Ex: 6.5

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1) Find limits

7 $\lim_{x\to 0} \frac{e^x - 1}{\sin x} \left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{e^x}{\cos x} = \frac{1}{1} = 1$

8 $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = \left(\frac{0}{0}\right)$

$\lim_{\theta\to 0} \frac{\sec^2\theta}{1} \Rightarrow 1$

9 $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x} \left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{2\cos 2x}{5\cos 5x} \Rightarrow \frac{2}{5}$

10 $\lim_{t\to 0} \frac{te^t}{1-e^t} \left(\frac{0}{0}\right)$

$\lim_{t\to 0} \frac{te^t + e^t}{-e^t} \Rightarrow \frac{-0+1}{1} = -1$

11 $\lim_{t\to \pi^+} \frac{\sin x}{x-\pi} \left(\frac{0}{0}\right)$

$\lim_{t\to \pi^+} \frac{\cos x}{1} \Rightarrow -1$

12 $\lim_{x\to 0^+} \frac{\sin x}{x^2} \left(\frac{0}{0}\right)$

$\lim_{x\to 0^+} \frac{\cos x}{2x} = +\infty$

13 $\lim_{x\to \infty} \frac{\ln x}{x} \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to \infty} \frac{1/x}{1} \Rightarrow 0$

14 $\lim_{x\to +\infty} \frac{e^{3x}}{x^2} \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to +\infty} \frac{3e^{3x}}{2x} \left(\frac{\infty}{\infty}\right)$

again L-Hop

$\lim_{x\to +\infty} \frac{9e^{3x}}{2} = +\infty$

15 $\lim_{x\to 0^+} \frac{\cot x}{\ln x} \left(\frac{0}{0}\right)$

$\lim_{x\to 0^+} \frac{-\operatorname{cosec}^2 x}{1/x}$

$\lim_{x\to 0^+} \frac{-x}{\sin^2 x}$ $\therefore \operatorname{cosec} x = \frac{1}{\sin x}$

Taking L-Hop again

$\lim_{x\to 0^+} \frac{-1}{2\sin x\cos x} \Rightarrow -\infty$

16 $\lim_{x\to 0^+} \frac{1-\ln x}{e^{1/x}} \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to 0^+} \frac{-1/\cancel{x}}{-\left(\frac{1}{x^2}\right)e^{1/\cancel{x}}}$

$\lim_{x\to 0^+} \frac{x}{e^{1/x}} \Rightarrow 0$

17 $\lim_{x\to +\infty} \frac{x^{100}}{e^x} \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to +\infty} \frac{100x^{99}}{e^x}$

$\lim_{x\to +\infty} \frac{9900x^{98}}{e^x}$

apply L-Hop till x'

$\therefore \lim_{x\to \infty} \frac{9900(98)\cdots(1)}{e^x} = 0$

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18 $\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)} \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to 0^+} \frac{\frac{1}{\sin x}(\cos x)}{\frac{1}{\tan x}(\sec^2 x)}$

$\lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}\left(\frac{1}{\cos^2 x}\right)} \Rightarrow \cos^2 x \Rightarrow 1$

19 $\lim_{x\to +\infty} x e^{-x} \left(\frac{\infty}{\infty}\right)$

$\frac{x}{e^x} \Rightarrow \frac{1}{e^x} = 0$

20 $\lim_{x\to \pi} (x-\pi)\tan\frac{1}{2}x$

$\lim_{x\to\pi} \frac{x-\pi}{\cot(x/2)}$

$\lim_{x\to\pi} \frac{1}{-(1/2)\operatorname{cosec}^2(x/2)} \Rightarrow -2$

21 $\lim_{x\to +\infty} x\sin\frac{\pi}{x}$

$\lim_{x\to +\infty} \frac{\sin(\pi/x)}{1/x}$

$\lim_{x\to +\infty} \frac{\cancel{(-\pi/x^2)}\cos(\pi/x)}{\cancel{-1/x^2}}$

$\lim_{x\to\infty} \pi\cos(\pi/x) \Rightarrow \pi$

22 $\lim_{x\to 0^+} \tan x \ln x$

$\lim_{x\to 0^+} \frac{\ln x}{\cot x}$ $\quad \frac{\sec^2 x}{\frac{x}{\cos^2 x}}$ $\frac{1}{x}$

$\lim_{x\to 0^+} \frac{1/x}{-\operatorname{cosec}^2 x}$

$\lim_{x\to 0^+} \frac{-\sin^2 x}{x}$

again L-Hop

$\lim_{x\to 0^+} \frac{-2\sin x\cos x}{1} = 0$

23 $\lim_{x\to \pi/2^-} \sec 3x \cos 5x$

$\lim_{x\to\pi/2^-} \frac{\cos 5x}{\cos 3x}$

$\lim_{x\to\pi/2^-} \frac{-5\sin 5x}{-3\sin 3x}$

$\Rightarrow \frac{-5(+1)}{-3(-1)} = \frac{-5}{3}$

24 $\lim_{x\to\pi} (x-\pi)\cot x$

$\lim_{x\to\pi} \frac{x-\pi}{\tan x}$

$\lim_{x\to\pi} \frac{1}{\sec^2 x} \Rightarrow 1$

25 $\lim_{x\to +\infty} (1-3/x)^x \,(1^\infty)$

apply ln

$\ln y = \lim_{x\to\infty} \ln(1-3/x)^x$

$\ln y = \lim_{x\to\infty} x\ln(1-3/x)$

$\ln y = \lim_{x\to\infty} \frac{\ln(1-3/x)}{1/x}$

$\ln y = \lim_{x\to\infty} \frac{\left(\frac{1}{1-3/x}\right)(-3)\cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{-1/x^2}}$

$\ln y = \lim_{x\to\infty} \frac{-3}{1-3/x}$

$\ln y = \lim_{x\to\infty} \Rightarrow \frac{-3}{1-0} \Rightarrow -3$

$y \Rightarrow e^{-3}$

26 $\lim_{x\to 0} (1+2x)^{-3/x}$

$\ln y = \lim_{x\to 0} -3/x \ln(1+2x)$

$\ln y = \lim_{x\to 0} = \frac{\frac{1}{1+2x}(-3)(2)}{1}$

$\ln y = \lim_{x\to 0} \frac{-6}{1+2x} \Rightarrow \frac{-6}{1+0}$

$\Rightarrow \ln y = -6$

$y = e^{-6}$

27 $\lim_{x\to 0} (e^x + x)^{1/x}$

$\ln y = \lim_{x\to 0} \frac{1}{x}\ln(e^x+x)$

$\ln y = \lim_{x\to 0} \frac{1}{e^x+x}(e^x+1)$

$\ln y = \lim_{x\to 0} \frac{e^x+1}{e^x+x} \Rightarrow 2$

$\ln y = 2$

$y = e^2$

28 $\lim_{x\to\infty} (1+a/x)^{bx}$

$\lim_{x\to\infty} \ln y = bx\ln(1+a/x)$

$\ln y = \lim_{x\to\infty} \frac{b\ln(1+a/x)}{1/x}$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1+a/x}\left(\cancel{-1/x^2}\right)(a)(b)}{\cancel{-1/x^2}}$

$\ln y = \lim_{x\to\infty} \frac{ab}{1+a/x}$

$\ln y = ab$

$y = e^{ab}$

29 $\lim_{x\to 1} (2-x)^{\tan(x\pi/2)}$

$\ln y = \lim_{x\to 1} \tan(x\pi/2)\ln(2-x)$

$\lim_{x\to 1} \frac{\ln(2-x)}{\cot(x\pi/2)}$

$\lim_{x\to 1} \frac{\frac{1}{2-x}(-1)}{-\csc^2(x\pi/2)(\pi)(1/2)}$

$\lim_{x\to 1} \frac{+2\sin^2(x\pi/2)}{\pi(2-x)} \Rightarrow \frac{2(1)}{\pi(1)}$

$\ln y = \frac{2}{\pi}$

$y = e^{2/\pi}$

30 $\lim_{x\to\infty} [\cos(2/x)]^{x}$

$\ln y = \lim_{x\to\infty} x^2\ln(\cos(2/x))$

$\ln y = \lim_{x\to\infty} \frac{\ln(\cos(2/x))}{1/x^2}$

$\ln y = \lim_{x\to\infty} \frac{\frac{-1(\sin(2/x))}{\cos(2/x)}(2)(-1/x^2)}{-2(1/x^{-3})}$

$\ln y = \lim_{x\to\infty} \frac{\tan(2/x)\cancel{(2/x^2)}}{-2/x^3}$

$\ln y = \lim_{x\to\infty} \frac{-\tan(2/x)}{1/x} \left(\frac{0}{0}\right)$

taking L-Hop again

$\ln y = \lim_{x\to\infty} \frac{\cancel{(2/x^2)}\sec^2(2/x)}{\cancel{-1/x^2}}$

$\ln y = \lim_{x\to\infty} -2\sec^2(2/x)$

$\ln y = -2(1)$

$y = e^{-2}$

31 $\lim_{x\to 0} (\csc x - 1/x)$

$\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right) = (\infty - \infty)$

$\lim_{x\to 0} \frac{x-\sin x}{(x\sin x)} \left(\frac{\infty}{\infty}\right)$

$\lim_{x\to 0} \frac{1-\cos x}{x\cos x + \sin x}$

again L-Hop

$\lim_{x\to 0} \frac{\sin x}{2\cos x - x\sin x}$

$= 0$

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32 $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{\cos 3x}{x^2} \right)$

$\lim_{x \to 0} \left(\frac{1 - \cos 3x}{x^2} \right) \quad \left(\frac{0}{0} \right)$

$\lim_{x \to 0} \frac{3 \sin 3x}{2x}$

again L-Hop

$\lim_{x \to 0} \frac{9 \cos 3x}{2}$

$9/2$

33 $\lim_{x \to \infty} \left(\sqrt{x^2 + x} - x \right) \quad (\infty - \infty)$

$\lim_{x \to \infty} \sqrt{x^2+x} - x \times \frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x}$

$\lim_{x \to \infty} \frac{(\cancel{x^2} + x) - \cancel{x^2}}{\sqrt{x^2+x} + x} \quad \left(\frac{\infty}{\infty} \right)$

$\lim_{x \to \infty} \frac{x}{\sqrt{x^2(1+\frac{1}{x})} + 1}$

$\lim_{x \to \infty} \frac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x}} + 1}$

$\lim_{x \to \infty} \frac{1}{\sqrt{1+\frac{1}{x}} + 1}$

apply limit

$\Rightarrow \frac{1}{\sqrt{1+0} + 1}$

$\Rightarrow \frac{1}{1+1}$

$\Rightarrow 1/2$

34 $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

$\lim_{x \to 0} \frac{e^x - 1 - x}{(e^x - 1)(x)}$

Taking L-Hop

$\lim_{x \to 0} \frac{e^x - 1}{xe^x + e^x - 1}$

again L-Hop

$\lim_{x \to 0} \frac{e^x}{xe^x + 2e^x}$

apply limit

$\Rightarrow 1/2$

35 $\lim_{x \to +\infty} \left[x - \ln(x^2+1) \right]$

$\lim_{x \to +\infty} \left[\ln e^x - \ln(x^2+1) \right]$

$\lim_{x \to \infty} \ln \left[\frac{e^x}{x^2+1} \right]$

$\ln \lim_{x \to \infty} \frac{e^x}{2x}$

again L-Hop

$\ln \lim_{x \to \infty} \frac{e^x}{2}$

apply limit

$\Rightarrow +\infty$

36 $\lim_{x \to \infty} \left[\ln x - \ln(1+x) \right]$

$\lim_{x \to \infty} \ln \left(\frac{x}{1+x} \right)$

$\ln \lim_{x \to \infty} \frac{1}{1}$

$\ln(1)$

0

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37 $\lim_{x\to 0^+} x^{\sin x}$

$\ln y = \sin x \ln x$

$\ln y = \lim_{x\to 0^+} \frac{\ln x}{\csc x}$

$\ln y = \lim_{x\to 0^+} \frac{1/x}{-\csc x \cot x}$

$\ln y = \lim_{x\to 0^+} \left(\frac{1}{\frac{1}{\sin x}}\right)\left(\frac{1}{x}\right)(-\tan x)$

$\ln y = \lim_{x\to 0^+} \frac{\sin x(-\tan x)}{x}$

$\ln y = \lim_{x\to 0^+} (1)(-\tan x)$

apply limit

$\ln y = 1(-0)$

$\ln y = 0$

$y = e^0$

$y = 1$

38 $\lim_{x\to 0^+} (e^{2x}-1)^x$

$\ln y = \lim_{x\to 0^+} x \ln(e^{2x}-1)$

$\ln y = \lim_{x\to 0^+} \frac{\ln(e^{2x}-1)}{1/x}$

$\ln y = \lim_{x\to 0^+} \frac{2e^{2x}\left(\frac{1}{e^{2x}-1}\right)}{-1/x^2}$

$\ln y = \lim_{x\to 0^+} \left(\frac{2e^{2x}(-x^2)}{e^{2x}-1}\right)$

apply limit $\Rightarrow \frac{0}{1}$

$\ln y = 0$

$y = e^0$

$y = 1$

39 $\lim_{x\to 0^+} \left[-\frac{1}{\ln x}\right]^x$

$\ln y = \lim_{x\to 0^+} x \ln\left[\frac{-1}{\ln x}\right]$

$\ln y = \lim_{x\to 0} \frac{\ln\left[\frac{-1}{\ln x}\right]}{1/x} =$

$\ln y = \lim_{x\to 0} \frac{\ln(-1) + \left(\ln\left(\frac{1}{\ln x}\right)\right)}{1/x}$

$\ln y = \lim_{x\to 0} \frac{0 - \ln(\ln x)}{1/x}$

$\ln y = \lim_{x\to 0} = \frac{-\frac{1}{\ln x} \cdot \frac{1}{x}}{(-x^{-2})}$

$\ln y = \lim_{x\to 0} \frac{x^{\cancel{2}}}{\cancel{x} \ln x}$

apply limit

$\ln y \Rightarrow 0$

$y = e^0$

$y = 1$

40 $\lim_{x\to +\infty} x^{1/x}$

$\ln y = \lim_{x\to +\infty} \frac{1}{x} \ln x$

$\ln y = \lim_{x\to \infty} \frac{\ln x}{x}$

$\ln y = \lim_{x\to \infty} \frac{1/x}{1}$

$\ln y = 0/1$

$\ln y = 0$

$y = e^0$

$y = 1$

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41 $\lim_{x\to\infty} (\ln x)^{1/x}$

$\ln y = \lim_{x\to\infty} \frac{1}{x} \ln(\ln x)$

$\ln y = \lim_{x\to\infty} \frac{\ln(\ln x)}{x}$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{\ln x}\left(\frac{1}{x}\right)}{1}$

apply limit

$\ln y = 0$

$y = e^0$

$y = 1$

42 $\lim_{x\to 0^+} (-\ln x)^x$

$\ln y = \lim_{x\to 0^+} x \ln(-\ln x)$

$\ln y = \lim_{x\to 0^+} \frac{\ln(-\ln x)}{1/x}$

$\ln y = \lim_{x\to 0^+} \frac{\frac{-1}{\ln x}\left(-\frac{1}{x}\right)}{(-x^{-2})}$

$\ln y = \lim_{x\to 0^+} \frac{-1}{\cancel{x}\ln x}(x^{\cancel{2}})$

$\ln y = \lim_{x\to 0^+} \frac{x}{\ln x}$

$\ln y = 0$

$y = e^0$

$y = 1$

43 $\lim_{x\to \pi/2^-} (\tan x)^{(\pi/2)-x}$

$\ln y = ((\pi/2)-x)\ln \tan x$

$\ln y = \lim_{x\to\pi/2} \frac{\ln \tan x}{1/(\pi/2-x)}$

$\ln y = \lim_{x\to\pi/2} \frac{\sec^2 x/\tan x}{1/(\pi/2-x)^2}$

$\ln y = \lim_{x\to\pi/2} \frac{\left(\frac{\pi}{2}-x\right)}{\cos x} \cdot \frac{\left(\frac{\pi}{2}-x\right)}{\sin x}$ $\cos x)$

$\ln y = \lim_{x\to\pi/2} \frac{(\pi/2-x)}{\cos x} \cdot \frac{(\pi/2-x)}{\sin x}$

apply limit

$\ln y = 1 \cdot 0 = 0$

$\ln y = 0$

$y = e^0$

$y = 1$