Ex: 2.8

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Q10 Suppose that $z = x^3y^2$ where both x and y are changing with time. At certain instant when $x = 1$ and $y = 2$, x is decreasing at the rate of 2 units/s and y is increasing at the rate of 3 unit/s. How fast is z changing at this instant? Is z increasing or dec.

Data

$$\left.\frac{dx}{dt}\right|_{x=1,\,y=2} = -2$$

$$\left.\frac{dy}{dt}\right|_{y=2,\,x=1} = 3$$

$$\left.\frac{dz}{dt}\right|_{x=1,\,y=2} = ?$$

Solution

$$z = x^3y^2$$

$$\frac{dz}{dt} = y^2 3x^2 \frac{dx}{dt} + 2y\frac{dy}{dt}x^3$$

$$\frac{dz}{dt} = (2)^2 3(1)^2(-2) + 2(2)(3)(1)^3$$

$$\frac{dz}{dt} = (-24) + (12)$$

$$\frac{dz}{dt} = -12 \text{ unit/s}$$

z is decreasing

Q11 The minute hand of a certain clock is 4 in long. Starting from the moment when the hand is pointing straight up, how fast is the area of the sector that is swept out by the hand increasing at any instant during the next revolution of hand.

Data

$$\frac{dA}{dt} = ?$$

$$\frac{d\theta}{dt} = \frac{2\pi}{60} = \frac{\pi}{30}$$

Solution

$$A = \tfrac{1}{2}r^2\theta$$

$$A = \tfrac{1}{2}(16)\theta$$

$$A = 8\theta$$

$$\frac{dA}{dt} = 8\frac{d\theta}{dt}$$

$$\frac{dA}{dt} = 8\left(\frac{\pi}{30}\right)$$

$$\frac{dA}{dt} = \frac{4\pi}{15} \text{ in}^2/\text{min}$$

Q12 A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate 3 ft/s. How rapidly is the area enclosed by the ripple increasing at the end of 10 s?

Data

$$\frac{dr}{dt} = 3 \text{ ft/s}$$

$$\left.\frac{dA}{dt}\right|_{t=10} = ?$$

Solution

radius at $t = 10$

$$r = 3 \times 10$$

$$r = 30 \text{ ft}$$

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$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(30)(3)$$

$$\frac{dA}{dt} = 180\pi \ ft^2/s$$

13 Q Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of $6 mi^2/h$. How fast is the radius of the spill increasing when the area is $9 mi^2$?

Data

$\frac{dA}{dt} = 6 mi^2/h$

$\frac{dr}{dt} = ?$

area $= 9 mi^2$

Solution

Area $= \pi r^2$

$9 = \pi r^2$

$r = 3/\sqrt{\pi}$

$dA/dt = 2\pi r \frac{dr}{dt}$ as $A = 2\pi r$

$6 = 2\pi\left(\frac{3}{\sqrt{\pi}}\right)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{6}{2\pi\left(\frac{3}{\sqrt{\pi}}\right)} = \frac{1}{\pi\left(\frac{1}{\sqrt{\pi}}\right)} = \frac{\sqrt{\pi}\cdot\sqrt{\pi}}{\pi\cdot\sqrt{\pi}} = \frac{1}{\sqrt{\pi}}$

or

$\frac{dr}{dt} = 0.5641 \ mi/h$

14 Q A spherical balloon is inflated so that its volume is increasing at the rate of $3ft^3/min$. How fast is the diameter of the balloon increasing when the radius is 1ft

Data

$\frac{dV}{dt} = 3$

$\left.\frac{dD}{dt}\right|_{r=1} = ?$

Solution

$D = 2r$

$D = 2(1)$

$D = 2$

$V = 4/3\pi r^3$

$V = 4/3\pi\left(\frac{D}{2}\right)^3$

$V = 1/6\pi D^3$

$\frac{dV}{dt} = \frac{1}{\cancel{6}_2}\pi \cancel{3}D^2\frac{dD}{dt}$

$\frac{dV}{dt} = 1/2\pi D^2\frac{dD}{dt}$

$\frac{dD}{dt} = \frac{2}{\pi D^2}\frac{dV}{dt}$

$\frac{dD}{dt} = \frac{2}{\pi(2)^2}(3)$

$\frac{dD}{dt} = \frac{3}{2\pi} \ ft/min$

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Q[15] A spherical balloon is deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?

Data

$\frac{dr}{dt} = -15$ cm/min

$\left.\frac{dV}{dt}\right|_{r=9} = ?$

Solution

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(9)^2(-15)$

$\frac{dV}{dt} = -4860\pi$ cm³/min

Q[16] A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s. How fast will top of ladder be moving down the wall when it is 8 ft above the ground?

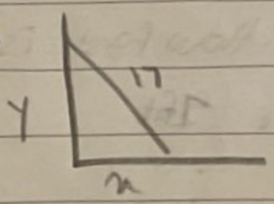
Data

let x denote distance of bottom of plank from wall

let y denote the wall

$\left.\frac{dy}{dt}\right|_{y=8} = ?$

$\frac{dx}{dt} = 5$



Solution

$x = \sqrt{17^2 - 8^2}$

$x = 15$

$x^2 + y^2 = 17^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-15}{8}(5)$

$\frac{dy}{dt} = \frac{-75}{8}$ ft/s

Q[17] A 13 ft ladder is leaning against a wall. if the top of the ladder slips down the wall at a rate of 2 ft/s. how fast will the foot be moving away from the wall when the top is 5 ft above ground

Data

size of ladder = 13 ft

let x denote distance of bottom from plank

let y denote wall

$\frac{dy}{dt} = -2$ ft, $\left.\frac{dx}{dt}\right|_{y=5} = ?$

Solution

$x = \sqrt{13^2 - 5^2}$

$x = 12$

$13^2 = x^2 + y^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-y}{x}\frac{dx}{dt}$

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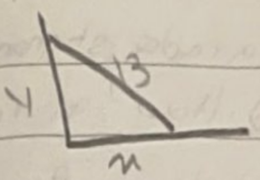


$$\frac{dy}{dt} = \frac{-5}{12}(-2)$$

$$\frac{dy}{dt} = 0.833 \text{ ft/s}$$

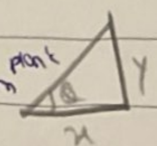
Q[18] A 10 ft plank is leaning against a wall. if at a certain instant the bottom of the plank is 2ft from the wall and is being pushed toward the wall at the rate of 6 in/s. how fast is the acute angle that the plank makes with the ground increasing?

$\cos\theta = \frac{2}{10}$

$\theta = 1.369$

Data

let x = distance of bottom from



let y = wall

θ = change of angle

$\frac{dx}{dt} = 6$ in/s $- \frac{6}{12} = -\frac{1}{2} = -0.5$ ft/s

(negative because x is decreasing when pushing)

$\left.\frac{d\theta}{dt}\right|_{x=2} = ?$

size of plank = 10 ft

Solution

$$\cos\theta = \frac{B}{H} = \frac{x}{10}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

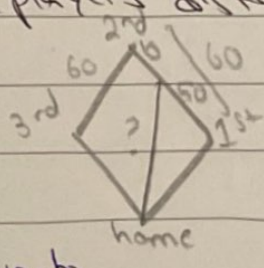
$$\frac{d\theta}{dt} = \frac{-1}{10\sin\theta}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-1}{10\sin(1.369)}\left(\frac{-1}{2}\right)$$

$$\frac{d\theta}{dt} = 0.051 \text{ rad/s}$$

Q[19] A softball diamond is a square whose sides are 60 ft long. suppose that a player running from first to second base has a speed of 25 ft/s at the instant when she is 10 ft from second base. At what rate is the player's distance from home plate changing at that instant

Data



Side = 60 ft

$\left.\frac{dx}{dt}\right|_{x=50} = 25$ ft/s

let x = distance from player to second plate

let y = distance from player to home plate

Solution

$$y = \sqrt{60^2 + (50)^2} = 10\sqrt{61}$$

$$y^2 = x^2 + s^2$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt} + 0$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{50}{10\sqrt{61}}(25)$$

$$\frac{dy}{dt} = 16.004 \text{ ft/s}$$

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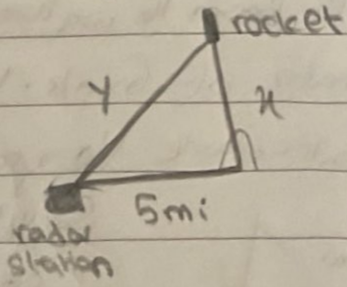
Q A rocket rising vertically is tracked by a radar station that is on the ground 5 mi from the launchpad. How fast is the rocket rising when it is 4mi high and its distance from the radar station is increasing a rate of 2000 mi/h

Data

$\left.\frac{dx}{dt}\right|_{x=4} = ?$

$\left.\frac{dy}{dt}\right|_{x=4} = 2000$



x = distance between radar and rocket

y = vertical distance

Solution

$y = \sqrt{4^2 + 5^2}$

$y = \sqrt{41}$

$y^2 = x^2 + 5^2$

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{\sqrt{41}}{4}(2000)$

$\frac{dx}{dt} = 500\sqrt{41}$ mi/h