2.3 Derivative.

(Q9) $f(x) = x^{-3} + \frac{1}{x^7}$

$= x^{-3} + x^{-7}$

$= -3x^{-4} + -7x^{-8}$

$= \frac{-3}{x^4} - \frac{7}{x^8}$

(10) $f(x) = \sqrt{x} + \frac{1}{x}$

$= x^{1/2} + x^{-1}$

$= \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

(11) $f(x) = -3x^{-8} + 2\sqrt{x}$

$= -3(-8)x^{-9} + \cancel{2}\frac{1}{\cancel{2}\sqrt{x}}$

$= \frac{24}{x^9} + \frac{1}{\sqrt{x}}$

(12) $f(x) = 7x^{-6} - 5\sqrt{x}$

$= -42x^{-7} - \frac{5}{2\sqrt{x}}$

$= \frac{-42}{x^7} - \frac{5}{2\sqrt{x}}$

(13) $f(x) = x^{\pi} + \frac{1}{x^{\sqrt{10}}}$

$= \pi x^{\pi-1} + x^{-\sqrt{10}}$

$= \pi x^{\pi-1} - \sqrt{10}\, x^{-\sqrt{10}-1}$

$= \boxed{\pi x^{\pi-1} - \frac{\sqrt{10}}{x^{\sqrt{10}+1}}}$

(14) $f(x) = \sqrt[3]{\frac{8}{x}}$

$= \frac{2}{\sqrt[3]{x}}$

$= 2x^{-1/3}$

$= \cancel{2}\left(\frac{-3}{\cancel{2}} x^{-4/3}\right)$

$= \boxed{\frac{-3}{x^{4/3}}}$

(15) $f(x) = (3x^2+1)^2$

$= 2(3x^2+1)$

$= \boxed{6x^2+2}$

(16) $f(x) = ax^3 + bx^2 + cx$

$= 3ax^2 + 2bx + c$

(Q 17)

$y = 5x^2 - 3x + 1$

$10x - 3$

$\boxed{= 7}$

(Q18) $y = \frac{x^{3/2} + 2}{x}$

$= \frac{x^{3/2}}{x} + \frac{2}{x}$

$\Rightarrow x^{1/2} + 2x^{-1}$

$= \boxed{\frac{1}{2\sqrt{x}} + -\frac{2}{x^2}}$

$= \frac{1}{2} - 2$

$= \boxed{\frac{-3}{2}}$

(Q19)

$x = t^2 - t$.

$\boxed{\dfrac{dx}{dt} = 2t - 1}$

(Q20) $x = \dfrac{t^2+1}{3t}$.

$= \dfrac{t^2}{3t} + \dfrac{1}{3t}$

$= \dfrac{t}{3} + \dfrac{1}{3t}$

$= \dfrac{t}{3} + \dfrac{t^{-1}}{3}$

$= \dfrac{1}{3}\left(t + \dfrac{1}{t}\right)$

$= \boxed{\dfrac{1}{3}\left(1 - \dfrac{1}{t^2}\right)}$ Ans

(Q)21

$y = 5$

(Q22) 6.

(Q23) $y = (1-x)(1+x)(1+x^2)(1+x^4)$

$(1^2 - x^2)(1+x^2)(1+x^4)$

$(1^2 - x^4)(1+x^4)$

$1 - x^{16}$.

$1 - 16x^{15}$.

$1 - 16$.

$\boxed{-15}$

(Q24) $24x^{23} + 24x^{11} + 24x^7 + 24x^5$

$= 24(1 + 1 + 1 + 1)$

$= \cancel{4}\ \boxed{96}$

41) (a) $y = 7x^3 - 5x^2 + x$

$\frac{dy}{dx} = 21x^2 - 10x.$

$\frac{d^2y}{dx^2} = 42x - 10.$

(d) $y = (5x^2 - 3)(7x^3 + x)$

$35x^5 + 5x^3 - 21x^3 - 3x$

$35x^5 + -16x^3 - 3x$

$\frac{dy}{dx} = 175x^4 - 48x^2 - 3$

$\frac{d^2y}{dx^2} = 700x^3 - 96x$

(b) $y = 12x^2 - 2x + 3$

$\frac{dy}{dx} = 24x - 2.$

$\frac{d^2y}{dx^2} = 24$

(c) $y = \frac{x}{\cancel{x}} + \frac{1}{x}$

$y = 1 + x^{-1}$

$\frac{dy}{dx} = \frac{-1}{x^2}.$

$\frac{d^2y}{dx^2} = 2x^{-3}$

$\boxed{= \frac{2}{x^3}}$

Q42) (a) $y = 4x^7 - 5x^3 + 2x$

$$\frac{dy}{dx} = 28x^6 - 15x^2 + 2.$$

$$\frac{d^2y}{dx^2} = \boxed{168x^5 - 30x}$$

(b) $y = 3x + 2$

$$\cancel{\frac{d^2y}{}}\ \frac{dy}{dx} = \boxed{3}$$

$$\frac{d^2y}{dx} = \boxed{0}$$

(c) $y = \frac{3x - 2}{5x}$

$$= \frac{3\cancel{x}}{5\cancel{x}} - \frac{2}{5x}$$

$$= \frac{-2}{3}x^{-1}$$

$$\frac{dy}{dx} = \frac{2}{3x^2}.$$

$$\boxed{\frac{d^2y}{dy^2} = \frac{-4}{3x^3}}$$

(d) $y = (x^3 - 5)(2x + 3)$

$$= 2x^4 + 3x^3 - 10x - 15$$

$$\frac{dy}{dx} = 8x^3 + 9x^2 - 10$$

$$\boxed{\frac{d^2y}{dx} = 24x^2 + 18x}$$

(Q 43)

(a) $y = x^{-5} + x^{5}$

$-5x^{-6} + 5x^{4}$.

$30x^{-7} + 20x^{3}$

$= \boxed{-210x^{-8} + 60x^{2}}$

(b) $y = \frac{1}{x}$. $\Rightarrow y = x^{-1}$

$-1x^{-2}$

$2x^{-3}$

$= \boxed{-6x^{-4}}$

(c) $y = ax^{3} + bx + c$

$3ax^{2} + b$.

$6ax$

$= \boxed{6a}$

(Q44)

a) $y = 5x^2 - 4x + 7$

$10x - 4$

$10.$

$y''' = \boxed{0}.$

(b) $y = 3x^{-2} + 4x^{-1} + x$

$= -6x^{-3} - 4x^{-2} + 1$

$= 18x^{-4} + 8x^{-3}$

$= -72x^{-5} - 24x^{-4}.$

$y''' = \boxed{\dfrac{-72}{x^5} - \dfrac{24}{x^4}}$

(c) $y = ax^4 + bx^2 + c$

$\Rightarrow 4ax^3 + 2bx$

$\Rightarrow 12ax^2 + 2b.$

$\Rightarrow \boxed{24ax}$

(Q45)

(a) $f'''(2)$ where $f(x) = 3x^2 - 2$.

$= 6x$

$= 6$

$= \boxed{0}$

(b) $\left.\dfrac{d^2y}{dx^2}\right|_{x=1}$ $\quad y = 6x^5 - 4x^2$.

$30x^4 - 8x$

$120x^3 - 8$.

$120 - 8$

$= \boxed{112}$

(c) $\left.\dfrac{d^4}{dx^4}\left[x^{-3}\right]\right|_{x=1}$.

$-3x^{-4}$

$12x^{-5}$

$-60x^{-6}$.

$+360x^{-7}$.

$= \boxed{360}$

(Q46)

(a) $y''''(0)$

$y = 4x^4 + 2x^3 + 3.$

$16x^3 + 6x^2.$

$48x^2 + 12x$

$96x + 12.$

$= \boxed{12}$

(b). $\left.\frac{d^4y}{dx^4}\right|_{x=1}$ where $y = \frac{6}{x^4}.$

$y = 6x^{-4}.$

$-24x^{-5}$

$120x^{-6}.$

$-720x^{-7}.$

$= \boxed{5040}$

(Q47) $y = x^3 + 3x + 1$

$$y''' + xy'' - 2y' = 0$$

$y' = 3x^2 + 3$

$y'' = 6x$

$y''' = 6$.

$$6 + x(6x) - 2(3x^2 + 3) = 0$$

$$\cancel{6} + \cancel{6x^2} - \cancel{6x^2} - \cancel{6} = 0$$

$$\boxed{0 = 0}$$

proved!

Ex 2.4

(Q1) $(x+1)(2x-1)$

$2x^2 - x + 2x - 1$

$2x^2 + x - 1$

$\frac{dy}{dx} = \boxed{4x+1.}$

$\frac{d}{dx}(u \cdot v) = (x+1)\frac{d}{dx}(2x-1) + (2x-1)\frac{d}{dx}(x+1)$

$= (x+1)(2) + (2x-1)(1)$

$= 2x + 2 + 2x - 1$

$= \boxed{4x+1}$

Proved!

(Q2). $(3x^2-1)(x^2+2)$

$= 3x^4 + 6x^2 - x^2 - 2$

$= 3x^4 + 5x^2 - 2.$

$= \boxed{12x^3 + 10x.}$

$(3x^2-1)\frac{d}{dx}(x^2+2) + (x^2+2)\frac{d}{dx}(3x^2-1)$

$(3x^2-1)(2x) + (x^2+2)(6x)$

$6x^3 - 2x + 6x^3 + 12x$

$\boxed{12x^3 + 10x}$

Proved!

1 (Q3) $f(x) = (x^2+1)(x^2-1)$

$= x^4 - 1$

$\frac{dy}{dx} = \boxed{4x^3}$

$(x^2+1)\frac{d}{dx}(x^2-1) + (x^2-1)\frac{d}{dx}(x^2+1)$

$(x^2+1)(2x) + (x^2-1)(2x)$

$2x^3 + \cancel{2x} + 2x^3 - \cancel{2x}$

$\boxed{4x^3}$

proved!

(Q4) $(x+1)(x^2-x+1)$

$\cancel{(x+1)}\ x(x^2-x+1) + 1(x^2-x+1)$

$x^3 - \cancel{x^2} + \cancel{x} + \cancel{x^2} - \cancel{x} + 1$

$= \frac{d}{dx}(x^3 - 1)$

$= \boxed{3x^2}$

$(x+1)\frac{d}{dx}(x^2-x+1) + (x^2-x+1)\frac{d}{dx}(x+1)$

$(x+1)(2x-1) + (x^2-x+1)(1)$

$(2x^2 - x + 2x - 1) + x^2 - x + 1$

$2x^2 + \cancel{x} - \cancel{1} + x^2 - \cancel{x} + \cancel{1}$

$\boxed{3x^2}$

proved!

Scanned with CamScanner

(Q5) $f(x) = (3x^2+6)(2x-\frac{1}{4})$

$(3x^2+6)\frac{d}{dx}(2x-1/4) + (2x-1/4)\frac{d}{dx}(3x^2+6)$

$(3x^2+6)(2) + (2x-1/4)(6x)$

$6x^2+12+12x^2-\frac{6}{4}x$

$= \boxed{18x^2-\frac{3}{2}x+12}$ ✓

confirmation

$6x^3-\frac{3x^2}{4}+12x-\frac{6}{4}$

$18x^2-\frac{6}{4}x+12$

$\boxed{18x^2-\frac{3}{2}x+12}$

(Q6) $f(x) = (2-x-3x^3)(7+x^5)$

$(2-x-3x^3)\frac{d}{dx}(7+x^5) + (7+x^5)\frac{d}{dx}(2-x-$

$(2-x-3x^3)(5x^4) + (7+x^5)(-1-9x^2)$

$10x^4-5x^5-15x^7+(-7-63x^2-x^5-9x$

$= \boxed{-24x^7-6x^5+10x^4-63x^2-7}$

Ans.

(Q7) $f(x) = (x^3+7x^2-8)(2x^{-3}+x^{-4})$

$(x^3+7x^2-8)\frac{d}{dx}(2x^{-3}+x^{-4})+(2x^{-3}+x^{-4})\frac{d}{dx}(x^3+7x^2-8)$

$(x^3+7x^2-8)(-6x^{-4}-4x^{-5})+(2x^{-3}+x^{-4})(3x^2+14x)$

$-6x^{-1}-4x^{-2}-42x^{-2}-28x^{-3}+48x^{-4}+32x^{-5}+4x^{-1}+28x^{-2}+2x^{-2}+14x^{-3}$

$\boxed{-2x^{-1}-16x^{-2}-14x^{-3}+48x^{-4}+32x^{-5}}$

(Q8) $f(x) = \left(\frac{1}{x}+\frac{1}{x^2}\right)(3x^3+27)$

$\left(\frac{1}{x}+\frac{1}{x^2}\right)\frac{d}{dx}(3x^3+27)+(3x^3+27)\frac{d}{dx}(x^{-1}+x^{-2})$

$\left(\frac{1}{x}+\frac{1}{x^2}\right)(9x^2)+(3x^3+27)(-x^{-2}-2x^{-3})$

$\left(\frac{x+1}{\cancel{x^2}}\right)(9\cancel{x^2}) + 3(x^3+9)\left(\frac{-1}{x^2}-\frac{2}{x^3}\right)$

$9(x+1) + 3(x^3+9)\left(\frac{-x-2}{x^3}\right)$

$\frac{9x^3(x+1)+(3x^3+27)(-x-2)}{x^3}$

$\frac{9x^4+\cancel{9x^3}+-\cancel{3x^3}-\cancel{6x^3}-27x-54}{x^3}$

$= \boxed{\frac{9x^4-27x-54}{x^3}}$

Scanned with CamScanner

(Q9) $f(x) = (x-2)(x^2+2x+4)$.

$\frac{d}{dx}(x-2)(x^2+2x+4)$

$(x-2)\frac{d}{dx}(x^2+2x+4) + (x^2+2x+4)\frac{d}{dx}(x-2)$

$(x-2)(2x+2) + (x^2+2x+4)(1)$

$2x^3+\cancel{2x}-\cancel{4x}-\cancel{4}+x^2\cancel{+2x}\cancel{+4}$

$= \boxed{2x^3+x^2}$

(Q10) $f(x) = (x^2+x)(x^2-x)$

$(x^2+x)\frac{d}{dx}(x^2-x) + (x^2-x)\frac{d}{dx}(x^2+x)$

$(x^2+x)(2x-1) + (x^2-x)(2x+1)$

$2x^3\cancel{-x^2}+\cancel{2x^2}-x+2x^3\cancel{+x^2}-\cancel{2x^2}-x$

$= \boxed{4x^3-2x}$

(Q11) $f(x) = \frac{3x+4}{x^2+1}$

$\Rightarrow \frac{(x^2+1)\frac{d}{dx}(3x+4) - (3x+4)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$.

$\Rightarrow \frac{(x^2+1)(3) - (3x+4)(2x)}{(x^2+1)^2}$

$\Rightarrow \frac{3x^2+3-6x^2-8x}{(x^2+1)^2} \Rightarrow \boxed{\frac{-3x^2-8x+3}{(x^2+1)^2}}$

$\Rightarrow \cancel{\boxed{\frac{3x^2+8x+3}{(x^2+1)^2}}}$

(Q12) $f(x) = \dfrac{x-2}{x^4+x+1}$

$$\frac{d}{dx}\left(\frac{x-2}{x^4+x+1}\right)$$

$$\Rightarrow \frac{(x^4+x+1)\frac{d}{dx}(x-2)-(x-2)\frac{d}{dx}(x^4+x+1)}{(x^4+x+1)^2}$$

$$\Rightarrow \frac{(x^4+x+1)(1)-(x-2)(4x^3+1)}{(x^4+x+1)^2}$$

$$\Rightarrow \frac{(x^4+x+1)-(4x^4+x-8x^3-2)}{(x^4+x+1)^2}$$

$$\Rightarrow \frac{x^4\cancel{+x}+1-4x^4\cancel{-x}+8x^3+2}{(x^4+x+1)^2}$$

$$\Rightarrow \boxed{\frac{-3x^4+8x^3+3}{(x^4+x+1)^3}}$$

(Q13) $f(x) = \dfrac{x^2}{3x-4}$

$$\frac{d}{dx}\left(\frac{x^2}{3x-4}\right)$$

$$\Rightarrow \frac{(3x-4)\frac{d}{dx}(x^2) - (x^2)\frac{d}{dx}(3x-4)}{(3x-4)^2}$$

$$\Rightarrow \frac{(3x-4)(2x) - x^2(3)}{(3x-4)^2}$$

$$\Rightarrow \frac{6x^2 - 8x - 3x^2}{(3x-4)^2}$$

$$\Rightarrow \boxed{\frac{3x^2 - 8x}{(3x-4)^2}}$$

(Q14) $f(x) = \dfrac{2x^2+5}{3x-4}$

$$\Rightarrow \frac{(3x-4)\frac{d}{dx}(2x^2+5) - (2x^2+5)\frac{d}{dx}(3x-4)}{(3x-4)^2}$$

$$\Rightarrow \frac{(3x-4)(4x) - (2x^2+5)(3)}{(3x-4)^2}$$

$$\Rightarrow \frac{14x^2 - 16x - 6x^2 - 15}{(3x-4)^2}$$

$$\Rightarrow \boxed{\frac{8x^2 - 16x - 15}{(3x-4)^2}}$$

(Q15) $f(x) = \dfrac{(2\sqrt{x}+1)(x-1)}{(x+3)}$

$$\Rightarrow \frac{d}{dx}\left(\frac{(2\sqrt{x}+1)(x-1)}{(x+3)}\right).$$

$$\Rightarrow \frac{(x+3)\frac{d}{dx}(2\sqrt{x}+1)(x-1) - (2\sqrt{x}+1)(x-1)\frac{d}{dx}(x+3)}{(x+3)^2}.$$

$$\Rightarrow \frac{-(2\sqrt{x}+1)(x-1) + (x+3)\{(2\sqrt{x}+1)\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(2\sqrt{x}+1)\}}{(x+3)^2}.$$

$$\Rightarrow \frac{-(2\sqrt{x}+1)(x-1) + (x+3)\{(2\sqrt{x}+1) + (x-1)\left(\frac{1}{\sqrt{x}}\right)}{(x+3)^2}$$

$$\Rightarrow \frac{-2\sqrt{x}\,x + 2\sqrt{x} - x + 1 + (x+3)\left(2\sqrt{x} + 1 + \sqrt{x} - 1/\sqrt{x}\right)}{(x+3)^2}.$$

$$\Rightarrow \frac{-2x^{3/2} + 2\sqrt{x} - x + 1 + (x+3)(3\sqrt{x} + 1 - 1/\sqrt{x})}{(x+3)^2}$$

$$\Rightarrow \frac{-2x^{3/2} + 2\sqrt{x} - \cancel{x} + 1 + 3x^{3/2} + \cancel{x} - \sqrt{x} + 9\sqrt{x} + 3 - 3/\sqrt{x}}{(x+3)^2}.$$

$$\Rightarrow \frac{x^{3/2} + \sqrt{x} + 9\sqrt{x} + 4 - 3/\sqrt{x}}{(x+3)^2}.$$

$$\Rightarrow \boxed{\frac{x^{3/2} + 10\sqrt{x} - 3/\sqrt{x} + 4}{(x+3)^2}} \Rightarrow \boxed{\frac{x^{3/2} + 10\sqrt{x} - 3x^{-1/2} + 4}{(x+3)^2}}$$

(Q16) $(2\sqrt{x}+1)\left(\dfrac{2-x}{x^2+3x}\right)$

$$\Rightarrow \left(\frac{2-x}{x^2+3x}\right)\frac{d}{dx}(2\sqrt{x}+1)+(2\sqrt{x}+1)\frac{d}{dx}\left(\frac{2-x}{x^2+3x}\right)$$

$$\Rightarrow \left(\frac{2-x}{x^2+3x}\right)\frac{1}{\sqrt{x}}+(2\sqrt{x}+1)\left\{\frac{(x^2+3x)\frac{d}{dx}(2-x)-(2-x)\frac{d}{dx}(x^2+3x)}{(x^2+3x)^2}\right\}$$

$$\Rightarrow \left(\frac{2-x}{x^2+3x}\right)\frac{1}{\sqrt{x}}+\frac{(2\sqrt{x}+1)}{(x^2+3x)^2}\left(x^2+3x(-1)-(2-x)(2x+3)\right)$$

$$\Rightarrow \frac{1}{\sqrt{x}}\left(\frac{2-x}{x^2+3x}\right)+\frac{(2\sqrt{x}+1)}{(x^2+3x)^2}\left(-x^2-3x-4x-6+2x^2-3x\right)$$

$$\Rightarrow \frac{1}{\sqrt{x}}\left(\frac{2-x}{x^2+3x}\right)+\frac{(2\sqrt{x}+1)}{(x^2+3x)^2}\left(x^2-10x-6\right)$$

$$\Rightarrow \frac{1}{\sqrt{x}}\left(\frac{2-x}{x^2+3x}\right)+\frac{2x^2\sqrt{x}-10x(2\sqrt{x})-6(2\sqrt{x})+x^2-10x-6}{(x^2+3x)^2}$$

$$\Rightarrow \frac{1}{\sqrt{x}}\left(\frac{2-x}{x^2+3x}\right)+\frac{2x^{5/2}-20x^{3/2}-12x^{1/2}+x^2-10x-6}{(x^2+3x)^2}$$

$$\Rightarrow \frac{(x^2+3x)(2-x)+\sqrt{x}(2x^{5/2}-20x^{3/2}-12x^{1/2}+x^2-10x-6)}{\sqrt{x}(x^2+3x)^2}$$

$$\Rightarrow \frac{2x^2+6x-x^3-3x^2+2x^3-20x^2-12+x^{5/2}-10x^{3/2}-6\sqrt{x}}{\sqrt{x}(x^2+3x)^2}$$

$$\Rightarrow \boxed{\frac{x^3 - 21x^2 + 6x - 6\sqrt{x} - 10x^{3/2} + x^{5/2} - 12}{\sqrt{x}\,(x^2+3x)^2}}$$

(Q17) $(2x+1)(1+x^{-1})(x^{-3}+7)$.

$$\{(2x+1)(1+x^{-1})\}\{(x^{-3}+7)\}$$

$$\{(2x+1)\frac{d}{dx}(1+x^{-1}) + (1+x^{-1})\frac{d}{dx}(2x+1)\}\{(1)\frac{d}{dx}(x^{-3}+7)+(x^{-3}+7)\frac{d}{dx}(1)$$

$$=\{(2x+1)(-x^{-2}) + (2+2x^{-1})\}\{-3x^{-4}+7\}$$

$$\{-\cancel{2x^{-1}} - x^{-2} + 2 + \cancel{2x^{-1}}\}\{-3x^{-4}+7\}$$

$$(2-x^{-2})(-3x^{-4}+7)$$

$$=\left(2-\frac{1}{x^2}\right)\left(\frac{-3}{x^4}+7\right)$$

$$\left(\frac{2x^2-1}{x^2}\right)\left(\frac{-3+7x^4}{x^4}\right).$$

$$= \boxed{\frac{-6x^2+14x^6+3-7x^4}{x^6}}$$

(Q17) $(2x+1)\left(1+\frac{1}{x}\right)(x^{-3}+7)$

$$\left(2x+\frac{\cancel{2x}}{\cancel{x}}+1+\frac{1}{x}\right)(x^{-3}+7)$$

$$\frac{d}{dx}\left(2x+3+\frac{1}{x}\right)(x^{-3}+7)$$

$$\left(2x+3+\frac{1}{x}\right)\frac{d}{dx}(x^{-3}+7)+(x^{-3}+7)\frac{d}{dx}(2x+3+x^{-1})$$

$$\left(2x+3+\frac{1}{x}\right)(-3x^{-4})+(x^{-3}+7)(2-x^{-2})$$

$$-6x^{-3}-9x^{-4}-3x^{-5}+2x^{-3}-x^{-5}+14-7x^{-2}$$

$$\Rightarrow \boxed{-4x^{-5}-9x^{-4}-4x^{-3}-7x^{-2}+14}$$

(Q18) $f(x) = x^{-5}(x^2+2x)(4-3x)(2x^9+1)$

$$x^{-5}(2x^9+1)(x^2+2x)(4-3x)$$

$$(2x^4+x^{-5})(4x^2-3x^3+8x-6x^2)$$

$$(2x^4+x^{-5})(-3x^3-2x^2+8x)$$

$$(-3x^3-2x^2+8x)\frac{d}{dx}(2x^4+x^{-5})+(2x^4+x^{-5})\frac{d}{dx}(-3x^3-2x^2+8x)$$

$$(-3x^3-2x^2+8x)(8x^3-5x^{-6})+(2x^4+x^{-5})(-9x^2-4x+8)$$

$$\Rightarrow 8x^3(-3x^3-2x^2+8x)-5x^{-6}(-3x^3-2x^2+8x)+2x^4(-9x^2-4x+8)+x^{-5}(-9x^2-4x+8)$$

$$\Rightarrow -24x^6-16x^5+64x^4+15x^{-3}+10x^{-4}-40x^{-5}-18x^6-8x^5+16x^4-9x^{-3}-4x^{-4}+8x^{-5}$$

$$\Rightarrow \boxed{-42x^6-24x^5+80x^4+6x^{-3}+6x^{-4}-32x^{-5}}$$

(Q19) $f(x) = (x^7 + 2x - 3)^3$

$3(x^7+2x-3)^2 \frac{d}{dx}(x^7+2x-3)$

$3(x^7+2x-3)^2(7x^6+2)$

$3\{(x^7)^2+(2x)^2+(-3)^2+2(2x^8 - 6x - 3x^7)\}(7x^6+2)$

$3\{\cancel{x^{14}+4x^4+9+2(4x^8-6}$

$3\{x^{14}+4x^2+9+4x^8-12x-6x^7\}.(7x^6+2)$

$(3x^{14}+12x^2+27+12x^8-36x-18x^7)(7x^6+2)$

$7x^6(3x^{14}+12x^2+27+12x^8-36x-18x^7)+2(3x^{14}+12x^2+27+12x^8$
$-36x-18x^7)$

$\Rightarrow 21x^{20}+84x^8+189+84x^{14}-252x^7-126x^{13}+6x^{14}+24x^2+54$
$+24x^8-72x-36x^7.$

$$\Rightarrow \boxed{21x^{20}+90x^{14}-126x^{13}+108x^8-36x^7+24x^2-334x+243}$$

Ans.

(20) $f(x) = (x^2+1)^4$.

$\Rightarrow 4(x^2+1)^3 \cdot 2x$

$\Rightarrow \boxed{8x(x^2+1)^3}$

(Q21)

$y = \left.\dfrac{2x-1}{x+3}\right|_{x=1}$.

$$\frac{dy}{dx} = \frac{(x+3)\frac{d}{dx}(2x-1) - (2x-1)\frac{d}{dx}(x+3)}{(x+3)^2}$$

$$= \frac{(x+3)2 - (2x-1)}{(x+3)^2}$$

at $x = 1$.

$\Rightarrow \dfrac{8-1}{16} = \boxed{\dfrac{7}{16}}$

(Q22) $y = \dfrac{4x+1}{x^2-5}$

$$\Rightarrow \frac{(x^2-5)\frac{d}{dx}(4x+1) - (4x+1)\frac{d}{dx}(x^2-5)}{(x^2-5)^2}$$

$$\Rightarrow \frac{(x^2-5)(4) - (4x+1)(2x)}{(x^2-5)^2}$$

Replace x by 1

$$\frac{(-4)(4) - (5)(2)}{(1-5)^2} = \frac{-16-10}{16} = \frac{-26}{16}$$

$$= \boxed{\frac{-13}{8}}$$

(23) $y = \left(\frac{3x+2}{x}\right)\left(x^{-5}+1\right)$

$= \left(\frac{3x+2}{x}\right)\left(\frac{1}{x^5}+1\right)$

$= \left(\frac{3x+2}{x}\right)\left(\frac{1+x^5}{x^5}\right)$

$= \frac{(3x^5+2x^4)(1+x^5)}{x^5}$

$= \frac{3x^5+2x^4+3x^{10}+2x^9}{x^5}$

Taking Derivative

$\Rightarrow \frac{x^5\frac{d}{dx}(3x^5+2x^4+3x^{10}+2x^9) - (3x^5+2x^4+3x^{10}+2x^9)\frac{d}{dx}x^5}{x^{10}}$

$\Rightarrow \frac{x^5(15x^4+8x^3+30x^9+18x^8) - 5x^4(3x^5+2x^4+3x^{10}+2x^9)}{x^{10}}$

Replace x by 1.

$\Rightarrow \frac{(15+8+30+18) - 5(3+2+3+2)}{1}$

$\Rightarrow \frac{71-50}{1} = \boxed{21}$ Ans

(Q24) $y = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$

$= \cancel{(x+1)(2x^7-(x^2)}$

$y = (2x^7 - x^2)\frac{d}{dx}\left(\frac{x-1}{x+1}\right) + \left(\frac{x-1}{x+1}\right)\frac{d}{dx}(2x^7 - x^2)$

$\left(\frac{x-1}{x+1}\right)(14x^6 - 2x) + \frac{(2x^7 - x^2)}{(x+1)^2}\left\{(x+1)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+1)\right\}$

$\frac{(x-1)}{(x+1)}(14x^6 - 2x) + \frac{(2x^7 - x^2)}{(x+1)^2}(\cancel{x}+1 - \cancel{x}+1)$

$\frac{(x-1)}{(x+1)}(14x^6 - 2x) + 2\frac{(2x^7 - x^2)}{(x+1)^2}$

Replacing x by 1.

$\frac{(1-1)}{(1+1)}(14(1)^6 - 2(1)) + 2\frac{(2(1)^7 - (1)^2)}{(1+1)^2}$

$0 + \frac{\cancel{2}}{\cancel{2}}(2-1)$

$= \boxed{1}$ Ans.

Ex 2.5

Q1) $f(x) = 4\cos x + 2\sin x$

$$\boxed{f(x)' = -4\sin x + 2\cos x}$$

Q2) $f(x) = \frac{5}{x^2} + \sin x$

$= 5x^{-2} + \sin x$

$$\boxed{f(x)' = \frac{-10}{x^3} + \cos x}$$

Q3) $\cancel{f(x) = -4x^2\cos x}$

$\cancel{f'(x) = +8x\sin x}$

Q4) $f(x) = 2\sin^2 x$

$= 4\sin x\cos x$

$$\boxed{f(x) = 2\sin 2\cancel{\theta}x}$$

Q3) $f(x) = -4x^2\cos x$

$= -4x^2(-\sin x) + \cos x(-8x)$

$$\boxed{= 4x^2\sin x - 8x\cos x}$$

Q5) $f(x) = \frac{5-\cos x}{5+\sin x}$

$\Rightarrow \frac{(5+\sin x)\frac{d}{dx}(5-\cos x) - (5-\cos x)\frac{d}{dx}(5+\sin x)}{(5+\sin x)^2}$

$\Rightarrow \frac{(5+\sin x)(\sin x) - (5-\cos x)(\cos x)}{(5+\sin x)^2}$

$\Rightarrow \frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$

$$\Rightarrow \boxed{\frac{5\sin x - 5\cos x + 1}{(5+\sin x)^2}}$$

Q6) $f(x) = \frac{\sin x}{x^2 + \sin x}$

$\frac{(x^2+\sin x)\frac{d}{dx}\sin x - \sin x\frac{d}{dx}(x^2+\sin x)}{(x^2+\sin x)^2}$

$\frac{\cos x(x^2+\sin x) - \sin x(2x+\cos x)}{(x^2+\sin x)^2}$

$\frac{x^2\cos x + \cancel{\sin x\cos x} - 2x\sin x + -\cancel{\cos x}\sin x}{(x^2+\sin x)^2}$

$$= \boxed{\frac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}}$$

(Q7) $f(x) = \sec x - \sqrt{2}\tan x$

$f'(x) = \sec x\tan x - \{\sqrt{2}\sec^2 x\} \cancel{+\tan x}$

$= \boxed{\sec x\tan x - \sqrt{2}\sec^2 x}$

Ans.

(Q8) $f(x) = (x^2+1)\sec x$

$f'(x) = (x^2+1)\sec x\tan x + \sec x(2x)$

$\boxed{= x^2\sec x\tan x + \sec x\tan x + 2x\sec x}$

Ans.

(Q9) $f(x) = 4\csc x - \cot x$

$f'(x) = 4(-\csc x\cot x) + \operatorname{cosec}^2 x$

$= \boxed{-4\operatorname{cosec} x\cot x + \operatorname{cosec}^2 x}$

(Q10) $f(x) = \cos x - x\csc x$

$= \boxed{-\sin x + x\csc x\tan x - \csc x}$

(Q11) ~~$f(x) = \sec x\tan x$~~ $f(x) = \sec x\tan x$

$\cancel{= \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}}$ $f'(x) = \sec x\sec^2 x + \tan x\sec x\tan x$

$\boxed{= \sec^3 x + \tan^2 x\sec x}$

$= \frac{\sin}{\cos^2 x}$

$\cancel{= \sec x(\sec^2 x + \tan^2 x)}$

$\cancel{= \sec x(1 - \tan^2 x + \tan^2 x)}$

$\cancel{= \boxed{\sec x}}$

$=$

(Q12) $f(x) = \csc x \tan x$

$\csc x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \csc x$

$\csc x (\sec^2 x) + \tan x (-\csc x \cot x)$

$\csc x \sec^2 x - \cancel{\tan x} \cot x \csc x$

$\boxed{\csc x (\sec^2 x - 1)}$

$\boxed{- \csc x \tan^2 x}$ Ans

$- \frac{1}{\cancel{\sin x}} \frac{\sin^{\cancel{2}} x}{\cos^2 x}$

$\boxed{- \sec^2 x \sin x}$ Ans.

(Q13) $f(x) = \frac{\cot x}{1 + \csc x}$

$\frac{(1+\csc x) \frac{d}{dx} \cot x - \cot x \frac{d}{dx} (1+\csc x)}{(1+\csc x)^2}$

$\frac{(1+\csc x)(-\csc^2 x) - \cot x (-\csc x \cot x)}{(1+\csc x)^2}$

$\frac{-\csc^2 x - \csc^3 x + \csc x \cot x}{(1+\csc x)^2}$

$\Rightarrow \boxed{\frac{-\csc x (\csc x + \csc^2 x + \cot x)}{(1+\csc x)^2}}$ Ans

(Q14) $f(x) = \frac{\sec x}{1+\tan x}$.

$$\frac{(1+\tan x)\frac{d}{dx}\sec x - \sec x \frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$\frac{(1+\tan x)(\sec x \tan x) - \sec x\,(\sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$$

$$= \boxed{\frac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}} \text{ Ans.}$$

(Q15) $f(x) = \sin^2 x + \cos^2 x$

$2\sin x\cos x + 2\cos x(-\sin x)$

$2\sin x\cos x - 2\cos x\sin x$

$= \boxed{0}$ Ans.

(Q16) $f(x) = \sec^2 x - \tan^2 x$

$= 2\sec x(\sec x\tan x) - 2\tan x(\sec^2 x)$

$= 2\sec^2 x\tan x - 2\tan x\sec^2 x$

$= \boxed{0}$ Ans.

(Q17) $\dfrac{\sin x \sec x}{1 + x\tan x}$

$$\dfrac{\sin x \dfrac{1}{\cos x}}{1 + x\tan x}$$

$$\dfrac{\tan x}{1 + x\tan x}$$

$$\dfrac{(1+x\tan x)\frac{d}{dx}\tan x - \tan x\, \frac{d}{dx}(1+x\tan x)}{(1+x\tan x)^2}$$

$$\dfrac{\sec^2 x(1+x\tan x) - \tan x(\tan x + x\sec^2 x)}{(1+x\tan x)^2}$$

$$\dfrac{\sec^2 x + \cancel{x\sec^2 x\tan x} - \tan^2 x - \cancel{x\sec^2 x \cdot \tan x}}{(1+x\tan x)^2}$$

$$= \boxed{\dfrac{\sec^2 x - \tan^2 x}{(1+x\tan x)^2}}$$

(Q18) $f(x) = \dfrac{(x^2+1)\cot x}{3 - \cos x \csc x}$

$$\frac{(3-\cos x\csc x)\frac{d}{dx}(x^2+1)\cot x - \cot x(x^2+1)\frac{d}{dx}(3-\cos x\csc x)}{(3-\cos x\csc x)^2}$$

$$\frac{(3-\cos x\csc x)\{\cot x\frac{d}{dx}(x^2+1)+(x^2+1)\frac{d}{dx}\cot x\} - \cot x(x^2+1)\{0-\frac{d}{dx}\cos x\csc x}{(3-\cos x\csc x)^2}$$

$$\frac{(3-\cos x\csc x)\{2x\cot x - \csc^2 x(x^2+1)\} - \cot x(x^2+1).\{-(\cos x\frac{d}{dx}\csc x + \csc x\frac{d}{dx}\cos x)\}}{(3-\cos x\csc x)^2}$$

$$= \frac{(3-\cos x\csc x)(2x\cot x - \csc^2 x\, x^2 - \csc^2 x) + \cot x(x^2+1)\{\cos x\csc x\cot x + \csc x(-\sin x)\}}{(3-\cos x\csc x)^2}$$

$$\frac{(3-\cos x\csc x)(2x\cot x - \csc^2 x\, x^2 - \csc^2 x) + \cot x(x^2+1)(-\cos x\csc x - \sin x}{(3-\cos x\csc x)^2}$$

$$\Rightarrow \frac{(3-\cos x\csc x)(2x\cot x - \csc^2 x^2 - \csc^2 x) + -\csc x\cot x(x^2+1)(\cos x + \sin x)}{(3-\cos x\csc x)^2}$$

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(Q19) $y = x\cos x$

$y' = x\frac{d}{dx}\cos x + \cos x\frac{dx}{dx}$

$= -x\sin x + \cos x.$

$y'' = -\sin x - \frac{d}{dx}x.\sin x$

$y'' = -\sin x - (x\cos x + \sin x)$

$\boxed{y'' = -\sin x - \sin x - x\cos x}$

(Q20) $y = \csc x.$

$\frac{dy}{dx} = -\csc x\cot x.$

$y'' = -\{\csc x\frac{d}{dx}\cot x + \cot x\frac{d}{dx}\csc x\}$

$= -\{\csc x(-\csc^2 x) + \cot x(-\csc x\cot x)\}$

$= -\{-\csc^3 x - \csc\cot^2 x\}$

$= \csc^3 x - \csc\cot^2 x$

$= \boxed{\csc x(\csc^2 x - \cot^2 x)}$

(Q21) $y = x\sin x - 3\cos x$.

$y' = 3\sin x + \frac{d}{dx}\, x\sin x$

$= 3\sin x + (\sin x + x\cos x)$

$= 3\sin x + \sin x + x\cos x$

$= 4\sin x + x\cos x$

$y'' = 4\cos x + \{-\sin x + \cancel{x}\cos x\}$

$= 4\cos x + \cos x - \sin x$

$= \boxed{5\cos x - \sin x}$

(Q22) $y = x^2\cos x + 4\sin x$

$y' = \cancel{+}4\cos x + \{x^2(-\sin x) + \cos x(2x)\}$

$y' = \cancel{+}4\cos x - x^2\sin x + 2x\cos x.$

$y'' = -4\sin x - \{x^2\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x^2\} + \cancel{2x}\, 2\{x\frac{d}{dx}\cos x + \cos x\frac{dx}{dx}$

$= -4\sin x - \{x^2\cos x + 2x\sin x\} + 2\{-x\sin x + \cos x\}.$

$= -4\sin x - x^2\cos x - 2x\sin x - 2x\sin x + 2\cos x$

$= \boxed{-4\sin x - 4x\sin x - x^2\cos x + 2\cos x}$

or.

$= \boxed{\cos x(2 - x^2) - 4\sin x(1+x)}$

(Q23) $y = \sin x \cos x$

$y' = \cos x(-\cos x) + \sin x(\sin x)$

$= -\cos^2 x + \sin^2 x$

$y'' = -2\cos x(-\sin x) + 2\sin x \cos x$

$= 2\cos x \sin x + 2\sin x \cos x$

$= \boxed{2(2\cos x \sin x)}$

$= \boxed{2\sin 2x}$ Ans.

(Q24) $y = \tan x$

$y' = \sec^2 x$

$y'' = 2\sec x \sec x \tan x$

$= \boxed{2\sec^2 x \tan x}$

$\boxed{\text{Ex } 2.6}$

(Q7) $f(x) = (x^3+2x)^{37}$

let $u = x^3+2x$.

$\frac{du}{dx} = 3x^2+2$.

$\frac{dy}{du} = \frac{d}{du} u^{37}$

$= 37u^{36}$

~~replace u by~~

$\frac{dy}{dx} = 37u^{36} \cdot 3x^2+2$

$\boxed{= 37(x^3+2x)^{36}(3x^2+2)}$

(Q8) $f(x) = (3x^2+2x-1)^6$.

let $u = 3x^2+2x-1$

$\frac{du}{dx} = 6x+2$

$y = u^6$.

$\frac{dy}{du} = 6u^5$.

$\frac{dy}{dx} = (6x+2) \cdot 6u^5$.

$= \boxed{6(3x^2+2x-1)^5 \cdot (6x+2)}$

(Q9) $f(x) = \left(x^3 - \frac{7}{x}\right)^{-2}$

$u = x^3 - \frac{7}{x}$

$\frac{du}{dx} = 3x^2 + \frac{7}{x^2}$

Let $y = u^{-2}$

$\frac{dy}{du} = -2u^{-3}$

$\left(3x^2 + \frac{7}{x^2}\right)\left(-2u^{-3}\right)$

$\boxed{-2\left(x^3 - \frac{7}{x}\right)^{-3}\left(3x^2 + \frac{7}{x^2}\right)}$

(Q10) $f(x) = \frac{1}{(x^5 - x + 1)^9}$

let $u = x^5 - x + 1$

$\frac{du}{dx} = 5x^4 - 1$

$y = \frac{1}{u^9} = u^{-9}$

$\frac{dy}{du} = -9u^{-10} = \frac{-9}{u^{10}}$

$\frac{dy}{dx} = (5x^4 - 1)\left(\frac{-9}{u^{10}}\right)$

Replace u

$\boxed{= -(5x^4 - 1)\left(\frac{9}{(x^5 - x + 1)^{10}}\right)}$

(Q 11) $f(x) = \frac{4}{(3x^2-2x+1)^3}$.

let $u = 3x^2-2x+1$

$\frac{du}{dx} = 6x-2$.

$y = \frac{4}{u^3} = 4u^{-3}$

$\frac{dy}{du} = \frac{-12}{u^4}$.

$\frac{dy}{dx} = \left(\frac{-12}{u^4}\right)(6x-2)$

$$\boxed{\frac{dy}{dx} = \frac{-12}{(3x^2-2x+1)^4} \cdot (6x-2)}$$

(Q12) $\sqrt{x^3-2x+5}$

let $u = x^3-2x+5$

$\frac{du}{dx} = 3x^2-2$.

$y = u^{1/2}$.

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{u}}(3x^2-2)$

$$\boxed{= \frac{1}{2\sqrt{x^3-2x+5}}(3x^2-2)}$$

(Q13) $\sqrt{4+\sqrt{3x}}$

let $u = 4+\sqrt{3x}$

$$\frac{du}{dx} = \frac{1}{2}(3x)^{-1/2}$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{3x}}$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}}.$$

$$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{u}}\right)\left(\frac{1}{2\sqrt{3x}}\right)$$

$$\boxed{\frac{dy}{dx} = \left(\frac{1}{2\sqrt{3x}}\right)\left(\frac{1}{2\sqrt{4+\sqrt{3x}}}\right)}$$ Ans.

(Q14) $f(x) = \sqrt[3]{12+\sqrt{x}}$

let $u = 12+\sqrt{x}$.

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$y = u^{1/3}$$

$$\frac{dy}{du} = \frac{1}{3}u^{-2/3} = \frac{1}{3u^{2/3}}$$

$$\frac{dy}{dx} = \frac{1}{3u^{2/3}} \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\cancel{2}6\sqrt{x}\,(12+\sqrt{x})^{2/3}}}$$ Ans

(Q15) $f(x) = \sin\left(\frac{1}{x^2}\right)$

let $u = \frac{1}{x^2}$

$\frac{du}{dx} = \frac{-2}{x^3}$

$y = \sin u$

$\frac{dy}{du} = \cos u.$

$\boxed{\frac{dy}{dx} = \frac{-2}{x^3} \cos(1/x^2)}$

(Q16) $f(x) = \tan\sqrt{x}$

let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}.$

$y = \tan u.$

$\frac{dy}{du} = \sec^2 u.$

$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \sec^2(\sqrt{x})}$

(Q17) $4\cos^5 x.$

let. $u = x.$

$\frac{du}{dx} = 1.$

$y = 4\cos^5 u.$

$\frac{dy}{du} = 20\cos^4 u \sin u.$

$\boxed{\frac{dy}{dx} = 20\cos^4(x)\sin(x)}$

OR.

let $u = \cos^5 x$

$\frac{du}{dx} = 5\cos^4 x \sin x$

$y = 4u.$

$\frac{dy}{du} = 4.$

$\frac{dy}{dx} = 4(5\cos^4 x \sin x)$

$\boxed{\frac{dy}{dx} = 20\cos^4 x \sin x}$

(Q18) $f(x) = 4x + 5\sin^4 x.$

let $u = x$

$\frac{du}{dx} = 1.$

$y = 4u + 5\sin^4 u.$

$\frac{dy}{du} = 4 + 5(4\sin^3 u)(\cos u)$

$\frac{dy}{du} = 4 + 20\sin^3 u \cos u$

$$\boxed{\frac{dy}{dx} = 4 + 20 \cdot \sin^3 x \cos x}$$

(Q19) $f(x) = \cos^2(3\sqrt{x})$

let $u = 3\sqrt{x}.$

$\frac{du}{dx} = \frac{3}{2\sqrt{x}}$

$y = \cos^2 x$

$\frac{dy}{du} = 2\cos u \sin u$

$\frac{dy}{dx} = (\cancel{2}\cos u \sin u)\left(\frac{3}{\cancel{2}\sqrt{x}}\right)$

$$\boxed{\frac{dy}{dx} = \frac{3\cos 3\sqrt{x}\sin 3\sqrt{x}}{\sqrt{x}}}$$ Ans.

(Q20) $f(x) = \tan^4(x^3)$

let $u = x^3$

$\frac{du}{dx} = 3x^2$.

$y = \tan^4 u$.

$\frac{dy}{du} = 4\tan^3 u \sec^2 u$.

$\frac{dy}{dx} = (4\tan^3 u \sec^2 u)(3x^2)$

$\boxed{= 12x^2 \tan^3(x^3)\sec^2(x^3)}$ Ans.

$\cancel{= 3x^2 (x^3)(\tan^3(x^3))(\sec^2(x^3))}$

$\cancel{= 3x^5 \tan^9 x^9 \sec^6 x}$

(Q21) $f(x) = 2\sec^2(x^7)$

let $u = x^7$

$\frac{du}{dx} = 7x^6$

$y = 2\sec^2 u$.

$\frac{dy}{du} = 4\sec u \sec u \tan u$

$= 4\sec^2 u \tan u$

$\frac{dy}{dx} = (4\sec^2 u \tan u)(7x^6)$

$\boxed{= 28x^6 \sec^2(x^7)\tan(x^7)}$

Ans

(Q22) $f(x) = \cos^3\left(\dfrac{x}{x+1}\right)$

let $u = \dfrac{x}{x+1}$

$$\frac{du}{dx} = \frac{(x+1)\frac{d}{dx}x + x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\frac{du}{dx} = \frac{(x+1)+x}{(x+1)^2} = \frac{2x+1}{(x+1)^2}$$

$y = \cos^3 u$

$$\frac{dy}{du} = 3\cos^2 u \cdot (-\sin u)$$

$$= -3\cos^2 u \sin u$$

$$\frac{dy}{dx} = -3\cos^2 u \sin u \left(\frac{2x+1}{(x+1)^2}\right)$$

$$= \boxed{-3\left(\frac{2x+1}{(x+1)^2}\right)\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}$$ Ans.

DERIVATIVE (2.6)

(Q23) $f(x) = \sqrt{\cos(5x)}$

let $u = \cos 5x$

$\frac{du}{dx} = -\sin 5x \, (5)$

$= -5 \sin 5x$

$y = \sqrt{u}$.

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$\frac{dy}{dx} = -5 \sin 5x \left(\frac{1}{2\sqrt{u}} \right)$

$\frac{dy}{dx} = \boxed{-5 \sin 5x \quad \frac{1}{2\sqrt{\cos 5u}}}$ Ans

(Q24) $f(x) = \sqrt{3x - \sin^2(4x)}$

let $u = 3x - \sin^2(4x)$

$\frac{du}{dx} = 3 - 2\sin 4x \cos 4x \, (4)$

$= 3 - 8 \sin 4x \cos 4x$

$y = \sqrt{u}$

$y = \frac{1}{2\sqrt{u}}$.

$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \, (3 - 8 \sin 4x \cos 4x)$

$= \boxed{\frac{1}{2\sqrt{3x - \sin^2(4x)}} \, (3 - 8 \sin 4x \cos 4x)}$ Ans

(Q25) $f(x) = \left[x + \csc(x^3+3)\right]^{-3}$

let $u = x + \csc(x^3+3)$

$$\frac{du}{dx} = 1 + \left(-\csc(x^3+3)\tan(x^3+3) \cdot 3x^2\right)$$

$$\frac{du}{dx} = 1 - 3x^2 \csc(x^3+3)\tan(x^3+3)$$

$$y = u^{-3}$$

$$\frac{dy}{du} = -3u^{-4}$$

$$= \frac{-3}{u^4}$$

$$\frac{dy}{dx} = \frac{-3}{u^4}\left(1 - 3x^2 \csc(x^3+3)\tan(x^3+3)\right)$$

$$\boxed{\frac{dy}{dx} = \frac{-3}{x + \csc(x^3+3)}\left(1 - 3x^2 \csc(x^3+3)\tan(x^3+3)\right)}$$

Ans.

(Q26) $f(x) = [x^4 - \sec(4x^2-2)]^{-4}$.

let $u = x^4 - \sec(4x^2-2)$

$$\frac{du}{dx} = 4x^3 - \sec(4x^2-2)\tan(4x^2-2)\ (8x)$$

$$= 4x^3 - 8x\ \sec(4x^2-2)\tan(4x^2-2)$$

$$y = u^{-4}$$

$$\frac{dy}{du} = \frac{-4}{4^5}$$

$$\frac{dy}{dx} = \frac{-4}{u^5}\ \left(4x^3 - 8x\ \sec(4x^2-2)\tan(4x^2-2)\right)$$

$$\boxed{\frac{dy}{dx} = \frac{-4}{[x^4 - \sec(4x^2-2)]} \cdot \left(4x^3 - 8x\ \sec(4x^2-2)\tan(4x^2-2)\right)}$$

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(Q27) $y = x^3 \sin^2(5x)$

Taking d/dx on b/s.

$$\frac{dy}{dx} = \frac{d}{dx}\left\{x^3 \cdot \sin^2(5x)\right\}.$$

$$\frac{dy}{dx} = x^3 \frac{d}{dx} \sin^2(5x) + \sin^2(5x) \frac{d}{dx} x^3$$

$$\frac{dy}{dx} = x^3 (2\sin(5x))(5) + \sin^2(5x)\,(3x^2)$$

$$\boxed{\frac{dy}{dx} = 10x^3 \sin(5x) + 3x^2 \sin^2(5x)}$$ Ans.

(Q28) $y = \sqrt{x} \tan^3(\sqrt{x})$

Taking d/dx on b/sides

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\sqrt{x} \tan^3(\sqrt{x})\right\}$$

$$\frac{dy}{dx} = \sqrt{x}\frac{d}{dx}\tan^3(\sqrt{x}) + \tan^3(\sqrt{x}) \frac{d}{dx} \sqrt{x}$$

$$\frac{dy}{dx} = \cancel{\sqrt{x}}\left(3\tan^2(\sqrt{x})\right)\left(\frac{1}{2\cancel{\sqrt{x}}}\right) + \tan^3(\sqrt{x})\frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{3}{2}\tan^2(\sqrt{x}) + \frac{\tan^3(\sqrt{x})}{2\sqrt{x}}}$$ Ans.

(Q29) $y = x^5 \sec(1/x)$

$$\frac{dy}{dx} = \sec(1/x)\frac{d}{dx}x^5 + x^5 \,{}^{d}/_{dx} \sec(x^{-1})$$

$$= 5x^4 \sec(1/x) + x^5 \sec(1/x)\tan(1/x)\left(\frac{-1}{x^2}\right)$$

$$= \boxed{5x^4 \sec(1/x) - x^3 \sec(1/x)\tan(1/x)}$$ Ans.

(Q30) $y = \dfrac{\sin x}{\sec(3x+1)}$

$$\frac{dy}{dx} = \frac{\sec(3x+1)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}\sec(3x+1)}{(\sec(3x+1))^2}$$

$$= \frac{\sec(3x+1)\cos x - 3\sin x \sec(3x+1)\tan(3x+1)}{(\sec(3x+1))^2}$$

$$= \frac{\cancel{\sec(3x+1)}\{\cos x - 3\sin x \tan(3x+1)\}}{(\sec(3x+1))^{\cancel{2}}}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x - 3\sin x \tan(3x+1)}{\sec(3x+1)}}$$ Ans.

(Q31) $y = \cos(\cos x)$

let $u = \cos x$

$$\frac{du}{dx} = -\sin x.$$

$y = \cos u.$

$$\frac{dy}{du} = -\sin u.$$

$$\frac{dy}{dx} = (-\sin u)(-\sin x)$$

$$\boxed{\frac{dy}{dx} = \sin(\cos x)\sin x}$$ Ans.

(Q32) $y = \sin(\tan 3x)$

let $u = \tan 3x$

$$\frac{du}{dx} = 3\sec^2(3x)$$

$y = \sin u.$

$$\frac{dy}{du} = \cos u.$$

$$\frac{dy}{dx} = \cos u \cdot 3\sec^2(3x)$$

$$= \cos(\tan 3x) \cdot 3\sec^2(3x)$$

$$\boxed{\frac{dy}{dx} = 3\sec^2(3x)\cos(\tan 3x)}$$ Ans

Q33) $y = \cos^3(\sin 2x)$

let $u = \sin 2x$

$$\frac{du}{dx} = 2.\cos 2x$$

$$y = \cos^3 u.$$

$$\frac{dy}{du} = 3\cos^2 u\,(-\sin u)$$

$$\frac{dy}{du} = -3\cos^2 u \sin u.$$

$$\frac{dy}{dx} = (-3\cos^2 u \sin u)(2\cos 2x)$$

$$= -3\cos^2(\sin 2u)\sin(\sin 2u)\ (2\cos 2x)$$

$$\boxed{\frac{dy}{dx} = -6\cos^2(\sin 2u)\sin(\sin 2u)(\cos 2x)}$$

Ans.

(Q 34) $y = \dfrac{1+\csc(x^2)}{1-\cot(x^2)}$

Taking d/dx on b/s.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\csc(x^2)}{1-\cot(x^2)}\right)$$

$$= \frac{1-\cot(x^2)\frac{d}{dx}\left(1+\csc(x^2)\right) - \left(1+\csc(x^2)\right)\frac{d}{dx}\left(1-\cot(x^2)\right)}{\left(1-\cot(x^2)\right)^2}$$

$$= \frac{\left(1-\cot(x^2)\right)\left(-\csc x^2\cot x^2\right)(2x) - \left(1+\csc(x^2)\right)\left(\csc^2(x^2)\right)(2x)}{\left(1-\cot(x^2)\right)^2}$$

$$\frac{dy}{dx} = \frac{-2x\left(1-\cot(x^2)\right)\left(-\csc x^2\cot x^2\right) - 2x\left(1+\csc(x^2)\right)\left(\csc^2(x^2)\right)}{\left(1-\cot(x^2)\right)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2x\csc(x^2)\left\{\left(1-\cot x^2\right)\cot x^2 - \left(1+\csc(x^2)\right)\csc(x^2)\right\}}{\left(1-\cot(x^2)\right)^2}}$$

Ans.

Q35) $y = (5x+8)^7 (1-\sqrt{x})^6$.

Taking d/dx on b/s.

$$\frac{dy}{dx} = \frac{d}{dx} (5x+8)^7 \cdot (1-\sqrt{x})^6$$

$$= (5x+8)^7 \frac{d}{dx} (1-\sqrt{x})^6 + (1-\sqrt{x})^6 \frac{d}{dx} (5x+8)^7$$

$$= (5x+8)^7 \left(\overset{3}{\cancel{6}} (1-\sqrt{x})^5 \left(\frac{-1}{\cancel{2}\sqrt{x}} \right) \right) + (1-\sqrt{x})^6 \left(7(5x+8)^6 \right)$$

$$\frac{dy}{dx} = -3 (5x+8)^7 (1-\sqrt{x})^5 + 7 (1-\sqrt{x})^6 (5x+8)^6.$$

$$\frac{dy}{dx} = (5x+8)^6 (1-\sqrt{x})^5 \{ -3(5x+8) + 7(1-\sqrt{x}) \}$$

$$\frac{dy}{dx} = (5x+8)^6 (1-\sqrt{x})^5 \{ -15x - 24 + 7 - 7\sqrt{x} \}$$

$$\boxed{\frac{dy}{dx} = (5x+8)^6 (1-\sqrt{x})^5 \{ -15x - 17 - 7\sqrt{x} \}}$$ Ans.

(Q36) $y = (x^2+x)^5 \sin^8 x.$

Taking d/dx on b/s.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2+x)^5 \cdot \sin^8 x.$$

$$= \sin^8 x \frac{d}{dx}(x^2+x)^5 + (x^2+x)^5 \frac{d}{dx}\sin^8 x$$

$$= \sin^8 x \left(5(x^2+x)^4(2x+1)\right) + (x^2+x)^5 (8\sin^7 x \cos x)$$

$$= 5\sin^8 x (x^2+x)^4(2x+1) + 8\sin^7 x \cos x (x^2+x)^5$$

$$\boxed{\frac{dy}{dx} = \sin^7 x (x^2+x)^4 \{5\sin x(2x+1) + 8\cos x(x^2+x)\}}$$

(Q37) $y = \left(\frac{x-5}{2x+1}\right)^3.$

let $u = \frac{x-5}{2x+1}$

$$\frac{du}{dx} = \frac{(2x+1)\frac{d}{dx}(x-5) - (x-5)\frac{d}{dx}(2x+1)}{(2x+1)^2}.$$

$$= \frac{(2x+1) - (x-5)(2)}{(2x+1)^2}$$

$$= \frac{\cancel{2x}+1 - \cancel{2x}+10}{(2x+1)^2}$$

$$\frac{du}{dx} = \frac{11}{(2x+1)^2}$$

37) continue

$$y = u^3$$

$$\frac{dy}{du} = 3u^2.$$

$$\frac{dy}{dx} = 3u^2 \left(\frac{11}{(2x+1)^2} \right)$$

$$\boxed{\frac{dy}{dx} = \frac{33}{(2x+1)^2} \left(\frac{x-5}{2x+1} \right)^2}$$ Ans.

$$\frac{dy}{dx} = \frac{33(x^2-10x+25)}{(2x+1)^4}.$$

$$\boxed{\frac{dy}{dx} = \frac{33x^2-330x+825}{(2x+1)^4}}$$

(Q38) $y = \left(\frac{1+x^2}{1-x^2}\right)^{17}$

let $u = \frac{1+x^2}{1-x^2}$

$$\frac{du}{dx} = \frac{(1-x^2)\frac{d}{dx}(1+x^2) - (1+x^2)\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$= \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}$$

$$= \frac{2x - \cancel{2x^3} + 2x + \cancel{2x^3}}{(1-x^2)^2}$$

$$\frac{du}{dx} = \frac{4x}{(1-x^2)^2}$$

$y = u^{17}$

$\frac{dy}{du} = 17u^{16}$

$$\frac{dy}{dx} = 17u^{16} \cdot \frac{4x}{(1-x^2)^2}$$

$$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \cdot \frac{4x}{(1-x^2)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{68x\,(1+x^2)^{16}}{(1-x^2)^{18}}}$$ Ans.

(Q39) $y = \dfrac{(2x+3)^3}{(4x^2-1)^8}$.

Taking d/dx on b/s.

$$\frac{dy}{dx} = \frac{(4x^2-1)^8 \frac{d}{dx}(2x+3)^3 - (2x+3)^3 \frac{d}{dx}(4x^2-1)^8}{((4x^2-1)^8)^2}$$

$$\Rightarrow \frac{(4x^2-1)^8 \left(3(2x+3)^2\right)(2) - (2x+3)^3\, 8(4x^2-1)^7 (8x)}{(4x^2-1)^{16}}$$

$$\Rightarrow \frac{6(4x^2-1)^8(2x+3)^2 - 64x(2x+3)^3(4x^2-1)^7}{(4x^2-1)^{16}}$$

$$\Rightarrow \frac{2(4x^2-1)^7(2x+3)^2\{3(4x^2-1) - 32x(2x+3)\}}{(4x^2-1)^{169}}$$

$$\Rightarrow \cancel{2(4} \frac{2(2x+3)^2}{(4x^2-1)^9}(12x^2-3-64x^2-96x)$$

$$\frac{dy}{dx} \Rightarrow \boxed{\frac{2(2x+3)^2}{(4x^2-1)^9}(-52x^2-96x-3)}$$

Ans.

(Q40) $y = [1 + \sin^3(x^5)]^{12}$

let $u = 1 + \sin^3(x^5)$

$\frac{du}{dx} = 3\sin^2(x^5)\cos(x^5)(5x^4)$

$\frac{du}{dx} = 15x^4\sin^2(x^5)\cos(x^5)$

$y = u^{12}$

$\frac{dy}{du} = 12u^{11}$

$\frac{dy}{dx} = 12u^{11}\left(15x^4\sin^2(x^5)\cos(x^5)\right)$

$\boxed{\frac{dy}{dx} = 12\left(1+\sin^3(x^5)\right)^{11}\left(15x^4\sin^2(x^5)\cos(x^5)\right)}$ Ans.