Ex: 7.5

9-34 DATE

Q Evaluate the integral

9. $\int \frac{dx}{x^2-3x-4}$

$\int \frac{dx}{(x-4)(x+1)} = \frac{A}{(x-4)} + \frac{B}{(x+1)}$ — ①

multiply $(x-4)(x+1)$ on B.S

$1 = A(x+1) + B(x-4)$

let $x+1=0$

$x=-1$

$1 = A(-1+1) + B(-1-4)$

$1 = 0 - 5B$

$B = -1/5$

let $x-4=0$

$x=4$

$1 = A(4+1) + B(4-4)$

$1 = 5A + 0$

$A = 1/5$

put both values in ①

$\int \frac{dx}{(x-4)(x+1)} = \frac{1}{5(x-4)} \; \frac{-1}{5(x+1)}$

$= \frac{1}{5}\int \frac{1}{(x-4)} - \frac{1}{5}\int \frac{1}{(x+1)}$

$= \frac{1}{5} \ln(x-4) - \frac{1}{5}\ln(x+1) + C$

$= \frac{1}{5} \ln \left| \frac{x-4}{x+1} \right| + C$

10 $\int \frac{dx}{(x^2-6x-7)}$

$\int \frac{dx}{(x-7)(x+1)} = \frac{A}{(x-7)} + \frac{B}{(x+1)}$ — ①

$1 = A(x+1) + B(x-7)$

let $x=-1$

$1 = A(-1+1) + B(-1-7)$

$1 = -8B$

$B = -1/8$

let $x=7$

$1 = A(7+1) + B(7-7)$

$1 = 8A$

$A = 1/8$

$$① \Rightarrow \int \frac{dx}{(x-7)(x+1)} = \frac{1}{8}\int \frac{1}{(x-7)} - \frac{1}{8}\int \frac{1}{(x+1)}$$

$$= \frac{1}{8}\ln(x-7) - \frac{1}{8}\ln(x+1) + C$$

$$= \frac{1}{8}\ln\left|\frac{x-7}{x+1}\right| + C$$

$$ii \int \frac{11x+17}{2x^2+7x-4}\,dx$$

$$\int \frac{11x+17}{2x^2+8x-x-4}\,dx$$

$$\int \frac{11x+17}{2x(x+4)-1(x+4)}$$

$$\int \frac{11x+17}{(x+4)(2x-1)}\,dx = \frac{A}{(x+4)} + \frac{B}{(2x-1)} \quad — ①$$

$$11x+17 = A(2x-1) + B(x+4)$$

for $x = 1/2$

$$11\left(\frac{1}{2}\right) + 17 = A\left(2\left(\frac{1}{2}\right)-1\right) + B\left(\frac{1}{2}+4\right)$$

$$\frac{45}{2} = 0 + B\left(\frac{9}{2}\right)$$

$$B = 45/9$$

for $x = -4$

$$11(-4)+17 = A(2(-4)-1) + B(\overset{+4}{4}$$

$$-27 = A(-9) + 0$$

$$A = 3$$

$$① \Rightarrow \int \frac{11x+17}{(x+4)(2x-1)}\,dx = \frac{3}{(x+4)} + \frac{45}{9(2x-1)}$$

$$= 3\int \frac{1}{(x+4)} + \frac{45}{9\times 2}\int \frac{2}{(2x-1)}$$

$$= 3\ln(x+4) + \frac{5}{2}\ln(2x-1)$$

12 $\int \frac{5x-5}{3x^2-8x-3}\,dx$

$\int \frac{5x-5}{3x^2-9x+x-3}$

$\int \frac{5x-5}{3x(x-3)+1(x-3)}$

$\int \frac{5x-5}{(3x+1)(x-3)}\,dx = \frac{A}{(3x+1)} + \frac{B}{(x-3)}$ — ①

$5x-5 = A(x-3) + B(3x+1)$

let $x = 3$

$5(3)-5 = A(3-3)+B(3(3)+1)$

$10 = 10B$

$B = 1$

let $x = -1/3$

$5\left(\frac{-1}{3}\right)-5 = A\left(\frac{-1}{3}-3\right)+B\left(3\frac{-1}{3}+1\right)$

$-\frac{20}{3} = -\frac{10}{3}A$

$A = +2$

① => $\int \frac{5x-5}{(3x+1)(x-3)} = \frac{2}{(3x+1)} + \frac{1}{(x-3)}$

$= \frac{2}{3}\int \frac{3}{(3x+1)} + \int \frac{1}{(x-3)}$

$= \frac{2}{3}\ln(3x+1) + \ln(x-3) + C$

13 $\int \frac{2x^2-9x-9}{x^3-9x}$

$\int \frac{2x^2-9x-9}{x(x^2-9)}$

$\int \frac{2x^2-9x-9}{x(x+3)(x-3)} = \frac{A}{x} + \frac{B}{(x+3)} + \frac{C}{(x-3)}$ — ①

$2x^2-9x-9 = A(x+3)(x-3) + B(x)(x-3) + C(x)(x+3)$

for $x=0$

$2(0^2)-9(0)-9 = A(0+3)(0-3)+0+0$

$-9 = -9A$

$A = 1$

for $x=-3$

$2(-3^2)-9(-3)-9 = 0+B(-3)(-3-3)+0$

$36 = 18B$

$B = 2$

for $x=3$

$2(3^2)-9(3)-9 = 0+0+C(3)(3+3)$

$-18 = 18C$

$C = -1$

put all values in ①

$$\int\frac{2x^2-9x-9}{x(x+3)(x-3)} = \int\frac{1}{x} + 2\int\frac{1}{(x+3)} - \int\frac{1}{(x-3)}$$

$$= \ln x + 2\ln(x+3) - \ln(x-3) + C$$

$$= \ln\left|\frac{x(x+3)^2}{(x-3)}\right| + C$$

14 $\displaystyle\int\frac{dx}{x(x^2-1)}$

$$\int\frac{dx}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \quad — ①$$

$$1 = A(x+1)(x-1) + B(x)(x-1) + C(x)(x+1)$$

for $x=0$

$1 = A(0+1)(0-1)+0+0$

$1 = -A$

$A = -1$

for $x=1$

$1 = 0+0+C(1)(1+1)$

$1 = 2C$

$C = 1/2$

for $x=-1$

$1 = 0+B(-1)(-1-1)+0$

$1 = 2B$

$B = 1/2$

$$① \Rightarrow \int\frac{dx}{x(x-1)(x+1)} = -\int\frac{1}{x} + \frac{1}{2}\int\frac{1}{(x+1)} + \frac{1}{2}\int\frac{1}{(x-1)}$$

$$= -\ln x + \frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x-1) + C$$

$$= \frac{1}{2}\left[-2\ln x + \ln(x+1) + \ln(x-1)\right] + C$$

$$= \frac{1}{2}\ln\left|\frac{(x+1)(x-1)}{x^2}\right| + C \Rightarrow \frac{1}{2}\ln\left|\frac{x^2-1}{x^2}\right| + C$$

long division => $Q + \frac{R}{D}$

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15 $\int \frac{x^2-8}{x+3}\,dx$

$x+3 \overline{)\,x^2 + -8}$ quotient $x-3$

$-\ x^2+3x$

$-3x-8$

$-\ -3x-9$

1

$\int\left(x-3+\frac{1}{x+3}\right)dx$

$\int x-3+\int\frac{1}{x+3}$

$\int x-3\int 1+\int\frac{1}{x+3}$

$\Rightarrow \frac{x^2}{2}-3x \quad +\ln|x+3|+C$

16 $\int\frac{x^2+1}{x-1}\,dx$

$x-1\overline{)\,x^2+1}$ quotient $x+1$

$-\ x^2-x$

$x+1$

$-\ x-1$

2

$\int\left(x+1+\frac{2}{(x-1)}\right)dx$

$\int x+1+\int\frac{2}{(x-1)}$

$\int x+\int 1+2\int\frac{1}{(x-1)}\,dx$

$\Rightarrow \frac{x^2}{2}+x+2\ln|x-1|+C$

17 $\int\frac{3x^2-10}{x^2-4x+4}\,dx$

$x^2-4x+4\overline{)\,3x^2-10}$ quotient 3

$-\ 3x^2-12x+12$

$12x-22$

$\int 3+\frac{12x-22}{(x-2)^2}$

$\int 3+\int\frac{12x-22}{(x-2)^2}$

$3x+\int\frac{12x-22}{(x-2)^2}$

consider $\int\frac{12x-22}{(x-2)^2}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}$ -①

$12x-22=A(x-2)+B$

for $x=2$.

$12(2)-22=0+B$

$B=2$

for A

$12x-22=Ax-2A+B$

compare coefficient of x.

$12=A$

① => $\int\frac{12x-22}{(x-2)^2}=12\int\frac{1}{(x-2)}+2\int\frac{1}{}$

$3x+12\ln(x-2)+2(-1)\frac{1}{(x-2)}+$

$3x+12\ln|x-2|-\frac{2}{(x-2)}+$

18 $\int \frac{x^2}{x^2-3x+2}\,dx$

$$\begin{array}{r} 1 \\ x^2-3x+2 \,\big)\, x^2 \\ -\;x^2-3x+2 \\ \hline 3x-2 \end{array}$$

$\int\left(1+\frac{3x-2}{x^2-3x+2}\right)dx$

$\int 1 + \int \frac{3x-2}{x^2-3x+2}\,dx$ — Ⓐ

consider $\int \frac{3x-2}{x^2-3x+2}\,dx$

$\int \frac{3x-2}{x^2-3x+2} = \frac{3x-2}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$ —①

$3x-2 = A(x-2) + B(x-1)$

for $x=2$ for $x=1$

$3(2)-2 = 0+B(2-1)$ $3(1)-2 = A(1-2)+0$

$B = 4$ $A = -1$

① => $\int \frac{3x-2}{x^2-3x+2}\,dx = -\int \frac{1}{(x-1)} + 4\int \frac{1}{(x-2)}$

$= -\ln|x-1| + 4(\ln(x-2)) + C$

Ⓐ => $x - \ln|x-1| + 4\ln|x-2| + C$

19 $\int \frac{2x-3}{x^2-3x-10}\,dx$

$\ln|x^2-3x+10| + C$

20 $\int \frac{3x+1}{3x^2+2x-1}\,dx$

$\frac{1}{2}\int \frac{2(3x+1)}{3x^2+2x-1}\,dx$

$\frac{1}{2}\ln|3x^2+2x-1| + C$

21 $\int \frac{x^5+x^2+2}{x^3-x}\,dx$

$$\begin{array}{r} x^2+1 \\ x^3-x \,\big)\, x^5+x^2+2 \\ -\;x^5-x^3 \\ \hline x^3+x^2+2 \\ -\;x^3-x \\ \hline +x+x^2+2 \end{array}$$

$\int x^2+1 + \frac{x^2+x-2}{x^3-x}$

consider

$\int \frac{x^2+x+2}{x^3-x}$

$\int \frac{x^2+x+2}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)}$

$x^2+x+2 = A(x+1)(x-1) + B(x)(x-1) + C(x)\overset{(x+1)}{}$

for $x=0$ for $x=1$ for $x=-1$

$-2 = A$ $4 = 2c$ $2 = 2B$

$C = 2$ $B = 1$

$\int \frac{x^2+x+2}{x(x+1)(x-1)} \Rightarrow -2\int\frac{1}{x} + \int\frac{1}{(x+1)} + 2\int\frac{1}{(x}$

$\int x^2 + \int 1 - 2\int\frac{1}{x} + \int\frac{1}{(x+1)} + 2\int\frac{1}{x+1)}$

$\frac{1}{3}x^3 + x - 2\ln x + \ln(x+1) + 2(\ln(x+1)$

$+C$

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22 $\int \frac{x^5-4x^3+1}{x^3-4x}\,dx$

$$x^3-4x \overset{x^2}{\overline{\smash{)}\cancel{x^5}-4\cancel{x^3}+1}}$$

$$\underline{-x^5-4x^3}$$

$$1$$

$\int x^2 + \frac{1}{x^3-4x}$

$\int x^2 + \frac{1}{x(x+2)(x-2)}$ — Ⓐ

consider

$\int \frac{1}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$ — ①

$1 = A(x+2)(x-2) + B(x)(x-2) + C(x)(x+2)$

For $x = 0$ for $x = 2$ for $x = -2$

$1 = -4A$ $1 = 0 + 0 + C(2)(4)$ $1 = 0 + B(-2)(-4)$

$A = -1/4$ $C = 1/8$ $B = 1/8$

$\Rightarrow \int \frac{1}{x(x+2)(x-2)}\,dx = -\frac{1}{4}\int \frac{1}{x} + \frac{1}{8}\int \frac{1}{(x+2)} + \frac{1}{8}\int \frac{1}{(x-2)}$

$= -\frac{1}{4}\ln x + \frac{1}{8}\ln(x+2) + \frac{1}{8}\ln(x-2)$

Ⓐ $\Rightarrow \frac{x^3}{3} - \frac{1}{4}\ln x + \frac{1}{8}\ln(x+2) + \frac{1}{8}\ln(x-2) + C$

23 $\int \frac{2x^2+3}{x(x-1)^2}$

$\int \frac{2x^2+3}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)^2}$

$2x^2+3 = A\,(x-1)^2 + B(x)(x-1) + C(x)$

for $x = 0$ for $x = 1$ for $x = -1$

$3 = +A$ $2+3 = C$ $2+3 = 2B+4($

$A = 3$ $C = 5$ $B = \frac{5-4(A)}{2}$

$B = \frac{+2}{2}$

$B = -1$

$\int \frac{2x^2+3}{x(x+1)(x-1)} = 3\int \frac{1}{x} - 1\int \frac{1}{(x-1)} + 5\int \frac{1}{(x-1)^2}$

$3\ln x - \ln|x-1| - \frac{5}{(x-1)} + C$

24 $\int \frac{3x^2-x+1}{x^3-x^2}\,dx$

$\int \frac{3x^2-x+1}{x^2(x-1)}\,dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}$

$3x^2-x+1 = A(x)(x-1)+B(x-1)+C(x^2)$

-c

for $x=0$ for $x=1$

$1=-B$ $3-1+1=C$

$B=-1$ $C=3$

for A

$3x^2-x+1 = Ax^2-Ax+Bx-B+Cx^2$

Taking coefficients of $x2$

$3=A+C$

$A=3-3$

$A=0$

$\int \frac{3x^2-x+1}{x^2(x-1)}\,dx = -\int \frac{1}{x^2} + 3\int \frac{1}{(x+1)}$

$= \frac{1}{x} + 3\ln(x+1) + C$

25 $\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}\,dx$

$\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}\,dx = \frac{A}{(x+1)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$

$2x^2-10x+4 = A(x-3)^2+B(x+1)(x-3)+C(x+1)$

for $x=-1$ for $x=3$

$2+10+4=16A$ $2(3^2)-10(3)+4=C(3+1)$

$A=1$ $18-30+4=4C$

$C=-2$

for B

$2x^2-10x+4 = Ax^2-6Ax+9A+Bx^2-3Bx-3B+Bx+Cx+C$

compare coefficient of $x2$

$2=A+B$

$B=2-1$

$B=1$

$\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}\,dx = \int \frac{1}{x+1} + \int \frac{1}{(x-3)} - 2\int \frac{1}{(x-3)^2}$

$\ln|x+1| + \ln|x-3| + \frac{2}{x-3} + C$

26 $\int \frac{2x^2-2x-1}{x^3-x^2}\,dx$

$\int \frac{2x^2-2x-1}{x^2(x-1)}\,dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}$

$2x^2-2x-1 = A(x)(x-1)+B(x-1)+C(x^2)$

for $x=0$ for $x=1$ for A

$-1=-B$ $2-2-1=C$ $2=A+C$

$B=1$ $C=-1$ $A=3$

$\int \frac{2x^2-2x-1}{x^3-x^2}\,dx = 3\int \frac{1}{x} + 1\int \frac{1}{x^2} - \int \frac{1}{(x-1)}$

$\Rightarrow 3\ln x - \frac{1}{x} - \ln|x-1| + C$

27 $\int \frac{x^2}{(x+1)^3}\,dx$

$\int \frac{x^2}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

$x^2 = A(x+1)^2+B(x+1)+C$

for $x=-1$

$1=C$

for B and A

$x^2 = Ax^2+2Ax+A+Bx+B+C$

compare x^2 compare x

$1=A$ $0=B+2A$

$-2(1)=B$

$B=-2$

$\int \frac{x^2}{(x+1)^3} = \int \frac{1}{(x+1)} - 2\int \frac{1}{(x+1)^2} + \int \frac{1}{(x+1)^3}$

$\Rightarrow \ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C$

28 $\int \frac{2x^2+3x+3}{(x+1)^3}\,dx$

$\int \frac{2x^2+3x+3}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

$2x^2+3x+3 = A(x+1)^2 + B(x+1) + C$

for $x = -1$

$2-3+3 = C$

$C = 2$

for A and B

$2x^2+3x+3 = Ax^2+2Ax+A+Bx+B+C$

compare x^2 compare x

$2 = A$ $3 = B+2A$

$B = 3-2(2)$

$B = -1$

$\therefore \int \frac{2x^2+3x+3}{(x+1)^3} = 2\int \frac{1}{(x+1)} - \int \frac{1}{(x+1)^2} + 2\int \frac{1}{(x+1)^3}$

$\Rightarrow 2\ln|x+1| + \frac{1}{(x+1)} - \frac{1}{(x+1)2} + C$

30 $\int \frac{dx}{x^3+2x}$

$\int \frac{dx}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$

$1 = A(x^2+2) + (Bx+C)(x)$

for $x = 0$

$1 = A(0+2)$

$A = 1/2$

for B and C

$1 = Ax^2+2A+Bx^2+Cx$

compare x^2 compare x

$1 = A+B$ $0 = C$

$B = -1/2$ $C = 0$

$\int \frac{1}{x(x^2+2)} = \frac{1}{2}\int \frac{1}{x} - \frac{1}{2}\int \frac{x+0}{x^2+2}$

$= \frac{1}{2}\ln x - \frac{1}{4}\ln|x^2+2| + C$

29 $\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$

$\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx = \frac{A}{(4x-1)} + \frac{Bx+C}{(x^2+1}$

$2x^2-1 = A(x^2+1) + Bx + C(4x$

for $x = 1/4$

$2(\frac{1}{4})^2 - 1 = A((\frac{1}{4})^2+1) + 0$

$-7/8 = \frac{17}{16}A$

$A = -\frac{14}{17}$

for B and C

$2x^2-1 = Ax^2+A+4Bx^2-Bx+4Cx-C$

compare x^2 compare x

$2 = A+4B$ $0 = -B+4C$

$2+\frac{14}{17} = 4B$ $\frac{12}{17} = 4C$

$B = \frac{12}{17}$ $C = 3/17$

So $\int \frac{2x^2-1}{(4x-1)(x^2+1)} = \frac{-14}{17}\int \frac{1}{4x-1} + \int \frac{12/17\,x+3}{(x^2+1}$

$= \frac{-14}{17}\int \frac{1}{4x-1} + \frac{1}{17}\int \frac{12x+3}{(x^2+1)}$

$= \frac{-14}{17\times 4}\int \frac{4}{4x-1} + \frac{6}{17}\int \frac{(2x+1/2)}{x^2+1}$

$\frac{-14}{68}\int \frac{4}{4x-1} + \frac{6}{17}\int \frac{2x}{x^2+1} + \frac{6}{17}\times\frac{1}{2}\int \frac{1}{x^2+1}$

$\frac{-7}{34}\ln|4x-1| + \frac{6}{17}\ln|x^2+1| + \frac{3}{17}\tan^{-1}$

$\frac{-7}{34}\ln|4x-1| + \frac{6}{17}\ln|x^2+1| + \frac{3}{17}\tan^{-1}x +$