Calculus Assignment #2

Date:

Q1 = Find dy/dx.

(1). $8 = 3x^2 + y^4$ (3) $x^2 + 2y^5 = 10xy$

Differentiating w.r.t x

$$0 = 6x + 4y^3 \frac{dy}{dx}$$

$$4y^3 \frac{dy}{dx} = -6x$$

$$\frac{dy}{dx} = \frac{-6x}{4y^3} \Rightarrow \frac{-3x}{2y^3}$$

(2) $\sin(2x - y) = 4x$

Differentiating w.r.t x

$$\cos(2x-y)\,(2x - dy/dx) = 4$$

$$2\cos(2x-y) - \frac{dy}{dx}\cos(2x-y) = 4$$

$$\frac{dy}{dx}\cos(2x-y) = 2\cos(2x-y) - 4$$

$$\frac{dy}{dx} = \frac{2\cos(2x-y) - 4}{\cos(2x-y)}$$

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(3) $x^2 + 2y^5 = 10xy$

Differentiating w.r.t x

$$2x + 10y^4 \frac{dy}{dx} = 10\left[x\frac{dy}{dx} + y(1)\right]$$

$$2x + 10y^4 \frac{dy}{dx} = 10x\frac{dy}{dx} + 10y$$

$$10y^4 \frac{dy}{dx} - 10x\frac{dy}{dx} = 10y - 2x$$

$$10\frac{dy}{dx}(y^4 - x) = 10y - 2x$$

$$\frac{dy}{dx} = \frac{10y - 2x}{10(y^4 - x)}$$

(4) Implicit Differentiation to find $\frac{d^2y}{dx^2}$

$7xy = 8$

Differentiating w.r.t x.

$$7\left[x\, dy/dx + y(1)\right] = 0$$

$$7x\frac{dy}{dx} + 7y = 0$$

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$$\cancel{7}x\frac{dy}{dx} = -\cancel{7}y$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Again differentiating w.r.t x

$$\frac{d^2y}{dx^2} = -\left[\frac{x\left[+dy/dx\right] - y(1)}{x^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-x\frac{dy}{dx} - y}{x^2}$$

Putting the value of $\frac{dy}{dx}$

$$\frac{d^2y}{dx^2} = \frac{-x\left(-\frac{y}{x}\right) + y}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{y + y}{x^2} \quad \#$$

$$\frac{d^2y}{dx^2} = \frac{2y}{x^2}$$

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(5) $e^x + y^2 = 4$

differentiating w.r.t x:-

$$e^x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-e^x}{2y}$$

Again differentiating w.r.t x:-

$$\frac{d^2y}{dx^2} = -\left[\frac{2y\,e^x - e^x\,2\frac{dy}{dx}}{4y^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-2ye^x + 2e^x\frac{dy}{dx}}{4y^2}$$

Putting value of dy/dx:-

$$\frac{d^2y}{dx^2} = \frac{-2y\,e^x + 2e^x\left(\frac{-e^x}{2y}\right)}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-2ye^x + e^{2x}/y}{4y^2} \Rightarrow \frac{\frac{2y^2e^x + e^{2x}}{y}}{4y^2}$$

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$$\frac{d^2y}{dx^2} = \frac{-2y^2e^x - e^{2x}}{4y^3}$$

Q6 = $x^2y + y^2 + 4 = 0$, $\left.\frac{dy}{dx}\right|_{x=2}$

For y at $x = 2$

$$(2)^2y + y^2 + 4 = 0$$
$$4y + y^2 + 4 = 0$$
$$(y)^2 + 2(y)(2) + (2)^2 = 0$$
$$(y+2)^2 = 0$$
$$\boxed{y = -2} \qquad (x,y) = (2,-2)$$

Differentiating w.r.t x:-

$$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx}(x^2 + 2y) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+2y}$$

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Putting x & y value.

$$\frac{dy}{dx} = \frac{-2(2)(-2)}{(2)^2 + 2(-2)} \Rightarrow \frac{8}{4-4}$$

$$\frac{dy}{dx} = \frac{8}{0}$$

(E) Non - Existent

Q7) = If $x^2 - y^2 = 5$, $\left.\frac{d^2y}{dx^2}\right|_{(3,2)} = ?$

$\cancel{2x - 2d}$

Differentiating w.r.t x

$$2x - 2y\frac{dy}{dx} = 5$$

$$2y\frac{dy}{dx} = 2x + 5$$

$$\frac{dy}{dx} = \frac{2x+5}{2y} \quad \text{—— (1)}$$

~~After putting values :-~~

$$\cancel{\frac{dy}{dx} = \frac{2(3)+5}{2(2)} \Rightarrow \frac{11}{4}}$$

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Q7 = $x^2 - y^2 = 5$, $\frac{d^2y}{dx^2}$ at $x, y = (3, 2)$

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = +\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(1) - x\,dy/dx}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - \frac{x^2}{y}}{y^2} \Rightarrow \frac{y^2 - x^2}{y^3}$$

$$\frac{d^2y}{dx^2} = \frac{(2)^2 - (3)^2}{(2)^3} = -\frac{5}{8}$$

(D) $-\frac{5}{8}$

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Q8 =

$$\left.\frac{dx}{dt}\right|_{x=1} = -30 \text{ mil/hr}$$

$$\left.\frac{dy}{dt}\right|_{y=0.4} = 15 \text{ mil/hr}$$

let D be the distance b/w them

$$D = \sqrt{x^2 + y^2}$$
$$D = \sqrt{1^2 + 0.4^2}$$
$$D = \frac{\sqrt{29}}{5}$$

$$D^2 = x^2 + y^2$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\cancel{2}\left[\frac{\sqrt{29}}{5}\right]\frac{dD}{dt} = \cancel{2}(1)[-30] + \cancel{2}(0.4)(15)$$

$$\frac{dD}{dt} = \frac{5}{\sqrt{29}}(-30+6)$$

$$\frac{dD}{dt} = \frac{-120}{\sqrt{29}} \Rightarrow -22.283 \text{ mil/hr}$$

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Q9 =

Let the Suface Area = S

$\frac{ds}{dt} = 0.2$

$S = 6s^2$

$s^2 = \frac{S}{6}$

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3\left(\frac{S}{6}\right)(0.2)$

$\frac{dV}{dt} = 0.1 S \text{ cm/s}$

(A) 0.1

Date:

Q10 = Inc / Dec, Concavity, Inflection, Critical and relative extrema

(a) $f(x) = x^5 - 5x^4$

$f'(x) = 5x^4 - 20x^3$

$f''(x) = 20x^3 - 60x^2$

① C.P :-

$f'(x) = 0$ & $f'(x) =$ undefined

Not possible for this question

$5x^4 - 20x^3 = 0$

$5x^3(x - 4) = 0$

$\boxed{x = 0}$ & $\boxed{x = 4}$

② Inc / Dec :-

$+ \quad - \quad +$

$-\infty \quad 0 \quad 4 \quad +\infty$

Inc : $(-\infty, 0] \cup [4, +\infty)$

Dec : $[0, 4]$

③ Inflection Points

$f''(x) = 20x^3 - 60x^2 = 0$ & $f''(x) =$ Undefined

Not Possible

$20x^2(x - 3) = 0$

$\boxed{x = 0}$ & $\boxed{x = 3}$

④ Concavity :-

$- \quad - \quad +$

$-\infty \quad 0 \quad 3 \quad +\infty$

concave up: $(-\infty, 3)$

concave down: $(3, +\infty)$

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⑤ Relative Extrema :-

At $x=0$:-

$f''(0) = 20(0)^3 - 60(0)^2$
$f''(0) = 0$

we cannot find the sign of $f''(x)$
So, it a inflection point

inflection point $= f(0) = (0)^5 - 5(0)^4$
$= 0$

inflection$(0,0)$

At $x = 4$:-

$f''(4) = 20(4)^3 - 60(4)^2$
$= 320 > 0$ Minima

So $f''(x)$ has minimum value at $x=4$

Minimum value $= f(4) = (4)^5 - 5(4)^4$
$= -256$

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(b)

Q10 = $f(x) = x^{1/5}$

$f'(x) = \frac{x^{-4/5}}{5} \Rightarrow \frac{1}{}$

$f''(x) = \frac{-4x^{-9/5}}{25}$

① Critical Points

$f'(x) = 0$

$\frac{1}{5x^{4/5}} = 0$

$1 = 0$ ✗

$f'(x) = $ undefined

$x^{4/5} = 0$

$\boxed{x = 0}$

② Inc/Dec

$-\infty \quad (-) \quad 0 \quad (+) \quad +\infty$

Inc : $[0, +\infty)$

Dec : $(-\infty, 0]$

③ Inflection Points :-

$f''(x) = \frac{-4x^{9/5}}{25} = 0$

$\frac{-4}{25x^{9/5}} = 0$

$-4 = 0$ ✗

$x^{9/5} = 0$

$\boxed{x = 0}$ ✓

④ Concavity

$-\infty \quad (+) \quad 0 \quad (-) \quad +\infty$

Concave Up. $(-\infty, 0)$

Concave Down : $(0, +\infty)$

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5 Relative Extrema:-

<u>At $x = 0$:-</u>

$f''(0) = -4$ $\not\exists''$

$\dfrac{}{25(0)^{9/5}}$

D.N.E

(c) $f(x) = (x+1)(x-1)^2$

$f'(x) = (x+1)\,2(x-1)(1) + (x-1)^2(1)$

$f'(x) = (x-1)[2x+2 + x-1]$

$f'(x) = (x-1)(3x+1)$

$f''(x) = (x-1)(3) + (3x+1)(1)$

$f''(x) = 3x-3 + 3x+1$

$f''(x) = 6x-2$

(1) Critical Point :-

$f'(x) = 0$ & $f'(x) =$ undefined

$(x-1)(3x+1) = 0$ D.N.E

$\boxed{x = 1}$ $\boxed{x = -\frac{1}{3}}$

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② Inc/Dec :-

$\oplus \quad \ominus \quad \oplus$

$-\infty \quad -1/3 \quad 1 \quad +\infty$

Inc: $(-\infty, -1/3) \cup (1, +\infty)$

Dec: $(-1/3, 1)$

③ Inflection Points :-

$f''(x) = 0$ & $f''(x) =$ undefined

$6x - 2 = 0$ D.N.E

$\boxed{x = \frac{1}{3}}$

④ Concavity :-

$\ominus \quad \oplus$

$-\infty \quad 1/3 \quad +\infty$

Concave Up: $(1/3, +\infty)$

Concave down: $(-\infty, 1/3)$

⑤ Relative Extrema :-

At $x = 1$:-	At $x = \frac{-1}{3}$:-
$f''(1) = 6(1) - 2$	$f''(-1/3) = 6\left(-\frac{1}{3}\right) - 2$
$= 4 > 0$ Minima	$= -2 - 2 = -4$
Minimum Value $= f(1)$	$= -4 < 0$ Maxima
$f(1) = (1+1)(1-1)^2$	Max Value $= f(-1/3)$:-
$= 0$	$f(-1/3) = (-1/3 + 1)(-1/3 - 1)^2$
	$= \frac{32}{27}$

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(d) $f(x) = \frac{x^2-3}{x-2}$

$$f'(x) = \frac{(x-2)(2x)-(x^2-3)}{(x-2)^2}$$

$$= \frac{2x^2-4x-x^2+3}{(x-2)^2}$$

$$f'(x) = \frac{x^2-4x+3}{(x-2)^2} \Rightarrow \frac{(x-1)(x-3)}{(x-2)^2}$$

$$f''(x) = \frac{(x-2)^2(2x-4)-(x^2-4x+3)(x-2)2}{(x-2)^4}$$

$$= \frac{\cancel{(x-2)}\{(x-2)(2x-4)-2(x^2-4x+3)\}}{(x-2)^{\cancel{4}3}}$$

$$= \frac{(x-2)(2x-4)-2x^2+8x-6}{(x-2)^3}$$

$$= \frac{2x^2-4x-4x+8-2x^2+8x-6}{(x-2)^3}$$

$$f''(x) = \frac{2}{(x-2)^3}$$

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① Critical Point :-

$(x-1)(x-3)=0$; $(x-2)^2=0$

$\boxed{x=1}$, $\boxed{x=3}$ $\boxed{x=2}$

② Inc / Dec :-

$-\infty \quad (+) \quad 1 \quad (-) \quad 2 \quad (-) \quad 3 \quad (+) \quad +\infty$

Inc : $(-\infty, 1] \cup [3, +\infty)$

Dec : $[1, 2] \cup [2, 3]$

or $(1, 3)$

③ Inflection Points :-

$f''(x) = 0$ & $f''(x) =$ undefined

$\dfrac{2}{(x-2)^3} = 0$ $(x-2)^3 = 0$

$2 = 0$ ✗ $x - 2 = 0$

$\boxed{x=2}$

④ Concavity :-

$-\infty \quad (-) \quad 2 \quad (+) \quad +\infty$

concave up : $(-\infty, 2]$

concave down : $[2, +\infty)$

⑤ Relative Extrema

At $x = 1$ $f''(1) = -2 < 0$ Maxima ~~Minima~~

~~Minimum~~ Maximum Value $= f(-2) = -\dfrac{1}{4}$

Teacher's Signature:

Date:

At $x = 3$:-

$f''(3) = 2 > 0$ Minima

Minimum value $= f(3) = 6$

At $x = 2$:-

$f''(2) =$ D.N.E

(e) $f(x) = x\sqrt{8-x^2}$

$f'(x) = \frac{1}{\cancel{2}\sqrt{8-x^2}}(-\cancel{2}x)(x) + \sqrt{8-x^2}$

$f'(x) = -\frac{x^2}{\sqrt{8-x^2}} + \sqrt{8-x^2}$

$f'(x) = \frac{x^2 + 8 - x^2}{\sqrt{8-x^2}} \Rightarrow \frac{8-2x^2}{\sqrt{8-x^2}}$

$f''(x) = \frac{\sqrt{8-x^2}(-4x) - (8-2x^2)\left(\frac{1}{2\sqrt{8-x^2}}\right)x}{(\sqrt{8-x^2})}$

$f''(x) = \frac{-4x(8-x^2) + x(8-2x^2)}{(8-x^2)^{3/2}}$

$= \frac{-24x + 2x^3}{(8-x^2)^{3/2}}$

$= \frac{2x(-12+x^2)}{(8-x^2)^{3/2}}$

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① Critical Points:-

$$\frac{8-2x^2}{\sqrt{8-x^2}} = 0 \qquad \sqrt{8-x^2} = 0$$

$8-2x^2=0$ | $8-x^2=0$

$x^2=4$ | $x^2=8$

$x=2 \;,\; x=-2$ | $x=\pm 2\sqrt{2}$

$x = 2\sqrt{2}, -2\sqrt{2}$

② Increasing / Decreasing:-

D.N.E, $\ominus$, $\oplus$, $\ominus$, D.N.E

$-\infty \quad -2\sqrt{2} \quad -2 \quad 2 \quad 2\sqrt{2} \quad +\infty$

Inc : $[-2, 2]$

Dec : $[-2\sqrt{2}, 2] \cup [2, 2\sqrt{2}]$

③ Inflection Points:-

$f''(x) = 0$ & $f''(x) =$ undefined

$$\frac{2x(-12+x^2)}{(8-x^2)^{3/2}} = 0 \qquad (8-x^2)^{3/2} = 0$$

$8-x^2=0$

$2x=0 \qquad x^2 = 12 \qquad x^2=8$

$\boxed{x=0} \qquad x = \pm 2\sqrt{3} \qquad \boxed{x = \pm 2\sqrt{2}}$

$x = 2\sqrt{3}, -2\sqrt{3} \qquad x = 2\sqrt{2}, -2\sqrt{2}$

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④ Concavity :-

$-\infty$	$-2\sqrt{3}$	$-2\sqrt{2}$	0	$2\sqrt{2}$	$2\sqrt{3}$	$+\infty$
	D.N.E	$\oplus$	$\ominus$	$\ominus$	D.N.E	D.N.E

Concave up: $[-2\sqrt{3}, -2\sqrt{2}]$

Concave down: $[2\sqrt{2}, 0] \cup [0, 2\sqrt{2}]$

or $(-2\sqrt{2}, 2\sqrt{2})$

⑤ Relative Extrema :-

At $x = 2$:-

$f''(2) = -4 < 0$

Relative Maxima

Maximum Value :-

$f(2) = 4$

At $x = -2$:-

$f''(-2) = 4 > 0$

Relative minima

Minimum Value :-

$f(-2) = -4$

(f) $f(x) = \dfrac{12}{x^2+12}$

$$f'(x) = \frac{(x^2+12)(0) - 12(2x)}{(x^2+12)^2}$$

$$f'(x) = \frac{-24x}{(x^2+12)^2}$$

$$f''(x) = (x^2+12)^2(-24) - 24x(2)(x^2+12)(2x)$$

$$f''(x) = \frac{(x^2+12)[-24(x^2+12) + 96x^2]}{(x^2+12)^4}$$

$$f''(x) = \frac{-24[x^2+12-4x^2]}{(x^2+12)^3}$$

$$f''(x) = \frac{-24(-3x^2+12)}{(x^2+12)^3}$$

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① Critical Point :-

$$\frac{-24x}{(x^2+12)^2} = 0 \qquad (x^2+12)^2 = 0$$

$-24x = 0$ $\qquad x^2 + 12 = 0$

$\boxed{x = 0}$ $\qquad x^2 = -12$

Complex Roots

② Inc / Dec :-

$\ominus$ $\oplus$

$-\infty \quad 0 \quad +\infty$

Inc : $(-\infty, 0]$

Dec : $[0, +\infty)$

③ Inflection Point :-

$$\frac{-24(-3x^2+12)}{(x^2+12)^3} = 0 \qquad (x^2+12)^3 = 0$$

$-3x^2 + 12 = 0$ $\qquad x^2 + 12 = 0$

$x^2 = 4$ $\qquad x = \pm\sqrt{-12}$

complex roots

$x = 2, -2$

④ Concavity :-

$\oplus$ $\ominus$ $\oplus$

$-\infty \quad -2 \quad 2 \quad +\infty$

concave up: $(-\infty, 2) \cup (2, +\infty)$

concave down: $(-2, 2)$

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(5) Relative Extrema :-

At $x = 0$:-

$$f''(0) = -24 \frac{(-3(0)^2 + 12)}{(0^2+12)^3} \Rightarrow -\frac{1}{6} < 0 \quad \text{Maxima}$$

$$f(0) = \frac{12}{(0)^2 + 12} \Rightarrow 1 \Rightarrow \text{Maximum Value}$$

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(9) = $f(x) = x^2 \ln x$

$$f'(x) = 2x \ln x + x$$

$$f''(x) = \frac{2x}{x} + 2\ln x + 1 \Rightarrow 2\ln x + 3$$

① Critical Points:-

$f'(x) = 0$

$2x \ln x + x = 0$

$x(2\ln x + 1) = 0$

$\boxed{x = 0}$ $\quad \ln x = -\frac{1}{2}$

$\boxed{x = e^{-1/2}}$

② Inc / Dec:-

D.N.E , ⊖ , ⊕

$-\infty \quad 0 \quad e^{-1/2} \quad +\infty$

Inc : $(e^{-1/2}, +\infty)$

Dec : $(0, e^{-1/2})$

③ Inflection Points

$f''(x) = 0$

$2\ln x + 3 = 0$

$\ln x = -\frac{3}{2}$

$\boxed{x = e^{-3/2}}$

and $\ln x$ is undefined at $x = 0$ so $x = 0$ is also inflection point

④ Concavity

D.N.E , $-$, $+$

$-\infty \quad 0 \quad e^{-3/2} \quad +\infty$

Concave Up: $(e^{-3/2}, +\infty)$

Concave down: $(0, e^{-3/2})$

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(5) Relative Extrema:-

At $x = 0$:-

$f''(0) = 2\ln(0) + 3$

At $x = e^{-1/2}$:-

$f''(e^{-1/2})\ 2\ln(e^{-1/2})+3$

$f''(e^{-1/2}) = 2 > 0$

Minima

Minimum Value:-

$f(e^{-1/2}) = (e^{-1/2})^2 \ln(e^{-1/2})$

$f(e^{-1/2}) = -0.183$

Date:

(h) $\cos x - 9x = f(x)$; $[0, 4\pi]$

$f'(x) = -\sin x - 9$

$f''(x) = -\cos x$

① Critical Points:-

$f'(x) = 0$ & $f'(x) = \text{undefined}$

$-\sin x - 9 = 0$ D.N.E

$\sin x = -9$

$x \in \emptyset$

(II) Increasing / Decreasing:-

$-\infty \longleftrightarrow +\infty$ Inc: $\emptyset$ or $\{\}$ Dec: $[0, 4\pi]$

(III) Inflection Points:-

$f''(x) = 0$ & $f''(x) = \text{undefined}$

$-\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2},$

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(IV) Concavity :-

$-\quad\quad +\quad\quad +\quad\quad +\quad\quad -$

$0\quad \pi/2\quad 3\pi/2\quad 5\pi/2\quad 7\pi/2\quad 4\pi$

Concave Up: $(\pi/2, 7\pi/2)$
Concave Down: $(0, \pi/2) \cup (7\pi/2, 4\pi)$

(V) Relative Extrema :-

Since, No critical points exist
So, Relative Extrema does not exists.

Date:

(i) $f(x) = \ln\sqrt{x^2+4}$

$$f'(x) = \frac{1}{\sqrt{x^2+4}}\left(\frac{1}{\cancel{2}\sqrt{(x^2+4)}}\right)(\cancel{2}x)$$

$$= \frac{x}{x^2+4}$$

$$f''(x) = \frac{(x^2+x)(1) - x(2x)}{(x^2+4)^2}$$

$$= \frac{(x^2+4-2x^2)}{(x^2+4)^2}$$

$$= \frac{4-x^2}{(x^2+4)^2}$$

(i) Critical Points :-

$$\frac{x}{x^2+4} = 0 \quad , \quad x^2+4=0$$

$\boxed{x=0}$ $\qquad x^2 = -4$

complex

(ii) Inc / Dec :-

$-\infty \quad \ominus \quad 0 \quad \oplus \quad +\infty$

inc : $(0, +\infty)$

dec : $(-\infty, 0)$

(iii) Inflection Point :

$4 - x^2 = 0$ & $x^2+4=0$

$\boxed{x = \pm 2}$ complex roots

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(i) $\ln\sqrt{x^2+4} = f(x)$

$$f'(x) = \frac{1}{\sqrt{x^2+4}}\left(\frac{1}{2\sqrt{x^2+4}}\right)(2x)$$

④ Concavity:-

concave up: $(-2, 2)$

concave down: $(-\infty, -2) \cup (2, +\infty)$

$$\underset{-\infty}{\longleftarrow} \ominus \underset{-2}{|} \oplus \underset{2}{|} \ominus \underset{+\infty}{\longrightarrow}$$

⑤ Relative Extrema:-

At $x = 0$:-

$$f''(0) = \frac{4-(0)^2}{[(0)^2+4]^2} \Rightarrow \frac{1}{4} > 0 \quad \text{Minima}$$

Minimum value $= f(0) = \ln\sqrt{0^2+4}$

$= \ln(4)$

$= 1.386$

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(j) $f(x) = xe^{-7x}$

$f'(x) = e^{-7x}(1) + xe^{-7x}(-7)$

$= e^{-7x}(1 - 7x)$

$f''(x) :-$

$= e^{-7x}(-7) + (1-7x)e^{-7x}(-7)$

$= e^{-7x}(-7 + 49x - 7)$

$= e^{-7x}(49x - 14)$

(I) Critical Points:-

$e^{-7x}(1-7x) = 0$

$e^{-7x} = 0$

D.N.E

$1 - 7x = 0$

$\boxed{x = \frac{1}{7}}$

(II) Inc / Dec :-

$-\infty \quad (+) \quad 1/7 \quad (-) \quad +\infty$

Inc: $(-\infty, 1/7)$

Dec: $(1/7, +\infty)$

(III) Inflection Points:-

$e^{-7x}(49x - 14) = 0$

$e^{-7x} = 0$

D.N.E

$49x - 14 = 0$

$\boxed{x = \frac{2}{7}}$

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(IV) Concavity:-

$$\underset{-\infty}{\longleftarrow} \quad \ominus \quad \underset{2/7}{|} \quad \oplus \quad \underset{+\infty}{\longrightarrow}$$

Concave Up: $(2/7, +\infty)$
Concave Down: $(-\infty, 2/7)$.

(V) Relative Extrema:-

At $x = \frac{1}{7}$:-

$$f''\left(\frac{1}{7}\right) = e^{-7\left(\frac{1}{7}\right)}\left[49\left(\frac{1}{7}\right) - 14\right]$$

$$= e^{-1}(-7)$$

$$= -\frac{7}{e} < 0 \quad \text{Maxima}$$

Maximum Value $= f(1/7)$

$$= \left(\frac{1}{7}\right) e^{-7\left(\frac{1}{7}\right)}$$

$$= \frac{1}{7e}$$