+6}\right)}{\frac{1}{9k}}$

$= \boxed{1}$ limit exists.

Since limit exists and it is diverging so the original also diverges.

7) $\sum_{k=1}^{\infty} \frac{5}{3^k+1}$

$a_n = \frac{5}{3^k+1}$

$b_n = (1/3)^k$. By comparision with geometric $r = 1/3$ when $r<1$ converges

$\lim_{k\to\infty} \frac{5 \cdot 3^k}{3^k+1}$

when k is so large so 1 ands does nt means so

$\lim_{k\to\infty} \left[\frac{3^k}{3^k}\right]$

$= 1$ limit exists

so the original also converges

8) $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$

$a_n = \frac{k(k+3)}{(k+1)(k+2)(k+5)}$

$b_n = \frac{k}{k^3} = \frac{1}{k^2}$

$\lim_{k\to\infty} \frac{a_n}{b_n} = \frac{\left(\frac{k(k+3)}{(k+1)(k+2)(k+5)}\right)}{\left(\frac{1}{k^2}\right)}$

$= \frac{k^3(k+3)}{(k+1)(k+2)(k+5)}$

$= \boxed{1}$

since limit exists and is converging so original is also converging.

9) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{8k^2-3k}}$

$a_n = \frac{1}{\sqrt[3]{8k^2-3k}}$

$b_n = \frac{1}{\sqrt[3]{8k^2}} = \frac{1}{2k^{2/3}}$

$\lim_{k\to\infty} \frac{a_n}{b_n} = \frac{\frac{1}{\sqrt[3]{8k^2-3k}}}{\frac{1}{2k^{2/3}}}$

$= \frac{2k^{2/3}}{\sqrt[3]{8k^2-3k}}$

$\boxed{= 1}$ $\quad p = \frac{2}{3} < 1$ diverges

Since $\sum_{k=1}^{\infty} \frac{1}{2k^{2/3}}$ diverges so the original also diverges.

10) $\sum_{k=1}^{\infty} \frac{1}{(2k+3)^{17}}$

$a_n = \frac{1}{(2k+3)^{17}}$

$b_n = \frac{1}{k^{17}}$

$\lim_{k\to\infty} \frac{a_n}{b_n} = \frac{k^{17}}{(2k+3)^{17}}$

$= \frac{1}{2^{17}}$ since $p = 17 > 1$

Since $\sum_{k=1}^{\infty} \frac{1}{k^{17}}$ converges so the original also converges.

Ratio Test (for factorials and n^{th} power)

11) $\sum_{k=1}^{\infty} \frac{3^k}{k!}$

$a_{k+1} = \frac{3^{k+1}}{(k+1)!}$

$a_k = \frac{3^k}{k!}$

$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{3^{k+1}}{(k+1)!} \cdot \frac{k!}{3k}$

$\lim_{k\to\infty} \frac{\cancel{3k} \cdot 3 \cdot \cancel{k!}}{\cancel{3k}(k+1)\cancel{k!}}$

$\lim_{k\to\infty} \frac{3}{k+1}$

applying limit.

$= \boxed{0} < 1$ converges

12) $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$.

$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{4^{k+1}}{(k+1)^2} \cdot \frac{k^2}{4^k}$

$= \frac{\cancel{4k} \cdot 4 \cdot k^2}{(k^2+1)^2 \cancel{4k}}$

$= \frac{4k^2}{(k+1)^2}$

$= \boxed{4} > 1$ diverges.

13) $\sum_{k=1}^{\infty} \frac{1}{5k}$.

$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{1}{5(k+1)} \cdot 5k$.

$= \boxed{1}$ might converge or inconclusive

14) $\sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k$.

$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = (k+1)\left(\frac{1}{2}\right)^{k+1} \cdot \frac{1}{(k)\left(\frac{1}{2}\right)^k}$

$\lim_{k\to\infty} \left(\frac{k+1}{k}\right) \frac{1}{2}$.

$= \lim_{k\to\infty} \frac{k+1}{2k}$

$= \boxed{\frac{1}{2}} < 1$ (converges)

15) $\sum_{k=1}^{\infty} \frac{k!}{k^3}$

$\lim_{k\to\infty} \frac{(k+1)!}{(k+1)^3} \cdot \frac{k^3}{k!}$

$= \frac{k^3(k+1)\cancel{k!}}{(k+1)^3 \cancel{k!}}$

applying limit

$= \boxed{\infty}$ converges

16) $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$

$\lim_{k\to\infty} \frac{k+1}{(k+1)^2+1} \cdot \frac{k^2+1}{k}$

$\lim_{k\to\infty} \frac{k^3+k+k^2+1}{k(k^2+2k+2)}$

$\lim_{k\to\infty} \frac{k^3+k^2+k+1}{k^3+2k^2+2k}$

$= \boxed{1}$ inconclusive or might be converging.

Root Test

$P = \lim_{n\to\infty} (a_n)^{1/k}$.

17) $\sum_{n=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$.

$\lim_{k\to\infty} (a_n)^{1/k} = \lim_{k\to\infty} \left(\frac{3k+2}{2k-1}\right)^{k \cdot 1/k}$

$= \frac{3}{2} > 1$. diverges.

18) $\sum_{n=1}^{\infty} \left(\frac{k}{100}\right)^k$.

$\lim_{k=\infty} (a_n)^{1/k} = \lim_{k\to\infty} \left(\frac{k}{100}\right)^{k \cdot 1/k}$

$= \infty$ diverges

19) $\sum_{k=1}^{\infty} \frac{k}{5^k}$.

$\lim_{k\to\infty} (a_n)^{1/k} = \lim_{k\to\infty} \left(\frac{k}{5^k}\right)^{1/k}$.

$= \frac{k^{1/k}}{5}$

applying power limit.

$\frac{1}{5} < 1$ converges.

20) $\sum_{k=1}^{\infty} \left(1 - e^{-k}\right)^{k}$

$\lim_{k\to\infty} (a_n)^{1/k} = \left(1-e^{-k}\right)^{k \cdot 1/k}$

$= \lim_{k\to\infty} 1-e^{-k}$

$= \lim_{k\to\infty} 1 - \frac{1}{e^{k}}$

$= 1 - 0.$

$= \boxed{1}$ inconclusive.

Ex # 9.3

Date:

Q3) $\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1}$

since it makes the structure of geometric series

$\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1}$ where

$r < 1$ so series converges.

for sum,

$a = 1$, $r = -\frac{3}{4}$.

$S_n = \frac{a}{1-r} = \frac{1}{1+3/4}$

$\boxed{S_n = +\frac{4}{7}}$

Q4) $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2}$.

$\frac{8}{27}, \frac{16}{81}, \frac{32}{243}, \ldots$

$\frac{8}{27}\left[1, \frac{2}{3}, \frac{4}{9} \ldots\right]$

Since it is geometric

$a = \frac{8}{27}$ and $r = \frac{2}{3}$

~~So~~ ~~Sn~~ $r < 1$ (converges)

$S_n = \frac{a}{1+r} = \frac{8}{27} \div \left(1 + \frac{2}{3}\right)$

$= \frac{8}{27} \cdot \frac{3}{5}$

$\boxed{S_n = 8/45}$

Q5) $\sum_{k=1}^{\infty}(-1)^{k-1}\left(\frac{7}{6^{k-1}}\right)$

$\sum_{k=1}^{\infty} 7\left(-\frac{1}{6}\right)^{k-1}$

since its geometric.

$a = 7$ and $|r| = |-1/6| < 1$

so converges

$S_n = \frac{a}{1-r}$.

$= \frac{7}{1-(-1/6)} = \frac{7 \cdot 6}{7}$

$\boxed{S_n = 6}$

Q6) $\sum_{k=1}^{\infty}\left(-\frac{3}{2}\right)^{k+1}$

$\frac{9}{4}, \frac{-27}{8}, \frac{81}{16}, \frac{-243}{32} \ldots$

since we dont know the last term so the sequence diverges. as 1, - are repeating.

OR.

$a = \frac{9}{2}$ and $r = \frac{-3}{2} < 1$.

$|r| \geq 1$

so diverges.

Date:

Q7) $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$

$\cancel{\frac{1}{3\cdot 4}} + \cancel{\frac{1}{4\cdot 5}} + \cancel{\frac{1}{5\cdot 6}}$

By partial fraction

$\frac{A}{(k+2)} + \frac{B}{(k+3)} = \frac{1}{(k+2)(k+3)}$

$A(k+3) + B(k+2) = 1$

For A $k=-2$

$\boxed{A=1}$

For B $k=-3$

$\boxed{B=-1}$

So

$\frac{1}{(k+2)(k+3)} = \frac{1}{(k+2)} - \frac{1}{(k+3)}$

$\left(\frac{1}{3} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) \ldots\ldots$

$\left(\cancel{\frac{1}{k-2}} - \cancel{\frac{1}{k-1}}\right) + \left(\cancel{\frac{1}{k-1}} - \frac{1}{k}\right)$

$S_n = \lim_{k\to\infty} \frac{1}{3} - \frac{1}{k}.$

$\lim_{k\to\infty} S_n = \boxed{\frac{1}{3}}$ $\quad 1/3 < 1$ conveyes

since series conveyes sum is also $1/3$.

Q8) $\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right)$

$\left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{8}}\right) \ldots$

$\left(\cancel{\frac{1}{2^{k-2}}} - \cancel{\frac{1}{2^{k-1}}}\right) + \left(\cancel{\frac{1}{2^{k-1}}} - \frac{1}{2^{k+1}}\right)$

$\lim_{k\to\infty} S_n = \lim_{k\to\infty} \frac{1}{2} - \frac{1}{2^k}$

$= \boxed{\frac{1}{2}} < 1$ (conveyes)

since the $\lim_{k\to\infty} S_n$ conveyes so the sum is also $1/2$.

Q9) $\sum_{k=1}^{\infty} \frac{1}{9k^2+3k-2}$

$\frac{1}{9k^2-3k+6k-2}$

$\frac{1}{3k(3k-1)+2(3k-1)}$

$\frac{1}{(3k+2)(3k-1)}$

By partial

$\frac{1}{(3k+2)(3k-1)} = \frac{A}{(3k+2)} + \frac{B}{(3k-1)}$

$1 = A(3k-1) + B(3k+2)$

For B, $k = 1/3$

$\boxed{B = 1/3}$

For A, $k = -2/3$

$\boxed{A = -1/3}$

Teacher's Sign.

So,

$\frac{-1}{3(3k+2)}$

$\frac{1}{3}\left[\frac{1}{3k-1}\right.$

$\frac{1}{3}\left(\left(\frac{1}{2} - \cancel{\frac{1}{5}}\right.\right.$

$\left(\cancel{\frac{1}{3(k-1)-1}} - \frac{1}{3(k-\right.$

$\lim_{n\to\infty} S_n =$

since th

10) $\sum_{k=2}^{\infty} \frac{1}{k^2-}$

$\frac{1}{(k+1)(k-}$

By partial

$\frac{1}{(k+1)(k-1)}$

$1 = A(k$

For A

$A = -1$

For B

$B = 1$

So,

$$\frac{-1}{3(3k+2)} + \frac{1}{3(3k-1)}$$

$$\frac{1}{3}\left[\frac{1}{3k-1} - \frac{1}{3k+2}\right].$$

$$\frac{1}{3}\left[\left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{8}}\right) \cdots\cdots\right.$$

$$\left(\cancel{\frac{1}{3(k-1)-1}} - \cancel{\frac{1}{3(k-1)+2}}\right) + \left(\cancel{\frac{1}{3(k)-1}} - \frac{1}{3k+2}\right)$$

$$\lim_{n\to\infty} S_n = \frac{1}{3}\lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{3k+2}\right)$$

$= 1/6 < 1$ (converges)

Since the sum is also $1/6$.

10) $\sum_{k=2}^{\infty} \frac{1}{k^2-1}$

$$\frac{1}{(k+1)(k-1)}$$

By partial.

$$\frac{1}{(k+1)(k-1)} = \frac{A}{(k+1)} + \frac{B}{(k-1)}$$

$1 = A(k-1) + B(k+1)$

For A, $k = -1$

$A = -1/2$

For B, $k = 1$

$B = 1/2$

So,

$$\frac{-1}{2(k+1)} + \frac{1}{2(k-1)}$$

$$\left(-\cancel{\frac{1}{6}} + \frac{1}{2}\right) + \left(-\cancel{\frac{1}{8}} + \frac{1}{4}\right) \cdots\cdots$$

$$\left(\cancel{\frac{1}{6}} - \frac{1}{10}\right) \cdots \left(\cancel{\frac{1}{2(k-1)}} - \frac{1}{2(k+1)}\right)$$

$\frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$ Since this is telescoping series so it will almost all cancelled

So then

$$\frac{1}{2}\left(1 + \frac{1}{2}\right) - \lim_{n\to\infty}\frac{1}{n+1}$$

$= \boxed{\frac{3}{4}}$ Ans.

Date:

11) $\sum_{k=3}^{\infty} \frac{1}{k-2}$.

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

Since it is harmonic series it alway diverge.

12) $\sum_{k=5}^{\infty} \left(\frac{e}{\pi}\right)^{k-1}$

Since it is geometric series and $\frac{e}{\pi} < 1$ so it converges. ~~diverges~~.

$a = \left(\frac{e}{\pi}\right)^4 \quad r = \frac{e}{\pi}$

$S_n = \frac{a}{1-r} = \frac{(e/\pi)^4}{1-\frac{e}{\pi}}$

$$\boxed{S_n = \frac{e^4}{\pi^3(\pi - e)}}$$

13) $\sum_{k=1}^{\infty} \frac{4^{k+2}}{7^{k-1}}$

$\sum_{k=1}^{\infty} \frac{4^{k+2+1-1}}{7^{k-1}}$

$\frac{4^{k-1} \cdot 4^3}{7^{k-1}}$

$\sum_{k=1}^{\infty} 64 \left(\frac{4}{7}\right)^{k-1}$

since $4/7 < 1$ converges

so $a = 64$, $r = 4/7$

$S_n = \frac{a}{1-r} = \frac{64}{1-4/7}$

$= 64 \cdot 7/3$

$$\boxed{S_n = 448/3}$$

14) $\sum_{k=1}^{\infty} 5^{3k} 7^{1-k}$

$\sum_{k=1}^{\infty} \frac{5^{3k} \cdot 7}{7^k}$

$\sum_{k=1}^{\infty} 7\left(\frac{125}{7}\right)^k$

$a = 125 \quad r = 125/7 > 1$.

diverges.

Date:

Q) Determine whether the series converges. (9-24)

9) $\sum_{k=1}^{\infty} \frac{1}{k+6}$.

$\frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$.

Its harmonic series
it always diverges

10) $\sum_{k=1}^{\infty} \frac{3}{5k}$.

$\left(\frac{3}{5}\right)\left(\frac{1}{k}\right)$

its ~~geometric~~ harmonic
so it always diverge

11) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+5}}$

By divergence Test

$\lim_{k\to\infty} \frac{1}{\sqrt{k+5}}$

$= 0$. inconclusive

By p-series test.

$\frac{1}{\sqrt{k}}$. as $\frac{1}{(k)^{p}}$.

$p = 1/2 < 1$

(diverges)

By using integral test

PCD

$\int_{1}^{\infty} \frac{1}{\sqrt{k+5}}\, dk$

$\lim_{a\to\infty} \int_{1}^{a} \frac{1}{\sqrt{k+5}}$

$\lim_{a\to\infty} 2\sqrt{k+5}\Big|_{1}^{a}$

$2\left[\lim_{a\to\infty} \sqrt{a+5} - \sqrt{6}.\right]$

$= \infty$ diverges

Teacher's Sign.

Date:

~~11) $\sum_{k=1}^{\infty} k^2 \sin^2\left(\frac{1}{k}\right)$~~

~~By integral test~~

~~$\int_1^{\infty} k^2 \sin^2 \frac{1}{k}$~~

12) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[k]{e}}$

By using divergence test

$\lim_{k\to\infty} \frac{1}{(e)^{1/k}}$

$= \boxed{1}$

as $a_k \neq 0$ (diverge)

13) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$

By using divergence

$\lim_{k\to\infty} \frac{1}{\sqrt{2(\infty)-1}}$

$= 0$ inconclusive.

By integral.

$\int_1^{\infty} \frac{1}{(2k-1)^{1/3}}$

$\frac{1}{2} \lim_{a\to\infty} \int_1^a \frac{2}{(2k-1)^{1/3}}$

$\frac{1}{2} \cdot \frac{3}{2} \lim_{a\to\infty} (2k-1)^{2/3} \Big|_1^a$

$\frac{3}{4}\left[(2(\infty)-1)^{2/3} - (2-1)\right]$

$= \boxed{\infty}$ diverging.

14) $\sum_{k=3}^{\infty} \frac{\ln k}{k}$

By integral test

$\int_3^{\infty} \frac{\ln k}{k}$

$\lim_{a\to\infty} \int_3^a \frac{1}{k} \ln k$

$\frac{1}{2} \lim_{a\to\infty} \ln(k)^2 \Big|_3^a$

$\frac{1}{2}\left(\ln|\infty| - \ln(3)\right)$

$= \boxed{\infty}$ diverging.

15) $\sum_{k=1}^{\infty} \frac{k}{\ln(k+1)}$

By divergence test

$\lim_{k\to\infty} \frac{k}{\ln(k+1)}$

By L-hopitol

$\frac{1}{1/(k+1)}$

$= \infty$ divegence

16) $\sum_{k=1}^{\infty} k e^{-k^2}$

By integral.

$-\frac{1}{2} \int_1^{\infty} -2k e^{-k^2}$

$\frac{-1}{2} \lim_{a\to\infty} e^{-k^2} \Big|_1^a$

$\frac{-1}{2}\left[\underbrace{e^{-\infty^2}}_{1/e^{\infty}} - e^{-1^2}\right] = \frac{-1}{2}\left[0 - e^{-1}\right]$

$= \boxed{\frac{e^{-1}}{2}}$ converging

7) $\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)$

By divergence

$\lim_{k\to\infty} \left(1 + \frac{1}{k}\right.$

$\lim_{k\to\infty} \frac{1}{(1+1/k)}$

$= \boxed{\frac{1}{e}}$ answ

8) $\sum_{k=1}^{\infty} \frac{k^2+1}{k^2+3}$

By diverge

$\lim_{k\to\infty} \frac{k^2+1}{k^2+3}$

$= \boxed{1}$ dive

19) $\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$

By integ

$\int_1^{\infty} \frac{\tan^{-1} k}{1+k^2}$

$\lim_{a\to\infty} \int_1^a \tan^{-}$

~~$\lim_{a\to\infty} \ln($~~

~~$\ln(\tan a)$~~

$\frac{1}{2} \lim_{a\to\infty} (\tan$

$\frac{1}{2}\left(\lim_{a\to\infty} \tan^{-1}(a)\right.$

$\frac{1}{2}\left(\left(\frac{\pi}{2}\right)\right.$

17) $\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{-k}$

By divergence test.

$\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^{-k}$

$\lim_{k\to\infty}\frac{1}{(1+1/k)^{k}}$

$=\boxed{\frac{1}{e}}$ ~~converging~~ diverges

18) $\sum_{k=1}^{\infty}\frac{k^2+1}{k^2+3}$

By divergence

$\lim_{k\to\infty}\frac{k^2+1}{k^2+3}$

$=\boxed{1}$ divergence

19) $\sum_{k=1}^{\infty}\frac{\tan^{-1}k}{1+k^2}$

By integral test.

$\int_1^{\infty}\frac{\tan^{-1}k}{1+k^2}$

$\lim_{a\to\infty}\int_1^{a}\tan^{-1}k\cdot\left(\frac{1}{1+k^2}\right)$

$\cancel{\lim_{a\to\infty}\ln(\tan k)\Big|_1^{a}}$

$\cancel{\ln(\tan a)-\ln(\tan 1)}$

$\frac{1}{2}\lim_{a\to\infty}(\tan^{-1}k)^2\Big|_1^{a}$

$\frac{1}{2}\left[\lim_{a\to\infty}\tan^{-1}(a)^2-\tan^{-1}(1)^2\right]$

$\frac{1}{2}\left[\left(\frac{\pi}{2}\right)^2-(\pi/4)^2\right]\Rightarrow\frac{1}{2}\left(\frac{3\pi^2}{16}\right)=\boxed{\frac{3\pi^2}{32}}\neq 0$ converges.

Teacher's Sign.

20) $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k^2+1}}$

By divergence test

$\lim_{k\to\infty}\frac{1}{\sqrt{k^2+1}}$

$=\boxed{0}$ inconclusive.

By integral test

$\int_1^{\infty}\frac{1}{\sqrt{k^2+1}}$

$\lim_{a\to\infty}\int_1^{a}\frac{1}{\sqrt{k^2+1}}$

$\lim_{a\to\infty}\ln\left|k+\sqrt{k^2+1}\right|\Big|_1^{a}$

$\ln\left|a+\sqrt{a^2+1}\right|-\ln\left|1+\sqrt{1+1}\right|$

$=\boxed{\infty}$ diverging.

~~21)~~ $\cancel{\sum_{k=1}^{\infty}k^2\sin^2\left(\frac{1}{k}\right)}$

By Divergence.

$\cancel{\lim_{k\to\infty}\frac{\sin^2(1/k)}{1/k^2}}$

$\cancel{\frac{2\sin(1/k)\cos(1/k)(-1/k^2)}{1/2k}}$

~~apply limit~~

$\cancel{-\frac{\sin 2(1/k)\cdot 2k}{4k^2}}$

~~apply limit~~

$=\boxed{0}$ ~~inconclusive.~~

Date:

21) $\sum_{k=1}^{\infty} k^2 \sin^2(1/k)$

Since small $1/k$, we can sin say approx.

$\sin x = x$.

So $\sin^2\left(\frac{1}{k}\right) = \left(\frac{1}{k}\right)^2$.

Substituting in main

$k^2\left(\frac{1}{k^2}\right)$

$= 1$

By divergence test $\neq 0$ diverges.

22) $\sum_{k=1}^{\infty} k^2 e^{-k^3}$

By integral test.

$\frac{1}{-3}\int_1^{\infty} -3k^2 e^{-k^3}$

$\frac{-1}{3}\lim_{a\to\infty} e^{-k^3}\Big|_1^a$

$\frac{-1}{3}\left[\lim_{a\to\infty} e^{-a^3} - e^{-(1)}\right]$

$\frac{-1}{3}\left[\frac{1}{e^{\infty}} - \frac{1}{e^1}\right]$

$\frac{-1}{3}\left[0 - \frac{1}{e}\right]$

$= \boxed{\frac{1}{3e}}$ converges.

23) $\sum_{n=1}^{\infty} 7k^{-1.01}$

$7\sum_{k=1}^{\infty} k^{-1.01}$

Since p-series $1.01 > 1$ converges.

24) $\sum_{k=1}^{\infty} \operatorname{sech}^2 k$.

By integral

$\int_1^{\infty} \operatorname{sech}^2 k$.

$\lim_{a\to\infty} \tanh\Big|_1^a$

$\lim_{a\to\infty} \tanh a - \tanh(1)$

$= \boxed{1-\tanh(1)}$

converges.

Q1) (a) {

A

a

lim
n→

(b) {

$a_n = (-$

a

(c) {4

$a_n =$

lim
n→∞

(d) $\{\frac{1}{2}$

$(\frac{1}{2})^n$ (

(e) $\{-\frac{1}{2}$

$a_n = \frac{(-1)}{n+}$

Date:

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Q1) (a) {

Ai

a

lim $n\to\infty$

(b) {2

$a_n = (-2$

dc

(c) {4,

$a_n =$

lim $n\to\infty$ a

a

(d) $\{\frac{1}{2}$

(e) $\{-\frac{1}{2}$

$a_n = \frac{(-1)^n}{n+1}$