Ex: 5.2

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Q Find the volume of the solid that results when the shaded region is revolved about the indicated axis

1 $y = \sqrt{3-x}$ $(-1, 3)$

$V = \int_{-1}^{3} \pi (\sqrt{3-x})^2$ (revolving around x-axis)

$V = \pi \int_{-1}^{3} (3-x)\,dx$

$V = 8\pi$

2 $y = x$ $y = 2-x^2$ $(0, 1)$

$y = 2-x^2$ (revolving around x-axis)
$0 \quad y = x \quad 1$

$V = \pi \int_{0}^{1} (2-x^2)^2 - x^2\,dx$

$V = \pi \int_{0}^{1} (4-5x^2+x^4)\,dx$

$V = 38\pi/15$

3 $y = 3-2x$ (revolving around y-axis)

$x = \frac{3-y}{2}$ $(0, 2)$ y axis

$V = \pi \int_{0}^{2} \left(\frac{3-y}{2}\right)^2 dy$

$= 13\pi/6$

(cavity)

4 $y = 1/x$ (revolving around y-axis)

$x = 1/y$ (inner radius) points wrt x

$x = 2$ (outer radius) $(1, 2)$

now body

$(1, 1/2)$

$V = \pi \int_{1/2}^{1} (2)^2 - \left(\frac{1}{y}\right)^2 dy = 9\pi/2$

5 $y = \sqrt{\cos x}$ $(\pi/4, \pi/2)$

$V = \pi \int_{\pi/4}^{\pi/2} (\sqrt{\cos x})^2 dx$ (revolving around x-axis)

$V = (1 - \sqrt{2}/2)\pi$

$V = 0.9201$

6 $y = x^2$, $y = x^3$ $(0, 1)$

$y = x^2$
$0 \quad y = x^3 \quad 1$ (revolving x-axis)

$V = \pi \int_{0}^{1} (x^2)^2 - (x^3)^2\,dx$

$V = \pi \int_{0}^{1} x^4 - x^6\,dx$

$V = 2\pi/35$

7 $x = \sqrt{1+y}$ $(-1, 3)$

$V = \pi \int_{-1}^{3} (\sqrt{1+y})^2\,dy$ (revolving y-axis)

$V = \pi(8)$

$V = 8\pi$

8 $y = x^2 - 1$ (cavity) (revolving w.r.t y)

$x = \sqrt{y+1}$ (inner radius)

$x = 2$ (outer radius)

$V = \pi \int_{0}^{3} 2^2 - (\sqrt{y+1})^2\,dy$

$V = \pi \int_{0}^{3} (4 - y + 1)\,dy$

$V = 9\pi/2$

6.2: #3

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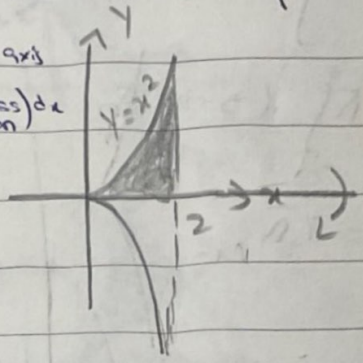
Q9 find the volume of the solid whose base is the region bounded b/w the curve $y = x^2$ and the x-axis from $x = 0$ to $x = 2$ and whose cross sections are taken perpendicular to the x-axis are squares

formula for volume for square cross section ⊥ to x-axis

$V = \int_a^b (\text{area of cross section})\,dx$

$\therefore V = \int_0^2 (x^2)^2\,dx$

$V = \frac{32}{5}$



Q14 find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis

11 $y = \sqrt{25 - x^2}$ $y = 3$

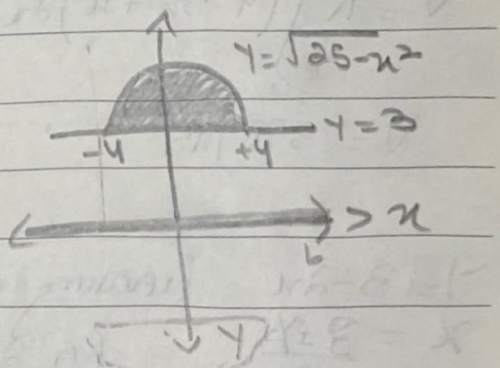
$3 = \sqrt{25 - x^2}$

$9 = 25 - x^2$

$x^2 = 16$

$x = \pm 4$

$y = \sqrt{25 - x^2}$ / $y = 3$ / -4 / $+4$



$V = \pi \int_{-4}^{4} (\sqrt{25 - x^2})^2 - 3^2\,dx$

$V = \frac{256\pi}{3}$

12 $y = 9 - x^2$ $y = 0$

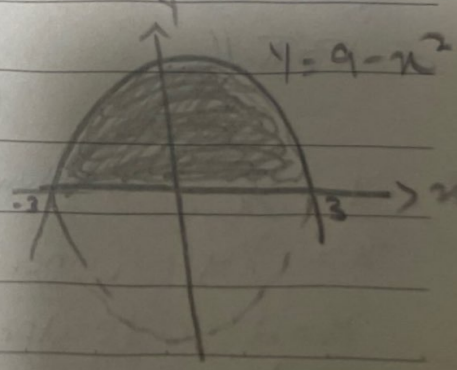
$0 = 9 - x^2$

$x^2 = 9$

$x = \pm 3$

$V = \pi \int_{-3}^{3} (9 - x^2)^2\,dx$

$= \frac{1299\pi}{5}$



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$x = \sqrt{y}$ $x = y/4$

about x-axis (given)

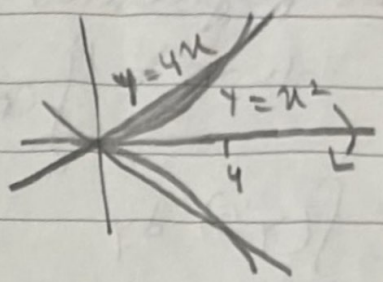
$\therefore y = x^2$ $y = 4x$

$4x = x^2$

$x = 4, \ x = 0$

$$V = \pi \int_0^4 (4x)^2 - (x^2)^2 \, dx$$

$$V = \frac{2048\pi}{15}$$



1 $y = \sin x$ $y = \cos x$ $x = 0$ $x = \pi/4$

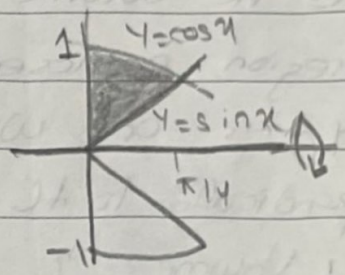
$y = \cos x$

0 $y = \sin x$ $\pi/4$

$$V = \pi \int_0^{\pi/4} (\cos x)^2 - (\sin x)^2 \, dx$$

$$V = \pi \int_0^{\pi/4} \cos 2x \, dx$$

$$V = \pi/2$$

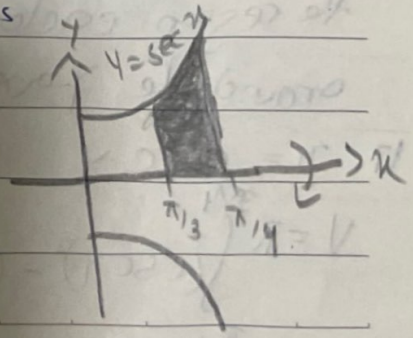


2) find the volume of the solid whose is the region bounded between the curve $y = \sec x$ and the x-axis from $x = \pi/4$ to $x = \pi/3$ and whose cross section taken perpendicular to the x-axis are squares formula for volume of a solid having square cross section perpendicular to x-axis

$$V = \int_a^b (\text{area of cross section}) \, dx$$

$$V = \int_{\pi/4}^{\pi/3} \sec^2 x \, dx$$

$$V = \sqrt{3} - 1 \qquad V = 0.7320\,5$$



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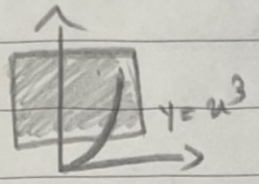
Q15 find the volume of the solid whose base is the region bounded b/w the curve $y = x^3$ and y-axis from $y=0$ to $y=1$ and whose cross sections taken perpendicular to the y-axis are squares

$y = x^3$
$x = y^{1/3}$
for $y=0$ $x=0$
for $y=1$ $x=1$

$$\text{Volume} = \int_a^b \text{Area Cross section}\, dx$$

$$V = \int_0^1 (y^{1/3})^2\, dy$$

$$V = 3/5$$



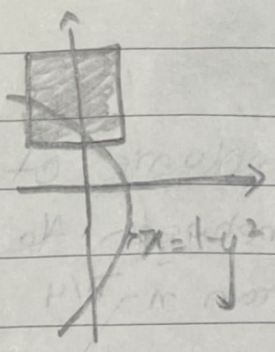
Q16 find the volume of the solid whose base is the region enclosed b/w the curve $x = 1 - y^2$ and the y-axis and whose cross sections taken perpendicular to the y-axis are squares

$x = 1 - y^2$
when $x = 0$
$y = \pm 1$

$$\text{Volume} = \int_a^b \text{area of cross section}\, dx$$

$$V = \int_{-1}^{1} (1-y^2)^2\, dy$$

$$V = \frac{16}{15}$$



Q17-20 find the volume of the solid that results when the region enclosed by the given curves is revolved around the y-axis

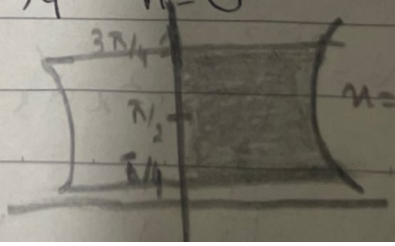
$x = \csc y$ $y = \pi/4$ $y = 3\pi/4$ $x = 0$

$x = \csc y$

$\pi/4$ $x=0$ $3\pi/4$

$$V = \pi \int_{\pi/4}^{3\pi/4} (\csc y)^2 - 0^2\, dy$$

$$V = 2\pi$$



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18 $y = x^2$, $x = y^2$

$x = \sqrt{y}$, $x = y^2$

$x = \sqrt{y}$

0 $x = y^2$ 1

$\sqrt{y} = y^2$

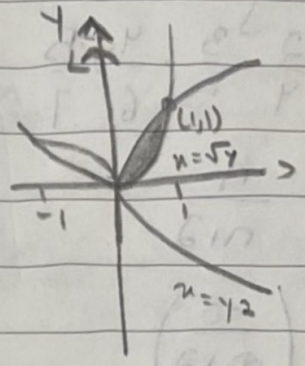
$y = y^4$

$1 = y^3$

$y = 1$, $y = 0$

$$V = \pi \int_0^1 (\sqrt{y})^2 - (y^2)^2 \, dy$$

$$V = \frac{3\pi}{10}$$



19 $x = y^2$, $x = y + 2$

$x = y + 2$

-1 $x = y^2$ 2

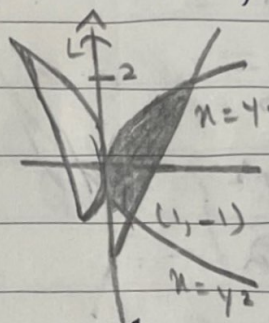
$y^2 = y + 2$

$y^2 - y - 2 = 0$

$y = 2$, $y = -1$

$$V = \pi \int_{-1}^{2} (y+2)^2 - (y^2)^2 \, dy$$

$$V = \frac{72\pi}{5}$$



20 $x = 1 - y^2$ $x = 2 + y^2$ $y = -1$ $y = 1$

$$V = \pi \int_{-1}^{1} (2+y^2)^2 - (1-y^2)^2 \, dy$$

$$V = 10\pi$$