

Date: _____

Assignment # 02

Name: Malik Zaryab Khan

Section: BSE-1A

Roll No : 24E-3055

Question: 01

$$v(t) = -5 \sin \omega t \sin(\omega t + \phi)$$

$$-5 = -5 \sin \phi$$

$$\phi = \sin^{-1} \left(\frac{-5}{-5} \right)$$

$$\phi = -0.927$$

Question: 02

$$(a) T^o E \text{ at equilibrium} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} (0.85)^2 (2)$$

$$T^o E = 0.7225 J$$

$$(b) T^o E = P.E + K.E$$

at extrem point $K.E = 0$

so $K.E$ will be null point

from it will return.

$$0.7225 = 0 + b x^2$$

$$x = \sqrt{\frac{0.7225}{200}}$$

$$x = 0.06 m$$

or

$$x = 6 cm$$

at $x = 15$

$$U = 200(0.15)^2$$

$$U = 4.5 J/m$$

it will turn

back as $P.E$

Question:03

$$m = 2 \text{ kg}$$

$$F = 20 \text{ N}$$

$$x_i = 0.200 \text{ m}$$

Sol:

$$(a) F = kx$$

$$k = \frac{F}{x} = \frac{20}{0.200}$$

$$k = 100 \text{ Nm}^{-1}$$

$$(e) \text{ Total energy} = \frac{1}{2} kx^2 = \frac{1}{2} (100)(0.2)^2$$

$$\boxed{T.E = 2 J}$$

$$(b) T = 2\pi \sqrt{\frac{m}{k}} = 2 \times 3.142 \sqrt{\frac{0.2}{100}}$$

$$(f) \text{ velocity when } x = \frac{1}{3}x_i$$

$$T = 0.888 \text{ sec}$$

$$\because V = -xw \Rightarrow V = -\frac{x}{3}w$$

$$f = \frac{1}{T} = \frac{1}{0.888}$$

$$V = -(1.414)/3$$

$$\boxed{f = 1.125 \text{ Hz}}$$

$$\boxed{V = -0.471 \text{ m/s}}$$

$$(g) \text{ acceleration when } x = \frac{1}{3}x_i$$

$$a = -\frac{x}{3}w^2$$

$$\boxed{a = -3.33 \text{ m/s}^2}$$

$$(c) v_{max} = -xw$$

$$= (0.200) \left(\sqrt{\frac{k}{m}} \right)$$

$$= -(0.2) (7.07) \quad \because \text{at the point when}$$

$$v_{max} = -1.414 \text{ m/s} \quad x=0, \text{ the velocity will be max}$$

$$(d) a_{max} = -xw^2$$

$$= -(0.2) \left(\sqrt{\frac{k}{m}} \right)^2$$

$$\boxed{a_{max} = -10 \text{ m/s}^2}$$

Date: _____

Question 04:

$$f = 3 \text{ hz}$$

$$m = 1450 \text{ kg}$$

$$k = ?$$

$$\text{no of spring} = 4$$

89c

$$(a) \text{ mass of 1 spring} = \frac{1450}{4} = 362.5$$

$$(b) \text{ mass of 5 passenger} \\ = 73 \times 5 = 365 \text{ kg}$$

$$w = \sqrt{\frac{k}{m}} = 3$$

$$\text{total mass} = 1450 + 365 \\ = 1815$$

$$2\pi f = \sqrt{\frac{k}{m}}$$

$$\text{mass on each spring} = \frac{1815}{4} = 453.75$$



$$\underline{\underline{ag}} \quad F = \frac{1}{2\pi f} \sqrt{k}$$

$$(2\pi f)^2 \times m = k$$

$$k = 128831.74 \text{ N/m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{128831.74}{453.75}}$$

$$f = 2.68 \text{ hz}$$

Date: _____

Question: 05

$$k = 7580 \text{ N/m}$$

$$m = 0.245 \text{ kg}$$

$$f = ?$$

Sol:-

$$\begin{aligned} k \text{ on whole system} &= 2 \times 7580 \\ &= 15160 \text{ N/m} \end{aligned}$$

$$\therefore f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\pi} \cdot \sqrt{\frac{15160}{0.245}}$$

$$f = 39.5 \text{ Hz}$$

Question: 06

$$\omega = 14 \text{ rad/s}$$

$$l = 0.450 \text{ m}$$

$$k = 120 \text{ N/m}$$

$$\theta = 400^\circ = 360^\circ - 40^\circ = 40^\circ$$

$$(b) T = 2\pi \sqrt{\frac{m}{c}} = 2\pi \sqrt{\frac{1.4}{120}}$$

$$T = 0.67 \text{ sec}$$

Sol:- (a) $F = kx$

$$F = w \sin \theta$$

$$w \sin \theta = kx$$

$$x = w \sin \theta = 14 \sin 40^\circ$$

$$\frac{k}{120}$$

$$x = 0.07 \text{ m}$$

total distance from equilibrium

$$\text{position} = 0.07 + 0.450$$

$$= 0.525 \text{ m}$$

Question: 07

$$T = 1.5 \text{ s}$$

$$\phi = \pi/6 \text{ rad}$$

$$f = \frac{1}{T} = 0.66 \text{ Hz}$$

$$t = 0.05 \text{ sec}$$

$$x_m = A$$

Sol:

$$x_2 = A_m \cos(\omega t + \phi)$$

$$= A_m \cos(2\pi(0.66)(0.5) + \pi/6)$$

$$= A_m \cos\left(\frac{6.2\pi}{1.5}\right)$$

$$= -A_m \cos(0.855)$$

$$x_2 = -0.855 A$$

$$\Delta x = x_1 - x_2$$

$$= -0.48A + 0.855A$$

$$x_1 = A_m \cos(\omega t + \phi)$$

$$\Delta x = 0.375A$$

$$= A \cos(2\pi(0.66)(0.5) + 0)$$

$$x_1 = -0.48A$$

$$v_1 = -\omega A_m \sin(\omega t)$$

$$v_2 = -\tilde{\omega} A_m \sin(\omega t + \phi)$$

$$= -\frac{2\pi}{1.5} A \sin\left(\frac{2\pi}{1.5} \cdot t\right)$$

$$= -\frac{2\pi}{1.5} A \sin\left(\frac{2\pi}{1.5}(0.5) + \frac{\pi}{6}\right)$$

$$= -\frac{2\pi}{1.5} A \sin\left(\frac{2\pi}{1.5} \cdot 0.5\right)$$

$$v_2 = -2.09 A \text{ m/s}$$

$$v_1 = 3.62 A \text{ m/s}$$

Question:08

- (a) position & velocity ($\pi/2$) cannot be same
 (b) velocity & acceleration ($\pi/2$) cannot be same
 (c) position & acceleration (π) always in opposite direction

There will be no change in total energy & potential energy as they both are not dependant on mass. Kinetic energy is dependant on mass so it can be changed.

Question:09

$$x = 5.00 \text{ cm} \cos(2t + \pi/6)$$

$t=0 ; \omega=2$

Solt:

(a) position $\Rightarrow x = 5.00 \cos\{2(0) + \pi/6\}$

$$x = 4.33 \text{ cm}$$

(b) velocity $v = 2(5.00) \sin(2t + \pi/6)$

$$v = 5 \text{ cm/s}$$

(c) acceleration $a = -\omega^2 x$

$$a = -(2)^2 (4.33)$$

$$a = -17.32 \text{ cm/s}^2$$

(d) amplitude 5 cm

(e) $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = 3.142 \text{ sec}$

Question: 10

$$m = 2.20 \text{ kg}$$

$$K = 250 \text{ N/m}$$

$$T = 0.615 \text{ sec}$$

Sol:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.615} = 20.216 \text{ rad/s}$$

$$b = \sqrt{mk} \\ = \sqrt{250 \times 2.2}$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$b = 23.45$$

System is under
damp.

$$(10.21)^2 = \frac{k}{m} - \frac{b^2}{4m^2}$$

$$104.37 = \frac{4k - b^2}{4m^2}$$

$$4(2.2)^2 (104.37) = 4(250)(2.20) - b^2$$

$$b^2 = 2200 - 2020.6$$

$$b = \sqrt{179.39}$$

$$\boxed{b = 13.39}$$

Question: 11

$$Fx = -kx$$

$$k = 580 \text{ N/m}$$

$$m = v = 1.66 \times 10^{-27} \text{ kg}$$

Sol:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m/2}} \quad \because v = m/f$$

$$f = \frac{1}{2\pi} \sqrt{\frac{580}{1.667 \times 10^{11}/2}}$$

$$\boxed{f = 1.33 \times 10^{10} \text{ Hz}}$$

Question: 12

$$m_0 = 1.50 \text{ kg}$$

$$m_B = 275 \text{ g} = 0.275 \text{ kg}$$

$$k = 185 \text{ N/m}$$

$$A = 15 \text{ cm} = 0.15 \text{ m}$$

$$(a) d_T = ?$$

$$(b) t = ?$$

$$(c) \Delta \theta = ?$$

Sol:

$$(a) F = kd$$

$$mg = kd$$

$$g = \frac{kd}{m}$$

$$d = \frac{mg}{k}$$

$$d = 1.715 \times 19.8$$

$$185$$

$$(b) x(t) = A \cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{185}{1.715}}$$

$$\omega = 10.2 \text{ rad/s}$$

$$\cos(\omega t) = \frac{x}{A}$$

$$\cos \omega t = \frac{9.4}{15}$$

$$\omega t = \cos^{-1} \left(\frac{9.4}{15} \right)$$

$$t = \frac{0.900}{10.2}$$

$$\boxed{d = 0.094 \text{ m}}$$

total weight

$$\Rightarrow d_T = 15 + 9.4 =$$

$$\boxed{d_T = 24.4 \text{ cm}}$$

$$\boxed{t = 0.221 \text{ sec}}$$

$$(c) T \cdot E = P \cdot E + K \cdot E$$

$$\frac{1}{2} kx^2 = \frac{1}{2} kx^2 + \frac{1}{2} mv^2$$

$$KA^2 = kx^2 + mv^2$$

$$mv^2 = KA^2 - kx^2$$

$$v = \sqrt{KA^2 - kx^2}$$

$$v = \sqrt{\frac{1.85}{1.775} (0.15^2 - 0.094^2)}$$

$$v = 1.19 \text{ m/s}$$

Question 13

$$x = 8.6 \text{ cm} = 0.086 \text{ m}$$

$$v(0) = 0.93 \text{ m/s}$$

$$a(0) = 48 \text{ m/s}^2$$

Sol:

(a) ω and f

For $f \Rightarrow$

$$f = \frac{\omega}{2\pi} = \frac{23.82}{2\pi}$$

By using acceleration

$$a(0) = -\omega^2 A \cos(\omega t + \phi)$$

$$\therefore a(0) = 0.086$$

$$f = 3.75 \text{ Hz}$$

$$a(0) = -\omega^2 (x(0))$$

$$48 = -\omega^2 (-0.086)$$

$$\omega = \sqrt{\frac{48}{0.086}} = 23.62 \text{ rad/sec}$$

$$\omega = 23.62 \text{ rad/sec}$$

(b) for ϕ

taking velocity

$$v(0) = -\omega A \sin(\omega t + \phi) \quad \text{--- (1)}$$

finding A from $v(0)$

$$A = \frac{|v(0)|}{\omega}$$

$$\cos(\omega t + \phi)$$

Replacing A in eq(1)

$$v(0) = -\omega \left(\frac{|v(0)|}{\cos(\omega t + \phi)} \right) \sin(\omega t + \phi)$$

substituting values

$$-0.93 = -23.62 (-0.086) \tan \phi$$

$$\phi = \tan^{-1} \left(\frac{-0.93}{23.62 \times 0.086} \right)$$

$$\phi = 0.429$$

(c) Amplitude

as we have

$$A \cos \phi = -0.086$$

$$A = \frac{-0.086}{\cos \phi}$$

$$A = 9.47 \text{ cm}$$

OR

$$A = 0.0947 \text{ m}$$