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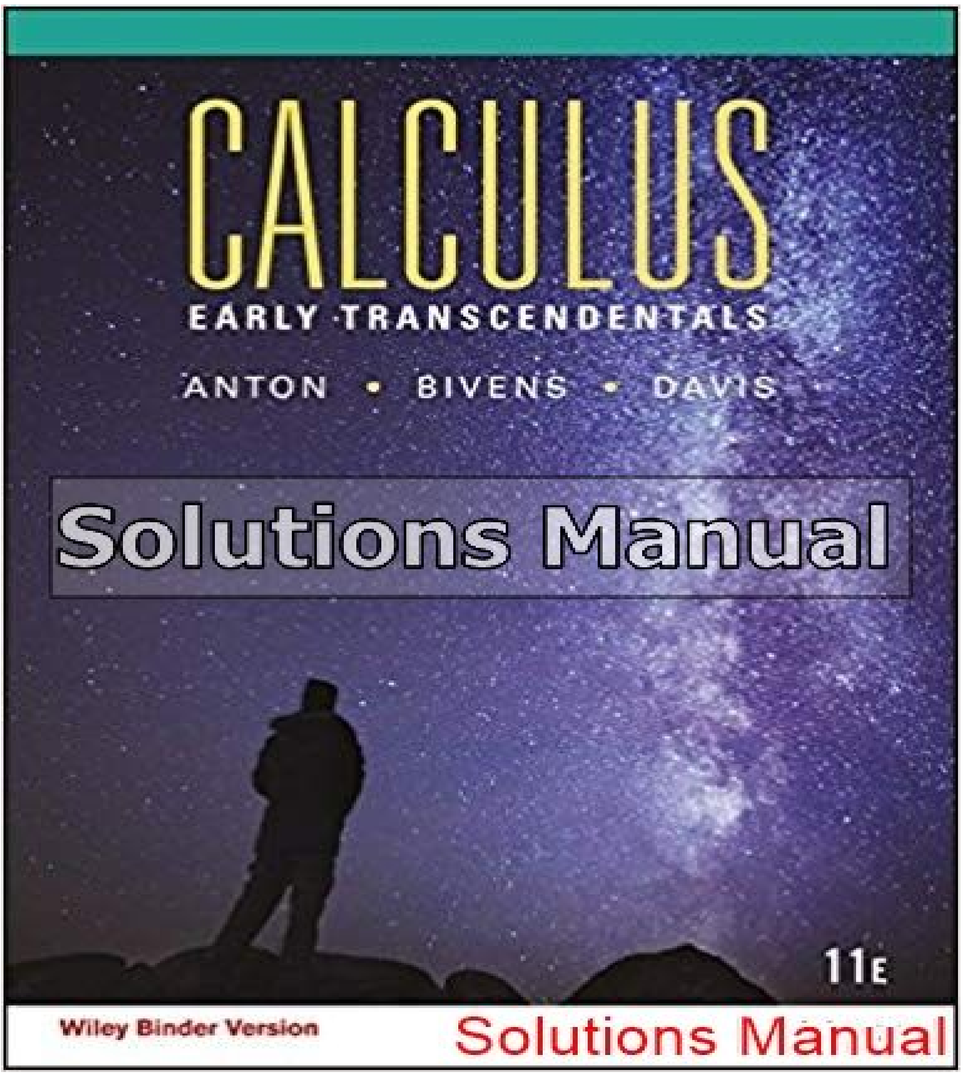

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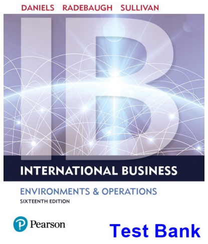

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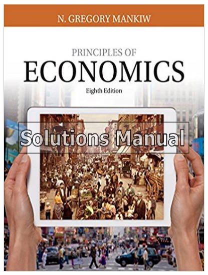

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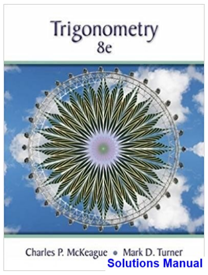

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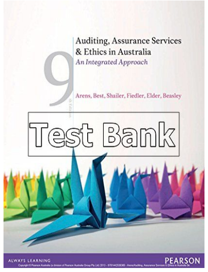

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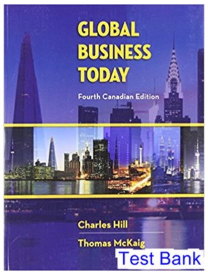

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Principles of Integral Evaluation

Exercise Set 7.1

1. $u = 4 - 2x$, $du = -2dx$, $\quad -\dfrac{1}{2}\displaystyle\int u^3\,du = -\frac{1}{8}u^4 + C = -\frac{1}{8}(4-2x)^4 + C.$

2. $u = 4 + 2x$, $du = 2dx$, $\quad \dfrac{3}{2}\displaystyle\int \sqrt{u}\,du = u^{3/2} + C = (4+2x)^{3/2} + C.$

3. $u = x^2$, $du = 2xdx$, $\quad \dfrac{1}{2}\displaystyle\int \sec^2 u\,du = \frac{1}{2}\tan u + C = \frac{1}{2}\tan(x^2) + C.$

4. $u = x^2$, $du = 2xdx$, $\quad 2\displaystyle\int \tan u\,du = -2\ln|\cos u| + C = -2\ln|\cos(x^2)| + C.$

5. $u = 2 + \cos 3x$, $du = -3\sin 3x dx$, $\quad -\dfrac{1}{3}\displaystyle\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln(2+\cos 3x) + C.$

6. $u = \dfrac{2}{3}x$, $du = \dfrac{2}{3}dx$, $\quad \dfrac{1}{6}\displaystyle\int \frac{du}{1+u^2} = \frac{1}{6}\tan^{-1}u + C = \frac{1}{6}\tan^{-1}\frac{2}{3}x + C.$

7. $u = e^x$, $du = e^x dx$, $\quad \displaystyle\int \sinh u\,du = \cosh u + C = \cosh e^x + C.$

8. $u = \ln x$, $du = \dfrac{1}{x}dx$, $\quad \displaystyle\int \sec u \tan u\,du = \sec u + C = \sec(\ln x) + C.$

9. $u = \tan x$, $du = \sec^2 x dx$, $\quad \displaystyle\int e^u\,du = e^u + C = e^{\tan x} + C.$

10. $u = x^2$, $du = 2xdx$, $\quad \dfrac{1}{2}\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u + C = \frac{1}{2}\sin^{-1}(x^2) + C.$

11. $u = \cos 5x$, $du = -5\sin 5x dx$, $\quad -\dfrac{1}{5}\displaystyle\int u^5\,du = -\frac{1}{30}u^6 + C = -\frac{1}{30}\cos^6 5x + C.$

12. $u = \sin x$, $du = \cos x\,dx$, $\quad \displaystyle\int \frac{du}{u\sqrt{u^2+1}} = -\ln\left|\frac{1+\sqrt{1+u^2}}{u}\right| + C = -\ln\left|\frac{1+\sqrt{1+\sin^2 x}}{\sin x}\right| + C.$

13. $u = e^x$, $du = e^x dx$, $\quad \displaystyle\int \frac{du}{\sqrt{4+u^2}} = \ln\left(u + \sqrt{u^2+4}\right) + C = \ln\left(e^x + \sqrt{e^{2x}+4}\right) + C.$

14. $u = \tan^{-1}x$, $du = \dfrac{1}{1+x^2}\,dx$, $\quad \displaystyle\int e^u\,du = e^u + C = e^{\tan^{-1}x} + C.$

15. $u = \sqrt{x-1}$, $du = \dfrac{1}{2\sqrt{x-1}}\,dx$, $\quad 2\displaystyle\int e^u\,du = 2e^u + C = 2e^{\sqrt{x-1}} + C.$

16. $u = x^2 + 2x$, $du = (2x+2)dx$, $\dfrac{1}{2}\displaystyle\int \cot u\, du = \dfrac{1}{2}\ln|\sin u| + C = \dfrac{1}{2}\ln\sin|x^2+2x| + C$.

17. $u = \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}\,dx$, $\displaystyle\int 2\cosh u\, du = 2\sinh u + C = 2\sinh\sqrt{x} + C$.

18. $u = \ln x$, $du = \dfrac{dx}{x}$, $\displaystyle\int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$.

19. $u = \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}\,dx$, $\displaystyle\int \frac{2\,du}{3^u} = 2\int e^{-u\ln 3}\,du = -\frac{2}{\ln 3}e^{-u\ln 3} + C = -\frac{2}{\ln 3}3^{-\sqrt{x}} + C$.

20. $u = \sin\theta$, $du = \cos\theta d\theta$, $\displaystyle\int \sec u\tan u\, du = \sec u + C = \sec(\sin\theta) + C$.

21. $u = \dfrac{2}{x}$, $du = -\dfrac{2}{x^2}\,dx$, $-\dfrac{1}{2}\displaystyle\int \operatorname{csch}^2 u\, du = \dfrac{1}{2}\coth u + C = \dfrac{1}{2}\coth\dfrac{2}{x} + C$.

22. $\displaystyle\int \frac{dx}{\sqrt{x^2-4}} = \ln\left|x + \sqrt{x^2-4}\right| + C$.

23. $u = e^{-x}$, $du = -e^{-x}dx$, $-\displaystyle\int \frac{du}{4-u^2} = -\frac{1}{4}\ln\left|\frac{2+u}{2-u}\right| + C = -\frac{1}{4}\ln\left|\frac{2+e^{-x}}{2-e^{-x}}\right| + C$.

24. $u = \ln x$, $du = \dfrac{1}{x}dx$, $\displaystyle\int \cos u\, du = \sin u + C = \sin(\ln x) + C$.

25. $u = e^x$, $du = e^x dx$, $\displaystyle\int \frac{e^x\,dx}{\sqrt{1-e^{2x}}} = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C = \sin^{-1} e^x + C$.

26. $u = x^{-1/2}$, $du = -\dfrac{1}{2x^{3/2}}\,dx$, $-\displaystyle\int 2\sinh u\, du = -2\cosh u + C = -2\cosh(x^{-1/2}) + C$.

27. $u = x^2$, $du = 2xdx$, $\dfrac{1}{2}\displaystyle\int \frac{du}{\csc u} = \frac{1}{2}\int \sin u\, du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$.

28. $2u = e^x$, $2du = e^x dx$, $\displaystyle\int \frac{2du}{\sqrt{4-4u^2}} = \sin^{-1} u + C = \sin^{-1}(e^x/2) + C$.

29. $4^{-x^2} = e^{-x^2\ln 4}$, $u = -x^2\ln 4$, $du = -2x\ln 4\,dx = -x\ln 16\,dx$, $-\dfrac{1}{\ln 16}\displaystyle\int e^u\,du = -\frac{1}{\ln 16}e^u + C = -\frac{1}{\ln 16}e^{-x^2\ln 4} + C = -\frac{1}{\ln 16}4^{-x^2} + C$.

30. $2^{\pi x} = e^{\pi x\ln 2}$, $\displaystyle\int 2^{\pi x}\,dx = \frac{1}{\pi\ln 2}e^{\pi x\ln 2} + C = \frac{1}{\pi\ln 2}2^{\pi x} + C$.

31. **(a)** $u = \sin x$, $du = \cos x\,dx$, $\displaystyle\int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$.

(b) $\displaystyle\int \sin x\cos x\,dx = \frac{1}{2}\int \sin 2x\,dx = -\frac{1}{4}\cos 2x + C = -\frac{1}{4}(\cos^2 x - \sin^2 x) + C$.

(c) $-\dfrac{1}{4}(\cos^2 x - \sin^2 x) + C = -\dfrac{1}{4}(1 - \sin^2 x - \sin^2 x) + C = -\dfrac{1}{4} + \dfrac{1}{2}\sin^2 x + C$, and this is the same as the answer in part (a) except for the constants.

32. **(a)** $\operatorname{sech} 2x = \dfrac{1}{\cosh 2x} = \dfrac{1}{\cosh^2 x + \sinh^2 x} = \dfrac{\operatorname{sech}^2 x}{1 + \tanh^2 x}$.

(b) $\displaystyle\int \operatorname{sech} 2x\, dx = \int \frac{\operatorname{sech}^2 x}{1 + \tanh^2 x}\, dx = \tan^{-1}(\tanh x) + C$, or, by substituting $u = 2x$, we obtain that $\displaystyle\int \operatorname{sech} x\, dx = 2\tan^{-1}(\tanh(x/2)) + C$.

(c) $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}} = \dfrac{2e^x}{e^{2x} + 1}$.

(d) $\displaystyle\int \operatorname{sech} x\, dx = 2\int \frac{e^x}{e^{2x} + 1}\, dx = 2\tan^{-1}(e^x) + C$.

(e) Using the identity $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x - y}{1 + xy}\right)$, the difference between the two functions obtained is the constant $\tan^{-1}(e^x) - \tan^{-1}(\tanh(x/2)) = \tan^{-1}\dfrac{e^{3x/2} + e^{-x/2}}{e^{3x/2} + e^{-x/2}} = \tan^{-1}(1) = \pi/2$.

33. **(a)** $\dfrac{\sec^2 x}{\tan x} = \dfrac{1}{\cos^2 x \tan x} = \dfrac{1}{\cos x \sin x}$.

(b) $\csc 2x = \dfrac{1}{\sin 2x} = \dfrac{1}{2\sin x \cos x} = \dfrac{1}{2}\dfrac{\sec^2 x}{\tan x}$, so $\displaystyle\int \csc 2x\, dx = \frac{1}{2}\ln\tan x + C$, then using the substitution $u = 2x$ we obtain that $\displaystyle\int \csc x\, dx = \ln(\tan(x/2)) + C$.

(c) $\sec x = \dfrac{1}{\cos x} = \dfrac{1}{\sin(\pi/2 - x)} = \csc(\pi/2 - x)$, so $\displaystyle\int \sec x\, dx = -\int \csc(\pi/2 - x)\, dx = -\ln\tan(\pi/4 - x/2) + C$.

Exercise Set 7.2

1. $u = x$, $dv = e^{-2x}dx$, $du = dx$, $v = -\dfrac{1}{2}e^{-2x}$; $\displaystyle\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$.

2. $u = x$, $dv = e^{3x}dx$, $du = dx$, $v = \dfrac{1}{3}e^{3x}$; $\displaystyle\int xe^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$.

3. $u = x^2$, $dv = e^x dx$, $du = 2x\, dx$, $v = e^x$; $\displaystyle\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx$. For $\displaystyle\int xe^x dx$ use $u = x$, $dv = e^x dx$, $du = dx$, $v = e^x$ to get $\displaystyle\int xe^x dx = xe^x - e^x + C_1$ so $\displaystyle\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + C$.

4. $u = x^2$, $dv = e^{-2x}dx$, $du = 2x\, dx$, $v = -\dfrac{1}{2}e^{-2x}$; $\displaystyle\int x^2 e^{-2x}dx = -\frac{1}{2}x^2 e^{-2x} + \int xe^{-2x}dx$. For $\displaystyle\int xe^{-2x}dx$ use $u = x$, $dv = e^{-2x}dx$ to get $\displaystyle\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$, so $\displaystyle\int x^2 e^{-2x}dx = -\frac{1}{2}x^2 e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$.

5. $u = x$, $dv = \sin 3x\, dx$, $du = dx$, $v = -\dfrac{1}{3}\cos 3x$; $\displaystyle\int x\sin 3x\, dx = -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\, dx = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$.

6. $u = x$, $dv = \cos 2x\, dx$, $du = dx$, $v = \dfrac{1}{2}\sin 2x$; $\displaystyle\int x\cos 2x\, dx = \frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\, dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$.

7. $u = x^2$, $dv = \cos x\,dx$, $du = 2x\,dx$, $v = \sin x$; $\int x^2\cos x\,dx = x^2\sin x - 2\int x\sin x\,dx$. For $\int x\sin x\,dx$ use $u = x$, $dv = \sin x\,dx$ to get $\int x\sin x\,dx = -x\cos x + \sin x + C_1$ so $\int x^2\cos x\,dx = x^2\sin x + 2x\cos x - 2\sin x + C$.

8. $u = x^2$, $dv = \sin x\,dx$, $du = 2x\,dx$, $v = -\cos x$; $\int x^2\sin x\,dx = -x^2\cos x + 2\int x\cos x\,dx$; for $\int x\cos x\,dx$ use $u = x$, $dv = \cos x\,dx$ to get $\int x\cos x\,dx = x\sin x + \cos x + C_1$ so $\int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$.

9. $u = \ln x$, $dv = x\,dx$, $du = \dfrac{1}{x}dx$, $v = \dfrac{1}{2}x^2$; $\int x\ln x\,dx = \dfrac{1}{2}x^2\ln x - \dfrac{1}{2}\int x\,dx = \dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2 + C$.

10. $u = \ln x$, $dv = \sqrt{x}\,dx$, $du = \dfrac{1}{x}dx$, $v = \dfrac{2}{3}x^{3/2}$; $\int \sqrt{x}\ln x\,dx = \dfrac{2}{3}x^{3/2}\ln x - \dfrac{2}{3}\int x^{1/2}dx = \dfrac{2}{3}x^{3/2}\ln x - \dfrac{4}{9}x^{3/2} + C$.

11. $u = (\ln x)^2$, $dv = dx$, $du = 2\dfrac{\ln x}{x}dx$, $v = x$; $\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \ln x\,dx$. Use $u = \ln x$, $dv = dx$ to get $\int \ln x\,dx = x\ln x - \int dx = x\ln x - x + C_1$ so $\int (\ln x)^2 dx = x(\ln x)^2 - 2x\ln x + 2x + C$.

12. $u = \ln x$, $dv = \dfrac{1}{\sqrt{x}}dx$, $du = \dfrac{1}{x}dx$, $v = 2\sqrt{x}$; $\int \dfrac{\ln x}{\sqrt{x}}dx = 2\sqrt{x}\ln x - 2\int \dfrac{1}{\sqrt{x}}dx = 2\sqrt{x}\ln x - 4\sqrt{x} + C$.

13. $u = \ln(3x-2)$, $dv = dx$, $du = \dfrac{3}{3x-2}dx$, $v = x$; $\int \ln(3x-2)dx = x\ln(3x-2) - \int \dfrac{3x}{3x-2}dx$, but $\int \dfrac{3x}{3x-2}dx = \int \left(1 + \dfrac{2}{3x-2}\right)dx = x + \dfrac{2}{3}\ln(3x-2) + C_1$ so $\int \ln(3x-2)dx = x\ln(3x-2) - x - \dfrac{2}{3}\ln(3x-2) + C$.

14. $u = \ln(x^2+4)$, $dv = dx$, $du = \dfrac{2x}{x^2+4}dx$, $v = x$; $\int \ln(x^2+4)dx = x\ln(x^2+4) - 2\int \dfrac{x^2}{x^2+4}dx$, but $\int \dfrac{x^2}{x^2+4}dx = \int \left(1 - \dfrac{4}{x^2+4}\right)dx = x - 2\tan^{-1}\dfrac{x}{2} + C_1$ so $\int \ln(x^2+4)dx = x\ln(x^2+4) - 2x + 4\tan^{-1}\dfrac{x}{2} + C$.

15. $u = \sin^{-1}x$, $dv = dx$, $du = 1/\sqrt{1-x^2}dx$, $v = x$; $\int \sin^{-1}x\,dx = x\sin^{-1}x - \int x/\sqrt{1-x^2}dx = x\sin^{-1}x + \sqrt{1-x^2} + C$.

16. $u = \cos^{-1}(2x)$, $dv = dx$, $du = -\dfrac{2}{\sqrt{1-4x^2}}dx$, $v = x$; $\int \cos^{-1}(2x)dx = x\cos^{-1}(2x) + \int \dfrac{2x}{\sqrt{1-4x^2}}dx = x\cos^{-1}(2x) - \dfrac{1}{2}\sqrt{1-4x^2} + C$.

17. $u = \tan^{-1}(3x)$, $dv = dx$, $du = \dfrac{3}{1+9x^2}dx$, $v = x$; $\int \tan^{-1}(3x)dx = x\tan^{-1}(3x) - \int \dfrac{3x}{1+9x^2}dx = x\tan^{-1}(3x) - \dfrac{1}{6}\ln(1+9x^2) + C$.

18. $u = \tan^{-1}x$, $dv = x\,dx$, $du = \dfrac{1}{1+x^2}dx$, $v = \dfrac{1}{2}x^2$; $\int x\tan^{-1}x\,dx = \dfrac{1}{2}x^2\tan^{-1}x - \dfrac{1}{2}\int \dfrac{x^2}{1+x^2}dx$, but $\int \dfrac{x^2}{1+x^2}dx = \int \left(1 - \dfrac{1}{1+x^2}\right)dx = x - \tan^{-1}x + C_1$ so $\int x\tan^{-1}x\,dx = \dfrac{1}{2}x^2\tan^{-1}x - \dfrac{1}{2}x + \dfrac{1}{2}\tan^{-1}x + C$.

19. $u = e^x$, $dv = \sin x\,dx$, $du = e^x dx$, $v = -\cos x$; $\int e^x\sin x\,dx = -e^x\cos x + \int e^x\cos x\,dx$. For $\int e^x\cos x\,dx$ use $u = e^x$, $dv = \cos x\,dx$ to get $\int e^x\cos x = e^x\sin x - \int e^x\sin x\,dx$, so $\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$,

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) + C_1$, $\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$.

20. $u = e^{3x}$, $dv = \cos 2x\,dx$, $du = 3e^{3x}dx$, $v = \frac{1}{2}\sin 2x$; $\int e^{3x}\cos 2x\,dx = \frac{1}{2}e^{3x}\sin 2x - \frac{3}{2}\int e^{3x}\sin 2x\,dx$. Use $u = e^{3x}$, $dv = \sin 2x\,dx$ to get $\int e^{3x}\sin 2x\,dx = -\frac{1}{2}e^{3x}\cos 2x + \frac{3}{2}\int e^{3x}\cos 2x\,dx$, so $\int e^{3x}\cos 2x\,dx = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x - \frac{9}{4}\int e^{3x}\cos 2x\,dx$, $\frac{13}{4}\int e^{3x}\cos 2x\,dx = \frac{1}{4}e^{3x}(2\sin 2x + 3\cos 2x) + C_1$, $\int e^{3x}\cos 2x\,dx = \frac{1}{13}e^{3x}(2\sin 2x + 3\cos 2x) + C$.

21. $u = \sin(\ln x)$, $dv = dx$, $du = \frac{\cos(\ln x)}{x}dx$, $v = x$; $\int \sin(\ln x)dx = x\sin(\ln x) - \int \cos(\ln x)dx$. Use $u = \cos(\ln x)$, $dv = dx$ to get $\int \cos(\ln x)dx = x\cos(\ln x) + \int \sin(\ln x)dx$ so $\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)dx$, $\int \sin(\ln x)dx = \frac{1}{2}x[\sin(\ln x) - \cos(\ln x)] + C$.

22. $u = \cos(\ln x)$, $dv = dx$, $du = -\frac{1}{x}\sin(\ln x)dx$, $v = x$; $\int \cos(\ln x)dx = x\cos(\ln x) + \int \sin(\ln x)dx$. Use $u = \sin(\ln x)$, $dv = dx$ to get $\int \sin(\ln x)dx = x\sin(\ln x) - \int \cos(\ln x)dx$ so $\int \cos(\ln x)dx = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)dx$, $\int \cos(\ln x)dx = \frac{1}{2}x[\cos(\ln x) + \sin(\ln x)] + C$.

23. $u = x$, $dv = \sec^2 x\,dx$, $du = dx$, $v = \tan x$; $\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx = x\tan x - \int \frac{\sin x}{\cos x}dx = x\tan x + \ln|\cos x| + C$.

24. $u = x$, $dv = \tan^2 x\,dx = (\sec^2 x - 1)dx$, $du = dx$, $v = \tan x - x$; $\int x\tan^2 x\,dx = x\tan x - x^2 - \int(\tan x - x)dx = x\tan x - x^2 + \ln|\cos x| + \frac{1}{2}x^2 + C = x\tan x - \frac{1}{2}x^2 + \ln|\cos x| + C$.

25. $u = x^2$, $dv = xe^{x^2}dx$, $du = 2x\,dx$, $v = \frac{1}{2}e^{x^2}$; $\int x^3e^{x^2}dx = \frac{1}{2}x^2e^{x^2} - \int xe^{x^2}dx = \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C$.

26. $u = xe^x$, $dv = \frac{1}{(x+1)^2}dx$, $du = (x+1)e^x\,dx$, $v = -\frac{1}{x+1}$; $\int \frac{xe^x}{(x+1)^2}dx = -\frac{xe^x}{x+1} + \int e^x dx = -\frac{xe^x}{x+1} + e^x + C = \frac{e^x}{x+1} + C$.

27. $u = x$, $dv = e^{2x}dx$, $du = dx$, $v = \frac{1}{2}e^{2x}$; $\int_0^2 xe^{2x}dx = \frac{1}{2}xe^{2x}\Big]_0^2 - \frac{1}{2}\int_0^2 e^{2x}dx = e^4 - \frac{1}{4}e^{2x}\Big]_0^2 = e^4 - \frac{1}{4}(e^4 - 1) = (3e^4 + 1)/4$.

28. $u = x$, $dv = e^{-5x}dx$, $du = dx$, $v = -\frac{1}{5}e^{-5x}$; $\int_0^1 xe^{-5x}dx = -\frac{1}{5}xe^{-5x}\Big]_0^1 + \frac{1}{5}\int_0^1 e^{-5x}dx = -\frac{1}{5}e^{-5} - \frac{1}{25}e^{-5x}\Big]_0^1 = -\frac{1}{5}e^{-5} - \frac{1}{25}(e^{-5} - 1) = (1 - 6e^{-5})/25$.

29. $u = \ln x$, $dv = x^2dx$, $du = \frac{1}{x}dx$, $v = \frac{1}{3}x^3$; $\int_1^e x^2\ln x\,dx = \frac{1}{3}x^3\ln x\Big]_1^e - \frac{1}{3}\int_1^e x^2dx = \frac{1}{3}e^3 - \frac{1}{9}x^3\Big]_1^e = \frac{1}{3}e^3 - \frac{1}{9}(e^3 - 1) = (2e^3 + 1)/9$.

30. $u = \ln x$, $dv = \frac{1}{x^2}dx$, $du = \frac{1}{x}dx$, $v = -\frac{1}{x}$; $\int_{\sqrt{e}}^e \frac{\ln x}{x^2}dx = -\frac{1}{x}\ln x\Big]_{\sqrt{e}}^e + \int_{\sqrt{e}}^e \frac{1}{x^2}dx = -\frac{1}{e} + \frac{1}{\sqrt{e}}\ln\sqrt{e} - \frac{1}{x}\Big]_{\sqrt{e}}^e =$

$-\frac{1}{e}+\frac{1}{2\sqrt{e}}-\frac{1}{e}+\frac{1}{\sqrt{e}}=\frac{3\sqrt{e}-4}{2e}$.

31. $u=\ln(x+2)$, $dv=dx$, $du=\frac{1}{x+2}dx$, $v=x$; $\int_{-1}^{1}\ln(x+2)dx=x\ln(x+2)\Big]_{-1}^{1}-\int_{-1}^{1}\frac{x}{x+2}dx=\ln 3+\ln 1-\int_{-1}^{1}\left[1-\frac{2}{x+2}\right]dx=\ln 3-[x-2\ln(x+2)]\Big]_{-1}^{1}=\ln 3-(1-2\ln 3)+(-1-2\ln 1)=3\ln 3-2$.

32. $u=\sin^{-1}x$, $dv=dx$, $du=\frac{1}{\sqrt{1-x^2}}dx$, $v=x$; $\int_0^{\sqrt{3}/2}\sin^{-1}x\,dx=x\sin^{-1}x\Big]_0^{\sqrt{3}/2}-\int_0^{\sqrt{3}/2}\frac{x}{\sqrt{1-x^2}}dx=\frac{\sqrt{3}}{2}\sin^{-1}\frac{\sqrt{3}}{2}+\sqrt{1-x^2}\Big]_0^{\sqrt{3}/2}=\frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right)+\frac{1}{2}-1=\frac{\pi\sqrt{3}}{6}-\frac{1}{2}$.

33. $u=\sec^{-1}\sqrt{\theta}$, $dv=d\theta$, $du=\frac{1}{2\theta\sqrt{\theta-1}}d\theta$, $v=\theta$; $\int_2^4\sec^{-1}\sqrt{\theta}d\theta=\theta\sec^{-1}\sqrt{\theta}\Big]_2^4-\frac{1}{2}\int_2^4\frac{1}{\sqrt{\theta-1}}d\theta=4\sec^{-1}2-2\sec^{-1}\sqrt{2}-\sqrt{\theta-1}\Big]_2^4=4\left(\frac{\pi}{3}\right)-2\left(\frac{\pi}{4}\right)-\sqrt{3}+1=\frac{5\pi}{6}-\sqrt{3}+1$.

34. $u=\sec^{-1}x$, $dv=x\,dx$, $du=\frac{1}{x\sqrt{x^2-1}}dx$, $v=\frac{1}{2}x^2$; $\int_1^2 x\sec^{-1}x\,dx=\frac{1}{2}x^2\sec^{-1}x\Big]_1^2-\frac{1}{2}\int_1^2\frac{x}{\sqrt{x^2-1}}dx=\frac{1}{2}[(4)(\pi/3)-(1)(0)]-\frac{1}{2}\sqrt{x^2-1}\Big]_1^2=2\pi/3-\sqrt{3}/2$.

35. $u=x$, $dv=\sin 2x\,dx$, $du=dx$, $v=-\frac{1}{2}\cos 2x$; $\int_0^\pi x\sin 2x\,dx=-\frac{1}{2}x\cos 2x\Big]_0^\pi+\frac{1}{2}\int_0^\pi\cos 2x\,dx=-\pi/2+\frac{1}{4}\sin 2x\Big]_0^\pi=-\pi/2$.

36. $\int_0^\pi(x+x\cos x)dx=\frac{1}{2}x^2\Big]_0^\pi+\int_0^\pi x\cos x\,dx=\frac{\pi^2}{2}+\int_0^\pi x\cos x\,dx$; $u=x$, $dv=\cos x\,dx$, $du=dx$, $v=\sin x$; $\int_0^\pi x\cos x\,dx=x\sin x\Big]_0^\pi-\int_0^\pi\sin x\,dx=\cos x\Big]_0^\pi=-2$, so $\int_0^\pi(x+x\cos x)dx=\pi^2/2-2$.

37. $u=\tan^{-1}\sqrt{x}$, $dv=\sqrt{x}dx$, $du=\frac{1}{2\sqrt{x}(1+x)}dx$, $v=\frac{2}{3}x^{3/2}$; $\int_1^3\sqrt{x}\tan^{-1}\sqrt{x}dx=\frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x}\Big]_1^3-\frac{1}{3}\int_1^3\frac{x}{1+x}dx=\frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x}\Big]_1^3-\frac{1}{3}\int_1^3\left[1-\frac{1}{1+x}\right]dx=\left[\frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x}-\frac{1}{3}x+\frac{1}{3}\ln|1+x|\right]_1^3=(2\sqrt{3}\pi-\pi/2-2+\ln 2)/3$.

38. $u=\ln(x^2+1)$, $dv=dx$, $du=\frac{2x}{x^2+1}dx$, $v=x$; $\int_0^2\ln(x^2+1)dx=x\ln(x^2+1)\Big]_0^2-\int_0^2\frac{2x^2}{x^2+1}dx=2\ln 5-2\int_0^2\left(1-\frac{1}{x^2+1}\right)dx=2\ln 5-2(x-\tan^{-1}x)\Big]_0^2=2\ln 5-4+2\tan^{-1}2$.

39. True.

40. False; choose $u=\ln x$.

41. False; e^x is not a factor of the integrand.

42. True; the column of $p(x)$ eventually has zero entries.

43. $t = \sqrt{x}$, $t^2 = x$, $dx = 2t\,dt$, $\displaystyle\int e^{\sqrt{x}}dx = 2\int te^t\,dt$; $u = t, dv = e^t dt, du = dt, v = e^t$, $\displaystyle\int e^{\sqrt{x}}dx = 2te^t - 2\int e^t\,dt =$ $2(t-1)e^t + C = 2(\sqrt{x}-1)e^{\sqrt{x}} + C$.

44. $t = \sqrt{x}$, $t^2 = x$, $dx = 2t\,dt$, $\displaystyle\int \cos\sqrt{x}\,dx = 2\int t\cos t\,dt$; $u = t, dv = \cos t dt, du = dt, v = \sin t$, $\displaystyle\int \cos\sqrt{x}\,dx =$ $\displaystyle 2t\sin t - 2\int \sin t dt = 2t\sin t + 2\cos t + C = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$.

45. Let $f_1(x), f_2(x), f_3(x)$ denote successive antiderivatives of $f(x)$, so that $f_3'(x) = f_2(x), f_2'(x) = f_1(x), f_1'(x) = f(x)$. Let $p(x) = ax^2 + bx + c$.

diff.		antidiff.
ax^2+bx+c		$f(x)$
	$\searrow +$	
$2ax+b$		$f_1(x)$
	$\searrow -$	
$2a$		$f_2(x)$
	$\searrow +$	
0		$f_3(x)$

Then $\displaystyle\int p(x)f(x)\,dx = (ax^2+bx+c)f_1(x) - (2ax+b)f_2(x) + 2af_3(x) + C$. Check: $\dfrac{d}{dx}[(ax^2+bx+c)f_1(x) - (2ax+b)f_2(x) + 2af_3(x)] = (2ax+b)f_1(x) + (ax^2+bx+c)f(x) - 2af_2(x) - (2ax+b)f_1(x) + 2af_2(x) = p(x)f(x)$.

46. Let I denote $\displaystyle\int e^x\cos x\,dx$. Then (Method 1)

diff.		antidiff.
e^x		$\cos x$
	$\searrow +$	
e^x		$\sin x$
	$\searrow -$	
e^x		$-\cos x$

and thus $I = e^x(\sin x + \cos x) - I$, so $I = \dfrac{1}{2}e^x(\sin x + \cos x) + C$.

On the other hand (Method 2)

diff.		antidiff.
$\cos x$		e^x
	$\searrow +$	
$-\sin x$		e^x
	$\searrow -$	
$-\cos x$		e^x

and thus $I = e^x(\sin x + \cos x) - I$, so $I = \dfrac{1}{2}e^x(\sin x + \cos x) + C$, as before.

47. Let I denote $\displaystyle\int (3x^2 - x + 2)e^{-x}\,dx$. Then

diff.		antidiff.
$3x^2-x+2$		e^{-x}
	$\searrow +$	
$6x-1$		$-e^{-x}$
	$\searrow -$	
6		e^{-x}
	$\searrow +$	
0		$-e^{-x}$

$$I = \int (3x^2 - x + 2)e^{-x} = -(3x^2 - x + 2)e^{-x} - (6x - 1)e^{-x} - 6e^{-x} + C = -e^{-x}[3x^2 + 5x + 7] + C.$$

48. Let I denote $\int (x^2 + x + 1)\sin x\, dx$. Then

diff.		antidiff.
$x^2 + x + 1$		$\sin x$
	$\searrow +$	
$2x + 1$		$-\cos x$
	$\searrow -$	
2		$-\sin x$
	$\searrow +$	
0		$\cos x$

$$I = \int (x^2+x+1)\sin x\, dx = -(x^2+x+1)\cos x + (2x+1)\sin x + 2\cos x + C = -(x^2+x-1)\cos x + (2x+1)\sin x + C.$$

49. Let I denote $\int 4x^4 \sin 2x\, dx$. Then

diff.		antidiff.
$4x^4$		$\sin 2x$
	$\searrow +$	
$16x^3$		$-\frac{1}{2}\cos 2x$
	$\searrow -$	
$48x^2$		$-\frac{1}{4}\sin 2x$
	$\searrow +$	
$96x$		$\frac{1}{8}\cos 2x$
	$\searrow -$	
96		$\frac{1}{16}\sin 2x$
	$\searrow +$	
0		$-\frac{1}{32}\cos 2x$

$$I = \int 4x^4 \sin 2x\, dx = (-2x^4 + 6x^2 - 3)\cos 2x + (4x^3 - 6x)\sin 2x + C.$$

50. Let I denote $\int x^3\sqrt{2x+1}\, dx$. Then

diff.		antidiff.
x^3		$\sqrt{2x+1}$
	$\searrow +$	
$3x^2$		$\frac{1}{3}(2x+1)^{3/2}$
	$\searrow -$	
$6x$		$\frac{1}{15}(2x+1)^{5/2}$
	$\searrow +$	
6		$\frac{1}{105}(2x+1)^{7/2}$
	$\searrow -$	
0		$\frac{1}{945}(2x+1)^{9/2}$

$$I = \int x^3\sqrt{2x+1}\, dx = \frac{1}{3}x^3(2x+1)^{3/2} - \frac{1}{5}x^2(2x+1)^{5/2} + \frac{2}{35}x(2x+1)^{7/2} - \frac{2}{315}(2x+1)^{9/2} + C.$$

51. Let I denote $\int e^{ax}\sin bx\,dx$. Then

diff.		antidiff.
e^{ax}		$\sin bx$
	$\searrow +$	
ae^{ax}		$-\dfrac{1}{b}\cos bx$
	$\searrow -$	
a^2e^{ax}		$-\dfrac{1}{b^2}\sin bx$

$I = \int e^{ax}\sin bx\,dx = -\dfrac{1}{b}e^{ax}\cos bx + \dfrac{a}{b^2}e^{ax}\sin bx - \dfrac{a^2}{b^2}I$, so $I = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$.

52. From Exercise 51 with $a=-3, b=5, x=\theta$, answer $= \dfrac{e^{-3\theta}}{34}(-3\sin 5\theta - 5\cos 5\theta) + C$.

53. **(a)** We perform a single integration by parts: $u=\cos x, dv=\sin x\,dx, du=-\sin x\,dx, v=-\cos x$, $\int \sin x\cos x\,dx = -\cos^2 x - \int \sin x\cos x\,dx$. This implies that $2\int \sin x\cos x\,dx = -\cos^2 x + C$, $\int \sin x\cos x\,dx = -\dfrac{1}{2}\cos^2 x + C$.

Alternatively, $u=\sin x, du=\cos x\,dx$, $\int \sin x\cos x\,dx = \int u\,du = \dfrac{1}{2}u^2 + C = \dfrac{1}{2}\sin^2 x + C$.

(b) Since $\sin^2 x + \cos^2 x = 1$, they are equal (although the symbol 'C' refers to different constants in the two equations).

54. **(a)** $u=x^2, dv=\dfrac{x}{\sqrt{x^2+1}}, du=2x\,dx, v=\sqrt{x^2+1}$, $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx = x^2\sqrt{x^2+1}\Big]_0^1 - \int_0^1 2x\sqrt{x^2+1}\,dx = \sqrt{2} - \frac{2}{3}(x^2+1)^{3/2}\Big]_0^1 = -\frac{1}{3}\sqrt{2}+\frac{2}{3}$.

(b) $u=\sqrt{x^2+1}, du=\dfrac{x}{\sqrt{x^2+1}}\,dx$, $\displaystyle\int_1^{\sqrt{2}}(u^2-1)\,du = \left(\frac{1}{3}u^3-u\right)\Big]_1^{\sqrt{2}} = \frac{2}{3}\sqrt{2}-\sqrt{2}-\frac{1}{3}+1 = -\frac{1}{3}\sqrt{2}+\frac{2}{3}$.

55. **(a)** $A = \displaystyle\int_1^e \ln x\,dx = (x\ln x - x)\Big]_1^e = 1$.

(b) $V = \pi\displaystyle\int_1^e (\ln x)^2 dx = \pi\Big[(x(\ln x)^2 - 2x\ln x + 2x)\Big]_1^e = \pi(e-2)$.

56. $A = \displaystyle\int_0^{\pi/2}(x - x\sin x)dx = \frac{1}{2}x^2\Big]_0^{\pi/2} - \int_0^{\pi/2} x\sin x\,dx = \frac{\pi^2}{8} - (-x\cos x + \sin x)\Big]_0^{\pi/2} = \pi^2/8 - 1$.

57. $V = 2\pi\displaystyle\int_0^{\pi} x\sin x\,dx = 2\pi(-x\cos x + \sin x)\Big]_0^{\pi} = 2\pi^2$.

58. $V = 2\pi\displaystyle\int_0^{\pi/2} x\cos x\,dx = 2\pi(\cos x + x\sin x)\Big]_0^{\pi/2} = \pi(\pi-2)$.

59. Distance $= \displaystyle\int_0^{\pi} t^3\sin t dt$;

diff.		antidiff.
t^3		$\sin t$
	$\searrow +$	
$3t^2$		$-\cos t$
	$\searrow -$	
$6t$		$-\sin t$
	$\searrow +$	
6		$\cos t$
	$\searrow -$	
0		$\sin t$

$\displaystyle\int_0^{\pi} t^3 \sin t\,dx = [(-t^3\cos t + 3t^2\sin t + 6t\cos t - 6\sin t)]\Big]_0^{\pi} = \pi^3 - 6\pi.$

60. $u = 2t, dv = \sin(k\omega t)dt$, $du = 2dt$, $v = -\dfrac{1}{k\omega}\cos(k\omega t)$; the integrand is an even function of t so $\displaystyle\int_{-\pi/\omega}^{\pi/\omega} t\sin(k\omega t)\,dt = 2\int_0^{\pi/\omega} t\sin(k\omega t)\,dt = -\frac{2}{k\omega}t\cos(k\omega t)\Big]_0^{\pi/\omega} + 2\int_0^{\pi/\omega}\frac{1}{k\omega}\cos(k\omega t)\,dt = \frac{2\pi(-1)^{k+1}}{k\omega^2} + \frac{2}{k^2\omega^2}\sin(k\omega t)\Big]_0^{\pi/\omega} = \frac{2\pi(-1)^{k+1}}{k\omega^2}.$

61. (a) $u = 1000 + 5t$, $dv = e^{0.12(20-t)}dt$, $du = 5dt$, $v = -\dfrac{e^{0.12(20-t)}}{0.12}$, $\displaystyle FV = \int_0^{20}(1000+5t)e^{0.12(20-t)}\,dt = -\frac{(1000+5t)e^{0.12(20-t)}}{0.12}\Big]_0^{20} + \int_0^{20}\frac{5e^{0.12(20-t)}}{0.12}\,dt = \frac{25000e^{2.4} - 27500}{3} - \frac{5e^{0.12(20-t)}}{0.12^2}\Big]_0^{20} \approx 86{,}173.41.$

(b) $u = 1000+5t$, $dv = e^{-0.12t}dt$, $du = 5dt$, $v = -\dfrac{e^{-0.12t}}{0.12}$, $\displaystyle PV = \int_0^{20}(1000+5t)e^{-0.12t}\,dt = -\frac{(1000+5t)e^{-0.12t}}{0.12}\Big]_0^{20} + \int_0^{20}\frac{5e^{-0.12t}}{0.12}\,dt = \frac{25000 - 27500e^{-2.4}}{3} - \frac{5e^{-0.12t}}{0.12^2}\Big]_0^{20} \approx 7817.48.$

(c) $86{,}173.41 \approx 7817.48e^{0.12\cdot 20}$

62. (a) $u = 1000 + 5t$, $dv = e^{0.12(20-t)}dt$, $du = 5dt$, $v = -\dfrac{e^{0.12(20-t)}}{0.12}$, $\displaystyle FV = \int_5^{20}(1000+5t)e^{0.12(20-t)}\,dt = -\frac{(1000+5t)e^{0.12(20-t)}}{0.12}\Big]_5^{20} + \int_5^{20}\frac{5e^{0.12(20-t)}}{0.12}\,dt = \frac{25625e^{1.8} - 27500}{3} - \frac{5e^{0.12(20-t)}}{0.12^2}\Big]_5^{20} \approx 44{,}260.76.$

(b) $u = 1000 + 5t$, $dv = e^{-0.12(t-5)}dt$, $du = 5dt$, $v = -\dfrac{e^{-0.12(t-5)}}{0.12}$, $\displaystyle PV = \int_5^{20}(1000+5t)e^{-0.12(t-5)}\,dt = -\frac{(1000+5t)e^{-0.12(t-5)}}{0.12}\Big]_5^{20} + \int_5^{20}\frac{5e^{-0.12(t-5)}}{0.12}\,dt = \frac{25625 - 27500e^{-1.8}}{3} - \frac{5e^{-0.12(t-5)}}{0.12^2}\Big]_5^{20} \approx 7316.25.$

(c) $44{,}260.76 \approx 7316.25e^{0.12(20-5)}$

63. (a) $u = 20000 - 200t^2$, $dv = e^{0.05(10-t)}dt$, $du = -400t\,dt$, $v = -20e^{0.05(10-t)}$, $\displaystyle FV = \int_0^{10}(20000-200t^2)e^{0.05(10-t)}\,dt = -20(20000-200t^2)e^{0.05(10-t)}\Big]_0^{10} - \int_0^{10}8000te^{0.05(10-t)}\,dt = 400000e^{0.5} - 8000\int_0^{10}te^{0.05(10-t)}\,dt$; $u = t$, $dv = e^{0.05(10-t)}dt$, $du = dt$, $v = -20e^{0.05(10-t)}$, $\displaystyle\int_0^{10}te^{0.05(10-t)}\,dt = -20te^{0.05(10-t)}\Big]_0^{10} + \int_0^{10}20e^{0.05(10-t)}\,dt = -200 - 400e^{0.05(10-t)}\Big]_0^{10} = -600 + 400e^{0.5}$, thus $FV = 400000e^{0.5} - 8000(-600 + 400e^{0.5}) \approx 183{,}580.44.$

(b) $u = 20000 - 200t^2$, $dv = e^{-0.05t}dt$, $du = -400t\,dt$, $v = -20e^{-0.05t}$, $\displaystyle PV = \int_0^{10}(20000-200t^2)e^{-0.05t}\,dt =$

$-20(20000-200t^2)e^{-0.05t}\Big]_0^{10} - \int_0^{10} 8000te^{-0.05t}\,dt = 400000 - 8000\int_0^{10} te^{-0.05t}\,dt$; $u = t$, $dv = e^{-0.05t}dt$, $du = dt$, $v = -20e^{-0.05t}$, $\int_0^{10} te^{-0.05t}\,dt = -20te^{-0.05t}\Big]_0^{10} + \int_0^{10} 20e^{-0.05t}\,dt = -200e^{-0.5} - 400e^{-0.05t}\Big]_0^{10} = -600e^{-0.5}+400$, thus $PV = 400000 - 8000(-600e^{-0.5}+400) \approx 111,347.17$.

(c) $183,580.44 \approx 111,347.17e^{0.05\cdot 10}$

64. (a) $u = 20000-200t^2$, $dv = e^{0.05(10-t)}dt$, $du = -400t\,dt$, $v = -20e^{0.05(10-t)}$, $FV = \int_2^{10}(20000-200t^2)e^{0.05(10-t)}\,dt = -20(20000-200t^2)e^{0.05(10-t)}\Big]_2^{10} - \int_2^{10} 8000te^{0.05(10-t)}\,dt = 384000e^{0.4} - 8000\int_2^{10} te^{0.05(10-t)}\,dt$; $u = t$, $dv = e^{0.05(10-t)}dt$, $du = dt$, $v = -20e^{0.05(10-t)}$, $\int_2^{10} te^{0.05(10-t)}\,dt = -20te^{0.05(10-t)}\Big]_2^{10} + \int_2^{10} 20e^{0.05(10-t)}\,dt = -200 + 40e^{0.4} - 400e^{0.05(10-t)}\Big]_2^{10} = -600+440e^{0.4}$, thus $FV = 384000e^{0.4} - 8000(-600+440e^{0.4}) \approx 121,637.75$.

(b) $u = 20000-200t^2$, $dv = e^{-0.05(t-2)}dt$, $du = -400t\,dt$, $v = -20e^{-0.05(t-2)}$, $PV = \int_2^{10}(20000-200t^2)e^{-0.05(t-2)}\,dt = -20(20000-200t^2)e^{-0.05(t-2)}\Big]_2^{10} - \int_2^{10} 8000te^{-0.05(t-2)}\,dt = 384000 - 8000\int_2^{10} te^{-0.05(t-2)}\,dt$; $u = t$, $dv = e^{-0.05(t-2)}dt$, $du = dt$, $v = -20e^{-0.05(t-2)}$, $\int_2^{10} te^{-0.05(t-2)}\,dt = -20te^{-0.05(t-2)}\Big]_2^{10} + \int_2^{10} 20e^{-0.05(t-2)}\,dt = -200e^{-0.4} + 40 - 400e^{-0.05(t-2)}\Big]_2^{10} = -600e^{-0.4}+440$, thus $PV = 384000 - 8000(-600e^{-0.4}+440) \approx 81,536.22$.

(c) $121,637.75 \approx 81,536.22e^{0.05(10-2)}$

65. (a) $u = 2000t$, $dv = e^{0.08(10-t)}dt$, $du = 2000dt$, $v = -\frac{25}{2}e^{0.08(10-t)}$, $FV = \int_0^{10}(2000t+400e^{-t})e^{0.08(10-t)}\,dt = \int_0^{10} 2000te^{0.08(10-t)}\,dt + 400\int_0^{10} e^{0.8-1.08t}\,dt = -\left(\frac{25}{2}\right)2000te^{0.08(10-t)}\Big]_0^{10} + \int_0^{10}\left(\frac{25}{2}\right)2000e^{0.08(10-t)}\,dt - \frac{400}{1.08}e^{0.8-1.08t}\Big]_0^{10} = -250000 - \frac{25000}{0.08}e^{0.08(10-t)}\Big]_0^{10} - (\frac{400}{1.08}e^{-10} - \frac{400}{1.08}e^{0.8}) = -250000 - \frac{25000}{0.08}(1-e^{0.8}) - \frac{400}{1.08}(e^{-10}-e^{0.8}) \approx 133,805.80$.

(b) $u = 2000t$, $dv = e^{-0.08t}dt$, $du = 2000dt$, $v = -\frac{25}{2}e^{-0.08t}$, $PV = \int_0^{10}(2000t+400e^{-t})e^{-0.08t}\,dt = \int_0^{10} 2000te^{-0.08t}\,dt + 400\int_0^{10} e^{-1.08t}\,dt = -\left(\frac{25}{2}\right)2000te^{-0.08t}\Big]_0^{10} + \int_0^{10}\left(\frac{25}{2}\right)2000e^{-0.08t}\,dt - \frac{400}{1.08}e^{-1.08t}\Big]_0^{10} = -250000e^{-0.8} - \frac{25000}{0.08}e^{-0.08t}\Big]_0^{10} - (\frac{400}{1.08}e^{-10.8} - \frac{400}{1.08}) = -250000e^{-0.8} - \frac{25000}{0.08}(e^{-0.8}-1) - \frac{400}{1.08}(e^{-10.8}-1) \approx 60,122.82$.

(c) $133,805.80 \approx 60,122.82e^{0.08\cdot 10}$

66. (a) $u = 2000t$, $dv = e^{0.08(25-t)}dt$, $du = 2000dt$, $v = -\frac{25}{2}e^{0.08(25-t)}$, $FV = \int_0^{25}(2000t+400e^{-t})e^{0.08(25-t)}\,dt = \int_0^{25} 2000te^{0.08(25-t)}\,dt + 400\int_0^{25} e^{2-1.08t}\,dt = -\left(\frac{25}{2}\right)2000te^{0.08(25-t)}\Big]_0^{25} + \int_0^{25}\left(\frac{25}{2}\right)2000e^{0.08(25-t)}\,dt - \frac{400}{1.08}e^{2-1.08t}\Big]_0^{25} = -625000 - \frac{25000}{0.08}e^{0.08(25-t)}\Big]_0^{25} - (\frac{400}{1.08}e^{-25} - \frac{400}{1.08}e^{2}) = -625000 - \frac{25000}{0.08}(1-e^2)$

$-\dfrac{400}{1.08}(e^{-25}-e^{2}) \approx 1,374,316.72.$

(b) $u = 2000t$, $dv = e^{-0.08t}dt$, $du = 2000dt$, $v = -\dfrac{25}{2}e^{-0.08t}$, $PV = \displaystyle\int_0^{25}(2000t+400e^{-t})e^{-0.08t}\,dt = \int_0^{25} 2000te^{-0.08t}\,dt + 400\int_0^{25} e^{-1.08t}\,dt = -\left(\frac{25}{2}\right)2000te^{-0.08t}\Big]_0^{25} + \int_0^{25}\left(\frac{25}{2}\right)2000e^{-0.08t}\,dt - \frac{400}{1.08}e^{-1.08t}\Big]_0^{25} = -625000e^{-2} - \frac{25000}{0.08}e^{-0.08t}\Big]_0^{25} - \left(\frac{400}{1.08}e^{-27} - \frac{400}{1.08}\right) = -625000e^{-2} - \frac{25000}{0.08}(e^{-2}-1) - \frac{400}{1.08}(e^{-27}-1) \approx 185,993.54.$

(c) $1,374,316.72 \approx 185,993.54e^{0.08\cdot 25}$

67. **(a)** The area of a ring of width Δx feet, x feet from the center is given by $A(x) = ((x+\Delta x)^2 - x^2)\pi = (2x\Delta x + (\Delta x)^2)\pi \approx 2x\Delta x\pi$, if we assume Δx is small. Thus the number of ants (in thousands) in this ring is about $2\pi x d(x)\Delta x = 6\pi x e^{-0.25x}\Delta x$.

(b) Using the result of the previous part, the total number of ants within six feet of the center of the colony is given by $\displaystyle\int_0^6 6\pi x e^{-0.25x}\,dx$; using $u = 6\pi x$, $dv = e^{-0.25x}dx$, $du = 6\pi dx$, $v = -4e^{-0.25x}$, we obtain $\displaystyle\int_0^6 6\pi x e^{-0.25x}\,dx = -24\pi x e^{-0.25x}\Big]_0^6 + \int_0^6 24\pi e^{-0.25x}\,dx = -144\pi e^{-3/2} - 96\pi e^{-0.25x}\Big]_0^6 = -240\pi e^{-3/2} + 96\pi \approx 133.357$ (thousand) ants.

68. **(a)** The area of a ring of width Δx feet, x feet from the center is given by $A(x) = ((x+\Delta x)^2 - x^2)\pi = (2x\Delta x + (\Delta x)^2)\pi \approx 2x\Delta x\pi$, if we assume Δx is small. Thus the number of ants (in thousands) in this ring is about $2\pi x d(x)\Delta x = 2\pi x(10-x)e^{-0.4x}\Delta x$.

(b) Using the result of the previous part, the total number of ants within six feet of the center of the colony is given by $\displaystyle\int_0^6 2\pi x(10-x)e^{-0.4x}\,dx$; using $u = 2\pi x(10-x)$, $dv = e^{-0.4x}dx$, $du = (20\pi - 4\pi x)dx$, $v = -2.5e^{-0.4x}$, we obtain $\displaystyle\int_0^6 2\pi x(10-x)e^{-0.4x}\,dx = -5\pi x(10-x)e^{-0.4x}\Big]_0^6 + \int_0^6 (50-10x)\pi e^{-0.4x}\,dx = -120\pi e^{-2.4} + \int_0^6 (50-10x)\pi e^{-0.4x}\,dx$; using $u = (50-10x)\pi$, $dv = e^{-0.4x}dx$, $du = -10\pi dx$, $v = -2.5e^{-0.4x}$, we get $\displaystyle\int_0^6 (50-10x)\pi e^{-0.4x}\,dx = -2.5(50-10x)\pi e^{-0.4x}\Big]_0^6 - \int_0^6 25\pi e^{-0.4x}\,dx = -2.5\cdot(-10)\pi e^{-2.4} + 2.5\cdot 50\pi + \frac{125}{2}\pi e^{-0.4x}\Big]_0^6 = \frac{175}{2}\pi e^{-2.4} + \frac{125}{2}\pi$; thus the population size is about $-120\pi e^{-2.4} + \dfrac{175}{2}\pi e^{-2.4} + \dfrac{125}{2}\pi \approx 187.087$ (thousand) ants.

69. **(a)** $\displaystyle\int \sin^4 x\,dx = -\frac{1}{4}\sin^3 x\cos x + \frac{3}{4}\int \sin^2 x\,dx = -\frac{1}{4}\sin^3 x\cos x + \frac{3}{4}\left(-\frac{1}{2}\sin x\cos x + \frac{1}{2}x\right) + C = -\frac{1}{4}\sin^3 x\cos x - \frac{3}{8}\sin x\cos x + \frac{3}{8}x + C.$

(b) $\displaystyle\int_0^{\pi/2} \sin^5 x\,dx = -\frac{1}{5}\sin^4 x\cos x\Big]_0^{\pi/2} + \frac{4}{5}\int_0^{\pi/2}\sin^3 x\,dx = \frac{4}{5}\left(-\frac{1}{3}\sin^2 x\cos x\Big]_0^{\pi/2} + \frac{2}{3}\int_0^{\pi/2}\sin x\,dx\right)$

$\displaystyle = -\frac{8}{15}\cos x\Big]_0^{\pi/2} = \frac{8}{15}.$

70. **(a)** $\displaystyle\int \cos^5 x\,dx = \frac{1}{5}\cos^4 x\sin x + \frac{4}{5}\int \cos^3 x\,dx = \frac{1}{5}\cos^4 x\sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x\sin x + \frac{2}{3}\sin x\right] + C = \frac{1}{5}\cos^4 x\sin x + \frac{4}{15}\cos^2 x\sin x + \frac{8}{15}\sin x + C.$

(b) $\int \cos^6 x\,dx = \frac{1}{6}\cos^5 x \sin x + \frac{5}{6}\int \cos^4 x\,dx = \frac{1}{6}\cos^5 x\sin x + \frac{5}{6}\left[\frac{1}{4}\cos^3 x\sin x + \frac{3}{4}\int \cos^2 x\,dx\right] = \frac{1}{6}\cos^5 x\sin x + \frac{5}{24}\cos^3 x\sin x + \frac{5}{8}\left[\frac{1}{2}\cos x\sin x + \frac{1}{2}x\right] + C$, so $\left[\frac{1}{6}\cos^5 x\sin x + \frac{5}{24}\cos^3 x\sin x + \frac{5}{16}\cos x\sin x + \frac{5}{16}x\right]_0^{\pi/2} = 5\pi/32$.

71. $u = \sin^{n-1} x$, $dv = \sin x\,dx$, $du = (n-1)\sin^{n-2} x\cos x\,dx$, $v = -\cos x$; $\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,(1-\sin^2 x)dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$, so $n\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx$, and $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$.

72. **(a)** $u = \sec^{n-2} x$, $dv = \sec^2 x\,dx$, $du = (n-2)\sec^{n-2} x\tan x\,dx$, $v = \tan x$; $\int \sec^n x\,dx = \sec^{n-2} x\tan x - (n-2)\int \sec^{n-2} x\tan^2 x\,dx = \sec^{n-2} x\tan x - (n-2)\int \sec^{n-2} x\,(\sec^2 x - 1)dx = \sec^{n-2} x\tan x - (n-2)\int \sec^n x\,dx + (n-2)\int \sec^{n-2} x\,dx$, so $(n-1)\int \sec^n x\,dx = \sec^{n-2} x\tan x + (n-2)\int \sec^{n-2} x\,dx$, and then

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx.$$

(b) $\int \tan^n x\,dx = \int \tan^{n-2} x\,(\sec^2 x - 1)\,dx = \int \tan^{n-2} x\sec^2 x\,dx - \int \tan^{n-2} x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$.

(c) $u = x^n$, $dv = e^x dx$, $du = nx^{n-1}dx$, $v = e^x$; $\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$.

73. **(a)** $\int \tan^4 x\,dx = \frac{1}{3}\tan^3 x - \int \tan^2 x\,dx = \frac{1}{3}\tan^3 x - \tan x + \int dx = \frac{1}{3}\tan^3 x - \tan x + x + C$.

(b) $\int \sec^4 x\,dx = \frac{1}{3}\sec^2 x\tan x + \frac{2}{3}\int \sec^2 x\,dx = \frac{1}{3}\sec^2 x\tan x + \frac{2}{3}\tan x + C$.

(c) $\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx = x^3 e^x - 3\left[x^2 e^x - 2\int x e^x dx\right] = x^3 e^x - 3x^2 e^x + 6\left[xe^x - \int e^x dx\right] = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C$.

74. **(a)** $u = 3x$, $\int x^2 e^{3x} dx = \frac{1}{27}\int u^2 e^u du = \frac{1}{27}\left[u^2 e^u - 2\int u e^u du\right] = \frac{1}{27}u^2 e^u - \frac{2}{27}\left[ue^u - \int e^u du\right] = \frac{1}{27}u^2 e^u - \frac{2}{27}ue^u + \frac{2}{27}e^u + C = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$.

(b) $u = -\sqrt{x}$, $\int_0^1 xe^{-\sqrt{x}}dx = 2\int_0^{-1} u^3 e^u du$, $\int u^3 e^u du = u^3 e^u - 3\int u^2 e^u du = u^3 e^u - 3\left[u^2 e^u - 2\int ue^u du\right] = u^3 e^u - 3u^2 e^u + 6\left[ue^u - \int e^u du\right] = u^3 e^u - 3u^2 e^u + 6ue^u - 6e^u + C$, so $2\int_0^{-1} u^3 e^u du = 2(u^3 - 3u^2 + 6u - 6)e^u\Big]_0^{-1} = 12 - 32e^{-1}$.

75. $u = x$, $dv = f''(x)dx$, $du = dx$, $v = f'(x)$; $\int_{-1}^1 x\,f''(x)dx = xf'(x)\Big]_{-1}^1 - \int_{-1}^1 f'(x)dx = f'(1) + f'(-1) - f(x)\Big]_{-1}^1 = f'(1) + f'(-1) - f(1) + f(-1)$.

76. **(a)** $\int u\,dv = uv - \int v\,du = x(\sin x + C_1) + \cos x - C_1 x + C_2 = x\sin x + \cos x + C_2$; the constant C_1 cancels out and hence plays no role in the answer.

(b) $u(v + C_1) - \int (v + C_1)du = uv + C_1 u - \int v\,du - C_1 u = uv - \int v\,du.$

77. $u = \ln(x+1), dv = dx, du = \dfrac{dx}{x+1}, v = x+1; \int \ln(x+1)\,dx = \int u\,dv = uv - \int v\,du = (x+1)\ln(x+1) - \int dx = (x+1)\ln(x+1) - x + C.$

78. $u = \ln(3x-2), dv = dx, du = \dfrac{3dx}{3x-2}, v = x - \dfrac{2}{3}; \int \ln(3x-2)\,dx = \int u\,dv = uv - \int v\,du = \left(x - \dfrac{2}{3}\right)\ln(3x-2) - \int \left(x - \dfrac{2}{3}\right)\dfrac{1}{x - 2/3}\,dx = \left(x - \dfrac{2}{3}\right)\ln(3x-2) - \left(x - \dfrac{2}{3}\right) + C.$

79. $u = \tan^{-1} x, dv = x\,dx, du = \dfrac{1}{1+x^2}\,dx, v = \dfrac{1}{2}(x^2+1) \int x\tan^{-1} x\,dx = \int u\,dv = uv - \int v\,du = \dfrac{1}{2}(x^2+1)\tan^{-1} x - \dfrac{1}{2}\int dx = \dfrac{1}{2}(x^2+1)\tan^{-1} x - \dfrac{1}{2}x + C.$

80. $u = \dfrac{1}{\ln x}, \quad dv = \dfrac{1}{x}\,dx, du = -\dfrac{1}{x(\ln x)^2}\,dx, v = \ln x, \int \dfrac{1}{x\ln x}\,dx = 1 + \int \dfrac{1}{x\ln x}\,dx.$ This seems to imply that $1 = 0$, but recall that both sides represent a function *plus an arbitrary constant*; these two arbitrary constants will take care of the 1.

Exercise Set 7.3

1. $u = \cos x, -\int u^3 du = -\dfrac{1}{4}\cos^4 x + C.$

2. $u = \sin 3x, \dfrac{1}{3}\int u^5\,du = \dfrac{1}{18}\sin^6 3x + C.$

3. $\int \sin^2 5\theta = \dfrac{1}{2}\int (1 - \cos 10\theta)\,d\theta = \dfrac{1}{2}\theta - \dfrac{1}{20}\sin 10\theta + C.$

4. $\int \cos^2 3x\,dx = \dfrac{1}{2}\int (1 + \cos 6x)dx = \dfrac{1}{2}x + \dfrac{1}{12}\sin 6x + C.$

5. $\int \sin^3 a\theta\,d\theta = \int \sin a\theta(1 - \cos^2 a\theta)\,d\theta = -\dfrac{1}{a}\cos a\theta + \dfrac{1}{3a}\cos^3 a\theta + C. \quad (a \neq 0)$

6. $\int \cos^3 at\,dt = \int (1 - \sin^2 at)\cos at\,dt = \int \cos at\,dt - \int \sin^2 at\cos at\,dt = \dfrac{1}{a}\sin at - \dfrac{1}{3a}\sin^3 at + C. \quad (a \neq 0)$

7. $u = \sin ax, \quad \dfrac{1}{a}\int u\,du = \dfrac{1}{2a}\sin^2 ax + C. \quad (a \neq 0)$

8. $\int \sin^3 x\cos^3 x\,dx = \int \sin^3 x(1 - \sin^2 x)\cos x\,dx = \int (\sin^3 x - \sin^5 x)\cos x\,dx = \dfrac{1}{4}\sin^4 x - \dfrac{1}{6}\sin^6 x + C.$

9. $\int \sin^2 t\cos^3 t\,dt = \int \sin^2 t(1 - \sin^2 t)\cos t\,dt = \int (\sin^2 t - \sin^4 t)\cos t\,dt = \dfrac{1}{3}\sin^3 t - \dfrac{1}{5}\sin^5 t + C.$

10. $\int \sin^3 x\cos^2 x\,dx = \int (1 - \cos^2 x)\cos^2 x\sin x\,dx = \int (\cos^2 x - \cos^4 x)\sin x\,dx = -\dfrac{1}{3}\cos^3 x + \dfrac{1}{5}\cos^5 x + C.$

11. $\int \sin^2 x \cos^2 x\, dx = \frac{1}{4}\int \sin^2 2x\, dx = \frac{1}{8}\int (1-\cos 4x)dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C.$

12. $\int \sin^2 x \cos^4 x\, dx = \frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)^2 dx = \frac{1}{8}\int (1-\cos^2 2x)(1+\cos 2x)dx$
$= \frac{1}{8}\int \sin^2 2x\, dx + \frac{1}{8}\int \sin^2 2x \cos 2x\, dx = \frac{1}{16}\int (1-\cos 4x)dx + \frac{1}{48}\sin^3 2x = \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{48}\sin^3 2x + C.$

13. $\int \sin 2x \cos 3x\, dx = \frac{1}{2}\int (\sin 5x - \sin x)dx = -\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + C.$

14. $\int \sin 3\theta \cos 2\theta d\theta = \frac{1}{2}\int (\sin 5\theta + \sin\theta)d\theta = -\frac{1}{10}\cos 5\theta - \frac{1}{2}\cos\theta + C.$

15. $\int \sin x \cos(x/2)dx = \frac{1}{2}\int [\sin(3x/2) + \sin(x/2)]dx = -\frac{1}{3}\cos(3x/2) - \cos(x/2) + C.$

16. $u = \cos x,\ -\int u^{1/3}du = -\frac{3}{4}\cos^{4/3} x + C.$

17. $\int_0^{\pi/2} \cos^3 x\, dx = \int_0^{\pi/2} (1-\sin^2 x)\cos x\, dx = \left[\sin x - \frac{1}{3}\sin^3 x\right]_0^{\pi/2} = \frac{2}{3}.$

18. $\int_0^{\pi/2} \sin^2(x/2)\cos^2(x/2)dx = \frac{1}{4}\int_0^{\pi/2} \sin^2 x\, dx = \frac{1}{8}\int_0^{\pi/2}(1-\cos 2x)dx = \frac{1}{8}\left(x - \frac{1}{2}\sin 2x\right)\Big]_0^{\pi/2} = \pi/16.$

19. $\int_0^{\pi/3} \sin^4 3x \cos^3 3x\, dx = \int_0^{\pi/3} \sin^4 3x(1-\sin^2 3x)\cos 3x\, dx = \left[\frac{1}{15}\sin^5 3x - \frac{1}{21}\sin^7 3x\right]_0^{\pi/3} = 0.$

20. $\int_{-\pi}^{\pi} \cos^2 5\theta\, d\theta = \frac{1}{2}\int_{-\pi}^{\pi}(1+\cos 10\theta)d\theta = \frac{1}{2}\left(\theta + \frac{1}{10}\sin 10\theta\right)\Big]_{-\pi}^{\pi} = \pi.$

21. $\int_0^{\pi/6} \sin 4x \cos 2x\, dx = \frac{1}{2}\int_0^{\pi/6}(\sin 2x + \sin 6x)dx = \left[-\frac{1}{4}\cos 2x - \frac{1}{12}\cos 6x\right]_0^{\pi/6} = [(-1/4)(1/2) - (1/12)(-1)] -$
$[-1/4 - 1/12] = 7/24.$

22. $\int_0^{2\pi} \sin^2 kx\, dx = \frac{1}{2}\int_0^{2\pi}(1-\cos 2kx)dx = \frac{1}{2}\left(x - \frac{1}{2k}\sin 2kx\right)\Big]_0^{2\pi} = \pi - \frac{1}{4k}\sin 4\pi k. \quad (k \neq 0)$

23. $u = 2x-1,\ du = 2dx,\ \frac{1}{2}\int \sec^2 u\, du = \frac{1}{2}\tan(2x-1) + C.$

24. $u = 5x,\ du = 5dx,\ \frac{1}{5}\int \tan u\, du = -\frac{1}{5}\ln|\cos 5x| + C.$

25. $u = e^{-x}, du = -e^{-x}\, dx;\ -\int \tan u\, du = \ln|\cos u| + C = \ln|\cos(e^{-x})| + C.$

26. $u = 3x,\ du = 3dx,\ \frac{1}{3}\int \cot u\, du = \frac{1}{3}\ln|\sin 3x| + C.$

27. $u = 4x,\ du = 4dx,\ \frac{1}{4}\int \sec u\, du = \frac{1}{4}\ln|\sec 4x + \tan 4x| + C.$

Exploring the Variety of Random Documents with Different Content

Continuity by Summation.—The idea of summation leads by another path to the same result. It is another form of the principle of continuity. A sum total of effects, obscure and indistinct in themselves, produces a phenomenon appreciable, perceptible, and distinct, apparently, but not really, heterogeneous in its components. The manifestations of atomic or molecular activity thus become manifestations of vital activity.

This is another consequence of the teaching of Leibniz. For, according to his philosophical theory, individual consciousness, like individual life, is the collective expression of a multitude of elementary lives or consciousnesses. These elements are inappreciable because of their low degree, and the real phenomenon is found in the sum, or rather the *integral*, of all these insensible effects. The elementary consciousnesses are harmonized, unified, integrated into a result that becomes manifest, just as "the sounds of the waves, not one of which would be heard if by itself, yet, when united together and perceived at the same instant, become the resounding voice of the ocean."

Ideas of the Philosophers as to Sensibility and Consciousness in Brute Bodies.—The philosophers have gone still further in the way of analogies, and have recognized in the play of the forces of brute matter, particularly in the play of chemical forces, a mere rudiment of the appetitions and tendencies that regulate, as they believe, the functional activity of living beings—a trace, as it were, of their sensibility. To them reactions of matter indicate the existence of a kind of *hedonic consciousness*—*i.e.*, a consciousness reduced simply to a distinction between comfort and discomfort, a desire for good and repulsion from evil, which they suppose to be the universal principle of all activity. This was the view held by Empedocles in antiquity; it was that of Diderot, of Cabanis, and, in general, of the modern materialistic school, eager to find, even in the lowest representatives of the inorganic world, the first traces of the vitality and intellectual life which blossom out at the top of the scale in the living world.

Similar ideas are clearly seen in the early history of all natural sciences. It was this same principle of appetition, or of love and of repulsion or hate that, under the names of affinity, selection, and incompatibility, was thought to direct the transformations of bodies when chemistry first began; when Boerhaave, for example, compared chemical combinations to voluntary and conscious alliances, in which the respective elements, drawn together by sympathy, contracted appropriate marriages.

General Principle of the Homogeneity of the Complex and its Components.—The assimilation of brute bodies to living bodies, and of the inorganic kingdom to the organic, was, in the mind of these philosophers, the natural consequence of positing *a priori* the principles of continuity and evolution. There is, however, a principle underlying these principles. This principle is not expressed explicitly by the philosophers; it is not formulated in precise terms, but is more or less unconsciously implied; it is everywhere applied. It, however, may be clearly seen behind the apparatus of philosophical argument It is the assertion that no arrangement or combination of elements can put forth any new activity essentially different from the activities of the elements of which it is composed. Man is living clay, say Diderot and Cabanis; and, on the other hand, he is a thinking being. *As it is impossible to produce that which thinks from that which does not think*, the clay must possess a rudiment of thought. But is there not another alternative? May not the new phenomenon, thought, be the effect of the arrangement of this clay? If we exclude this alternative, we must then consider arrangement and organization as incapable of producing in arranged and organized matter a new property different from that which it presented before such arrangement. Living protoplasm, says another, is merely an assemblage of brute elements; "these brute elements must therefore possess a rudiment of life." This is the same implied supposition which we have just considered; if life is not the basis of each element, it cannot result from their simple assemblage.

Man and animals are combinations of atoms, says M. le Dantec. It is more natural to admit that human consciousness is the result of the elementary consciousness of the constituent atoms than to consider it as resulting from construction by means of elements with no consciousness. "Life," says Haeckel, "is universal; we could not conceive of its existence in certain aggregates of matter if it did not belong to their constituent elements." Here the postulate is almost expressed.

The argument is always the same; even the same words are used: the fundamental hypothesis is the same; only it remains more or less unexpressed, more or less unperceived. It may be stated as follows:—Arrangement, assemblage, construction, and aggregation are powerless to bring to light in the complex anything new and essentially heterogeneous to what already exists in the elements. Reciprocally, grouping reveals in a complex a property and character which is the gradual development of an analogous property and character in the elements. It is in this sense that there exists a collective soul in crowds, the psychology of which has been discussed by M. G. Le Bon. In the same way, many sociologists, adopting the views advanced by P. de Lilienfeld in 1865, attribute to nations a formal individuality, after the type of that possessed by each of their constituent members. M. Izolet considers society as an organism, which he calls a "hyperzoan." Herbert Spencer has developed the comparison of the collective organism with the individual organism, insisting on their resemblances and differences. Th. Ribot has dwelt, in particular, on the resemblances.

The postulate that we have clearly stated here is accepted by many as an axiom. But it is not an axiom. When we say that there is nothing in the complex that cannot be found in the parts, we think we are expressing a self-evident truth; but we are, in fact, merely stating an hypothesis. It is assumed that arrangement, aggregation, and complicated and skilful grouping of elements can produce nothing really new in the order of phenomena. And this is an assertion that requires verification in each particular case.

The Principle of Continuity, a Consequence of the Preceding.—Let us apply this principle to the beings in nature. All beings in nature are, according to current ideas, arrangements, aggregates, or groupings of the same universal matter, that is to say, of the same simple chemical bodies. It results from the preceding postulate that their activities can only differ in degree and form, and not fundamentally. There is no essential difference of nature between the activities of various categories of beings, no heterogeneity, no discontinuity. We may pass from one to another without coming to an hiatus or impassable gulf. The law of continuity thus appears as a simple consequence of the fundamental postulate. And so it is with the law of evolution, for evolution is merely continuity of action.

Such are the origins of the philosophical doctrine which universalizes life and extends it to all bodies in nature.

It may be remarked that this doctrine is not confined to any particular school or sect. Leibniz was by no means a materialist, and he endowed his mundane elements, his *monads*, not only with a sort of life, but even with a sort of soul. Father Boscovitch, Jesuit as he was, and professor in the college of Rome, did not deny to his *indivisible points* a kind of inferior vitality. St. Thomas, too, the angelical doctor, attributed, according to M. Gardair, to inanimate substances a certain kind of activity, inborn inclinations, and a real appetition towards certain acts.

CHAPTER II.
ORIGIN OF BRUTE MATTER IN LIVING MATTER.

Spontaneous generation: an episode in the history of the globe—Verification of the identity between brute and living matter—Slow identification—Rapid identification—Contrary opinion—Hypothesis of cosmozoa; cosmic panspermia—Hypothesis of pyrozoa.

There should be two ways of testing the doctrine of the essential identity of brute and living matter—one slow and more laborious, the other more rapid and decisive.

Identification of the Two Matters, Brute and Living.—The laborious method, which we will be obliged to follow, consists in the attentive examination of the various activities by which life is manifested, and in finding more or less crude equivalents for them in all brute beings, or in certain of them.

Rapid Verification. Spontaneous Generation.—The rapid and decisive method, which, unhappily, is beyond our resources, would consist in showing unquestionable, clearly marked life, the superior life, arising from the kind of inferior life that is attributed to matter in general. It would be necessary completely to construct in all its parts, by a suitable combination of inorganic materials, a single living being, even the humblest plant or the most rudimentary animal. This would indeed be an irrefutable proof that the germs of all vital activity are contained in the molecular activity of brute bodies, and that there is nothing essential to the latter that is not found in the former.

Unhappily this demonstration cannot be given. Science furnishes no example of it, and we are forced to have recourse to the slow method.

The question here involved is that of spontaneous generation. It is well known that the ancients believed in spontaneous generation, even for animals high in the scale of organization. According to Van Helmont, mice could be born by some incomprehensible fermentation in dirty linen mixed with wheat. Diodorus speaks of animal forms which were seen to emerge, partly developed, from the mud of the Nile. Aristotle believed in the spontaneous birth of certain fishes. This belief, though rejected as to the higher forms, was for a long time held with regard to the lower forms of animals, and to insects—such as the bees which the shepherd of Virgil saw coming out from the flanks of the dead bullock—flies engendered in putrefying meat, fruit worms and intestinal worms; finally, with regard to infusoria and the most rudimentary vegetables. The hypothesis of the spontaneous generation of the living being at the expense of the materials of the ambient medium has been successively driven from one classificatory group to another. The history of the sciences of observation is also a history of the confutation of this theory. Pasteur gave it the finishing stroke, when he showed that the simplest microorganisms obeyed the general law which declares that the living being is formed only by *filiation*—that is to say, by the intervention of a pre-existing living organism.

Spontaneous Generation an Episode in the History of the Globe.—Though we have been unable to effect spontaneous generation up to the present, it has been referred by Haeckel to a more or less distant past, to the time when the cooling of the globe, the solidification of its crust, and the condensation of aqueous vapour upon its surface created conditions compatible with the existence of living beings similar to those with which we are acquainted. Lord Kelvin has fixed these geological events as occurring from twenty to forty million years ago. Then circumstances became propitious for the appearance of the first organisms, whence were successively derived those which now people the earth and the waters.

Circumstances favourable to the appearance of the first beings apparently occurred only in a far distant past; but most physiologists admit that if we knew exactly these circumstances, and could reproduce them, we might also expect to produce their effect—namely, the creation of a living being, formed in all its parts, developed from the inorganic kingdom. To all those who held this view the impotence of experiment at the present time is purely temporary. It is comparable to that of primitive men before the time of Prometheus; they, not knowing how to produce fire, could only get it by transmitting it from one to another. It is due to the inadequacy of our knowledge and the weakness of our means; it does not contradict the possibility of the fact.

Contrary Opinion. Life did not Originate on our Globe.—But all biologists do not share this opinion. Some, and not the least eminent, hold it to be an established fact that it is impossible for life to arise from a concurrence of inorganic materials and forces. This was the opinion of Ferdinand Cohn, the great botanist; of H. Richter, the Saxon physician, and of W. Preyer, a physiologist well known from his remarkable researches in biological chemistry. According to these scientists, life on the surface of the globe cannot have appeared as a result of the reactions of brute matter and the forces that continue to control it.

According to F. Cohn and PI. Richter, life had no beginning on our planet. It was transported to the earth from another world, from the cosmic medium, under the form of cosmic germs, or *cosmozoa*, more or less comparable to the living cells with which we are acquainted. They may have made the journey either enclosed in meteorites, or floating in space in the form of cosmic dust. The theory in question has been presented in two forms:—*The Hypothesis of Meteoric Cosmozoa*, by a French writer, the Count de Salles-Guyon; and that of *cosmic panspermia* brought forward in 1865 and 1872 by F. Cohn and H. Richter.

Hypothesis of the Cosmozoa.—The hypothesis of the cosmozoa, living particles, protoplasmic germs emanating from other worlds and reaching the earth by means of aerolites, is not so destitute of probability as one might at first suppose. Lord Kelvin and Helmholtz gave it the support of their high authority. Spectrum analysis shows in cometary nebulæ the four or five lines characteristic of hydrocarbons. Cosmic matter, therefore, contains compounds of carbon, substances that are especially typical of organic chemistry. Besides, carbon and a sort of humus have been found in several meteorites. To the objection that these aerolites are heated while passing through our atmosphere, Helmholtz replies that this elevation of temperature may be quite superficial and may allow micro-organisms to subsist in their interior. But other objections retain their force:—First, that of M. Verworn, who considers the hypothesis of cosmic germs as inconsistent with the laws of evolution; and that of L. Errera, who denies that the conditions necessary for life exist in interplanetary bodies.

Hypothesis of Cosmic Panspermia.—Du Bois-Reymond has given the name of *cosmic panspermia* to a theory very similar to the preceding, formulated by F. Cohn in 1872. The first living germs arrived on our globe mingled with the cosmic dust that floats in space and falls slowly to the surface of the earth. L. Errera observes that if they escape by this gentle fall the dangerous heating of meteorites, they still remain exposed to the action of the photic rays, which is generally destructive to germs.

Hypothesis of Pyrozoa.—W. Preyer declined to accept this cosmic transmigration of the simplest living beings, nor would he allow the intervention of other worlds into the history of our own. Life, according to him, must have existed from all time, even when the globe was an incandescent mass. But it was not the same life as at present. Vitality must have undergone many profound changes in the course of ages. The *pyrozoa*, the first living beings, vulcanians, were very different from the beings of the present day that are destroyed by a slight elevation of temperature. No doubt this theory of pyrozoa, proposed by W. Preyer in 1872, seems quite chimerical, and akin to Kepler’s dreamy visions. But in a certain way it accords with contemporary ideas concerning the life of *matter*. It is related to them by the evolution which it implies in the materials of the terrestrial globe.

According to Preyer, primitive life existed in fire. Being igneous masses in fusion, the pyrozoa lived after their own manner; their vitality, slowly modified, assumed the form which it presents to-day. Yet, in this profound transformation their number has not varied, and the total quantity of life in the universe has remained unchanged.

Here we recognize the ideas of Buffon. These cosmozoa, these pyrozoa, have a singular resemblance to the *organic molecules* of "live matter" of the illustrious naturalist—distributed everywhere, indestructible, and forming living structures by their concentration.

But we must leave these scientific or philosophical theories, and come to arguments based upon facts.

It is in a spirit quite different from that of the poets, the metaphysicians, and the more or less philosophical scientists that the science of our days looks at the more or less obscure vitality of inanimate bodies. It claims that we may recognize in them, in a more or less rudimentary state, the action of the factors which intervene in the case of living beings, the manifestation of the same fundamental properties.

CHAPTER III.
ORGANIZATION AND CHEMICAL COMPOSITION OF LIVING AND BRUTE MATTER.

Laws of the organization and of the chemical composition of living beings—Relative value of these laws; vital phenomena in crushed protoplasm—Vital phenomena in brute bodies.

Enumeration of the Principal Characters of Living Beings.—The programme which we have just sketched compels us to look in the brute being for the properties of living beings. What, then, are, in fact, the characteristics of an authentic, complete, living being? What are its fundamental properties? We have enumerated them above as follows:—A certain chemical composition, which is that of living matter; a structure or organization; a specific form; an evolution which has a duration, that of life, and an end, death; a property of growth or nutrition; a property of reproduction. Which of these characters counts for most in the definition of life? Are they all equally necessary? If some of them were wanting, would that justify the transference of a being, who might possess the rest, from the animate world to that of minerals? This is precisely the question that is under consideration.

Organization and Chemical Composition of Living Beings.—All that we know concerning the constitution of living matter and its organization is summed up in the laws of the *chemical unity* and the *morphological unity of living beings* (v. Book III.). These laws seem to be a legitimate generalization from all the facts observed. The first states that the phenomena of life are manifested only in and through living matter, protoplasm—*i.e.*, in and through a substance which has a certain chemical and physical composition. Chemically it is a proteid complexus with a hexonic nucleus. Physically it shows a frothy structure analogous to that resulting from the mixture of two granular, immiscible liquids, of different viscosities. The second law states that the phenomena of life can only be maintained in a protoplasm which has the organization of the complete cell, with its cellular body and nucleus.

Relative Value of these Laws. Exceptions.—What is the signification of these laws of the chemical composition and organization of living beings? Evidently that life in all its plenitude can only exist and be perpetuated under their protection. If these laws were absolute, if it were true that no life were possible but in and through albuminous protoplasm, but in and through the cell, the problem of "the life of matter" would be decided in the negative.

May it not happen, however, that fragmentary and incomplete vital manifestations, progressive traces of a true life, may occur under different conditions; for example, in matter which is not protoplasm, and in a body which has a structure differing from that of a cell—that is to say, in a being which would be neither animal nor plant? We must seek the answer to this question by an appeal to experiment.

Without leaving the animal and vegetable kingdoms—*i.e.*, real living beings—we already see less rigour in the laws governing chemical constitution and cellular organization.

Experiments in merotomy—*i.e.*, in amputation—carried out on the nervous element by Waller, on infusoria by Brandt, Gruber, Balbiani, Nussbaum, and Verworn, show us the necessity of the presence of the cellular body and the nucleus—*i.e.*, of the integrity of the cell. But they also teach us that when that integrity no longer exists death does not immediately follow. A part of the vital functions continues to be performed in denucleated protoplasm, in a cell which is mutilated and incomplete.

Vital Phenomena in Crushed Protoplasm.—It is true also that grinding and crushing suppress the greater part of the functions of the cell. But tests with pulps of various organs and with those of certain yeasts also show that protoplasm, even though ground and disorganized, cannot be considered as inert, and that it still exhibits many of its characteristic phenomena; for example, the production of diastases, the specific agents of vital chemistry. Finally, while we do not know enough about the actions of which the secondary elements of protoplasm—its granulations, its filaments—are capable, which this or that method of destruction may bring to light, at least we know that actions of this kind exist.

To sum up, we are far from being able to deny that rudimentary, isolated vital acts may be produced by the various bodies that result from the dismemberment of protoplasm. The integrity of the cellular organization, even the integrity of protoplasm itself, are therefore not indispensable for these partial manifestations of vitality.

Besides, biologists admit that there exist within the protoplasm aliquot parts, elements of an inferior order, which possess special activities. These secondary elements must have the principle of their activity within themselves. Such are the *biophors* to which Weismann attributes the vital functions of the cell, nutrition, growth, and multiplication. If there are biophors within the cell, we may imagine them outside the cell, and since they carry within themselves the principle of their activity they may exercise it in an independent manner. Unhappily the biophors, and other constituent elements of that kind, are purely hypothetical. They are like Darwin's gemmules, Altmann's bioblasts, and the pangens of De Vries. They have no relation to facts of observation and to real existence.

Vital Phenomena in Brute Bodies.—There is no doubt that certain phenomena of vitality may occur outside of the cellular atmosphere. And carrying this further, we may admit that they may be produced in certain slightly organized bodies (crushed cells), and then in certain unorganized bodies in certain brute beings. In every case it is certain that effects are produced at any rate similar to those which are characteristic of living matter. It is for observation and experiment to decide as to the degree of similarity, and their verdict is that the similarity is complete. The crystals and the crystalline germs studied by Ostwald and Tammann are the seat of phenomena which are quite comparable to those of vitality.

CHAPTER IV.
EVOLUTION AND MUTABILITY OF LIVING MATTER AND BRUTE MATTER.

Supposed immobility of brute bodies—Mobility and mutability of the sidereal world.—§ 1. The movement of particles and molecules in brute bodies—The internal movements of brute bodies—Kinetic conception of molecular motion—Reality of the motion of particles—Comparison of the activity of particles with vital activity.—§ 2. Brownian movement—Its existence—Its character—Its independence of the nature of the bodies and of the nature of the environment—Its indefinite duration—Its independence of external conditions—The Brownian movement must be the first stage of molecular motion.—§ 3. Motion of particles—Migration of material particles—Migration under the action of weight; of diffusion; of electrolysis; of mechanical pressure.—§ 4. Internal activity of alloys—Their structure—Changes produced by deforming agencies—Slow return to equilibrium—Residual effect—Effect of annealing; effect of stretching—Nickel steel—Colour photography—Conclusion—Relations of the environment to the living or brute matter.

One of the most remarkable characteristics of a living being is its evolution. It undergoes a continuous change. It starts from something very small; it assumes a configuration and grows; in most cases it declines and disappears, having followed a course which may be predicted—a sort of ideal trajectory.

Supposed Immobility of Brute Bodies.—It may be asked whether this evolution, this directed mobility, is so exclusively a feature of the living being as it appears, and if many brute bodies do not present something analogous to it. We may answer in no uncertain tones.

Bichat was wrong when he contrasted in this respect brute bodies with living bodies. Vital properties, he said, are temporary; it is their nature to be exhausted; in time they are used up in the same body. Physical properties, on the contrary, are eternal. Brute bodies have neither a beginning nor an inevitable end, neither age, nor evolution; they remain as immutable as death, of which they are the image.

Mobility and Mutability of the Sidereal World.—This is not true, in the first place, of the sidereal bodies. The ancients held the sidereal world to be immutable and incorruptible. The doctrine of the incorruptibility of the heavens prevailed up to the seventeenth century. The observers who at that epoch directed towards the heavens the first telescope, which Galileo had just invented, were struck with astonishment at discovering a change in that celestial firmament which they had hitherto believed incorruptible, and at perceiving a new star that appeared in the constellation Ophiuchus. Such changes no longer surprise us. The cosmogonic system of Laplace has become familiar to all cultivated minds, and every one is accustomed to the idea of the continual mobility and evolution of the celestial world. "The stars have not always existed," writes M. Faye; "they have had a period of formation; they will likewise have a period of decline, followed by final extinction."

Thus all the bodies of inanimate nature are not eternal and immutable; the celestial bodies are eminently susceptible of evolution, slow indeed with that we observe on the surface of our globe; but this disproportion, corresponding to the immensity of time and of cosmic spaces as compared with terrestrial measurements, should not mislead us as to the fundamental analogy of the phenomena.

§ 1. The Movement of Particles and Molecules in Brute Bodies.

It is not only in celestial spaces that we must search for that mobility of brute matter which imitates the mobility of living matter. In order to find it we have only to look about us, or to inquire from physicists and chemists.

As far as geologists are concerned, M. le Dantec tells us somewhere of one who divided minerals into *living rocks*—rocks capable of change of structure, of evolution under the influence of atmospheric causes; and *dead rocks*—rocks which, like clay, have found at the end of all their changes a final state of repose. Jerome

Cardan, a celebrated scientist of the sixteenth century, at once mathematician, naturalist, and physician, declared not only that stones live, but that they suffer from disease, grow old, and die. The jewellers of the present day use similar language of certain precious stones; the torquoise, for example.

The alchemists carried these ideas to an extreme. It is not necessary here to recall the past, to evoke the hermetic beliefs and the dreams of the alchemists, who held that the different kinds of matter lived, developed, and were transmuted into each other.

I refer to precise and recent data, established by the most expert investigators, and related by one of them, Charles Edward Guillaume, some years ago, before the *Société helvétique des Sciences naturelles*. These data show that determinate forms of matter may live and die, in the sense that they may be slowly and continuously modified, always in the same direction, until they have attained an ultimate and definitive state of eternal repose.

The Internal Movements of Bodies.—Swift's reply to an idle fellow who spoke slightingly of work is well known. "In England," said the author of *Gulliver's Travels*, "men work, women work, horses work, oxen work, water works, fire works, and beer works; it is only the pig who does nothing at all; he must, therefore, be the only gentleman in England." We know very well that English gentlemen also work. Indeed, everybody and everything works. And the great wit was nearer right than he supposed in comparing men and things in this respect. Everything is at work; everything in nature strives and toils, at every stage, in every degree. Immobility and repose in the case of natural things are usually deceptive; the seeming quietude of matter is caused by our inability to appreciate its internal movements. Because of their minuteness we do not perceive the swarming particles that compose it, and which, under the impassible surface of the bodies, oscillate, displace each other, move to and fro, and group themselves into forms and positions adapted to the conditions of the environment. In comparison with these microscopic elements we are like Swift's giant among the Lilliputians; and this is far from being a sufficiently forcible comparison.

Kinetic Conception of Molecular Motion.—The idea of this peculiar form of motion is by no means new to us. We were familiarized with it in scientific theories during our school days. The atomic theory teaches us that matter behaves, from a chemical point of view, as if it were divided into molecules and atoms. The kinetic theory explains the constitution of gases and the effects of heat by supposing that these particles are endowed with movements of rotation and displacement. The wave theory explains photic phenomena by supposing peculiar vibratory movements in a special medium—the ether. But these are merely hypotheses which are not at all necessary; they are the images of things, not the things themselves.

Reality of the Motion of Particles.—Here there is no question of hypotheses. This internal agitation, this interior labour, this incessant activity of matter are positive facts, an objective reality. It is true that when the chemical or mechanical equilibrium of bodies is disturbed it is only restored more or less slowly. Sometimes days and years are required before it is regained. Scarcely do they attain this relative repose when they are again disturbed, for the environment itself is not fixed; it experiences variations which react in their turn upon the body under consideration; and it is only at the end of these variations, at the end of their respective periods, that they will attain together, in a universal uniformity, an eternal repose.

We shall see that metallic alloys undergo continual physical and chemical changes. They are always seeking a more or less elusive equilibrium. Physicists in modern times have given their attention to this internal activity of material bodies, to the pursuit of stability. Wiedemann, Warburg, Tomlinson, MM. Duguet, Brillouin, Duhem, and Bouasse have revived the old experimental researches of Coulomb and Wertheim on the elasticity of bodies, the effects of pressures and thrusts, the hammering, tempering, and annealing of metals.

The internal activity manifested under these circumstances presents quite remarkable characteristics which cannot but be compared to the analogous phenomena presented by living bodies. Thus have arisen even in physics, a figurative terminology, and metaphorical expressions borrowed from biology.

Comparison of the Activity of Particles with Vital Activity.—Since Lord Kelvin first spoke of the *fatigue* of metals, or the *fatigue* of elasticity, Bose has shown in these same bodies the fatigue of electrical contact. The term *accommodation* has been employed in the study of torsion, and according to Tomlinson for the very phenomena which are the inverse of those of fatigue. The phenomena presented by glass when acted on by an external force which slowly bends it, have been called facts of adaptation. The manner in which a bar of steel resists wire-drawing has been compared to *defensive* processes against threatened rupture. And M. C. E. Guillaume speaks somewhere of "the heroic resistance of the bar of nickel-steel." The term "defence" has also been applied to the behaviour of chloride or iodide of silver when exposed to light.

There has been no hesitation in using the term "memory" concurrently with that of hysteresis to designate the behaviour of bodies acted on by magnetism or by certain mechanical forces. It is true that M. H. Bouasse protests in the name of the physico-mathematicians against the employment of these figurative expressions. But has he not himself written "a twisted wire is a wound-up watch," and elsewhere, "the properties of bodies depend at every moment upon all anterior modifications"? Does not this imply that they retain in some manner the impression of their past evolution? Powerful deformative agencies leave a trace of their action; they modify the body's condition of molecular aggregation, and some physicists go so far as to say that they even modify its chemical constitution. With the exception of M. Duhem, the disciples of the mechanical school who have studied elasticity admit that the effect of an external force upon a body depends upon the forces which have been previously acting on it, and not merely upon those which are acting on it at the present moment. Its present state cannot be anticipated, it is the recapitulation of preceding states. The effect of a torsional force upon a new wire will be different from that of the same force upon a wire previously subjected to torsions and detorsions. It was with reference to actions of this kind that Boltzmann, in 1876, declared that "a wire that has been twisted or drawn out remembers for a certain time the deformations which it has undergone." This memory is obliterated and disappears after a certain definite period. Here then, in a problem of static equilibrium, we find introduced an unexpected factor—time.

To sum up, it is the physicists themselves who have indicated the correspondence between the condition of existence in many brute bodies and that in many living bodies. It cannot be expected that these analogies will in any way serve as explanations. We should rather seek to derive the vital from the physical phenomenon. This is the sole ambition of the physiologist. To derive the physical from the vital phenomenon would be unreasonable. We do not attempt to do this here. It is nevertheless true that analogies are of service, were it only to shake the support which, from the time of Aristotle, has been accorded to the division of the bodies of nature into *psuchia* and *apsuchia*—*i.e.*, into living and brute bodies.

§ 2. The Brownian Movement.

The Existence of the Brownian Movement.—The simplest way of judging of the working activity of matter is to observe it when the liberty of the particles is not interfered with by the action of the neighbouring particles. We approximate to this condition when we watch, through the microscope, grains of dust suspended in a liquid, or globules of oil suspended in water. Now what we see is well known to all microscopists. If the granulations are sufficiently small, they seem to be never at rest. They are animated by a kind of incessant tremor; we see the phenomena called the "Brownian movement." This movement has struck all observers since the invention of the magnifying glass or simple microscope. But the

English botanist, Brown, in 1827, made it the object of special research and gave it his name. The exact explanation of it remained for a long time obscure. It was given in 1894 by M. Gouy, the learned physicist of the Faculty of Lyons.

The observer who for the first time looks through the microscope at a drop of water from the river, from the sea, or from any ordinary source—that is to say, water not specially purified—is struck with surprise and admiration at the motion revealed to him. Infusoria, microscopic articulata, and various micro-organisms people the microscopic field, and animate it by their movements; but at the same time all sorts of particles are also agitated, particles which cannot be considered as living beings, and which are, in fact, nothing but organic detritus, mineral dust, and debris of every description. Very often the singular movements of these granulations, which simulate up to a certain point those of living beings, have perplexed the observer or led him to erroneous conclusions, and the bodies have been taken for animalcules or for bacteria.

Characters of this Movement.—But it is as a rule quite easy to avoid this confusion. The Brownian movement is a kind of oscillation, a stationary, dancing to-and-fro movement. It is a Saint Vitus's dance on one and the same spot, and is thus distinguished from the movements of displacement customary with animate beings. Each particle has its own special dance. Each one acts on its own account, independently of its neighbour. There is, however, in the execution of these individual oscillations a kind of common and regular character which arises from the fact that their amplitudes differ little from each other. The largest particles are the slowest; when above four thousandths of a millimetre in diameter, they almost cease to be mobile. The smallest are the most active. When so small as to be barely visible in the microscope, the movement is extremely rapid, and can only occasionally be perceived. It is probable that it would be still more accelerated in smaller objects; but the latter will always escape our observation.

Its Independence of the Nature of the Bodies and of the Environment.—M. Gouy remarked that the movement depends neither on the nature nor on the form of the particles. Even the nature of the liquid has but little effect. Its degree of viscosity alone comes into play. The movements are, indeed, more lively in alcohol or ether, which are very mobile liquids; they are slow in sulphuric acid and in glycerine. In water, a grain one two-thousandth of a millimetre in diameter traverses, in a second, ten or twelve times its own length.

The fact that the Brownian movement is seen in liquors which have been boiled, in acids and in concentrated alkalies, in toxic solutions of all degrees of temperature, shows conclusively that the phenomenon has no vital significance; that it is in no way connected with vital activity so called.

Its Indefinite Duration.—The most remarkable character of this phenomenon is its permanence, its indefinite duration. The movement never ceases, the particles never attain repose and equilibrium. Granitic rocks contain quartz crystals which, at the moment of their formation, include within a closed cavity a drop of water containing a bubble of gas. These bubbles, contemporary with the Plutonian age of the globe, have never since their formation ceased to manifest the Brownian movement.

Its Independence of External Conditions.—What is the cause of this eternal oscillation? Is it a tremor of the earth? No! M. Gouy saw the Brownian movement far away from cities, where the mercurial mirror of a seismoscope showed no subterranean vibration. It does not increase when the vibrations occur and become quite appreciable. Neither is it changed by variation in light, magnetism, or electric influences; in a word, by any external occurrences. The result of observation is to place before us the paradox of a phenomenon which is kept up and indefinitely perpetuated in the interior of a body without known external cause.

The Brownian Movement must be the First Stage of Molecular Motion.—When we take in our hands a sheet of quartz containing a gaseous inclusion, we seem to be holding a perfectly inert object. When we have placed it upon the stage of the microscope, and have seen the agitation of the bubble, we are convinced that this seeming inertia is merely an illusion.

Repose exists only because of our limited vision. We see the objects as we see from afar a crowd of people. We perceive them only as a whole, without being able to discern the individuals or their movements. A visible object is, in the same way, a mass of particles. It is a molecular crowd. It gives us the impression of an indivisible mass, of a block in repose.

But as soon as the lens brings us near to this crowd, as soon as the microscope enlarges for us the minute elements of the brute body, then they appear to us, and we perceive the continual agitation of those elements which are less than four thousandths of a millimetre in diameter. The smaller the particles under consideration, the more lively are their movements. From this we infer that if we could perceive molecules, whose probable dimensions are about one thousand times less, their probable velocity would be, as required by the kinetic theory, some hundreds of metres per second. In the case of objects we can only just see, the Brownian velocity is only a few thousandths of a millimetre per second. No doubt, concludes M. Gouy, the particles that show this velocity are really enormous when compared with true molecules. From this point of view the Brownian movement is but the first degree, and a magnified picture of the molecular vibrations assumed in the kinetic theory.

§ 3. The Internal Activity of Bodies.

Migration of Material Particles.—In the Brownian movement we take into account only very small, isolated masses, small free fragments—*i.e.*, material particles which are not hampered by their relations to neighbouring particles. Any one but a physicist might believe that in true solids endowed with cohesion and tenacity, in which the molecules were bound one to the other, in which form and volume are fixed, there could be no longer movements or changes. This is a mistake. Physics teaches us the contrary, and, in late years especially, has furnished us characteristic examples. There are real migrations of material particles throughout solid bodies—migrations of considerable extent. They are accomplished through the agency of diverse forces acting externally—pressures, thrusts, torsions; sometimes under the action of light, sometimes under the action of electricity, sometimes under the influence of forces of diffusion. The microscopic observation of alloys by H. and A. Lechatelier, J. Hopkinson, Osmond, Charpy, J. R. Benoit; researches into their physical and chemical properties by Calvert, Matthiessen, Riche, Roberts Austen, Lodge, Laurie, and C. E. Guillaume; experiments on the electrolysis of glass, and the curious results of Bose upon electrical contact of metals, show in a striking manner the chemical and kinetic evolutions which occur in the interior of bodies.

Migration under the Action of Weight.—An experiment by Obermeyer, dating from 1877, furnishes a good example of the motions of solid bodies through a hardened viscid mass, taking place under the influence of weight. The black wax that shoemakers and boatbuilders use, is a kind of resin extracted from the pine and other resinous trees, melted in water, and separated from the more fluid part which rises from it. Its colour is due to the lampblack produced by the combustion of straw and fragments of bark. At an ordinary temperature it is a mass so hard that it cannot always be easily scratched by the finger-nail; but if it is left to itself in a receptacle, it finally yields, spreads out as if it were a liquid, and conforms to the shape of the vessel. Suppose we place within a cavity hollowed out of a piece of wood a portion of this substance, and keep it there by means of a few pebbles, having previously placed at the bottom of the cavity a few fragments of some light substance, such as cork. The piece of wax is thus between a light body below and a heavy body above. If we wait a few days, this order is reversed—the wax has filled the cavity by conforming to it; the cork has passed through the wax and appears on the surface, while the stones are at the bottom. We have here the celebrated experiment of the flask with the three elements, in which are seen the liquids mercury, oil, and water superposed in the order of their density, but in this case demonstrated with solid bodies.

Influence of Diffusion.—Diffusion, which disseminates liquids throughout each other, may also cause solids to pass through other solids. Of this W. Roberts Austen gave a convincing proof. This ingenious physicist placed a little cylinder of lead upon a disc of gold, and kept the whole at the temperature of boiling water. At this temperature both metals are perfectly solid, for the melting point of gold is 1,200° C., and of lead is 330°. Still, after this contact has been prolonged for a month and a half, analysis shows that the gold has become diffused through the top of the cylinder of lead.

Influence of Electrolysis.—Electrolysis offers another no less remarkable means of transportation. By its means we may force metals, such as sodium or lithium, through glass walls. The experiment may be performed as indicated by M. Charles Guillaume. A glass bulb containing mercury is placed in a bath of sodium amalgam, and a current is then made to pass from within outward. After some time it can be shown that the metal has penetrated the wall of the bulb, and has become dissolved within it.

Influence of Mechanical Pressure.—Mechanical pressure is also capable of causing one metal to pass into another. We need not recall the well-known experiment of Cailletet, who, by employing considerable pressure, caused mercury to sweat through a block of iron. In a more simple manner W. Spring showed that a disc of copper could be welded to a disc of tin by pressing them strongly one against the other. Up to a certain distance from the surfaces of contact a real alloy is formed; a layer of bronze of a certain thickness unites the two metals, and this could not take place did not the particles of both metals mutually interpenetrate.

§ 4. Internal Activity of Alloys.

Structure of Alloys.—Metallic alloys have a remarkable structure, which is essentially mobile, and which we have only now begun to understand by the aid of the microscope. Microscopical examination justifies to a certain degree Coulomb's conjecture. That illustrious physicist explained the physical properties of metals by imagining them to be formed of two kinds of elements—integral particles, to which the metal owes its elastic properties, and a *cement* which binds the particles, and to which it owes its coherence. M. Brillouin has also taken up this hypothesis of duality of structure. The metal is supposed to be formed of very small, isolated, crystalline grains, embedded in an almost continuous network of viscous matter. A more or less compact mass surrounding more or less distinct crystals is the conception which may be formed of an alloy.

Changes of Structure produced by Deforming Agencies.—It has been shown that profound changes of crystalline structure can be produced by various mechanical means, such as hammering, and the stretching of metallic bars carried to the point of rupture. Some of these changes are very slow, and it is only after months and years that they are completed, and the metal attains the definite equilibrium corresponding to the conditions to which it is exposed. Though there may be discussions concerning the extent of the transformations to which it is subjected, though some believe they affect the chemical condition of the alloy, while others limit its power to physical effects, it is nevertheless true—and this brings us back to our subject—that the mass of these metals is at work, and that it only slowly attains the phase of complete repose.

The Slow Re-establishment of Equilibrium. Residual Effect.—These operations by which the physical characters of metals are changed, and by which they are adapted to a variety of industrial needs—compression, hammering, rolling, stretching, and torsion—have an immediate, very apparent effect; but they have also a consecutive effect, slowly produced, much less marked and less evident. This is the "residual effect," or "Nachwirkung" of the Germans. It is not without importance, even in practical applications.

Heat also creates a kind of *forced equilibrium*. This becomes but slowly modified, so that a body may remain for a long time in a state which is, however, not the most stable for the conditions under which it is considered. The number of these bodies *not in equilibrium* is as great as that of the substances which have been exposed to fusion. All the Plutonic rocks are in this condition. Glass presents a condition of the same kind. Thermometers placed in melting ice do not always mark the zero Centigrade. This displacement of the zero point falsifies all records if care is not taken to correct it. The correction usually requires prolonged observation. The theory of the displacement of the thermometric zero is not entirely established; but we may suppose, with the author of the *Traité de Thermométrie*, that in glass, as in alloys, are to be found compounds which vary according to the temperature. At each temperature glass tends to assume a determinate composition and a corresponding state of equilibrium; but the previous temperature to which it has been subjected clearly has an influence on the rapidity with which it attains its state of repose. The effect of variation is more marked when we observe glass of more complicated composition. We can understand that those which contain comparable quantities of the two alkalies, soda and potash, may be more subject to these modifications than those having a more simple composition based on a single alkali.

Effects of Annealing.—A piece of brass wire that has been drawn and then heated is the scene of certain very remarkable internal changes, and these have been only recently recognized. The violent treatment of the metallic thread in forcing it through the hole in the die has crushed the crystalline particles; the interior state of the wire is that of broken crystals embedded in a granular mass. Heating changes all that. The crystals separate, repair themselves, and are built up again; they are then hard, geometrical bodies, in an amorphous, relatively soft and plastic mass; their number keeps on increasing; equilibrium is not established until the entire mass is crystallized. We may imagine how many displacements, enormous when compared with their dimensions, the molecules have to undergo when passing through the resisting mass, and arranging themselves in definite places in the crystalline structures.

In the same way, too, in the manufacture of steel, the particles of coal at first applied to the surface pass through the iron.

This *faculty of molecular displacement* enables the metal in some cases to modify its state at one point or another. The use made of this faculty under certain circumstances is very curious, greatly resembling the adaptation of an animal to its environment, or the methods of defence against agents that might destroy it.

Effect of Stretching. Hartmann's Experiment.—When a cylindrical rod of metal, held firmly at either end—a test-piece, as it is called in metallurgy—is pulled sufficiently hard, it often elongates considerably, part of the elongation disappearing as soon as the strain ceases, and the other part remaining. The total elongation is thus the sum of an "elastic elongation," which is temporary, and a "permanent elongation." If we continue the stretching, there appears at some point of the rod a local extension with contraction of sectional area. It is here that the rod will break.

But in place of continuing the stretching, Mr. Hartmann suspends it. He stops, as if to give the "metal-being" time to rally. During this delay it would seem that the molecules hasten to the menaced point to reinforce and harden the weak part. In fact the metal, which was soft at other points, at this spot looks like tempered metal. It is no longer extensible.

When the experimenter begins the stretching again after this rest, and after the narrowed bar has been rolled and become cylindrical again, the local extension and sectional contraction is forced to occur at another point. If another rest is given at this point the metal will also become hardened.

If we repeat the experiment a sufficient number of times, we shall find a total transformation of the rod, which becomes hardened throughout its entire extent. It will break rather than elongate if the stretching is sufficiently severe.

Nickel Steels—their "Heroic" Resistance.—Nickel steels present this phenomena in an exaggerated degree. The alternation of operations which we have just described, bringing the various parts of an ordinary steel rod into a tempered state, is not necessary with nickel steel. The effect is produced in the course of a single trial. As soon as there is any tendency to contraction the alloy hardens at that precise place; the contraction is hardly noticeable; the movement is stopped at this point to attack another weak point, stops there again and attacks a third, and so on; and, finally, the paradoxical fact appears that a rod of metal which was in a soft state and could be considerably elongated has now become throughout its whole extent as hard, brittle, and inextensible as tempered steel. It is in connection with this point that M. C. E. Guillaume spoke of "heroic resistance to rupture." It would seem, in fact, as if the ferro-nickel bar had reinforced each weak point as it was threatened. It is only at the end of these efforts that the inevitable catastrophe occurs.

Effect of Temperature.—When the temperature changes, it is seen that these ferro-nickel bars elongate or retract, modifying at the same time their chemical constitution. But these effects, like those which occur in the glass bulb of a thermometer, do not occur at once. They are produced rapidly for one part, and more slowly for a small remaining portion. Bars of ferro-nickel which have been kept at the same temperature change gradually in length in the course of a year. Can we find a better proof of internal activity occurring in a substance differing so greatly from living matter?

Nature of the Activity of Particles.—These are examples of the internal activity that occurs in brute bodies. Besides, these facts that we are quoting merely to refute Bichat's assertion relative to the immutability of brute bodies, and to show us their activity, also afford us another proof. They show that this activity, like that of animals, wards off foreign intervention, and that this parrying of the attack, again like that of animals, is adapted for the defence and preservation of the brute mass. So that if we consider of special importance the adaptative, teleological characteristic of vital phenomena, a characteristic which is so easily made too much of in biological interpretations, we may also find it again in the inanimate world. To this end we may add to the preceding examples one more which is no less remarkable. This is the famous case of Becquerel's process for colour-photography.

Colour-Photography.—A greyish plate, treated with chloride or iodide of silver and exposed to a red light, rapidly becomes red. It is then exposed to green light, and after passing through dull and obscure tints it becomes green. To explain this remarkable phenomenon, we cannot improve on the following statement:—The silver salt protects itself against the light that threatens its existence; that light causes it to pass through all kinds of stages of coloration before reducing it; the salt stops at the stage which protects it best. It stops at red, if it is red light that assails it, because in becoming red by reflection it best repels that light—*i.e.*, it absorbs it the least.

It may then be advantageous, for the comprehension of natural phenomena, to regard the transformation of inanimate matter as manifestations of a kind of internal life.

Conclusion. Relations of the Surrounding Medium to the Living Being and the Brute Body.—Brute bodies, then, are not immutable any more than are living bodies. Both depend on the medium that surrounds them, and they depend upon it in the same way. Life brings together, brings into conflict, an appropriate organism and a suitable environment. Auguste Comte and Claude Bernard have taught us that vital phenomena result from the reciprocal action of these two factors which are in close correlation. It is also from the reciprocal action of the environment and the brute body that inevitably result the phenomena which that body presents. The living body is sometimes more sensitive to variations of the ambient medium than is the brute body, but at other times the reverse is the case. For example, there is no living organism as impressionable to any kind of stimulus whatever as the bolometer is to the slightest variations of temperature.

There can only be, then, one chemically immutable body—namely, the atom of a simple body, since, by its very definition, it remains unaltered and intangible in combinations. This notion of an unalterable atom has, however, itself been attacked by the doctrine of the ionization of particles due to Sir J. J. Thomson; and besides, with very few exceptions—those of cadmium, mercury, and the gases of the argon series—the atoms of simple bodies cannot exist in a free state.

Thus, as in the vital struggle, the ambient medium by means of alimentation furnishes to the living being, whether whole or fragmentary, the materials of its organization and the energies which it brings into play. It also furnishes to brute bodies their materials and their energies.

It is also said that the ambient medium furnishes to the living being a third class of things, the *stimuli* of its activities—*i.e.*, its "provocation to action." The protozoon finds in the aquatic environment which is its habitat the stimuli which provoke it to move and to absorb its food. The cells of the metazoon encounter in the same way in the lymph, the blood, and the interstitial liquids which bathe them, the shock, the stimulus which brings their energies into play. They do not derive from themselves, by a mysterious spontaneity without parallel in the rest of nature, the capricious principle which sets them in motion.

Vital spontaneity, so readily accepted by persons ignorant of biology, is disproved by the whole history of the science. Every vital manifestation is a response to a stimulus, a provoked phenomenon. It is unnecessary to say this is also the case with brute bodies, since that is precisely the foundation of the great principle of the inertia of matter. It is plain that it is also as applicable to living as to inanimate matter.