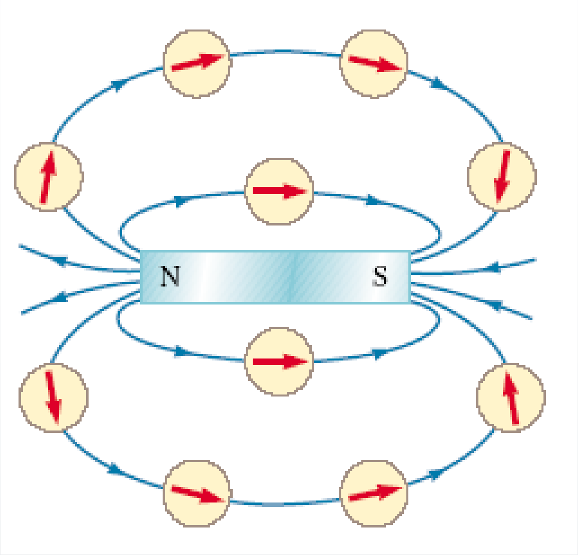
THE MAGNETIC FIELD

An electric field surrounds any stationary or moving electric charge.
In addition to an electric field, the region of space surrounding any *moving electric charge also contains a magnetic field.*
A magnetic field also surrounds any magnetic substance.
The symbol B has been used to represent a magnetic field.

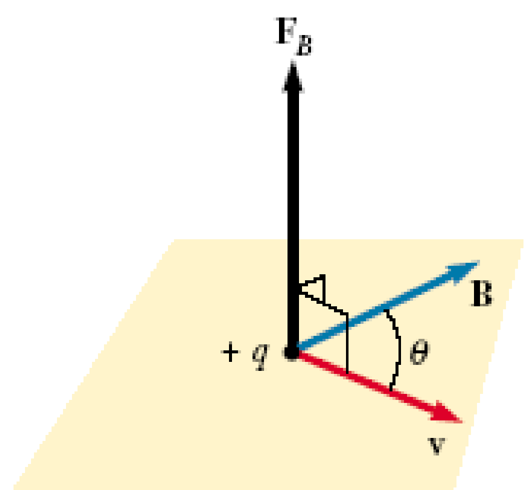


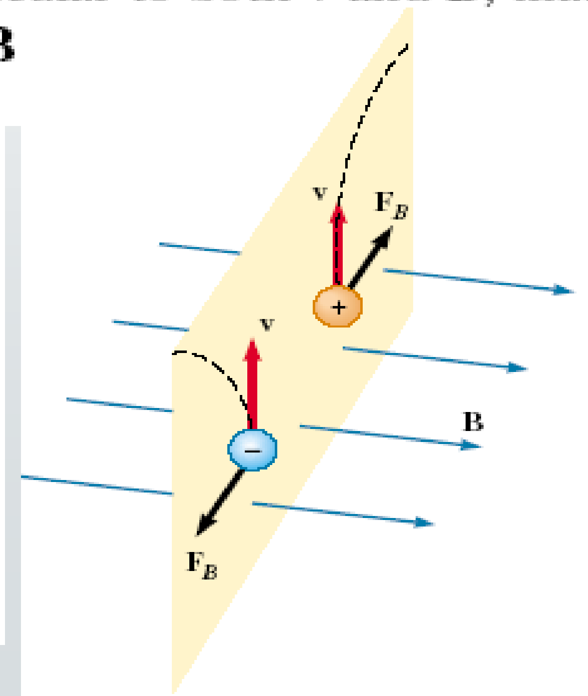
We can define a magnetic field **B** at some point in space in terms of the magnetic force $\mathbf{F}_B$ that the field exerts on a test object, for which we use a charged particle moving with a velocity **v**. For the time being, let us assume that no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

Properties of the magnetic force on a charge moving in a magnetic field **B**

- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.

- The magnitude and direction of $\mathbf{F}_B$ depend on the velocity of the particle and on the magnitude and direction of the magnetic field **B**.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and **B**; that is, $\mathbf{F}_B$ is perpendicular to the plane formed by **v** and **B**





- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. 29.3b).
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle the particle's velocity vector makes with the direction of $\mathbf{B}$.

We can summarize these observations by writing the magnetic force in the form

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

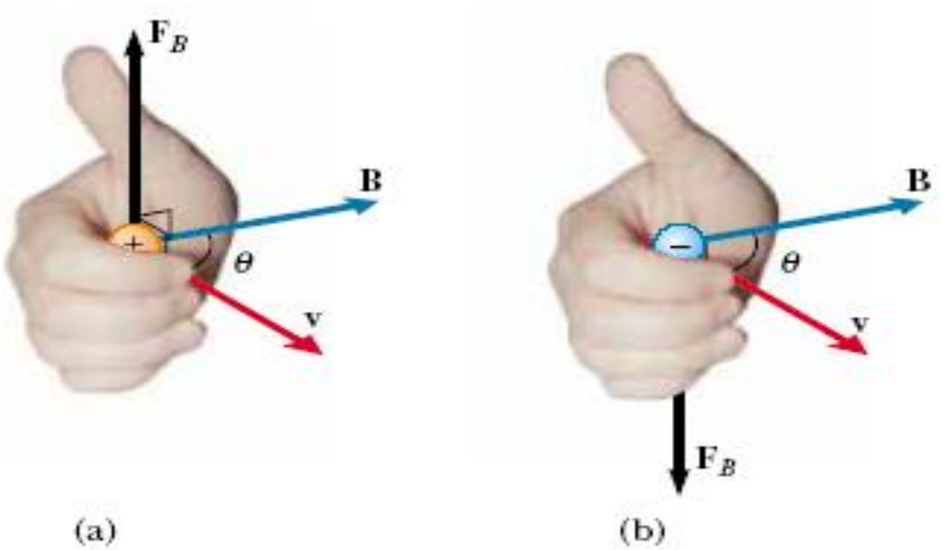


Figure 29.4 The right-hand rule for determining the direction of the magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ acting on a particle with charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. The direction of $\mathbf{v} \times \mathbf{B}$ is the direction in which the thumb points. (a) If q is positive, $\mathbf{F}_B$ is upward. (b) If q is negative, $\mathbf{F}_B$ is downward, antiparallel to the direction in which the thumb points.

Magnitude of the magnetic force on a charged particle moving in a magnetic field

$$F_B = |q|vB\sin\theta$$

where θ is the smaller angle between $\mathbf{v}$ and $\mathbf{B}$. From this expression, we see that F is zero when $\mathbf{v}$ is parallel or antiparallel to $\mathbf{B}$ ($\theta = 0$ or $180°$) and maximum ($F_{B,\max} = |q|vB$) when $\mathbf{v}$ is perpendicular to $\mathbf{B}$ ($\theta = 90°$).

Differences between electric and magnetic forces

There are several important differences between electric and magnetic forces:

- The electric force acts in the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced.

A magnetic field cannot change the speed of a particle

when a charged particle moves with a velocity **v** through a magnetic field, the field can alter the direction of the velocity vector but cannot change the speed or kinetic energy of the particle.

http://fys.kuleuven.be/pradem/applets/suren/MovChgMag/MovChgMag.html

MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

A positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field. Let us assume

that the direction of the magnetic field is into the page. Figure shows that the particle moves in a circle in a plane perpendicular to the magnetic field. The particle moves in this way because the magnetic force F_B *is at right angles* to v and B and has a constant magnitude *qvB. As the force deflects the particle,* the directions of v and F_B *change continuously, as Figure shows. Because* F_B always points toward the center of the circle, it changes only the direction of v and not its magnitude.
If *q were negative, the rotation would be clockwise.*

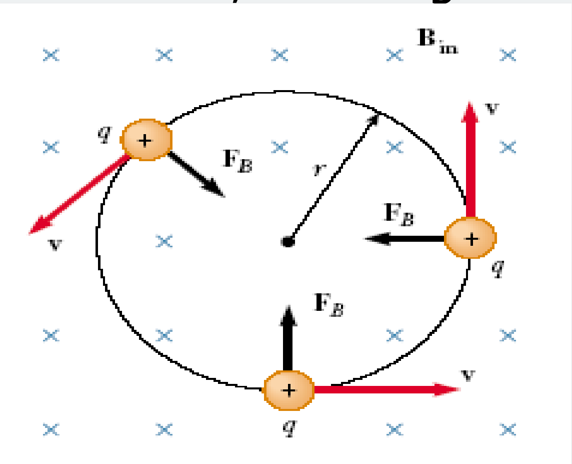
$$\sum F = ma_r$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$



These results show that the angular speed of the particle and the period of the circular motion do not depend on the linear speed of the particle or on the radius of the orbit. The angular speed is often referred to as the cyclotron frequency because charged particles circulate at this angular speed in the type of accelerator called a *cyclotron,*

APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

Lorentz force

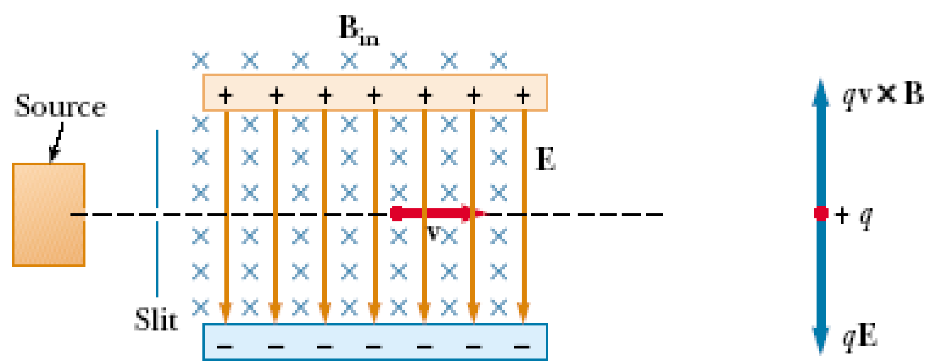
A charge moving with a velocity **v** in the presence of both an electric field **E** and a magnetic field **B** experiences both an electric force $q\mathbf{E}$ and a magnetic force $q\mathbf{v} \times \mathbf{B}$. The total force (called the Lorentz force) acting on the charge is

$$\sum \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Velocity Selector

$$qE = qvB,$$

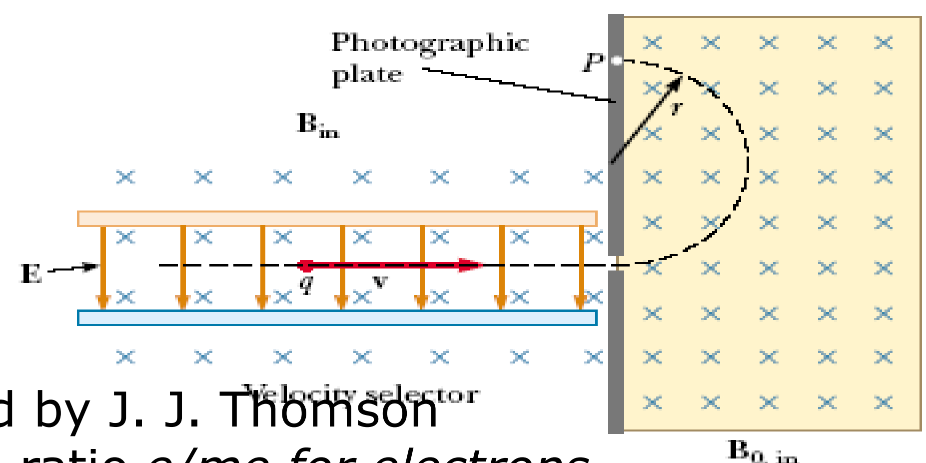
$$v = \frac{E}{B}$$



The Mass Spectrometer

$$\frac{m}{q} = \frac{rB_0}{v}$$

$$\frac{m}{q} = \frac{rB_0 B}{E}$$



A variation of this technique was used by J. J. Thomson (1856–1940) in 1897 to measure the ratio *e/me for electrons.*

THE HALL EFFECT

When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon, first observed by Edwin Hall (1855–1938) in 1879, is known as the ***Hall effect***. The Hall effect gives information regarding the sign of the charge carriers and their density; it can also be used to measure the n fields.

$$qv_dB = qE_H$$

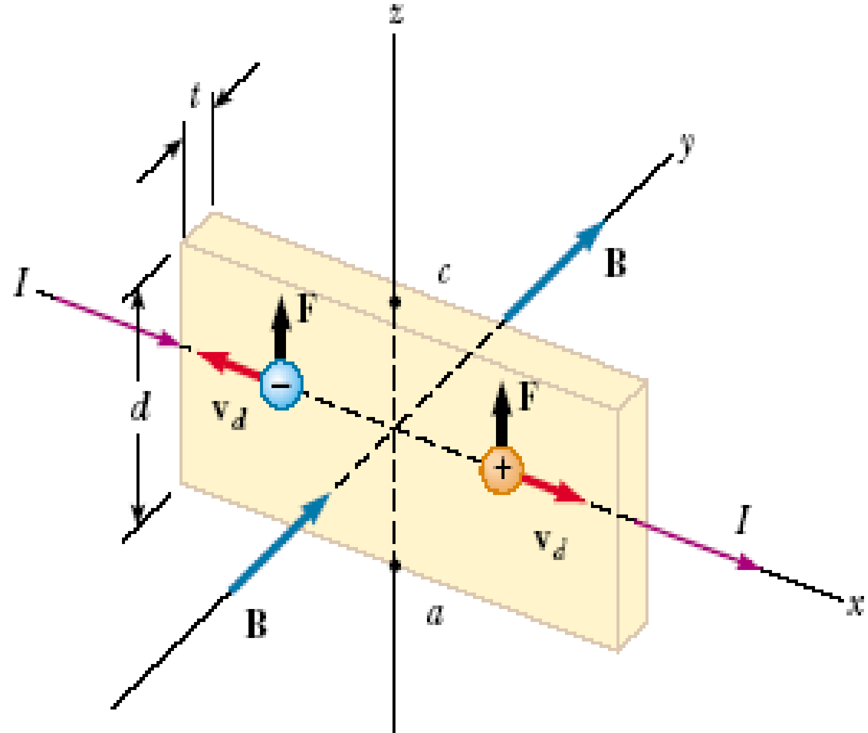
$$E_H = v_dB$$

$$\Delta V_H = E_H d = v_d B d$$

$$v_d = \frac{I}{nqA}$$

$$\Delta V_H = \frac{IBd}{nqA}$$

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$



where $R_H = 1/nq$ is the **Hall coefficient.**

GAUSS'S LAW IN MAGNETISM

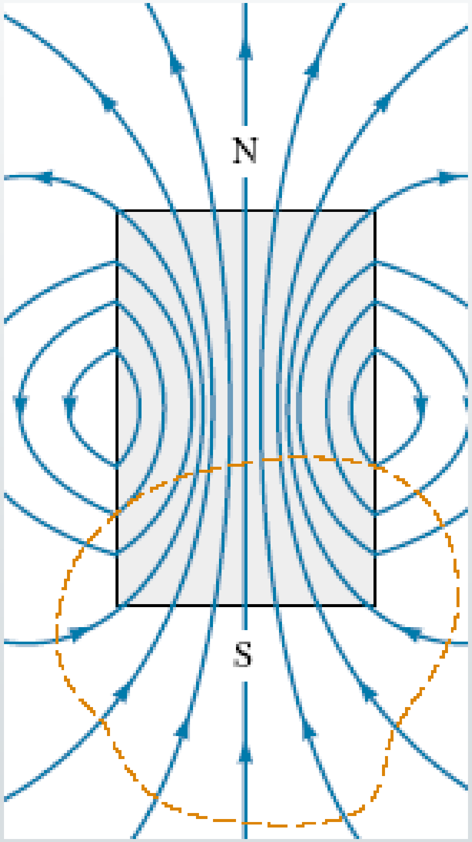
Gauss's law in magnetism states that

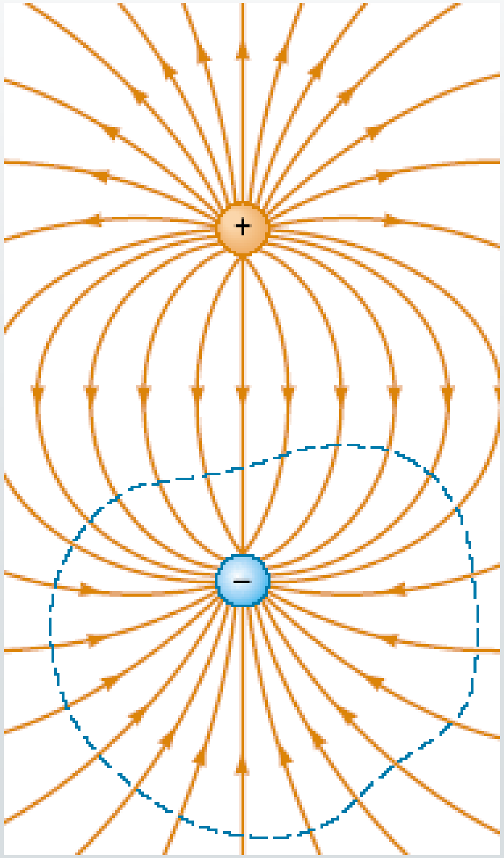
the net magnetic flux through any closed surface is always zero:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

An isolated magnetic poles (monopoles) have never been detected and perhaps do not exist.

The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through the closed surface (dashed red line) surrounding one of the poles (or any other closed surface) is zero.
The electric field lines surrounding an electric dipole begin on the positive charge





Classification of Magnetic Substances

Substances can be classified as belonging to one of three categories, depending on their magnetic properties. Paramagnetic and ferromagnetic materials are those

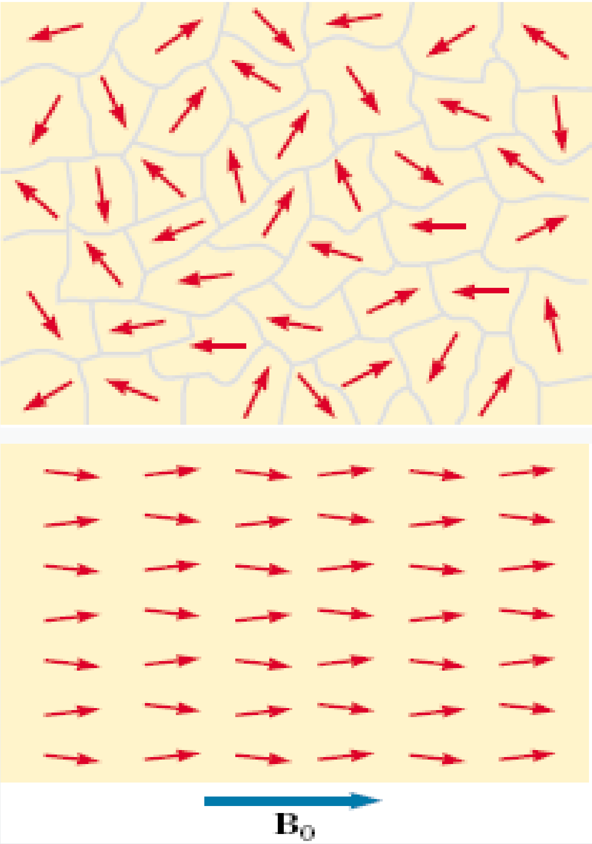
Substances may be classified in terms of how their magnetic permeability μ_m compares with μ_0 (the permeability of free space), as follows:

$$\text{Paramagnetic} \qquad \mu_m > \mu_0$$

$$\text{Diamagnetic} \qquad \mu_m < \mu_0$$

Because χ is very small for paramagnetic and diamagnetic substances (see Table 30.2), μ_m is nearly equal to μ_0 for these substances. For ferromagnetic substances, however, μ_m is typically several thousand times greater than μ_0 (meaning that χ is very great for ferromagnetic substances).

Ferromagnetism



A small number of crystalline substances in which the atoms have permanent magnetic moments exhibit strong magnetic effects called ferromagnetism.

Some examples of ferromagnetic substances are iron, cobalt, nickel, gadolinium, and dysprosium.

All ferromagnetic materials are made up of microscopic regions called **domains**, regions within which all magnetic moments are aligned.

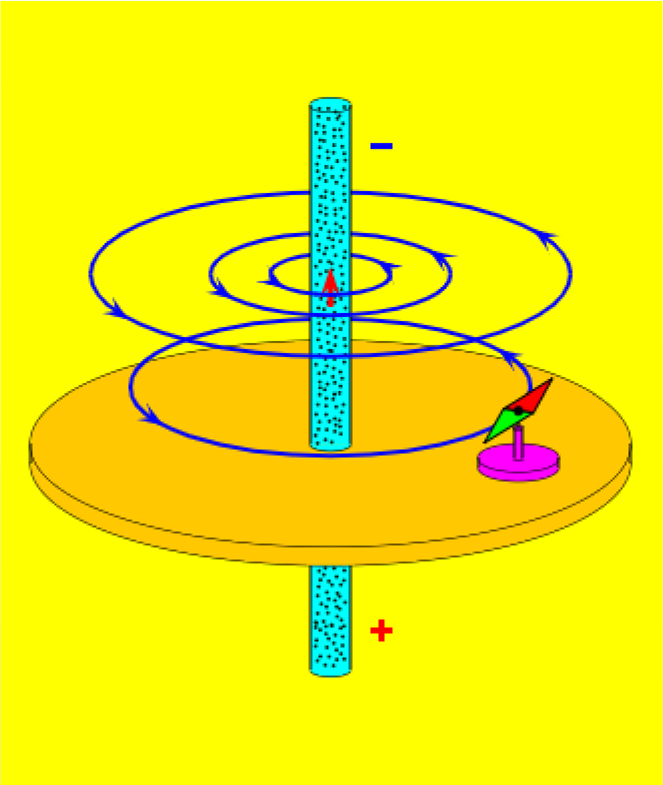
These domains have volumes of about 10^12 to 108 m3 and contain 10^17 to 10^21 atoms. The boundaries between the various domains having different orientations are called **domain walls**.

Paramagnetism

Paramagnetic substances have a small but positive magnetic susceptibility ($0 < \chi \ll 1$) resulting from the presence of atoms (or ions) that have permanent magnetic moments. These moments interact only weakly with each other and are randomly oriented in the absence of an external magnetic field. When a paramagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field. However, this alignment process must compete with thermal motion, which tends to randomize the magnetic moment orientations.

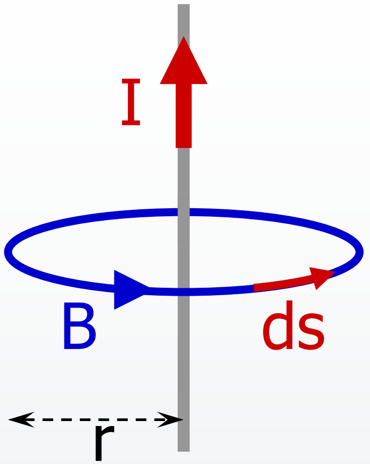
Diamagnetism

When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field. This causes diamagnetic substances to be weakly repelled by a magnet. Although diamagnetism is present in all matter, its effects are much smaller than those of paramagnetism or ferromagnetism, and are evident only when those other effects do not exist.



- magnetic field of long straight wire:

$$B = \frac{\mu_0 I}{2\pi r}$$ winds around the wire



Line integral of B over a closed circular path around wire:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r)$$

$$\vec{B} \parallel d\vec{s}$$

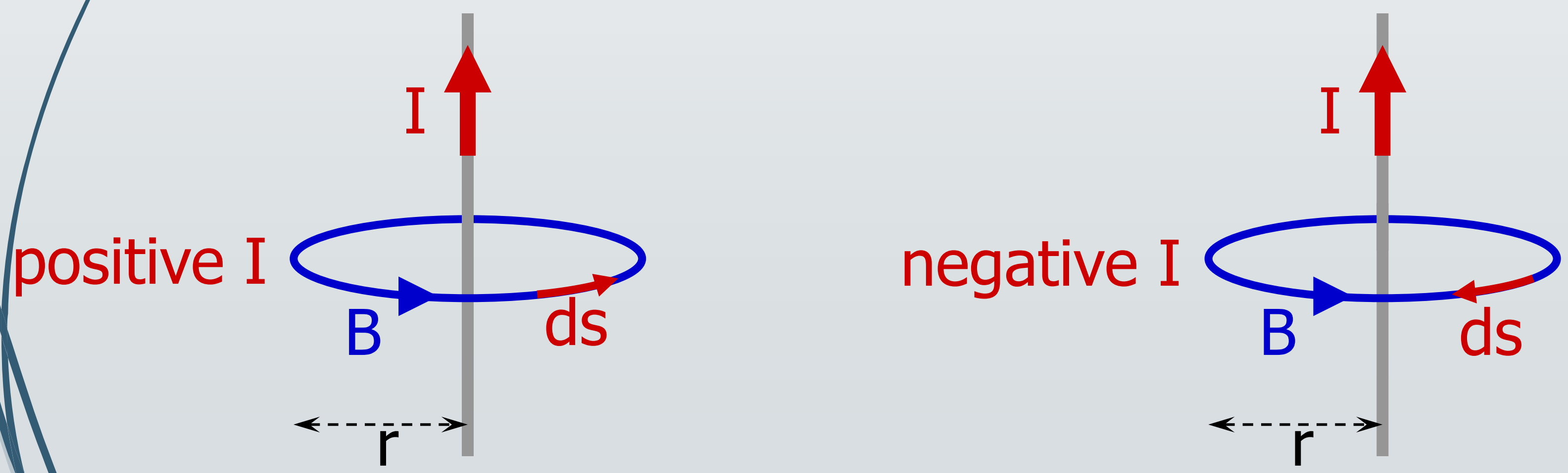
$$\oint \vec{B} \cdot d\vec{s} = \left(\frac{\mu_0 I}{2\pi r} \right) (2\pi r) = \mu_0 I$$

Is this an accident, valid only for this particular situation?

Ampere's Law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\, I_{encl}$$ Ampere's Law

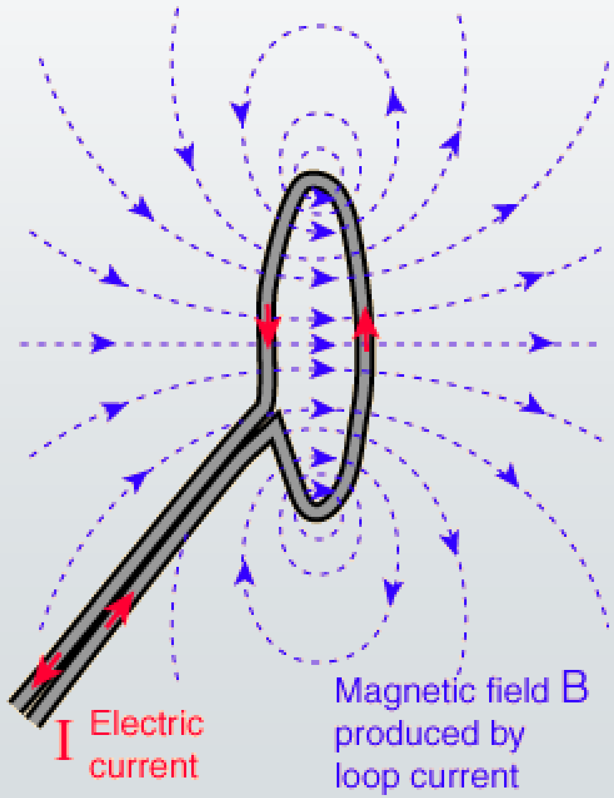
- I_{encl} is total current that passes through surface bounded by closed path of integration.
- **law of nature:** holds for any closed path and any current distribution
- current I counts positive if integration direction is the same as the direction of $\vec{B}$ from the right hand rule



Magnetic Field of a Solenoid

A solenoid is made of many loops of wire, packed closely to form long cylinder.

Single loop:

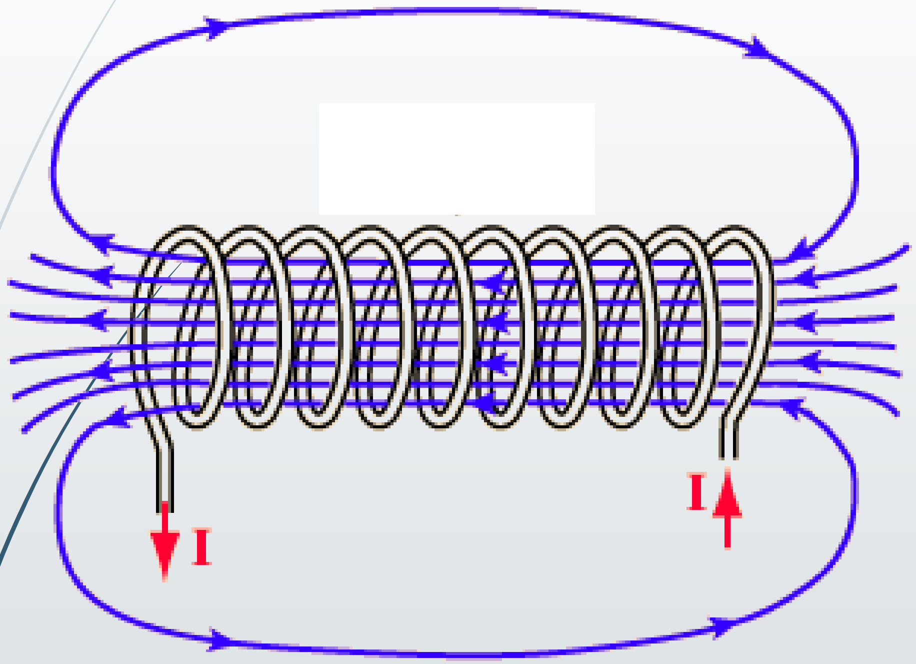


images from hyperphysics.

*But not so closely that you can use

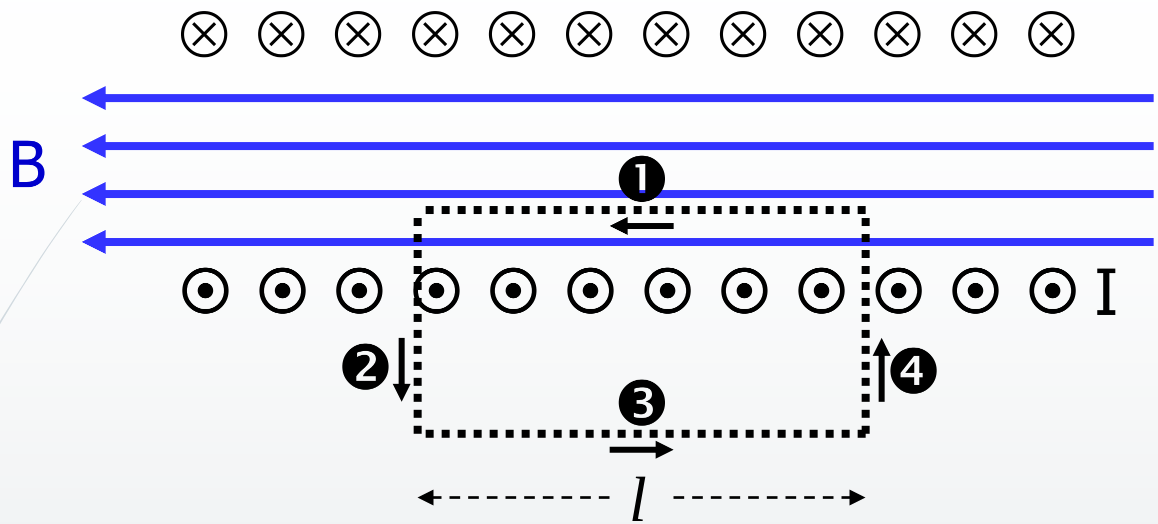
$$B = \frac{\mu_0 N I}{2a}$$

Stack many loops to make a solenoid:



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

Ought to remind you of the magnetic field of a bar magnet.



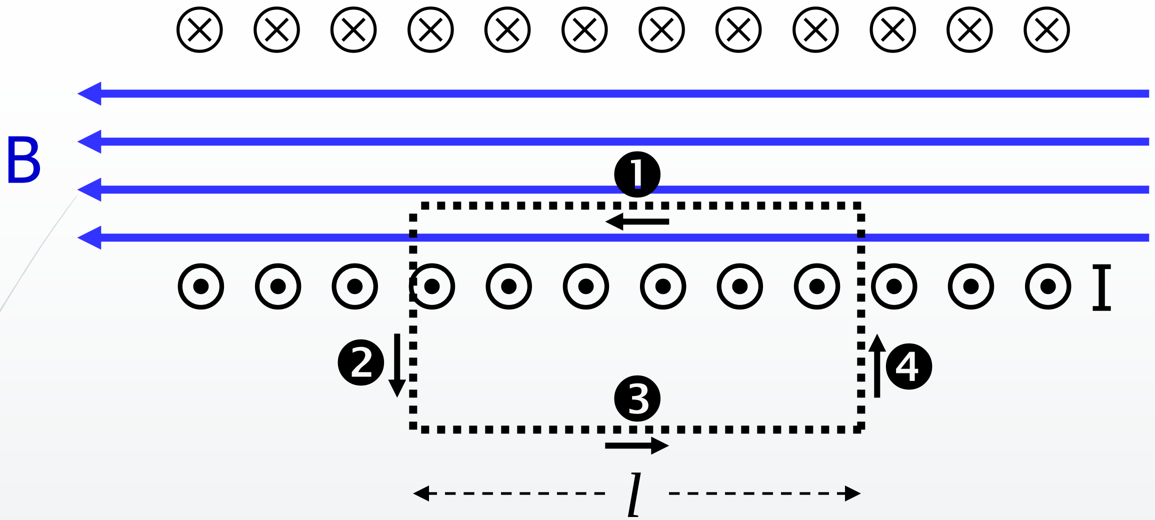
Use Ampere's law to calculate the magnetic field of a solenoid:

$$\oint \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

$$\oint \vec{B}\cdot d\vec{s} = B\ell + 0 + 0 + 0 = \mu_0 I_{enclosed}$$

$$B\ell = \mu_0 N I$$

N is the number of loops enclosed by our surface.



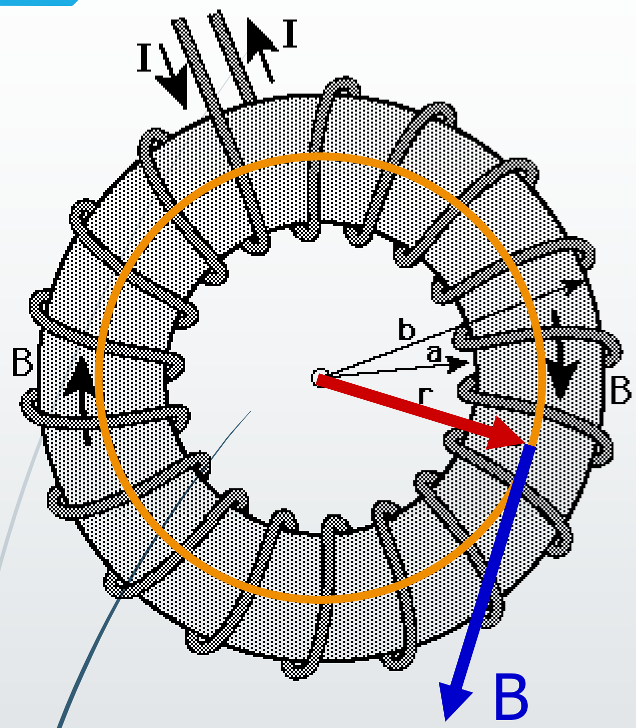
$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \mu_0 n I$$

Magnetic field of a solenoid of length l , N loops, current I. $n=N/l$ (number of turns per unit length).

The magnetic field inside a long solenoid does not depend on the position inside the solenoid (if end effects are neglected).

A toroid* is just a solenoid "hooked up" to itself.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed} = \mu_0 N I$$

$$\oint \vec{B} \cdot d\vec{s} = B \int ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Magnetic field inside a toroid of N loops, current I.

The magnetic field inside a toroid is not subject to end effects, but is not constant inside (because it depends on r).

*Your text calls this a "toroidal solenoid."

Example: a thin 10-cm long solenoid has a total of 400 turns of wire and carries a current of 2 A. Calculate the magnetic field inside near the center.

$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \left(4\pi \times 10^{-7}\ \frac{T \cdot m}{A}\right) \frac{(400)}{(0.1\ m)} (2\ A)$$

$$\boxed{B = 0.01\ T}$$

"Help! Too many similar starting equations!"

$$B = \frac{\mu_0 I}{2\pi r}$$ long straight wire

$$B = \mu_0 \frac{N}{\ell} I$$ solenoid, length l, N turns

$$B = \mu_0 n I$$ solenoid, n turns per unit length

$$B = \frac{\mu_0 N I}{2\pi r}$$ toroid, N loops