Date:

Assignment: 02

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Find dy/dx

1) $8 = 3x^2 + y^4$

diff wrt x both side

$$\frac{d}{dx}(8) = 3\frac{d}{dx}x^2 + \frac{d}{dx}y^4$$

$$0 = 3(2x) + 4y^3\frac{dy}{dx}$$

$$-6x = 4y^3\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-\cancel{6}x}{\cancel{4}y^3}$$

$$\frac{dy}{dx} = \frac{-3x}{2y^3}$$

2) $\sin(2x-y) = 4x$

diff wrt 'x' both side

$$\frac{d}{dx}\left(\sin(2x-y)\right) = \frac{d}{dx}(4x)$$

$$\cos(2x-y)\frac{d}{dx}(2x-y) = 4$$

$$\cos(2x-y)\left\{\frac{d}{dx}(2x) - \frac{dy}{dx}\right\} = 4$$

$$2 - \frac{dy}{dx} = \frac{4}{\cos(2x-y)}$$

$$2 - \frac{4}{\cos(2x-y)} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2\cos(2x-y) - 4}{\cos(2x-y)}$$

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3) $x^2 + 2y^5 = 10xy$

diff wrt 'x' on b/s

$$\frac{d}{dx}(x^2) + 2\frac{d}{dx}y^5 = 10\frac{d}{dx}(x \cdot y)$$

$$2x + 2(5y^4)\frac{dy}{dx} = 10\left\{y\frac{dx}{dx} + x\frac{dy}{dx}\right\}$$

$$2x + 10y^4\frac{dy}{dx} = 10y + 10x\frac{dy}{dx}$$

$$10y^4\frac{dy}{dx} - 10x\frac{dy}{dx} = 10y - 2x$$

$$\frac{dy}{dx}(10y^4 - 10x) = 10y - 2x$$

$$\frac{dy}{dx} = \frac{10y - 2x}{10y^4 - 10x}$$

4) $7xy = 8$

diff wrt 'x' b/s

$$7\frac{d}{dx}(x \cdot y) = \frac{d}{dx}(8)$$

$$7\left\{x\frac{dy}{dx} + y\right\} = 0$$

$$x\frac{dy}{dx} + y = 0$$

$$x\frac{dy}{dx} = -y$$

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$$\frac{dy}{dx} = -\frac{y}{x}$$

diff w.r.t to x

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{x}{y}\right)$$

$$= \frac{y\frac{d}{dx}x - x\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2}$$

$$= \frac{\frac{y^2 - x^2}{y}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}$$

Ans

again diffrentiatg w.r.t 'x'

$$\frac{d^2y}{dx^2} = -\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$= -\left\{\frac{x\frac{dy}{dx} - y\frac{dx}{dx}}{x}\right\}$$

$$= -\left\{\frac{x\left(\frac{-y}{x}\right) - y}{x}\right\}$$

$$= -\left\{\frac{-y - y}{x}\right\}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2y}{x}}$$

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5) $e^x + y^2 = 4$

diff wrt 'x' on b/s.

$$\frac{d}{dx}\left\{e^x + y^2\right\} = \frac{d(4)}{dx}$$

$$e^x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -e^x$$

$$\frac{dy}{dx} = \frac{-e^x}{2y}$$

diff again wrt 'x'

$$\frac{d^2y}{dx^2} = \frac{-1}{2}\frac{d}{dx}\left(\frac{e^x}{y}\right)$$

$$= \frac{-1}{2}\left\{\frac{y\frac{d}{dx}e^x - e^x\frac{dy}{dx}}{y^2}\right\}$$

$$= \frac{-1}{2}\left\{\frac{e^x y - e^x\left(\frac{-e^x}{2y}\right)}{y^2}\right\}$$

$$= \frac{-1}{2}\left\{\frac{e^x y + \frac{e^{2x}}{2y}}{y^2}\right\}$$

$$= \frac{-1}{2}\left\{\frac{\frac{2y^2e^x + e^{2x}}{2y}}{y^2}\right\}$$

$$= \frac{-1}{2}\left\{\frac{2e^xy^2 + e^{2x}}{2y^3}\right\}$$

$$\frac{d^2y}{dx^2} = \frac{-2e^xy^2 - e^{2x}}{2y^3}$$

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Q6: $x^2y + y^2 + 4 = 0$ $\quad \frac{dy}{dx}$ at $x=2$ $=2$

diff wrt 'x'

$$\frac{d}{dx}\{x^2y + y^2 + 4\} = 0$$

$$\left\{x^2\frac{dy}{dx} + y(2x)\right\} + 2y\frac{dy}{dx} + 0 = 0$$

$$\frac{dy}{dx}\{x^2 + 2y\} = -2xy$$

$$\frac{dy}{dx} = -\frac{2xy}{x^2+2y}$$

replace $x=2$ & $y=-2$

$$\frac{dy}{dx} = -\frac{2(2)(-2)}{(2)^2 + 2(-2)}$$

$$= \frac{-8}{4+(-4)}$$

$$\boxed{\frac{dy}{dx} = \frac{-8}{0}}$$ ANS

ANSER ⇒ DNE

does not exist

put $x=2$

$x^2y + y^2 + 4 = 0$

$2^2y + y^2 + 4 = 0$

$y^2 + 4y + 4 = 0$

$\boxed{y = -2}$

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Q7) $x^2 - y^2 = 5$ $\quad \frac{d^2y}{dx^2}$ at $(3,2)$?

diff wrt 'x'

$$\frac{d}{dx}(x^2 - y^2) = \frac{d}{dx}(5)$$

$$2x - 2y\frac{dy}{dx} = 0$$

$$\cancel{+}\cancel{2}y\frac{dy}{dx} = \cancel{+}\cancel{2}x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

again diff wrt 'x'

$$\frac{d^2y}{dx^2} = \frac{y\frac{dx}{dx} - x\frac{dy}{dx}}{y^2}$$

$$= \frac{y - x\left(\frac{dy}{dx}\right)}{y^2}$$

$$= \frac{y - x\left(\frac{x}{y}\right)}{y^2}$$

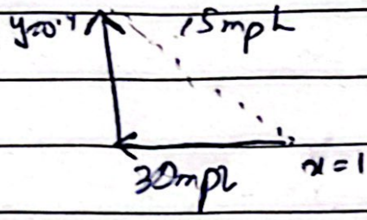
$$= \frac{\frac{y^2 - x^2}{y}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}$$

$$\frac{d^2y}{dx^2}\Bigg|_{\substack{\text{at } x=3 \\ y=2}} = \frac{2^2 - 3^2}{2^3} = \frac{-5}{8}$$

Date:

Question :- 08



$\frac{dx}{dt}\Big|_{x=1} = -30$ mph

$\frac{dy}{dt}\Big|_{y=0.4} = 15$ mph

$\frac{ds}{dt} = ?$

Sol: By using pythagorus theorem

$$s^2 = x^2 + y^2$$

$$s = \sqrt{(1)^2 + (0.4)^2}$$

$$\boxed{s = 1.07 \text{ miles}}$$

Again

$$s^2 = x^2 + y^2$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$2(1.07)\frac{ds}{dt} = 2(1)(-30) + 2(0.4)(15)$$

$$= \frac{-48}{2.15}$$

$$\boxed{\therefore \frac{ds}{dt} = -22.28 \text{ mph}}$$

So, the distance b/w them is decreasing at a rate of -22.28 mph.

Question : 09

let 's' be the side of a cube,

$\frac{ds}{dt} = 0.2$ cm/s

Sol: surface area is given by

$$S = 6s^2$$

$$s^2 = \frac{S}{6}$$

Now,

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$$

$$= 3\left(\frac{S}{6}\right)(0.2)$$

$$\boxed{\frac{dV}{dt} = 0.15}$$

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Date:

Question: 10

a) $f(x) = x^5 - 5x^4$

$f'(x) = 5x^4 - 20x^3$

$f''(x) = 20x^3 - 60x^2$

CRITICAL POINTS:-

Let $f'x = 0$ & $f'(x) =$ undefined

$5x^4 - 20x^3 = 0$ DNE

$5x^3(x-4) = 0$ $5x^4 = 20x^3$

$\boxed{x=0}$ $\boxed{x=4}$ $x = \frac{20}{5}$

$\boxed{x=4}$ $\boxed{x=0}$

INCREASING / DECREASING POINT:-

inc $\Rightarrow$ $(-\infty, 0] \cup [4, +\infty)$

dec $\Rightarrow$ $(-0, 4)$

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INFLECTION POINTS:

→ let

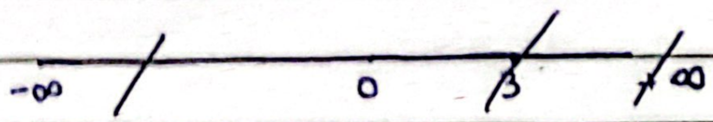
$f''(x) = 0$ & $f''(x) =$ undefined one

$20x^3 - 60x^2 = 0$ $\qquad$ $20x^2(x-3) = 0$

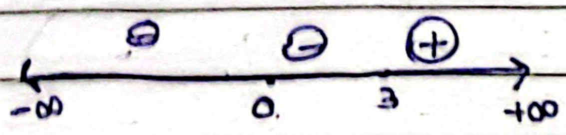
$\cancel{20}x^3 = \overset{3}{\cancel{60}}\cancel{x^2}$ $\qquad$ $\boxed{x=3}$, $\boxed{x=0}$

$\boxed{x=3}$

~~Increasing | decreasing:~~



CONCAVITY:



concave down ⇒ $(-\infty, 3)$

concave up ⇒ $(3, +\infty)$

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Relative Extremas

$f''(x = 0) \Rightarrow$

$f''(0) = 0 \longrightarrow$ inflection

$f''(4) = 320 > 0$

its minima

Minimum Value:

$$\boxed{f(4) = -256}$$

Q (b) $f(x) = x^{\frac{1}{5}}$

$f(x) = \frac{1}{5} x^{\frac{1}{5}-1}$

$= \frac{1}{5} x^{-\frac{4}{5}}$

$f'(x) = \frac{1}{5x^{\frac{4}{5}}}$

$f''(x) = \frac{1}{5} \cdot \frac{-4}{5} x^{-\frac{4}{5}-1}$

$= \frac{-4}{25 x^{\frac{9}{5}}}$

$f''(x) = \frac{-4}{25x^{\frac{9}{5}}}$

CRITICAL POINTS:-

$f'(x) = 0$

$\frac{1}{5} x^{-\frac{4}{5}} = 0$

$x^{-\frac{4}{5}} = 0$

$f'x =$ undefined

$x^{\frac{4}{5}} = 0$

$\boxed{x = 0}$

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Date:

$\boxed{x=0}$

INC/DEC POINTS:-

$\oplus$ $\oplus$

0

increasing: ~~[illegible]~~ DNE

decreasing: $(-\infty, 0)$

INFLECTION POINTS:-

$f''(x)=0$ $\qquad$ $f''x = \text{undefined}$

$\frac{-4}{25} x^{-\frac{9}{5}}=0$ $\qquad$ $25x^{\frac{9}{5}}=0$

$\boxed{x=0}$ $\qquad$ $\boxed{x=0}$

CONCAVITY:-

$\oplus$ $\ominus$

$-\infty$ 0 $+\infty$

concave up $= (-\infty, 0)$

concave down $= (0, \infty)$

RELATIVE EXTREMAS:-

$f''(x == C.P)$

$f''(x) = =0) = \frac{-4}{25(0)}$

$= \infty$

DNE

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(c) $f(x) = (x+1)(x-1)^2$

$f'(x) = \frac{d}{dx} (x+1)(x-1)^2$

$= (x+1)\,2(x-1) + (x-1)^2(1)$

$= 2(x+1)(x+1) + (x-1)^2$

$f'(x) = 2(x+1)(x-1) + (x-1)^2$

$f'(x) = 2(x^2-1) + (x-1)^2$

~~$f''(x) = 2\{(x+1)(1) + (x-1)$~~

$f'(x) = 2x^2 - 2 + (x-1)^2$

$f''(x) = 4x - 0 + 2(x-1)$

$f''(x) = 2(x-1) + 4x$

C.P:-

$f'(x) = 0$ — $f(x) =$ undefined

DNE

$2x^2 - 2 + (x-1)^2 = 0$

$2x^2 - 2 + x^2 - 2x + 1 = 0$

$3x^2 - 2x - 1 = 0$

$\boxed{x = 1}$, $\boxed{x = -1/3}$

INC/DEC:

$\ominus \quad -1/3 \quad \oplus \quad \oplus$

$-\infty \quad 1 \quad +\infty$

Increasing $= (-1/3, +\infty)$

decreasing $= (-\infty, -1/3)$

Scanned with CamScanner

Date:

INFLECTION POINTS:-

$f''(x) = 0$ $\qquad$ $f''(x) =$ undefined

$2(x-1)+4x=0$ $\qquad$ dne

$2x-2+4x=0$

$6x-2=0$

$\boxed{x = 1/3}$

Relative Extremae

$f''(x==1) = 4 > 0$
minima

$f''(-1/3) = -4 < 0$
maxima

minima at 1 i.e 45

maxima at 1/3 i.e -75

CONCAVITY:-

$\ominus$ $\qquad$ $\oplus$

$-\infty \qquad 1/3 \qquad +\infty$

concave up:- $(1/3, +\infty)$

concave down: $(-\infty, 1/3)$

(d) $f(x) = \dfrac{x^2-3}{x-2}$

$f'(x) = \dfrac{(x-2)(2x) - (x^2-3)(1)}{(x-2)^2}$

$= \dfrac{2x^2-2x - x^2+3}{(x-2)^2}$

$f(x) = \dfrac{x^2-2x+3}{(x-2)^2}$

$f(x) = \dfrac{(x-1)(x-3)}{(x-2)^2}$

$f''(x) = \dfrac{(x-2)^2(2x-2) - (x^2-2x+3)(2(x-2))}{(x-2)^4}$

$f''(x) = \dfrac{(x-2)^2(2x-2) - (x^2-2x+3)(2x-}{(x-2)^4}$

$f''(x) = \dfrac{\cancel{(x-2)}[(x-2)(2x-4) - (x^2-4x+3)]}{(x-2)^{\cancel{4}\,3}}$

$= \dfrac{2x^2-4x-4x+8-2x^2-8x}{(x-2)^3}$

$f''(x) = \dfrac{2}{(x-2)^3}$

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C.P:-

$f'(x) = 0$

$\frac{x^2 - 4x + 3}{(x-2)^2} = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\boxed{x=1}$ $\boxed{x=3}$

$f'(x) = \text{undefined}$

$(x-2)^2 = 0$

$x - 2 = 0$

$\boxed{x=2}$

INC/DEC:-

$+ \quad + \quad - \quad +$

$-\infty \quad 1 \quad 2 \quad 3 \quad +\infty$

increasing:- $(-\infty, +\infty)$

decreasing:- $(2, 3)$

Inflection Points:

let $f''(x) = 0$

$\frac{2}{(x-2)^3} = 0$

dne

$\boxed{x=2}$

$2/(x-2)^3 = \text{undefined}$

$(x-2)^3 = 0$

$\boxed{x=2}$

Relative Extrema:-

$f''(1) = -2 < 0$

minima at 2

$f''(2)$ - dne

$f''(3) = 2$ maxima at 0

CONCAVITY:-

$\ominus \quad \oplus$

$-\infty \quad 2 \quad +\infty$

concave up = $(2, +\infty)$

concave down = $(-\infty, 2)$

Date:

(e) $f(x) = x\sqrt{8-x^2}$

$$f'(x) = x\left(\frac{1}{2}\cdot\frac{1}{\sqrt{8-x^2}}\right)(-2x) + \sqrt{8-x^2}$$

$$= \frac{-x^2}{\sqrt{8-x^2}} + \sqrt{8-x^2}$$

$$= \frac{-x^2 + 8 - x^2}{\sqrt{8-x^2}}$$

$$f'(x) = \frac{8-2x^2}{\sqrt{8-x^2}}$$

$$f''(x) = \frac{(\sqrt{8-x^2})(-4x) - (8-2x^2)\left(\frac{1}{2\sqrt{8-x^2}}\cdot(-2x)\right)}{(\sqrt{8-x^2})^2}$$

$$= \frac{-4x\sqrt{8-x^2} - \left((8-2x^2)\cdot\frac{-x}{\sqrt{8-x^2}}\right)}{8-x^2}$$

$$= \frac{-4x\sqrt{8-x^2} + \frac{x(8-2x^2)}{\sqrt{8-x^2}}}{8-x^2}$$

$$= \frac{-4x(8-x^2) + x(8-2x^2)}{(8-x^2)(\sqrt{8-x^2})}$$

$$= \frac{-32x + x^3 + 8x - 2x^3}{(8-x^2)^{\frac{3}{2}}}$$

$$= \frac{-24x - x^3}{(8-x^2)^{3/2}}$$

$$f''(x) = \frac{2x(x^2 - 12)}{(8-x^2)^{3/2}}$$

Date:

C.P:

$f'(x)=0$ $\quad$ $f'(x)$ = undefined

$\frac{8-2x^2}{\sqrt{8-2x^2}}=0$ $\quad$ $\sqrt{8-x^2}=0$

$8-2x^2=0$ $\quad$ $8-x^2=0$

$2x^2=8$ $\quad$ $x^2=8$

$x^2=4$ $\quad$ $x=\pm\sqrt{8}$

$x=\pm 2$

Relative Extermas.

$f''(2)=-2.17$

its maxima at 2

$f''(-2)=4$

its minima at -2

$f''(\sqrt{8})$ = dne

$f''(-\sqrt{8})$ dne.

INC/DEC:

$\ominus$ $\quad$ $\oplus$ $\quad$ $\ominus$

$-\infty \quad -\sqrt{8} \quad -2 \quad\quad +2 \quad \sqrt{8} \quad +\infty$

INC ⇒ $(-2, +2)$

DEC ⇒ $(-\sqrt{8}, -2) \cup (2, \sqrt{8})$

~~CONCAVITY:~~

INFLECTION POINTS:

$f''(x)=0$ $\quad$ $f''(x)$ = undefined

$\frac{2x(x^2-12)}{(8-x^2)^{\frac{3}{2}}}=0$ $\quad$ $(8-x^2)^{\frac{3}{2}}=0$

$8-x^2=0$

$x^2=8$

$x=\pm\sqrt{8}$

$2x=0$, $x^2-12=0$

$x=0$, $x^2=12$

$x=\pm\sqrt{12}$

Date:

(f) $f(x) = \frac{12}{x^2+12}$

$f'(x) = 12\left\{\frac{d}{dx}(x^2+12)^{-1}\right\}$

$= 12\left((-1)\cdot(x^2+12)^{-2}(2x)\right)$

$f'(x) = \frac{-24x}{(x^2+12)^2}$

$f''(x) = \frac{(x^2+12)^2(-24) - [(-24x)\,2(x^2+12)(2x)]}{(x+12)^2}$

$f''(x) = \frac{\cancel{(x^2+12)}\{(x^2+12)(-24) + 24x(4x)\}}{(x^2+12)^{4\,3}}$

$f''(x) = \frac{-24\{x^2+12-4x^2\}}{(x^2+12)^3}$

$f''(x) = \frac{-24(-3x^2+12)}{(x^2+12)^3}$

C.P.:

$f'(x)=0$

$\frac{-24x}{(x^2+12)^2}=0$

$-24x=0$

$\boxed{x=0}$

$f'(x) = \text{undefined}$

$(x^2+12)^2=0$

$x^2+12=0$

$x^2=-12$

complex

INC/DECS.

$+ \qquad -$

$-\infty \qquad 0 \qquad +\infty$

increasing $\Rightarrow (-\infty, 0)$

decreasing $\Rightarrow (0, +\infty)$

Inflection Points.

$f''(x)=0$

$\frac{-24(-3x^2+12)}{(x^2+12)^3}=0$

$-24(12-3x^2)=0$

~~$-288+72x^2=$~~

$-(12-3x^2)=0$

$3x^2-12=0$

$x^2=\frac{12}{3}$

$\boxed{x=\pm 2}$

$f''(x)=\text{undefined}$

$(x^2+12)^3=0$

$x^2=-12$

complex

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Concavity

$$\underset{-\infty}{}\quad + \quad \underset{-2}{} \quad - \quad \underset{+2}{} \quad + \quad \underset{+\infty}{}$$

concave up → $(-\infty, -2) \cup (2, +\infty)$

concave down → $(-2, 2)$

Relative Extrema

$$f''(0) = \frac{-24\{12 - 3(0)^2\}}{((0)^2 + 12)^3}$$

$$= \frac{-288}{1728}$$

$$f''(0) = \frac{-1}{6} < 0$$

minima at 0

(g) $f(x) = x^2 \ln x$

Sol:

$$f'(x) = 2x \ln x + \frac{1}{\cancel{x}} x^{\cancel{2}}$$

$$= 2x \ln x + x$$

$$f'(x) = x(2\ln x + 1)$$

$$f''(x) = (1)(1 + 2\ln x) + x\left[\frac{2}{\cancel{x}}\right]$$

$$= 1 + 2\ln x + 2$$

$$f''(x) = 3 + 2\ln x$$

C.P:

$f'(x) = 0$

$x(2\ln x + 1) = 0$

$\boxed{x = 0}$ $\quad 1 + 2\ln x = 0$

$$\ln x = \frac{-1}{2}$$

$$\boxed{x = e^{-1/2}}$$

INC / DEC

$$\underset{-\infty}{}\quad \text{no sol} \quad \underset{0}{} \quad - \quad \underset{e^{-1/2}}{} \quad + \quad \underset{+\infty}{}$$

increasing: $(e^{-1/2}, +\infty)$

decreasing: $(0, e^{-1/2})$

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Inflection point.

$f''(x)=0$

$3+2\ln x=0$

$\ln x=\frac{-3}{2}$

$\boxed{\therefore x=e^{-3/2}}$

Concavity,

$+$

$-\infty \quad e^{-3/2} \quad +\infty$

concave up: $(e^{-3/2},+\infty)$

Relative Extrema:

$f''(0)=3+2\ln(0)$

dne

$f''(e^{-1/2})=3+2\ln(e^{-1/2})$

$=3+\cancel{2}\left(\frac{-1}{\cancel{2}}\right)$

$f''(e^{-1/2})=2>0$

minima at $e^{-1/2}$

(h) $f(x)=\cos x-9x;\ (0,4\pi)$

$f'(x)=-\sin x-9$

$f''(x)=-\cos x$

C.P:-

$f'(x)=0$

$\sin x-9=0$

$-\sin x=9$

$x=\sin^{-1}(-9)$

dne

Inc/Dec:

$\ominus$

$0 \quad 4\pi$

decreasing: $(0,4\pi)$

Increasing: None.

I.P:-

$f''(x)=0$

$\cos x=0$

$x=\cos^{-1}0$

$x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$

concavity,

$-\quad+\quad-\quad+\quad-$

$0 \quad \pi/2 \quad 3\pi/2 \quad 5\pi/2 \quad 7\pi/2 \quad 4\pi$

concave up $=(\pi/2,3\pi/2)\cup(5\pi/2,7\pi/2)$

concave down $=(0,\pi/2)\cup(3\pi/2,5\pi/2)\cup(7\pi/2,4\pi)$

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Relative extrema

minima ⇒ none

maxima ⇒ none.

(i) $f(x) = \ln\sqrt{x^2+4}$

$f'(x) = \frac{1}{\sqrt{x^2+4}} \cdot \frac{1}{2\sqrt{x^2+4}} (2x)$

$f(x) = \frac{x}{x^2+4}$

$f''(x) = \frac{(x^2+4)(1) - x(2x)}{(x^2+4)^2}$

$= \frac{x^2+4-2x^2}{(x^2+4)^2}$

$f''(x) = \frac{4-x^2}{(x^2+4)^2}$

C.P

$f'(x)=0$ $f(x)$ undefined

$\frac{x}{x^2+4} = 0$ $x^2+4=0$

complex

$\boxed{x=0}$

INC/Dec-

$-\infty$ ⊖ 0 ⊕ $+\infty$

increasing ⇒ $(-\infty, 0)$

decreasing ⇒ $(0, +\infty)$

Inflection Points-

$f''(x)=0$ $f''(x)$ = undefined

$4-x^2=0$ complex

$x = \pm 2$

Concavity:

$-\infty$ ⊖ -2 ⊕ 2 ⊖ $+\infty$

concave up ⇒ $(-2, 2)$

concave down ⇒ $(-\infty, -2) \cup (2, +\infty)$

Relative Extrema:

$f''(0) = \frac{4-0}{(0+4)^2}$

$f''(0) = \frac{1}{4} > 0$

minima at 0.

maxima ⇒ none.

Date:

(i) $f(x) = x e^{-7x}$

$f'(x) = x e^{-7x}(-7) + e^{-7x}$

$f'(x) = e^{-7x}(1-7x)$

$f''(x) = e^{-7x}(-7)(1-7x) + e^{-7x}(-7)$

$= \cancel{-7(e^{-7x} \quad 7e^{-7x} \quad 7e^{-7x}}$

$= -7e^{-7x}(1-7x+1)$

$f''(x) = -7e^{-7x}(2-7x)$

C.P.

$f'(x) = 0$

$e^{-7x}(1-7x) = 0$

$e^{-7x} = 0 \qquad 1-7x=0$

$\ln e^{-7x} = \ln(0) \qquad \boxed{x = \frac{1}{7}}$

$-7x = \infty$

dne

INC/DEC

+	−

$-\infty \qquad 1/7 \qquad +\infty$

increasing $= (-\infty, 1/7)$

decreasing $= (1/7, +\infty)$

I.P.

$f''(x) = 0$

$-7e^{-7x} = 0, \quad 2-7x=0$

dne $\quad , \quad x = \frac{2}{7}$

Concavity:

−	+

$-\infty \qquad 2/7 \qquad +\infty$

concave up: $(2/7, \infty)$

concave down: $(-\infty, 2/7)$

Relative Extrema:

$f''(1/7) = -7e^{-7(1/7)}\{2-7(1/7)\}$

$= -1.72 \times 10^{-19} < 0$

maxima at $1/7$

minima ⇒ None.

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