Ex: 7.1

1-30

Q Evaluate the integrals by making appropriate u-substitution and applying the formulas reviewed in this section.

1 $\int (4-2x)^3 \, dx$

$\int 4\,dx - \int 2x^3\,dx$

$4x - \frac{2x^4}{4} + C$

$4x - \frac{1}{2}x^4 + C$

2 $\int 3\sqrt{4+2x}\,dx$

let $u = 4+2x$

$du = 2\,dx$

$dx = du/2$

$\int 3\sqrt{u}\,\frac{du}{2}$

$\frac{3}{2}\int \sqrt{u}$

$\frac{3}{2}\,\frac{u^{1/2+1}}{1/2+1} + C$

$\frac{\cancel{3}}{\cancel{2}}\,\frac{u^{3/2}}{\cancel{3}/\cancel{2}} + C$

$u^{3/2} + C$

$(4+2x)^{3/2} + C$

3 $\int x\sec^2(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$\frac{1}{2}\int \sec^2(u)\,du$

$\frac{1}{2}\tan u + C$

$\Rightarrow \frac{1}{2}\tan(x^2) + C$

4 $\int 4x\tan(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$x\,dx = du/2$

$\frac{4}{2}\int \tan(u)\,du$

$2\ln|\sec u| + C$

$2\ln|\sec x^2| + C$

5 $\int \frac{\sin 3x}{2+\cos 3x}\,dx$

$u = 2+\cos 3x$

$du = -3\sin 3x\,dx$

$\sin 3x\,dx = -du/3$

$-\frac{1}{3}\int \frac{du}{u}$

$-1/3 \ln u + C$

$-1/3 \ln(2+\cos 3x) + C$

6 $\int \frac{1}{9+4x^2}\,dx$

$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right) + C$

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7 $\int e^x \sinh(e^x)\,dx$

let $u = e^x$

$du = e^x\,dx$

$\int \sinh(u)\,du$

$\cosh u + C$

$\cosh e^x + C$

8 $\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,dx$

$u = \ln x$

$du = 1/x\,dx$

$\int \sec u \tan u\,du$

$\sec u + C$

$\sec(\ln x) + C$

9 $\int e^{\tan x}\sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int e^u\,du$

$e^u + C$

$e^{\tan x} + C$

10 $\int \frac{x}{\sqrt{1-x^4}}\,dx$

$u = x^2$

$\frac{u}{2} = x\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

$1/2 \sin^{-1}(u) + C$

$1/2 \sin^{-1}(x^2) + C$

11 $\int \cos^5 5x \sin 5x\,dx$

$u = \cos 5x$

$du = -5\sin 5x\,dx$

$-\frac{du}{5} = \sin 5x\,dx$

$-\frac{1}{5}\int u^5\,du$

$-\frac{1}{5}\ \frac{u^6}{6} + C$

$-\frac{1}{30}u^6 + C$

$-\frac{1}{30}\cos^6 5x + C$

12 $\int \frac{\cos x}{\sin x\sqrt{\sin^2 x+1}}\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int \frac{du}{u\sqrt{u^2+1}}$

Using formula

$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$

$-\ln\left|\frac{1+\sqrt{1+u^2}}{u}\right| + C$

$-\ln\left|\frac{1+\sqrt{1+\sin^2 x}}{\sin x}\right| + C$

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13 $\int \frac{e^x}{\sqrt{4+e^{2x}}} dx$

$u = e^x$

$du = e^x dx$

$\int \frac{du}{\sqrt{4+u^2}}$

Using formula

$\int \frac{du}{\sqrt{a^2+u^2}} = \ln(u + \sqrt{u^2+a^2}) + C$

$\ln(u + \sqrt{u^2+4}) + C$

$\ln(e^x + \sqrt{e^{2x}+4}) + C$

14 $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

$u = \tan^{-1}x$

$du = \frac{1}{1+x^2} dx$

$\int e^u du$

$e^u + C$

$e^{\tan^{-1}x} + C$

15 $\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}} dx$

$u = \sqrt{x-1}$

$du = \frac{1}{2\sqrt{x-1}} dx$

$2du = \frac{1}{\sqrt{x-1}} dx$

$2\int e^u du$

$2e^u + C \Rightarrow 2e^{\sqrt{x-1}} + C$

16 $\int (x+1)\cot(x^2+2x) dx$

$u = x^2+2x$

$du = 2x+2 \, dx$

$du = 2(x+1) dx$

$dx(x+1) = du/2$

$\frac{1}{2}\int \cot u \, du$

$\frac{1}{2}\ln|\sin u| + C$

$\frac{1}{2}\ln \sin|x^2+2x| + C$

17 $\int \frac{\cosh\sqrt{x}}{\sqrt{x}} dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$2du = \frac{1}{\sqrt{x}} dx$

$2\int \cosh u \, du$

$2\sinh u + C$

$2\sinh(\sqrt{x}) + C$

18 $\int \frac{dx}{x(\ln x)^2}$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int \frac{du}{u^2}$

$-\frac{1}{u} + C$

$-\frac{1}{\ln x} + C$

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19 $\int \frac{dx}{\sqrt{x}\,3^{\sqrt{x}}}$

$U=\sqrt{x}$

$dU = \frac{1}{2\sqrt{x}}\,dx$

$2\,dU = \frac{1}{\sqrt{x}}\,dx$

$2\int \frac{dU}{3^{U}}$

Taking e and ln

$2\int \frac{dU}{e^{U\ln 3}}$

$2\int e^{-U\ln 3}\,dU$

$-\frac{2}{\ln 3}\,e^{-U\ln 3} + C$

removing / cancelling e and ln

$-\frac{2}{\ln 3}\,3^{-U} + C$

$-\frac{2}{\ln 3}\,3^{-\sqrt{x}} + C$

20 $\int \sec(\sin\theta)\tan(\sin\theta)\cos\theta\,d\theta$

$U = \sin\theta$

$dU = \cos\theta\,d\theta$

$\int \sec U \tan U\,dU$

$\sec U + C$

$\sec(\sin U) + C$

21 $\int \frac{\operatorname{csch}^2(2/x)}{x^2}\,dx$

$U = 2/x$

$dU = -2/x^2\,dx$

$-\frac{dU}{2} = \frac{dx}{x^2}$

$-\frac{1}{2}\int \operatorname{csch}^2(U)\,dU$

$-\frac{1}{2}\coth U + C$

$-1/2\,\coth\left(\frac{2}{x}\right) + C$

22 $\int \frac{dx}{\sqrt{x^2-4}}$

apply formula

$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + C$

$\ln\left|x+\sqrt{x^2+}\right| + C$

23 $\int \frac{e^{-x}}{4-e^{-2x}}\,dx$

$U = e^{-x}$

$dU = -e^{-x}\,dx$

$-dU = e^{-x}\,dx$

$-\int \frac{dU}{4-U^2}$

apply formula

$\int \frac{dU}{a^2-U^2} = \frac{1}{2a}\ln\left|\frac{a+U}{a-U}\right| + C$

$-\frac{1}{4}\ln\left|\frac{2+e^{-x}}{2-e^{-x}}\right| + C$

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24 $\int \frac{\cos(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \cos u\,du$

$+\sin u + C$

$\sin(\ln x) + C$

25 $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$u = e^x$

$du = e^x\,dx$

$\int \frac{du}{\sqrt{1-u^2}}$

$\sin^{-1} u + C$

$\sin^{-1}(e^x) + C$

26 $\int \frac{\sinh(x^{-1/2})}{x^{3/2}}\,dx$

$u = x^{-1/2}$

$du = \frac{-1}{2x^{3/2}}\,dx$

$-2du = \frac{1}{x^{3/2}}\,dx$

$-2\int \sinh(u)\,du$

$-2\cosh u + C$

$-2\cosh(x^{-1/2}) + C$

27 $\int \frac{x}{\csc(x^2)}\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{du}{\csc u}$

$\frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos u + C$

$-1/2\cos(x^2) + C$

28 $\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$

$u = e^x$

$du = e^x\,dx$

$\int \frac{u}{\sqrt{4-u^2}}$

$\sin^{-1}\left(\frac{u}{2}\right) + C$

$\sin^{-1}\left(\frac{e^x}{2}\right) + C$

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29 $\int x 4^{-x^2} dx$

Consider 4^{-x^2}

apply ln and e

$e^{-x^2 \ln 4}$

now $U = -x^2 \ln 4$

$dU = -2x \ln 4 \, dx$

$dU = -x \ln 4^2 \, dx$

$dU = -x \ln 16 \, dx$

$-\frac{dU}{\ln(16)} = x \, dx$

$-\frac{1}{\ln(16)} \int e^U \, dU$

$-\frac{1}{\ln(16)} e^U + C$

$-\frac{1}{\ln(16)} e^{-x^2 \ln 4} + C$

Substitute back

$-\frac{1}{\ln 16} 4^{-x^2} + C$

30 $\int 2^{\pi x} dx$

Consider $2^{\pi x}$

apply ln and e

$e^{x\pi \ln 2}$

$U = x\pi \ln 2$

$dU = \pi \ln 2 \, dx$

$\frac{dU}{\pi \ln 2} = dx$

$\frac{1}{\pi \ln 2} \int e^U \, dU$

$\frac{1}{\pi \ln 2} e^U + C$

$\frac{1}{\pi \ln 2} e^{x\pi \ln 2} + C$

$\frac{1}{\pi \ln 2} 2^{\pi x} + C$