CS1231S Midterm Cheatsheet 24/25
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Sets (Slide 1.1.3)		
$\mathbb{N}$	Set o All Natural Numbers	{0, 1, 2, 3, 4, ...}
$\mathbb{Z}$	Set of All Integers	{..., -1, 0, 1, ...}
$\mathbb{Z}^+$	Set of All Positive Integers	{1, 2, 3, 4, ...}
$\mathbb{Q}$	Set of All Rational Numbers	{..., -0.5, 0, 0.5, ...}
$\mathbb{R}$	Set of All Real Numbers	{..., -1, π, $\sqrt{2}$, 4.5, ...}
$\mathbb{C}$	Set of All Complex Numbers	{..., $-i$, 0, i, ...}

Conjunction & Disjunction / Tautology & Contradiction	
Conjunction of p and q: $(p \wedge q)$	Disjunction of p and q: $(p \vee q)$
Tautology: Always true	Contradiction: Always false

Logical Equivalence ≡	
$p \equiv q$	p & q have identical truth values ∀ (possible substitutions)

Implication Law		
Statement Type	Statement	Equivalence
Conditional	$p \to q$	$\sim p \vee q$
Negation	$\sim(p \to q)$	$p \wedge \sim q$
Converse	$q \to p$	$\sim q \vee p$
Inverse	$\sim p \to \sim q$	$p \vee \sim q$
Contrapositive	$\sim q \to \sim p$	$q \vee \sim p$

If, Only If, Biconditional	
p if q / if q then p / p is a necessary condition for q	$q \to p$
p only if q / only if q then p / p is a sufficient condition for q	$\sim q \to \sim p$ $p \to q$
p iff q / p if and only if q / p is a necessary and sufficient condition for q	$p \leftrightarrow q$ $(p \to q) \wedge (q \to p)$ $(\sim q \to \sim p) \wedge (\sim p \to \sim q)$

Order of Operations (Left to Right)				
()	~	R	$\wedge \vee$	$\to \leftrightarrow$

Theorem 2.1.1		
Commutative Laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative Laws	$p \wedge q \wedge r \equiv (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $p \vee q \vee r \equiv (p \vee q) \vee r \equiv p \vee (q \vee r)$	
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	
Identity Laws	$p \wedge true \equiv p$	$p \vee false \equiv p$
Negation Laws	$p \wedge \sim p \equiv false$	$p \vee \sim p \equiv true$
Double Negative Laws	$\sim(\sim p) \equiv p$	
Idempotent Laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Universal Bound Laws	$p \vee true \equiv true$	$p \wedge false \equiv false$
De Morgan's Laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Absorption Laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Variant Absorption Laws (Assignment 1)	$p \vee (\sim p \wedge q) \equiv p \vee q$	$p \wedge (\sim p \vee q) \equiv p \wedge q$

Negation of T and F	$\sim true \equiv false$	$\sim false \equiv true$

Arguments (Valid and Sound)	
Valid	All premises true ∧ Conclusion true Conclusions in all critical row is true
Invalid	There is a critical row which the conclusion is false
Critical Row: All premises are true	
Sound	Valid ∧ All premises true
Unsound	Not sound ie $\sim Valid \vee$ Contradictory $Premise$
Valid & Unsound: Contradictory Premise (Vacuously True)	

Rules of Inference			
Inference	Premise 1	Premise 2	Conclusion
Modus Ponens	$p \to q$	p	$\therefore q$
Modus Tollens	$p \to q$	$\sim q$	$\therefore \sim p$
Generalisation	p q		$\therefore p \vee q$ $\therefore p \vee q$
Specialisation	$p \wedge q$ $p \wedge q$		$\therefore p$ $\therefore q$
Conjunction	p	q	$\therefore p \wedge q$
Elimination	$p \vee q$ $p \vee q$	$\sim p$ $\sim q$	$\therefore q$ $\therefore p$
Transitivity	$p \to q$	$q \to r$	$\therefore p \to r$
Contradiction	$\sim p \to false$		$\therefore p$
Proof by Division into Cases	$p \vee q$	$(p \to r) \wedge (q \to r)$	$\therefore r$

Predicate Logic	
Predicate	In the form of $P(x)$, becomes statement when x has value
Domain	Set of values that substitute x
Truth Set	$\{x \in D \mid P(x)\}$ where D is Domain of x

Universal Statements & Existential Statements	
Universal Statement	$\forall x \in D, P(x)$
Existential Statement	$\exists x \in D, P(x)$
Unique Existential Statement	$\exists! x \in D, P(x)$
Negation of Universal Statement	$\exists x \in D(\sim P(x))$
Negation of Existential Statement	$\forall x \in D(\sim P(x))$
Negation of UE Statement	$(\forall x \in D, \sim P(x)) \vee (\lvert\{x \in D, P(x)\}\rvert \geq 2)$

Rules of Inference for Quantified Statements		
Name	Premise	Conclusion
Universal Instantiation	$\forall x \in D(P(x))$	$\therefore P(a)$ if $a \in D$
Universal Generalization	$P(a)$ for every $a \in D$	$\therefore \forall x \in D(P(x))$
Existential Instantiation	$\exists x \in D(P(x))$	$\therefore P(a)$ for some $a \in D$
Existential Generalization	$P(a)$ for some $a \in D$	$\therefore \exists x \in D(P(x))$

Direct Proof & Counterexample
Proving existential statements by constructive proof.
An existential statement: $\exists x \in D(Q(x))$ is true iff $Q(x)$ is true for at least one x in D. To prove such statement, we may use constructive proofs of existence: 1) Find an x in D that makes $Q(x)$ true; or 2) Give a set of directions for finding such an x.
Disproving universal statements by counterexample.
Given a universal (conditional) statement: $\forall x \in D(P(x) \to Q(x))$. Showing this statement is false is equivalent to showing that its negation is true. The negation of the above statement is an existential statement: $\exists x \in D(P(x) \wedge \sim Q(x))$. Find a value of x in D for which the hypothesis $P(x)$ is true but the conclusion $Q(x)$ is false. Such an x is called a counterexample.
Proving universal statements by exhaustion.
Given a universal conditional statement: $\forall x \in D\ (P(x) \to Q(x))$. When D is finite or when only a finite number of elements satisfy $P(x)$, we may prove the statement by the method of exhaustion.
Proving universal statements by generalizing from the generic particular (arbitrarily chosen element).
To show that every element of a set satisfies a certain property, suppose x is a particular but arbitrarily chosen element of the set, and show that x satisfies the property.

Indirect Proof
Proof by contradiction
1) Suppose the statement to be proved, S, is false. That is, the negation of the statement, $\sim S$, is true. 2) Show that this supposition leads logically to a contradiction. 3) Conclude that the statement S is true.
Proof by contraposition (Contrapositive of $p \to q$ is $\sim q \to \sim p$)
1) Statement to be proved: $\forall x \in D(P(x) \to Q(x))$. 2) Rewrite the statement into its contrapositive form: $\forall x \in D(\sim Q(x) \to \sim P(x))$. 3) Prove the contrapositive statement by a direct proof. 3.1) Suppose x is an (particular but arbitrarily chosen) element of D s.t. $Q(x)$ is false. 3.2) Show that $P(x)$ is false. 4) Therefore, the original statement $\forall x \in D\ (P(x) \to Q(x))$ is true.

Proven (Methods of Proof)	
n is even	$\exists k \in \mathbb{Z}(n = 2k)$
n is odd	$\exists k \in \mathbb{Z}(n = 2k + 1)$
n is prime	$(n > 1) \wedge \forall r, s \in \mathbb{Z}^+$ $(n = rs \Rightarrow (r = 1 \wedge s = n) \vee (r = n \wedge s = 1))$
n is prime (alt)	$(n > 1) \wedge \left(\forall r, s \in \mathbb{Z}((r > 1) \wedge (s > 1) \to rs \neq n)\right)$
n is prime (alt) (Lec 4 Slide 7)	$(n \neq 1) \wedge \forall y, z \in \mathbb{Z}\left(x = yz \to ((y = x) \vee (y = 1))\right)$
n is composite	$\exists r, s \in \mathbb{Z}^+(n = rs \wedge (1 < r < n) \wedge (1 < s < n))$
n is rational	$\exists a, b \in \mathbb{Z}^+\left(n = \frac{a}{b} \wedge b \neq 0\right)$
$d \mid n\ (d, n \in Z)$	$\exists k \in \mathbb{Z}(n = dk)$

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Lecture 4 Slide 16 (Example #4)	The sum of any two even integers is even.
Lecture 4 Slide 19 (Theorem 4.2.1) (5th: 4.3.1)	Every integer is a rational number.
Lecture 4 Slide 20 (Theorem 4.2.2) (5th: 4.3.2)	The sum of any two rational numbers is rational.
Lecture 4 Slide 21 (Corollary 4.2.3) (5th: 4.2.3)	The double of a rational number is rational. (Corollary: Simple deduction from theorem.)
Lecture 4 Slide 24 (Theorem 4.3.1) (5th: 4.4.1)	For all positive integers, a and b, if $a\|b$, then $a \le b$.
Lecture 4 Slide 25 (Theorem 4.3.2) (5th: 4.4.2)	The only divisors of 1 are 1 and -1.
Lecture 4 Slide 26 (Theorem 4.3.3) (5th: 4.4.3)	For all integers a, b and c, if $a\|b$ and $b\|c$ then $a\|c$.
Lecture 4 Slide 29 (Theorem 4.6.1) (5th: 4.7.1)	There is no greatest integer.
Lecture 4 Slide 32 (Proposition 4.6.4) (5th: 4.7.4)	For all integers n, if n^2 is even then n is even.
Tutorial 1 Q10	The product of any two odd integers is an odd integer.
Tutorial 1 Q11	For all integers n, n^2 is odd iff n is odd.
Tutorial 2 Q11	If n is a product of two positive integers a and b, then $a < n^{1/2}$ or $b < n^{1/2}$.
Tutorial 2 Q4	Rational numbers are closed under addition.
Tutorial 2 Q8	$\forall x \in \mathbb{R}((x^2 > x) \to (x < 0) \vee (x > 1))$

Sets			
Set-Roster Notation	$\{1, 2, 3, \dots\}$		
Set-Builder Notation	$\{x \in U \| P(x)\}$		
Replacement Notation	$\{t(x) \| x \in A\}$		
Membership of Set ($\in$)	$x \in S$: x is an element of S		
Cardinality of Set ($\|S\|$)	$\|S\|$: Size of set S		
$A \subseteq B$	$\forall x\,(x \in A \Rightarrow x \in B)$	$A \subsetneq B$	$A \subseteq B \wedge A \neq B$
$A \not\subseteq B$	$\exists x\,(x \in A \wedge x \notin B)$	$A = B$	$\{\forall x \in A \Leftrightarrow x \in B\}$
$A \times B$	$\{(a, b) \| a \in A \wedge b \in B\}$	$A \setminus B$	$\{x \in U \| x \in A \wedge x \notin B\}$
$A \cup B$	$\{x \in U \| x \in A \vee x \in B\}$	$A \cap B$	$\{x \in U \| x \in A \wedge x \in B\}$
$\bar{A}$	$\{x \in U \| x \notin A\}$	$\mathcal{P}(A)$	$\{S \subseteq U \| S \subseteq A\}$
$\bigcup_{i=0}^{n} A_i$	$\{x \in U \| x \in A_i \text{ for at least one } i = 0, 1, \dots, n\}$		
$\bigcap_{i=0}^{n} A_i$	$\{x \in U \| x \in A_i \text{ for all } i = 0, 1, \dots, n\}$		
Lecture 5 Slide 34 (Cardinality of Power Set of a Finite Set)	Let A be a finite set where $\|A\| = n$, then $\|\mathcal{P}(A)\| = 2^n$		

Lecture 5 Slide 35 (Theorem 6.3.1)	Suppose A is a finite set with n elements, the $\mathcal{P}(A)$ has 2^n elements. In other words, $\|\mathcal{P}(A)\| = 2^{\|A\|}$.
Equality of Ordered n-tuples, $\forall n \in \mathbb{Z}^+$ $(x_1, x_2, \dots, x_n) = (y_1, y_2, \dots, y_n)$	$(x_1 = y_1) \wedge (x_2 = y_2) \wedge \dots \wedge (x_n = y_n)$
Cartesian Product of Sets $A_1, A_2, \dots, A_n$ If A is a set, then	$A_1 \times A_2 \times \dots \times A_n = \{a_1, a_2, \cdots, a_n: a_1 \in A_1 \wedge a_2 \in A_2 \wedge \dots \wedge a_n \in A_n\}$ $A^n = A \times A \times \dots \times A$ (n many A's)
Disjoint Sets	$A \cap B = \emptyset$
Pairwise Disjoint Sets	$A_i \cap A_j = \emptyset$ whenever $i \neq j$
Partition of Set	$\{A_1, A_2, \dots, A_n\}$ where $A_1, A_2, \dots, A_n$ are mutually disjoint subsets of A and $\bigcup_{i=0}^{n} A_i = A$

Theorem 6.2.1 (Some Subset Relations) *For all sets A, B and C			
Inclusion of Intersection	(a) $A \cap B \subseteq A$ (b) $A \cap B \subseteq B$		
Inclusion in Union	(a) $A \subseteq A \cup B$ (b) $B \subseteq A \cup B$		
Transitive Property of Subsets	$A \subseteq B \wedge B \subseteq C \to A \subseteq C$		
$a \in X \cup Y$	$a \in X \vee a \in Y$	$a \in X \cap Y$	$a \in X \wedge a \in Y$
$a \in X \setminus Y$	$a \in X \wedge a \notin Y$	$a \in \bar{X}$	$a \notin X$
$(a, b) \in X \times Y$	$a \in X \wedge b \in Y$		

Theorem 6.2.2 Set Identities *For all sets A, B and C		
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	
Identity Laws	$A \cup \emptyset = A$	$A \cap U = A$
Complement Laws	$A \cup \bar{A} = U$	$A \cap \bar{A} = \emptyset$
Double Complement Laws	$\bar{\bar{A}} = A$	
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Universal Bounds Laws	$A \cup U = U$	$A \cap \emptyset = \emptyset$
De Morgan's Laws	$\overline{A \cup B} = \bar{A} \cap \bar{B}$	$\overline{A \cap B} = \bar{A} \cup \bar{B}$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complements of U and $\emptyset$	$\bar{U} = \emptyset$	$\bar{\emptyset} = U$
Set Difference Law	$A \setminus B = A \cap \bar{B}$	

* Note that Logical Equivalence and Set Properties are similar, as they are special cases of the same general structure Boolean Algebra

Proven (Sets)	
Lecture 5 Slide 22	$\{x \in \mathbb{Z} \| x^2 = 1\} = \{1, -1\}$
Lecture 5 Slide 31 (Theorem 4.4.1)	Quotient-Remainder Theorem $\forall n \in \mathbb{Z}, d \in \mathbb{Z}^+ (\exists!\, q, r \in \mathbb{Z}\ ((n = dq + r) \wedge (0 \le r < d)))$
Lecture 5 Slide 46	$(A \cap B) \cup (A \setminus B) = A$ for all sets A, B
Tutorial 3 Qn 3	Let $\|A\| = n$, $\|B\| = k$, then $\|\mathcal{P}(A \times B)\| = 2^{nk}$
Tutorial 3 Qn 5	$A \cap B \setminus C = (A \cap B) \setminus C$
Tutorial 3 Qn 6	$A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$
Tutorial 3 Qn 7	$A \otimes B = (A \setminus B) \cup (B \setminus A)$
Tutorial 3 Qn 9	$A \otimes B = (A \cup B) \setminus (A \cap B)$

Tutorial 3 Qn 8	$A \subseteq B \Leftrightarrow A \cup B = B$

Relations	
Relation (binary) xRy "x is R-related to y"	$\forall (x, y) \in A \times B (xRy \Leftrightarrow (x, y) \in R)$ $\forall (x, y) \in A \times B (x\not{R}y \Leftrightarrow (x, y) \notin R)$
Domain of R: $Dom(R)$	$\{a \in A \| aRb \text{ for some } b \in B\}$
Co-Domain of R: $coDom(R)$	B
Range of R: $Range(R)$	$\{b \in B \| aRb \text{ for some } a \in A\}$
Inverse Relation: R^{-1}	$\{(y, x) \in B \times A: (x, y) \in R\}$ $\forall x \in A, \forall y \in B ((y, x) \in R^{-1} \Leftrightarrow (x, y) \in R)$
"Divides" Relations: |	$\{\forall (d, n) \in \mathbb{Z} \times \mathbb{Z} \| (d\|n \Leftrightarrow \exists k \in \mathbb{Z}(n = dk))\}$
Relation on a set A	$\forall (a_1, a_b) \in A \times A$
A^n	$A^n = A \times A \times \dots \times A$ (n times)
Composition of R with S $R \subseteq A \times B, S \subseteq B \times C$	$\forall x \in A, \forall z \in C$ $(xS \circ Rz \Leftrightarrow (\exists y \in B(xRy \wedge ySz)))$
Composition is Associative	$T \circ (S \circ R) = (T \circ S) \circ R = T \circ S \circ R$
Inverse of Composition	$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Properties of Relations (Relation R on set A)		
Reflexive	$\forall x \in A(xRx)$	
Symmetric	$\forall x, y \in A(xRy \Rightarrow yRx)$	
Transitive	$\forall x, y, z \in A(xRy \wedge yRz \Rightarrow xRz)$	
Equivalence (~)	Reflexive, Symmetric, Transitive	
Class of a ($[a]$)	$[a]_\sim = \{x \in A \| a \sim x\}$	$\forall x \in A(x \in [a]_\sim \Leftrightarrow a \sim x)$
Quotient of set A by ~	Set of all equivalence classes wrt ~ $A/\sim = \{[x]_\sim : x \in A\}$	
Antisymmetric	$\forall x, y \in A(xRy \wedge yRx \Rightarrow x = y)$	
~ Antisymmetric	$\exists x, y \in A(xRy \wedge yRx \wedge x \neq y)$	
Asymmetric	$\forall x, y \in A(xRy \Rightarrow y\not{R}x)$	
Tutorial 4 Qn 8	Every Asymmetric relation is Antisymmetric	
Partial Order	Reflexive, Antisymmetric, Transitive $((\mathbb{R}, \le), (U, \subseteq))$ $((\mathbb{Z}^+, \|)$: Proven)	

Transitive Closure of R: R^t		
Relation obtained with least ordered pairs added to ensure transitivity in relation.		
Transitive	$R \subseteq R^t$	$\forall S \in U((S \text{ is transitive}) \wedge (R \subseteq S) \to R^t \subseteq S)$

Reflexive Closure of R (Tutorial 4 Qn 7)		
The smallest relation on A that is reflexive and contains R as a subset. $\forall x, y \in A(xSy \Leftrightarrow (x = y) \vee (xRy))$		
Reflexive	$R \subseteq S$	$\forall S' \in U((S' \text{is reflexive}) \wedge (R \subseteq S') \to S \subseteq S')$

Partition of a set A
C is a partition of a set A if the following hold: (1) C is a set of which all elements are non-empty subsets of A, i.e., $\emptyset \neq S \subseteq A$ for all $S \in$ C. (2) Every element of A is in exactly one element of C, i.e., $\forall x \in A\ \exists S \in$ C $(x \in S)$ and

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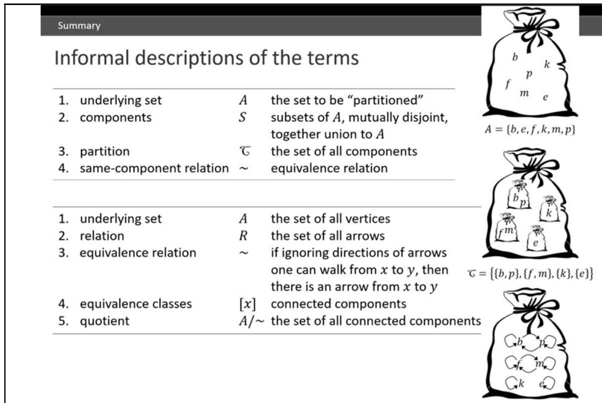
$\forall x \in A\ \forall S1, S2 \in C\ (x \in S1 \wedge x \in S2 \Rightarrow S1 = S2)$.	
Partition Definition	$\forall x \in A\ \exists! S \in C(x \in S)$
Induced Relation by partition	$\forall x, y \in A(xRy \Leftrightarrow \exists S \in C(x, y \in S))$ R is reflexive, symmetric, and transitive (proven)

Partial Ordered Set (Poset)	
A set A is called a partially ordered set (or poset) with respect to a partial order relation R on A, denoted by (A, R).	
Lecture 6 Slide 71 (Notation $\preccurlyeq$)	Because of the special paradigmatic role played by the ≤ relation in the study of partial order relations, the symbol $\preccurlyeq$ is often used to refer to a general partial order, and the notation $x \preccurlyeq y$ is read "x is curly less than or equal to y".
Hasse Diagram	Let $\preccurlyeq$ be a partial order on a set A. A Hasse diagram of $\preccurlyeq$ satisfies the following condition for all distinct $x, y, m \in A$: If $x \preccurlyeq y$ and no $m \in A$ is such that $x \preccurlyeq m \preccurlyeq y$, then x is placed below y with a line joining them, else no line joins x and y.
x, y comparable	$(x \preccurlyeq y) \vee (y \preccurlyeq x)$
x, y noncomparable	$\sim(x \preccurlyeq y) \wedge \sim(y \preccurlyeq x)$
x, y compatible	$\exists z \in A((x \preccurlyeq z) \wedge (y \preccurlyeq z))$
Tutorial 5 Qn 11	Any two comparable elements are compatible. Any two compatible elements are not always comparable. Eg $(\mathbb{Z}^+, \vert)$: 2,3
Maximal Element	$\forall x \in A(c \preccurlyeq x \Rightarrow c = x)$
Minimal Element	$\forall x \in A(x \preccurlyeq c \Rightarrow c = x)$
Largest Element	$\forall x \in A(x \preccurlyeq c)$
Smallest Element	$\forall x \in A(c \preccurlyeq x)$
Largest Element = Greatest Element = Maximum Smallest Element = Least Element = Minimum	
Chain	$(C \subseteq A) \wedge (\forall x, y \in C(x \preccurlyeq y \vee\ y \preccurlyeq x))$
Maximal Chain	$\text{Chain}(C) \wedge (t \notin C \Rightarrow \sim\text{Chain}(C) \cup \{t\})$
Total Order Relation	R is a partial order and $\forall x, y \in A(xRy \vee yRx)$
Linearization	Let $\preccurlyeq$ be a partial order on a set A. A linearization of $\preccurlyeq$ is a total order $\preccurlyeq*$ on A: $\forall x, y \in A(x \preccurlyeq y \Rightarrow x \preccurlyeq^* y)$
Well-Ordered Set	$\forall S \in \mathcal{P}(A), S \neq \emptyset \Rightarrow (\exists x \in S \forall y \in S(x \preccurlyeq y))$ Ie Every non-empty subset of A has a smallest element. Eg $(\mathbb{N}, \leq)$ is well-ordered, $(\mathbb{Z}, \leq)$ is not.

Kahn's Algorithm to finding Linearization on a partial order set
Input: A finite set A and a partial order $\preccurlyeq$ on A. 1. Set $A_0 \coloneqq A$ and $i \coloneqq 0$. 2. Repeat until $A_i = \emptyset$ 2.1. find a minimal element c_i of A_i wrt $\preccurlyeq$ 2.2. set $A_i + 1 = A_i \setminus \{c_i\}$ 2.3. set $i \coloneqq i + 1$
Output: A linearization $\preccurlyeq*$ of $\preccurlyeq$ defined by setting, for all indices i, j, $c_i \preccurlyeq* c_j \Leftrightarrow i \leq j$.

Proven (Relations)	
Tutorial 4 Qn 2	(i) R is symmetric, ie $\forall x, y \in A(xRy \Rightarrow yRx)$ (ii) $\forall x, y \in A(xRy \Leftrightarrow yRx)$ (iii) $R = R^{-1}$
Tutorial 4 Qn 5	(i) R is an equivalence relation (ii) $R^{-1} \circ R = R \circ R^{-1}$ (iii) $R \subseteq R \circ R$ (iv) $R \circ R \subseteq R$ (v) $R \circ R^{-1} = R$
Tutorial 4 Qn 6	R is an equivalence relation $\Leftrightarrow\ R \circ R = R$
Tutorial 4 Qn 9	$S = \{(m, n) \in \mathbb{Z}^2 \colon \mathrm{m}^3 + \mathrm{n}^3\}$ $S^{-1} = S$ $S \circ S = S$ $S \circ S^{-1} = S$
Tutorial 4 Qn 10	$\forall a, b \in \mathbb{Z} \setminus \{0\}(\mathrm{a} \sim \mathrm{b} \Leftrightarrow \mathrm{ab} > 0)$ $\sim$ is an equivalence relation.
Lecture 6 Slide 27 (Example #12)	Congruence module 3 defined as: $\forall x, y \in z(xRy \Leftrightarrow 3 \vert (x - y))$ is reflexive, symmetric and transitive.
Lecture 6 Slide 39 (Theorem 8.3.1)	Relation Induced by a Partition is reflexive, symmetric and transitive.
Lecture 6 Slide 47 (Lemma Rel.1 Equivalence classes)	(i) $x \sim y$ equivalent $\forall x, y \in A$ (ii) $[x] = [y]$ (iii) $[x] \cap [y] \neq \emptyset$
Lecture 6 Slide 50 (Theorem 8.3.4)	If A is a set and R is an equivalence relation on A, then the distinct equivalence classes of R form a partition of A; that is, the union of the equivalence classes is A, and the intersection of any two distinct classes is empty.
Lecture 6 Slide 52	Congruence module n *relation*: $\forall x, y \in \mathbb{Z}\ (xRy \Leftrightarrow n \vert (x - y))$ iff $a \equiv b (\mathrm{mod}\ n)$
Lecture 6 Slide 54 (Proposition)	Congruence-mod n is an equivalence relation on $\mathbb{Z}$ for every $n \in \mathbb{Z}^+$
Lecture 6 Slide 57 (Theorem Rel.2)	Equivalence classes for a partition. ie $A/\sim$ is a partition of A.



Lecture 6 Slide 69 (Example #20)	$\vert$ is a partial order relation on $A \in \mathbb{Z}^+$
Lecture 6 Slide 83	Consider a partial order $\preccurlyeq$ on a set A. Any smallest element is minimal. Likewise, any largest element is maximal.