24K - 3007
BSE - 1A
Date:

Assignment #03

Q1) (a) $\{1, 2, 3, 4 \dots\}$

Airthmatic series

$a_n = n$

$\lim_{n \to \infty} n = \infty$ diverging.

Sum of all natural No.

$S_n = \frac{n(n+1)}{2}$

$\lim_{S_n \to \infty} S_n = \boxed{\infty}$ diverging

(b) $\{2, -2, 2, -2 \dots\}$

$a_n = (-2)^{n+1}$

diverging as we dont know the last term.

(c) $\{4, 7, 10, 13 \dots\}$

$a_n = 4 + (n-1)3$.

$\lim_{n \to \infty} a_n = 4 + 3(n-1)$

appyling limit.

$= \boxed{\infty}$ diverges

(d) $\{\frac{1}{2}, \frac{-1}{4}, \frac{1}{8}, \frac{-1}{16} \dots\}$

Theoretical Bases:-

$\frac{(-1)^{n+1}}{(2)^n} = a_n$.

diverging as we don't know the last term

OR. (calculative Base).

$-1\left(\frac{-1}{2}\right)^n$ its. geometric

$r = -1/2$

$|r| < 1$ (converges)

(e) $\{\frac{-1}{2}, \frac{2}{3}, \frac{-3}{4}, \frac{4}{5} \dots\}$.

$a_n = \frac{(-1)^n n}{n+1}$

diverging as we don't know the last term.

1) $\sum_{n=1} \frac{1}{2n}$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}.$

as it is harmonic
its diverging

2) $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$

By diverging test.

$\lim_{n\to\infty} \frac{n+1}{2n-3}$

$1/2 \neq 0$ diverges

3) $\sum_{n=1}^{\infty} \frac{n^2}{n^2-1}$

By diverging test

$\lim_{n\to\infty} \frac{n^2}{n^2-1}$

$= 1 \neq 0$ diverging.

4) $\sum_{n=1}^{\infty} \frac{n(n+2)}{(n+3)^2}$.

By divergence test.

$\lim_{n\to\infty} \frac{n^2+2n}{n^2+6n+9}$

$1 \neq 0$ diverging.

5) $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

$(\frac{1}{3})^n + (\frac{2}{3})^n$

By divergence test.

~~as it~~

$\lim_{n\to\infty} (1/3)^n + \lim_{n\to\infty} (2/3)^n$

$0 + 0 = 0$ inconclusive

Applying Ratio test

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\lim_{n\to\infty} \frac{\frac{1+2^{n+1}}{3^{n+1}}}{\frac{1+2^n}{3^n}}$

$\frac{1+2^{n+1}}{3\cdot\cancel{3^n}} \cdot \frac{\cancel{3^n}}{1+2^n}$

$\lim_{n\to\infty} \frac{\cancel{2^n}(1/2^n + 2)}{\cancel{2^n}(3 + 3/2^n)}$

apply limit

$\frac{2}{3} < 1$

(series converges)

7) $\sum_{n=1}^{\infty} \sqrt[n]{2}$

By divergence test.

$\lim_{n\to\infty} (2)^{1/n}$.

apply limit

$2^{1/\infty}$.

$= \boxed{1}$ (diverges)

as $U_k \neq 0$.

8) $\sum_{n=1}^{\infty} 0.8^{n-1} - 0.3^{n}$

~~By divergence test.~~

$\cancel{\lim_{n\to\infty} 0.8^{n-1} - 0.3^{n}}$.

$\cancel{\infty - \infty}$

$(0.8)^{n-1} - (0.3)^{n}$.

as both are geometric so.

$a = 0.8$ $a = 0.3$

$r = 0.8$. $r = 0.3$

Since both are geometric and.

$|r| < 1$ $|r| < 1$

(converges)

9) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$

By divergence test.

$\lim_{n\to\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$

$\lim_{n\to\infty} \ln\left[\frac{\cancel{n^2}(1+1/n^2)}{\cancel{n^2}(2+1/n^2)}\right]$.

applying limit.

$\ln\left(\frac{1}{2}\right) \neq 0$.

(diverging).

10) $\sum_{n=1}^{\infty} \cos^n(1)$

By Root test.

$\lim_{n\to\infty} (a_n)^{1/n} = (\cos(1)^n)^{1/n}$

$\Rightarrow \cos(1) < 1$ (converges)

11) $\sum_{n=1}^{\infty} \tan^{-1}(n)$

By divergence test.

$\lim_{n\to\infty} \tan^{-1}(\infty)$

$\therefore \pi/2 \neq 0$

(diverging).

12) $\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n} \right)$

$\frac{3}{5}$ + $\frac{3}{5^n} + \frac{2}{n}$.

$3\left(\frac{1}{5}\right)^n + \frac{2}{n}$

$a = 3$

$r = 1/5$

$|r| < 1$

converges.

harmonic always diverges.

The whole series diverges.

13) $\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)} \right)$

$\frac{1}{e^n} \Rightarrow \left(\frac{1}{e}\right)^n$

$a = 1 , r = 1/e$

$|r| < 1$ (converges)

$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

By partial , $\boxed{A = 1 , B = -1}$

$A(n+1) + Bn = 1$.

For A , $n = 0$. , for B , $n = -1$

$\boxed{A = 1}$ $\boxed{B = -1}$

$\frac{1}{n} + \frac{(-1)}{n+1}$

$\left(\frac{1}{n} - \frac{1}{n+1}\right)$ - Telescoping series.

$\left(1 - \cancel{1/2}\right) + \left(\cancel{1/2} - \cancel{1/3}\right) \dots \left(\cancel{1/n} - 1/n+1\right)$

$S_n = \left(1 - 1/n+1\right)$

$S_n = \left(1 - \frac{1}{n+1}\right)$

$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(1 - \frac{1}{n+1}\right)$

$= \boxed{1}$ converges

∴ The series is converging

14) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

Method 1:-

By divergence test.

$\lim_{n\to\infty} \frac{e^n}{n^2}$

$\lim_{n\to\infty} \frac{e^n}{2n}$

$\lim_{n\to\infty} \frac{e^n}{2}$

applying limit

$= \infty$ (diverges)

Method 2:-

By Ratio Test.

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{e^{n+1}}{(n+1)^2} \div \frac{e^n}{n^2}$

$= \frac{\cancel{e^n} \cdot e}{(n+1)^2} \cdot \frac{n^2}{\cancel{e^n}}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{en^2}{(n+1)^2}$

applying limit

$\boxed{= e}$

$e > 1$ (diverges)

(~~converges~~).

$\frac{2}{n^2-1}$

$(n-1)$.

$\frac{A}{(n+1)} + \frac{B}{(n-1)}$

$+ B(n+1)$

$= 1$

-1

15) $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

$a_n = \frac{2}{n^2-1}$

$b_n = 1/n^2$.

By limit comparison

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2}{n^2-1} \cdot n^2$.

$\lim_{n\to\infty} \frac{2n^2}{n^2-1}$

$= 2$

Since $1/n^2$ is p-series $p=2>1$ so converges.

Since b_n converges so orininal also converges.

16) $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$

By limit comparision

$a_n = \frac{2}{n^2+4n+3}$

$b_n = 1/n^2$.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2}{n^2+4n+3} \cdot n^2$

$= \frac{2n^2}{n^2+4n+3}$

applying limit.

$= 2$

since $1/n^2$ is p-series and $p=2>1$ (converges)

Since bn converges so original also converges.

17) $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$.

$a_n = \frac{3}{n^2+3n}$

$b_n = 1/n^2$.

By limit comparision

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{3}{n^2+3n} \cdot n^2$.

applying limit.

$= \boxed{3}$

as $1/n^2$ is p and $p=2>1$ converges

as bn converges so original also converges.

another Method:-

$\frac{3}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}$.

$3 = A(n+3) + (n)B$.

for B, $n=-3$

$\boxed{B=-1}$

for A, $n=0$

$\boxed{A=1}$

$\left(\frac{1}{n} + \frac{(-1)}{n+3}\right)$

$\left(1 - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right)\left(\frac{1}{3} - \cancel{\frac{1}{6}}\right)\left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right)\ldots$

$\ldots\left(\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+3}}\right)$.

$S_n = \left(1 + 1/2 + 1/3 + 1/n\right)$

Date:

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{n}\right)$$

$$= \frac{11}{6}.$$

(converges).

18) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ - Telescoping.

By divergence test

$\lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right)$

Applying L-hopital

$\ln(1) = 0.$

(inconclusive)

as $\ln\left(\frac{n}{n+1}\right) = \ln(n) - \ln(n+1).$

So,

$(\ln 1 - \cancel{\ln 2}) + (\cancel{\ln 2} - \cancel{\ln 3}) \dots \cancel{\ln(n)} - \ln(n+1)$

$\lim_{n\to\infty} \ln 1 - \ln(n+1)$

$0 - \infty$

∞ (diverges)

21) $\sum_{n=1}^{\infty} 6(0.9)^{n-1}$

$a = 6$, $r = 0.9$.

$|r| < 1$

converges.

19) $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$ Telescoping.

$(e^1 - \cancel{e^{1/2}}) + (\cancel{e^{1/2}} - \cancel{e^{1/3}}) + \dots$

$(\cancel{e^{1/n}} - e^{1/(n+1)})$

$\lim_{n\to\infty} e - e^{1/(n+1)}$

$e - e^0$

$= e - 1$

(converging)

20) $\sum_{n=1}^{\infty} \left(\cos\left(\frac{1}{n^2}\right) - \cos\left(\frac{1}{(n+1)^2}\right)\right)$

(Telescoping)

$\left(\cos(1) - \cancel{\cos\left(\frac{1}{(2)^2}\right)}\right)$

$\left(\cancel{\cos\frac{1}{(2)^2}} - \cancel{\cos\left(\frac{1}{(3)^2}\right)}\right) \dots\dots$

$\cancel{\cos\left(\frac{1}{n^2}\right)} - \cos\left(\frac{1}{(n+1)^2}\right)$

$\lim_{n\to\infty} \cos(1) - \cos\left(\frac{1}{(n+1)^2}\right)$

applying limit.

$= 0.99 - 1$

$= -1$ (converges)