Ex: 7.6

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Q@ make a u-substitution to convert the integrand to a rational fraction of u and then evaluate the integral. if you have a CAS, use it to evaluate the integral (no substitution) and then confirm that the result is equivalent to that in part (a)

65 $\int \frac{dx}{1+\sin x+\cos x}$

$u = \tan\left(\frac{x}{2}\right)$

$\tan^{-1} u = \frac{x}{2}$

$2\tan^{-1} u = x$

$dx = \frac{2}{1+u^2}\, du$

$\therefore \sin x = \frac{2u}{u^2+1}$

$\cos x = \frac{1-u^2}{u^2+1}$

put all values in question

$\int \frac{\frac{2}{1+u^2}\, du}{1+\left(\frac{2u}{u^2+1}\right)+\left(\frac{1-u^2}{u^2+1}\right)}$

$\int \frac{2\, du}{u^2+1+2u+1-u^2}$

$\int \frac{2\, du}{2u+2}$

$\int \frac{du}{u+1}$

$\ln(u+1) + C$

$\ln\left[\tan\left(\frac{x}{2}\right)+1\right] + C$

66 $\int \frac{dx}{2+\sin x}$

$\int \frac{\frac{2}{1+u^2}\, du}{2+\frac{2u}{1+u^2}}$

$\int \frac{\frac{2}{1+u^2}\, du \div \frac{2[(1+u^2)+2u]}{1+u^2}}{}$

$\int \frac{du}{1+u^2+u}$

$\int \frac{du}{(u+1/2)^2-\frac{1}{4}+1}$ $\quad \therefore u^2+u+1 = (u)^2+2(u)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2+1$

$\int \frac{du}{(u+1/2)^2+3/4}$

$\int \frac{du}{(u+1/2)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

formula

$\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$

$\frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{u+1/2}{\sqrt{3}/2}\right)+C$

$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2u+1}{\sqrt{3}}\right)+C$

$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\left(\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{3}}\right)+C$

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67 $\int \frac{d\theta}{1-\cos\theta}$

$\int \frac{\frac{2}{1+u^2}\,du}{1-\frac{1-u^2}{u^2+1}}$

$\int \frac{2\,du}{u^2+1-1+u^2}$

$\int \frac{2\,du}{2u^2}$

$\int \frac{du}{u^2}$

$\int u^{-2}\,du$

$-u^{-1}+C$

$-\frac{1}{u}+C$

$-\frac{1}{\tan(\frac{\theta}{2})}+C$

$-\cot(\theta/2)+C$

68 $\int \frac{dx}{4\sin x-3\cos x}$

$u=\tan\left(\frac{x}{2}\right)$

$2\tan^{-1}u=x$

$\frac{2}{1+u^2}\,du=dx$

$\sin x=\frac{2u}{u^2+1}$

$\cos x=\frac{1-u^2}{u^2+1}$

$\int \frac{\frac{2}{1+u^2}\,du}{4\left(\frac{2u}{u^2+1}\right)-3\left(\frac{1-u^2}{u^2+1}\right)}$

$\int \frac{2\,du}{8u-3+3u^2}$

$3\left(u^2+2(u)\left(\frac{8}{6}\right)+\left(\frac{8}{6}\right)^2-\left(\frac{8}{6}\right)^2-1\right)$

$3\left\{(u+4/3)^2-\frac{25}{9}\right\}$

$\int \frac{2\,du}{3\left(u^2+\frac{8}{3}u-1\right)}$

$\frac{2}{3}\int \frac{du}{(u+\frac{4}{3})^2-\frac{25}{9}}$

$\frac{2}{3}\int \frac{du}{(u+\frac{4}{3})^2-(\frac{5}{3})^2}$

$\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$

$\frac{2}{3}\,\frac{1}{2(5/3)}\ln\left|\frac{u+4/3-5/2}{u+4/3+5/2}\right|+$

$\frac{1}{5}\ln\left|\frac{u-1/3}{u+3}\right|+C$

$\frac{1}{5}\ln\left|\frac{\tan(\frac{x}{2})-1/3}{\tan(x/2)+3}\right|+C$

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69 $\int \frac{dx}{\sin x + \tan x}$

$\int \frac{dx}{\sin x + \frac{\sin x}{\cos x}}$

$\int \frac{dx}{\sin x\left(1 + \frac{1}{\cos x}\right)}$

$\int \frac{dx}{\sin x\left(\frac{\cos x + 1}{\cos x}\right)}$

$\int \frac{\cos x\, dx}{\sin x(\cos x + 1)}$

$\int \frac{\left(\frac{1-u^2}{1+u^2}\right)\left(\frac{2}{1+u^2}\right) du}{\left(\frac{2u}{1+u^2}\right)\left(\frac{1-u^2}{1+u^2} + 1\right)}$

$\int \frac{\frac{2-2u^2}{(1+u^2)^2}\, du}{\left(\frac{2u}{1+u^2}\right)\left(\frac{1-u^2+1+u^2}{1+u^2}\right)}$

$\int \frac{2-2u^2\ du}{2u(2)}$

$\int \frac{2(1-u^2)du}{4u}$

$\frac{1}{2}\int \frac{1-u^2\, du}{u}$

$\frac{1}{2}\int \frac{1}{u} du - \int u\, du$

$\frac{1}{2}\ln|u| - \frac{1}{4}u^2 + C$

$\frac{1}{2}\ln\tan\left(\frac{x}{2}\right) - \frac{1}{4}\tan^2\left(\frac{x}{2}\right) + C$

70 $\int \frac{\sin x}{\sin x + \tan x}\, dx$

$\int \frac{\sin x\, dx}{\sin x + \frac{\sin u}{\cos x}}$

$\int \frac{\sin x\, dx}{\sin x\left(1 + \frac{1}{\cos x}\right)}$

$\int \frac{dx}{\left(1 + \frac{1}{\cos x}\right)}$

$\int \frac{\cos x\, dx}{\cos x + 1}$

$\int \frac{\left(\frac{1-u^2}{u^2+1}\right)\left(\frac{2}{1+u^2}\right) du}{\frac{1-u^2}{1+u^2} + 1}$

$\int \frac{1-u^2\left(\frac{2}{1+u^2}\right) du}{1-u^2+1+u^2}$

$\int \frac{1-u^2}{2} \cdot \frac{2}{1+u^2} du$

$\int \frac{1-u^2}{1+u^2} du$

$-\int \frac{u^2-1}{1+u^2} du$

$-\int \frac{u^2+1-1-1}{u^2+1}$

$-\int \frac{u^2+1-2}{u^2+1}$

$-\int \frac{u^2+1}{u^2+1} du + 2\int \frac{1}{u^2+1} du$

$-u + 2\tan^{-1}(u) + C$

$2\tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right) - \tan\left(\frac{x}{2}\right) + C$

$x - \tan\left(\frac{x}{2}\right) + C$