MA1521 Summary 24/25

Teh Xu An

Real Numbers and Functions	
Absolute	$\lvert xy\rvert = \lvert x\rvert\lvert y\rvert$, $\lvert x+y\rvert \le \lvert x\rvert + \lvert y\rvert$, $\lvert\lvert x\rvert - \lvert y\rvert\rvert \le \lvert x-y\rvert$
Function $f: A \to B$	Domain: A, Codomain: B Range: $\{f(x) \in B \mid x \in A\}$
Compose	$g \circ f(x) = g(f(x))$
Bijection	Inj: 1 x to 1 y +, Sur: all y has a x f^{-1} exists iff f is a bijective function
Polynomial	$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$
Partial Fraction	$\frac{P(x)}{Q(x)} = \begin{cases} \frac{A}{ax+b}, & ax+b \\ \frac{A}{ax+b} + \frac{B}{(ax+b)^2}, & (ax+b)^2 \\ \frac{Ax+B}{ax^2+bx+c}, & ax^2+bx+c, b^2-4ac<0 \end{cases}$
Root of $p(x)$	$p(root) = 0$
Rational Fun	$f(x) = p(x)/q(x)$, where $q(x) \neq 0$
Trig Fun	sin, cos, tan, csc, sec, cot
Exp/Log Fun	Exp a: $f(x) = a^x, a > 0$, Log base a: $f(x) = log_a x$

Limits and Continuity	
Limits	$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = \lim_{x\to c} f(x)$
Continuity	f is continuous in interval I iff $\lim_{x\to c} f(x)$ exists $\forall c \in I$
Useful Results	$\lim_{x\to c}(f(x) \pm g(x)) = lim_{x\to c}f(X) \pm lim_{x\to c}g(x)$
	$\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$, where k is a constant
	$\lim_{x\to c} f(x)g(x) = \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right)$
	$\lim_{x\to c} f(x)/g(x) = \lim_{x\to c} f(x) / \lim_{x\to c} g(x)$
Asymptote	$y = c \in R$, where $\lim_{x\to\infty} f(x) = c$ or $\lim_{x\to\infty} f(x) = c$
Indeterminate	$\lim_{x\to c} \mathrm{f(x)/g(x)} = 0/0$ or ∞/∞
Limit of Poly at Infinity	$\lim_{x\to\pm\infty} \frac{P(x)}{Q(x)} = \lim_{x\to\pm\infty} \frac{Ax^\alpha+\ldots}{Bx^\beta+\ldots} = \begin{cases} 0, \alpha<\beta \\ A/B, \alpha=\beta \\ \infty\, or -\infty, \alpha>\beta \end{cases}$
Useful Results	$\lim_{x\to c} g(x) = 0 \Rightarrow \lim_{x\to c} \frac{\sin(g(x))}{g(x)} = \lim_{x\to c} \frac{g(x)}{\sin(g(x))} = 1$ $\lim_{x\to c} g(x) = 0 \Rightarrow \lim_{x\to c} \frac{\tan(g(x))}{g(x)} = \lim_{x\to c} \frac{g(x)}{\tan(g(x))} = 1$
Squeeze Theorem	$\forall x \in I, g(x) \le f(x) \le h(x))$ $(\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L) \Rightarrow \lim_{x\to c} f(x) = L$
Intermediate Value Thm	$f$continuous on $[a,b]) \wedge (k$ between $f(a)$ and $f(b))$ $\Rightarrow \exists c \in [a,b](f(c) = k)$
Precise Limit	$\lim_{x\to c} f(x) = L \Leftrightarrow \forall\epsilon \in \mathrm{R}^+ \exists\delta \in \mathrm{R}^+$ $st\ 0 < \lvert x-c\rvert < \delta \Rightarrow \lvert f(x) - L\rvert < \epsilon$

Differentiability			
Derivative	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$		
Thm 2.1	Differentiability implies continuity		
Rules of Differentiation			
Constant	$\frac{d}{dx}(c) = 0$	Const Multi	$\frac{d}{dx}(cu) = c\frac{du}{dx}$
Sum	$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$	Product	$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$

Chain	$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$	Quotient	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$
Implicit	$\frac{d}{dx}g(y) = g'(y)\frac{dy}{dx}$	Inv Diff	$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$
Parametric Equation	$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}, \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{y'(t)}{g'(t)}\right)}{x'(t)} = \frac{x''(t)y'(t) - x'(t)y''(t)}{y'(t)^3}, \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$		
Eclipse	$x = a\cos t + x_0$ and $y = b\sin t + y_0$ where $a, b > 0$		
Circles	$x = r\cos t + x_0$ and $y = r\sin t + y_0$, where $r > 0$		
Hyperbola	$x = a\sec t + x_0$ and $y = b\tan t + y_0$, where $a, b > 0$ $x = a\tan t + x_0$ and $y = b\sec t + y_0$, where $a, b > 0$		
$y = f(x)^{g(x)}$	$\frac{d}{dx}ln(f(x)^{g(x)}) = \frac{1}{y}\frac{dy}{dx} = f'(x) \cdot g(x) + f(x) \cdot g'(x)$		
Change base	$log_a x = \ln x / \ln a$, $a > 0$ and $a \neq 1$		

Applications of Differentiation	
Graph Tangent	$y = f(x) \Rightarrow y - f(x_0) = m(x - x_0)$ $y(t), x(t) \Rightarrow y - y(t_0) = m(x - x(t_0))$
Graph Normal	$y = f(x) \Rightarrow y - f(x_0) = -(1/m)(x - x_0)$ $y(t), x(t) \Rightarrow y - f(t_0) = -(1/m)(x - x(t_0))$
Theorem 3.1	f is increasing on $[a,b]$ if $\forall x \in (a,b)\ (f'(x) > 0)$ f is decreasing on $[a,b]$ if $\forall x \in (a,b)\ (f'(x) < 0)$
Theorem 3.2	f concaved upwards on $(c, f(c))$ if $f''(c) > 0$ f concaved downwards on $(c, f(c))$ if $f''(c) < 0$
Inflection Point	$(c, f(c))$ is inflection point and $f(c)\exists \Rightarrow f'(c) = 0$
Extreme Value Theorem	f is continuous on $[a,b]$ then f has an absolute max and absolute min at some point in $[a,b]$
Critical Point	Not end-point and $f'(c) = 0$ or $f'(c)$ does not exist
First Derivative Test	Abs Max @c: $(\forall x < c(f'(x) > 0)) \wedge (\forall x < c(f'(x) < 0))$ Abs Min @c: $(\forall x < c(f'(x) < 0)) \wedge (\forall x < c(f'(x) > 0))$ Loc Max @c: f' changes $+$ve to $-$ve at $x = c$ Loc Min @c: f' changes $-$ve to $+$ve at $x = c$
Second Derivative Test	Loc Max @c: $f'(c) = 0$ and $f''(c) < 0$ Loc Min @c: $f'(c) = 0$ and $f''(c) > 0$ No conclusion can be drawn if $f''(c) = 0$
L'Hopital's Rule	$\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$ or $\pm\infty \Rightarrow \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$
Rolle's Theorem	f continuous on $[a,b]$ & $f(a) = f(b) \Rightarrow \exists c \in (a,b)\ (f'(c) = 0)$
Mean Value Theorem	f continuous on $[a,b]$ and differentiable on (a,b) $\Rightarrow \exists c \in (a,b)\left(f'(c) = \frac{f(b)-f(a)}{b-a}\right)$

Integrals		
Antiderivative	$\forall x \in I(F'(x) = f(x)) \Rightarrow F$ antiderivative of f on I	
Commutative	$\int \alpha f(x) + \beta g(x)dx = \alpha\int f(x)dx + \beta\int g(x)dx$	
Substitution	$\int f(g(x))g'(x) = \int f(u)du$, where $u = g(x)$	
By Parts	$\int f'(x)g(x)dx = f(x) \cdot g(x) - \int f(x)g'(x)dx$	
Rule of Thumbs		
Algebraic Func	Power functions x^a, Polynomials	Differentiate
Trig Func	sin,cos,$\tan^{-1}(ax+b)$	Differentiate
	csc,sec,$\cot^{-1}(ax+b)$	Integrate
Inv Trig Func	$\sin^{-1}, \cos^{-1}, \tan^{-1}(ax+b)$	Differentiate
Exp Func	e^{ax+b}	Integrate
Log Func	$\ln(ax+b)^n, n > 0$	Differentiate

Reimann Sums			
Definite	$\int_a^b f(x)dx$	Reimann Sum	$\sum_{k=1}^n \left(\frac{b-a}{n}\right) f\left(a + k\left(\frac{b-a}{n}\right)\right)$
Definite Integral	$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^n \left(\frac{b-a}{n}\right) f\left(a + k\left(\frac{b-a}{n}\right)\right)$		
	$\int_a^b f(x)dx = \sum_{k=1}^n \left(\frac{b-a}{n}\right) f\left(a + k\left(\frac{b-a}{n}\right)\right)$, @ large n		
FTC 1	$\int_b^a F'(x)dx = F(b) - F(a)$	FTC 2	$\frac{d}{dx}\int_a^x f(t)dt = f(x)$
Improper Type I	$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$		$\int_{-\infty}^a f(x)dx = \lim_{b\to-\infty}\int_b^a f(x)dx$
	$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^\infty f(x)dx$		
Improper Type II	$\int_a^b f(x)dx = \lim_{c\to a^+}\int_c^b f(x)dx$		$\int_a^b f(x)dx = \lim_{c\to b^-}\int_a^c f(x)dx$
	$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$		

Applications of Integration	
Area	$A = \int_a^b \lvert y_f(x) - y_g(x)\rvert dx = \int_c^d \lvert x_f(y) - x_g(y)\rvert dy$
Volume Revolution	$V = \pi\int_a^b \lvert y_f(x)^2 - y_g(x)^2\rvert dx = \pi\int_c^d \lvert x_f(y)^2 - x_g(y)^2\rvert dy$ $V = 2\pi\int_a^b x\lvert y_f(x) - y_g(x)\rvert dx = 2\pi\int_c^d y\lvert x_f(y) - x_g(y)\rvert dy$
Arc Length	$l = \int_a^b \sqrt{1 + y_f'(x)^2}dx$, $l = \int_c^d \sqrt{1 + x_f'(y)^2}dy$

Sequences and Series					
Infinite Seq	$D_f = Z^+$	Converge	$\lim_{n\to\infty} a_n = L$	Diverge	$\lim_{n\to\infty} a_n = \pm\infty$

Limit Laws for Sequences	
$\lim_{n\to\infty} ca_n = c\lim_{n\to\infty} a_n$	$\lim_{n\to\infty} a_n \pm b_n = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$
$\lim_{n\to\infty} a_n b_n = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$	$\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$, $if \lim_{n\to\infty} b_n \neq 0$
Squeeze Thm	$\forall n\ a_n \le b_n \le c_n$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L \Rightarrow \lim_{n\to\infty} b_n = L$

Series	
Geometric	Form: $\sum_{n=1}^\infty ar^{n-1}, (a \neq 0) \Rightarrow \sum_{n=1}^\infty ar^{n-1} = \frac{a}{1-r}, \lvert r\rvert < 1$
Theorem 6.3	$\sum_{n=1}^\infty a_n$and $\sum_{n=1}^\infty b_n$ are convergent $\Rightarrow \sum_{n=1}^\infty c_a a_n + c_b b_n = c_a\sum_{n=1}^\infty a_n + c_b\sum_{n=1}^\infty b_n$
Lemma 6.4	$\sum_{n=1}^\infty a_n$ is convergent $\Rightarrow \lim_{n\to\infty} a_n = 0$
n[th] Term Test	$\lim_{n\to\infty} a_n$ not exist or $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^\infty a_n$ is divergent
Theorem 6.6	$\sum_{n=1}^\infty a_n, a_n \ge 0$ converges $\Leftrightarrow \exists K\forall n(S_n < K)$
Integral Test	$convergence(\int_1^\infty f(x)dx) \Leftrightarrow convergence(\sum_{n=1}^\infty a_n)$
p-series	Form: $\sum_{n=1}^\infty \frac{1}{n}, \sum_{n=1}^\infty \frac{1}{n^p}$ is convergent $\Leftrightarrow p > 1$
Comparison Test	$\sum_{n=1}^\infty a_n(a_n \ge 0)$ and $\sum_{n=1}^\infty b_n(b_n \ge 0)$ $(0 \le a_n \le b_n \forall n)$ $\Rightarrow convergence(\sum_{n=1}^\infty b_n) \Rightarrow convergence(\sum_{n=1}^\infty a_n)$
Ratio Test	$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = L \begin{cases} \sum_{n=1}^\infty \lvert a_n\rvert \text{ is convergent}, 0 \le L < 1 \\ \sum_{n=1}^\infty a_n \text{ is divergent}, L > 1 \\ \text{inclonclusive}, L = 1 \end{cases}$
Root Test	$\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert} = L \begin{cases} \sum_{n=1}^\infty \lvert a_n\rvert \text{ is convergent}, 0 \le L < 1 \\ \sum_{n=1}^\infty a_n \text{ is divergent}, L > 1 \\ \text{inclonclusive}, L = 1 \end{cases}$

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Alternating	$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$	Harmonic	$\sum_{n=1}^{\infty}\left\lvert(-1)^{n-1}\frac{1}{n}\right\rvert = \sum_{n=1}^{\infty}\frac{1}{n}$
Alternating Series Test	b_n decreasing (ie $b_n \ge b_{n+1}$ and $\lim_{n\to\infty} b_n = 0$) $\Rightarrow \sum_{n=1}^{\infty}(-1)^{n-1}b_n$ and $\sum_{n=1}^{\infty}(-1)^{n}b_n$ are convergent		
Abs Conv	$convergence(\sum_{n=1}^{\infty}\lvert a_n\rvert) \Rightarrow convergence(\sum_{n=1}^{\infty} a_n)$		
Cond Conv	$convergence(\sum_{n=1}^{\infty} a_n)$ and $divergence(\sum_{n=1}^{\infty}\lvert a_n\rvert)$		
Power Series	Form: $\sum_{n=0}^{\infty} c_n(x-a)^n$		
Power Series Convergence	$\begin{cases}\text{converges at } x=1\\ \text{converges at all } x\end{cases}$	$\begin{cases}\text{converges abs at } \lvert x-a\rvert<R\\ \text{diverges at } \lvert x-a\rvert>R\end{cases}$	
Interval of Convergence $I=(a\pm R)$	$\sum_{n=0}^{\infty} c_n(x-a)^n$, $c_n \neq 0$ for all n, $\left(\left.\begin{matrix}\lim_{n\to\infty}\left\lvert\frac{c_{n+1}}{c_n}\right\rvert\\ \lim_{n\to\infty}\sqrt[n]{\lvert c_n\rvert}\end{matrix}\right\} = L\begin{cases}real\\ \infty\end{cases}\right) \Rightarrow R=\frac{1}{L}$		
Power Series Representat^n	$\sum_{n=0}^{\infty} c_n(x-a)^n = f(x)$, $R>0$ $\begin{cases}f'(x)=\sum_{n=1}^{\infty} nc_n(x-a)^{n-1}\\ \int f(x)dx=\sum_{n=0}^{\infty} c_n\frac{(x-a)^{n+1}}{n+1}\end{cases}, \lvert x-a\rvert<R$		
Taylor	$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$	Maclaurin	$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$
Precise Seq	$\forall\epsilon\in R^+\exists N\in Z\forall n(n>N\Rightarrow\lvert a_n-L\rvert<\epsilon)\Rightarrow\lim_{n\to\infty}a_n=L$		

Vectors and Geometry of Space			
Distance (P_1 to P_2)	$\lvert P_1P_2\rvert=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$		
Eq of Sphere	$(x-h)^2+(y-k)^2+(z-l)^2=r^2$		
Vector $\vec{v}/v$	$v=\langle v_1,v_2,v_3\rangle$, where v_1,v_2,v_3 are components of v		
Vector $\overrightarrow{AB}$	$\overrightarrow{AB}=\langle x_B-x_A,y_B-y_A,z_B-z_A\rangle$		
Length $\Vert v\Vert$	$\Vert v\Vert=\sqrt{v_1^2+v_2^2+v_3^2}$	Unit Vector u	$u=\frac{v}{\Vert v\Vert}, \Vert u\Vert=1$
Dot Product	$a\cdot b=a_1b_1+a_2b_2+a_3b_3$		
Angle θ	$a\cdot b=\Vert a\Vert\Vert b\Vert\cos\theta$	$\Vert a\times b\Vert=\Vert a\Vert\Vert b\Vert\sin\theta$	
Orthogonal	$Orthogonal(a,b)\Leftrightarrow a\cdot b=0$		
Scalar Proj	Scalar Projection of b onto a, $comp_a b=\Vert b\Vert\cos\theta=\frac{a\cdot b}{\Vert a\Vert}$		
Vector Proj	Vector Projection of b onto a, $proj_a b=comp_a b\times\frac{a}{\Vert a\Vert}=\frac{a\cdot b}{a\cdot a}a$		
Distance (P to plane)	$P(x_P,y_P,z_P)$ to $ax+by+cz=d$: $\frac{\lvert ax_P+by_P+cz_P-d\rvert}{\sqrt{a^2+b^2+c^2}}$		
Cross Product $(a\times b)\cdot a=0$	$\boldsymbol{a}\times\boldsymbol{b}=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}=(a_2b_3-a_3b_2)\boldsymbol{i}-(a_1b_3-a_3b_1)\boldsymbol{j}+(a_1b_2-a_2b_1)\boldsymbol{k}$		
Para Eq of Line	$x=x_0+at, y=y_0+bt, z=z_0+ct$	Skew: Non-Parallel	
Vec Eq of Plane	$n\cdot(r-r_0=0), n=\langle a,b,c\rangle, r=\langle x,y,z\rangle, r_0=\langle x_0,y_0,z_0\rangle$		
Lin Eq of Plane	$ax+by+cz+d=0$, where $d=-(ax_0+by_0+cz_0)$		

Functions of Several Variables			
Vector-valued Function	$r(t)=f(t)i+g(t)j+h(t)k=\langle f(t),g(t),h(t)\rangle$		
Component Function	The scalar functions f,g,h in r		
Derivative of r	$r'(a)=\langle f'(a),g'(a),h'(a)\rangle$ whr f',g',h' exists at $t=a$		
Arc Length of s	$s=\int_a^b\sqrt{f'(t)^2+g'(t)^2+h'(t)^2}dt=\int_a^b\Vert r'(t)\Vert dt$		
Function of 2 Var	$f:R^2\to R\Rightarrow f(x,y)$		
Level Curve	$f(x,y)=$ constant k	Contour Plot	Graph of n level curves
Cylinder	$\exists P$ st $\forall p_\parallel(P\cap p_\parallel=C)$, for some curve C		
Quadric Surface (2[nd] Degree)	$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$, where $A,B,\ldots,J$ are constants		
Elliptic Paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$, where $c>0$ for symmetry about z-axis		

Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ where at $a=b=c$ gives a sphere		
Double Cone	$\frac{z^2}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Hyperboloid of 2 Sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperbolic Paraboloid	$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Hyperboloid of 1 Sheets	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Fun of 3 Var	$f:R^3\to R\Rightarrow f(x,y,z)$	Level Surface	$f(x,y,z)=$ constant k
Partial Derivative	$f_x(x,y)=\lim_{h\to0}\frac{f(x+h,y)-f(x,y)}{h}, f_y(x,y)=\lim_{h\to0}\frac{f(x,y+h)-f(x,y)}{h}$		
Higher Order PD	$f_{xx}=(f_x)_x=\frac{\partial^2f}{\partial x^2}, f_{yy}=(f_y)_y=\frac{\partial^2f}{\partial y^2}$ $f_{xy}=(f_x)_y=\frac{\partial^2f}{\partial y\partial x}=f_{yx}=(f_y)_x=\frac{\partial^2f}{\partial x\partial y},$		
Normal Vector	$@(a,b,f(a,b))\Rightarrow\langle f_x(a,b),f_y(a,b),-1\rangle$		
Tangent Plane	$f_x(a,b)(x-a)+f_y(a,b)(y-b)-(z-f(a,b))=0$ $z=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$		
Chain Rule for $z=f(x,y)$	$\mathrm{x=g(t),y=h(t)}\Rightarrow\frac{dz}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}$ $x=g(s,t),y=h(s,t)\Rightarrow\frac{\partial z}{\partial(s/t)}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial(s/t)}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial(s/t)}$		
Chain Rule General	$u=f(x_1,\ldots,x_n),x_i(t_1,\ldots,t_m)\Rightarrow\frac{\partial u}{\partial t_i}=\frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i}+\cdots+\frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}$		
Implicit Diff, 2 Independent Var	$F_z(x,y,z)\neq0\Rightarrow\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)},\frac{\partial z}{\partial y}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}$		
Increment in z	$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$		
Differential	$dx=\Delta x,dy=\Delta y,dz=f_x(x,y)dx+f_y(x,y)dy$		
Theorem 8.10	$\Delta x,\Delta y$ small $\Rightarrow\Delta z\approx f_x(a,b)dx+f_y(a,b)dy=f_x(a,b)\Delta x+f_y(a,b)\Delta y$		
Directional Diff	$D_vf(\mathrm{x_0,y_0,z_0})=\nabla\mathrm{f}(\mathrm{x_0,y_0,z_0})\cdot\frac{\mathrm{v}}{\Vert\mathrm{v}\Vert},\nabla f=\langle f_x,f_y,f_z\rangle$		
Directional Diff (max,min)	$D_vf=\Vert\nabla f\Vert\cos\theta\therefore D_vf_{xtrm}=\pm\Vert\nabla f\Vert$		
Level Curve & ∇f	$\nabla f(x_0,y_0)\neq0\Rightarrow\nabla f(x_0,y_0)\perp f(x,y)=k$ at point $(x_0,y_0),f(x_0,y_0)=k$		
Level Surface & ∇f	$F(x,y,z)=k,r(t_0)=\langle x_0,y_0,z_0\rangle\Rightarrow\nabla F(x_0,y_0,z_0)\cdot r'(t_0)=0$		
Tangent Plane to Level Surface	$\nabla F(x_0,y_0,z_0)\cdot\langle x-x_0,y-y_0,z-z_0\rangle=0$ $F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0$		
Critical Point	$f_x(a,b)=f_y(a,b)=0$		
Saddle Point @ $P(a,b)$	$CriticalPoint(P(a,b))\land\forall$ Open Disk D $(\exists(x,y)\in D(f(x,y)<f(a,b))\land\exists(x,y)\in D(f(x,y)>f(a,b)))$		
Discriminant	$D=D(a,b)=f_{xx}(a,b)f_{yy}(a,b)-[f_{xy}(a,b)]^2$		
Second Derivative Test	$CriticalPoint(a,b)=\begin{cases}\text{Local Min,} & D>0,f_{xx}(a,b)>0\\ \text{Local Max,} & D>0,f_{xx}(a,b)<0\\ \text{Saddle Point,} & D<0\\ \text{Inconclusive,} & D=0\end{cases}$		

Double Integrals	
Reimann Sum	$\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i^*)\Delta x=\int_a^bf(x)dx, \Delta x=\frac{b-a}{n}$
Volume	$V=\lim_{m,n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n}f(x_{ij}^*,y_{ij}^*)\Delta A=\int\int_Rf(x,y)dA$
Iterated Integral	$A(x)=\int_c^df(x,y)dy\Rightarrow\int_a^bA(x)dx=\int_a^b\left[\int_c^bf(x,y)dy\right]dx$
Fubini's Theorem	$\int_a^b\int_c^df(x,y)dydx=\int_c^d\int_a^bf(x,y)dxdy$ $\int\int_Rg(x)h(y)dA=\left(\int_a^bg(x)dx\right)\left(\int_c^dh(y)dy\right)$
Type I Domain Double Integral	$D=\{(x,y):a\le x\le b,g_1(x)\le y\le g_2(x)\}$ $\Rightarrow\int\int_Df(x,y)dA=\int_a^b\int_{g_1(x)}^{g_2(x)}f(x,y)dydx$
Type II Domain Double Integral	$D=\{(x,y):c\le y\le d,h_1(y)\le x\le h_2(y)\}$ $\Rightarrow\int\int_D\mathrm{f(x,y)dA}=\int_c^d\int_{h_1(y)}^{h_2(y)}\mathrm{f(x,y)dxdy}$
Rect to Polar Coord	$r^2=x^2+y^2,x=r\cos\theta,y=r\sin\theta$
Polar Rectangle	$R:\{(r,\theta):a\le r\le b,\alpha\le\theta\le\beta\}$
Polar Coord DI	$\int\int_Rf(x,y)dA=\int_\alpha^\beta\int_a^bf(r\cos\theta,r\sin\theta)rdrd\theta$
Polar Coord SA	Surface Area $=\int\int_DdS=\int\int_D\sqrt{f_x^2+f_y^2+1}\,dA$

Ordinary Differential Equations (ODE)	
First Order ODE	$\frac{dy}{dx}=f(x)g(y)\Rightarrow\frac{1}{g(y)}dy=f(x)dx\Rightarrow\int\frac{1}{g(y)}dy=\int f(x)dx+C$
Reduction to Separable Form	$y'=g\left(\frac{y}{x}\right),v=\frac{y}{x}\Rightarrow v+xv'=g(v)\Rightarrow v'=\frac{g(v)-v}{x}$ $y'=f(ax+by),u=ax+by\Rightarrow u'=g(y')\Rightarrow g^{-1}(u')=f(u)$
Linear FO ODE	$\frac{dy}{dx}+P(x)y=Q(x),I(x)=e^{\int P(x)dx}$ $\Rightarrow\frac{d}{dx}\left(ye^{\int P(x)dx}\right)=Q(x)e^{\int P(x)dx}\Rightarrow y\cdot I(x)=\int Q(x)\cdot I(x)dx$
Bernoulli Eq	$y'+p(x)y=q(x)y^n,u=y^{1-n}$ $\Rightarrow u'+(1-n)p(x)u=(1-n)q(x)$