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A Model of the Probability of a Cross-Median Crash When a Vehicle Fully Crosses the Median

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Abstract

The consequences of cross-median crashes are often catastrophic but crashes into median barriers can also be severe. Wide medians provide traversable space where vehicles can recover or stop but sometimes even wide medians can be crossed over. Determining if a median barrier is needed at a particular site involves balancing the risks associated with crossing completely over the median and striking a vehicle in the opposing lanes with the risks of an errant vehicle striking a median barrier. Median crossover crashes can be viewed as a conditional probability model: first the vehicle must enter the median; second, the vehicle must cross completely over the median; third, a vehicle must be in the opposing lanes where it is struck and, finally, there is a chance of a severe or fatal injury if all these conditions are met. All the elements of this conditional probability model are documented in the literature except the third; the probability of a crash with an opposing-direction vehicle given that a vehicle has completely traversed the median. Estimating this probability is the subject of this paper.

For several decades there has been interest in the roadside safety community in developing selection and placement guidance for the multiple test levels of median barriers. The large variety of median widths and terrains combined with evolving testing specifications and lack of conclusive data on median cross-over crashes have been significant obstacles to developing more realistic guidance. The current implementation of the *Manual for Assessing Safety Hardware* (MASH) (1) combined with new data collection efforts and the availability of new analysis tools have overcome some of these obstacles to developing median barrier guidance.

Variables like the presence or absence of a barrier, the offset from the lanes to the barrier, the type and test level of the barrier, the highway and median characteristics, and the traffic characteristics can each be assessed using updated tools such as the third version of the roadside safety analysis program (RSAPv3) by any transportation agency to develop guidance for median barriers (2). RSAPv3 contains a great deal of internal data used to model these relationships. While many of these supporting parameters are already available in RSAPv3, one element unique to the problem of medianrelated-events (MREs) that was lacking in the literature is an understanding of how an MRE becomes a cross-mediancrash (CMC).

The probability of a CMC - P(CMC) – is assumed to be a conditional probability in which a vehicle first fully crosses the median and then strikes another vehicle in the opposing direction. Given an MRE has occurred and the vehicle's

trajectory extends the full width of the median, the P(CMC) is a function of another vehicle being present in the opposing lanes. The crash severity distribution of head-on crashes has been found in other studies and is represented in RSAPv3 (2). Developing a model to represent the probability of a CMC given an MRE [i.e., P(CMC|MRE)] was the objective of this study and is documented in this paper.

Model Considerations

The words median and median barrier have minor, but different definitions among the AASHTO and state Department of Transportation literature (3–5). NTSB has asked FHWA and NHTSA to provide a more consistent definition for CMCs, citing a lack of consistency throughout the states (i.e., H-98-13, H-11-28). It is helpful, therefore, to first establish the definitions of terms used consistently in this paper. These definitions are provided in Table 1.

The definitions for CMEs, MREs, and CMCs are interrelated. Appreciating the importance of inter-relationship at the onset of this effort is crucial to ensuring proper data management and reliable analysis outcomes. This P(CMC) model will be used as part of the encroachment probability model,

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Table I. Definitions of Terms

Median	The portion of a divided highway separating the traveled ways for traffic in opposite directions.
Median barrier	A longitudinal barrier system intended to reduce the risk of an errant vehicle crossing the highway median. Median barriers are designed to be impacted from either direction of travel.
Median-Related-Event (MRE)	Any event in which an errant vehicle enters the median. MREs represent all vehicles that encroach left into the median, regardless of the outcome (i.e., crash or no crash).
Cross-Median Event (CME)	An event in which an errant vehicle fully crosses the median and may or may not collide with another vehicle from the opposite direction. CMEs are a subset of MREs.
Cross-Median Crash (CMC)	A cross-median crash is one in which an errant vehicle crosses the median of a highway and strikes or is struck by a vehicle from the opposite direction. CMCs are a subset of CMEs, which are, in turn, a subset of MREs.

which is a conditional probability model. The encroachment probability model first predicts the probability of a median-related-event, P(MRE). Given an MRE, the encroachment probability model then predicts the probability that the errant vehicle's trajectory will extend to the other side of the median and become a cross-median-event (CME), P(CME|MRE). Given a CME has occurred, the encroachment probability model then determines the probability that the CME will become a CMC, P(CMC|CME).

Traffic volume in the opposing direction is thought to affect P(CMC|CME). The conditional probability of CMC given CME, P(CMC|CME) is defined by this relationship:

$$P(CMC|CME) = \frac{P(CMC \cap CME)}{P(CME)}$$

In other words, the probability of both a CMC and CME, $P(CMC \cap CME)$ compared with the P(CME) determines the conditional probability of a CMC, which is the object of this study. This modeling technique is founded on the assumption that P(CME) is greater than zero, a reasonable assumption considering the many observed CMCs and the definitions used to establish these variables.

Avoiding Measuring the Same Variable Twice

The effect of median width is captured through modeling P(CME), so care should be taken to not measure the effect of median width when modeling P(CMC|CME). It was desirable, therefore, to model what happens when a vehicle reaches the far edge of the median, absent the influence of median width and terrain. This was accomplished by considering cross-over crashes (COCs) on undivided roadways. An undivided roadway can be viewed as essentially a divided roadway with a median width equal to the distance between the double yellow lines (i.e., typically 1 ft) and flat median terrain. Head-on and opposite direction side-swipe crashes that occurred on undivided roadways were used to develop the data set upon which these efforts are based to remove the influence of the median width and median terrain confounders. In essence, on two-lane undivided roadways the P(CME|MRE) is essentially unity.

For roadways with a median width equal to essentially zero (i.e., undivided roadways), P(CME) = P(MRE) = 1.00 because all left encroachments cross both yellow lines at the center of the roadway at the start of the encroachment event by definition.

The conditional probability of CMC given CME, which was established above, can be rewritten for this exercise as follows:

$$P\big(CMC|CME\big) = P\big(COC|MRE\big) = \frac{P\big(COC \cap MRE\big)}{P\big(MRE\big)}$$

In other words, the probability of both a COC and MRE compared with the P(MRE) is a surrogate for the conditional probability of a CMC that eliminates the effect of median width and terrain.

P(MRE) is equal to the frequency of left-encroaching vehicles compared with the average annual daily traffic (AADT) of the segment of interest, $P(MRE) = FREQ_{enc} / AADT_{seg}$; whereas the $P(COC \cap MRE)$ is equal to the frequency of a COC and left-encroaching vehicle compared with the AADT of the segment of interest, $P(COC \cap MRE) = FREQ_{coc \& enc} / AADT_{seg i}$. The above relationship can be further simplified as follows:

$$\begin{split} P\big(CMC|CME\big) &= P\big(COC|MRE\big) = \frac{P\big(COC \cap MRE\big)}{P\big(MRE\big)} \\ &= \frac{FREQ_{coc\,\&enc} \ / \ AADT_{seg\ i}}{FREQ_{enc} \ / \ AADT_{seg\ i}} = \frac{FREQ_{coc\,\&enc}}{FREQ_{enc}} \end{split}$$

The frequency of left-encroaching vehicles on divided highways is known based on other studies (2, 6). The frequency of COCs when a left encroachment over the centerline has occurred, however, is not known. The focus of model development is to determine the frequency of COCs using the same units the frequency of encroachment is tabulated by to develop this piece of the encroachment probability model, such that the results can be incorporated into RSAPv3. The $FREQ_{COC}$ model itself must control for the remaining confounding factors.

Method

The practice for modeling count data such as highway crashes is to fit a negative binomial model, usually with a Poisson-gamma-mixture distribution. "In statistics, count data refer to observations that have only nonnegative integer values ranging from zero to some greater undetermined value" (7). In highway safety, zero counts of crashes are particularly important and represent areas in which crashes were not observed. One approach to accounting for zero counts as well as the non-zero counts is to track crashes by highway segment. This approach has the added benefit of allowing the consideration of the influence of segment characteristics like horizontal curvature, grade, lane width, etc. on crash frequency.

The crash count becomes the response variable and the segment characteristics such as AADT, percent trucks (PT), highway geometrics, and area type become the explanatory variables that explain the occurrence of the crashes. Each segment is associated with each of the predictor variables and the number of crashes that occur on that segment during the study period. The characteristics of a segment are used to explain why each segment experiences more or fewer crashes than other segments.

Ideally, all possible predictor variables would be known. This ideal situation, however, remains unrealized, so it is common to consider the known predictor variables when developing a model to ensure the effect of the predictor variable of interest is not misrepresented.

Roadway characteristics that may also modify the P(CMC|CME) such as PT, highway geometrics, and land use (i.e., urban and rural) are recognized confounding factors, but are accounted for elsewhere in the encroachment probability model. Ensuring that the final representation of P(CMC|CME) when implemented in the encroachment probability model does not double-count the effect of these confounders is equally important to controlling for their effect. Controlling for these confounding variables was attempted using two different approaches: 1) explicitly modeling confounding variables, and 2) limiting the data set to segments in which the confounders contained measurements within the base conditions of the previously developed models. The latter is ultimately recommended, as discussed below.

Under both approaches, a negative binomial regression model of COCs was estimated using the COUNT package available in R (8, 9). The model relates the explanatory variables to the response variable using the method of maximum likelihood to quantify the magnitude of each predictor's relationship. The fit statistics are presented along with each model. The p-value is a measure of how probable the result observed may have occurred by chance. A low p-value indicates the results are statistically significant and were unlikely to have occurred by chance (e.g., p<0.05). A higher p-value only indicates that the results have not proven the null hypothesis false, not that the null hypothesis is true. The p-value cannot be relied on alone.

The pseudo-R² statistic was determined for each model. The pseudo-R² is not interpreted the same way as the coefficient of determination for an ordinary least squares regression. A low value pseudo-R² can indicate lack of fit, whereas higher values carry no such indication. There is no definition for a low value. The Akaike Information Criterion (AIC) fit statistic provides comparative information, with lower values indicating a better fitting model then the model it is compared with. The Bayesian Information Criterion (BIC) is interpreted the same way. Both are calculated from the likelihood function (10).

Negative binomial model parameters are estimated using maximum likelihood in which the parameters of the probability distribution that characterize the data are estimated. The log of the likelihood function (LL) is used to determine which parameters make the model most likely to be the case when the data is considered. Through an iterative process, the derivative of the LL function is taken and set to zero to estimate the parameters. When the difference between iterative values is less than a specified tolerance (e.g., 10^{-6}), the iterations stop and the values are at the maximum likelihood estimated values. The LL is also reported with the models, however, it is only useful when calculating other fit statistics (e.g., AIC and BIC).

Any measurement has uncertainty, which should be communicated. This uncertainty in statistical analyses can be conveyed through noting the standard error or the confidence interval along with the measurements. The standard error is a measure of how much the estimate could change within the model. The 95% confidence interval is essentially the same type of statistic as standard error: the 95% confidence interval limits indicate that the analysis is 95% confident the true value of the coefficient is within the stated range. It is important to note that the 95% confidence interval is equal to twice the standard error for normally distributed errors, and negative binomial models are assumed to have normally distributed errors.

Data Used for Modeling

The original intent was to use the models developed under NCHRP 794, "Median Cross-Section Design for Rural Divided Highways" (11). Unfortunately, there appears to be a typographical error in the model printed in NCHRP Report 794, as both the CMC+CME and CMC models shown are identical. Instead, the Highway Safety Information System database of Ohio and Washington highway crashes was used, which could be linked to highway segment information such as AADT, percentage of heavy vehicles (PT), segment length (SegL), area type, speed limit (spd_limt), number of lanes (no_lanes), lane width (lanewid), vehicle type and the crash severity distribution as represented by the KABCO scale. This database was requested for Ohio for 2002 through 2010 and for Washington from 2002 through 2007.

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Table 2. Descriptive Statistics for P(CMC) Data S
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Continuous						
variables	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
L	0.1	0.13	0.19	0.31	0.35	2
AADT	10	1,110	2,280	3,611	4,760	68,336
PT	0.0	3.8	5.7	7.00	8.6	67
Lane width	7	10	10	10.97	12	41
Shoulder width	0	2	3	3.59	4	30
DOC	0	0	0	0.52	0	76
PG	0	0	0	1.52	0	20
PC_KABCOU	0	0	0	0.0345	0	6
HV_KABCOU	0	0	0	0.0075	0	3
MC_KABCOU	0	0	0	0.0008	0	2
KABCOU	0	0	0	0.0445	0	7

	Categorical			Categorical		
Categorical variables	Count	variables	Count	variables	Count	
PSL = 20	68	PSL = 40	5,528	PSL = 60	1,764	
PSL = 25	5,783	PSL = 45	20,958	PSL = 65	229	
PSL = 30	392	PSL = 50	6,868	Rural	185,414	
PSL = 35	30,921	PSL = 55	149,064	Urban	35,561	

Note: PSL = posted speed limit (mph); LW = lane width (ft).

Preparing the Data to Develop the Model

A data set of homogenous segments was merged with crossover and opposite direction side-swipe crashes (i.e., acctype field codes 1 and 4). The crashes were counted by crash severity and vehicle type and assigned to the appropriate homogenous segment using the recorded route and milepost of each crash. The resulting data set included a list of segments. Each segment had a field for AADT, PT, SegL, area type, spd_limt, no_lanes, and lanewid. Each segment also contained a field for the count of crashes occurring on the segment by each crash severity and vehicle type (e.g., passenger car fatal crash = PC_K; heavy vehicle serious crash = HV_A, etc.). The data set includes 1,204,084 segments, some with crashes and some without.

In some instances, the segments included fields for which the information was not available (NA) or the field contained a nonsense value (e.g., AADT=0). Prior to working with the data set, segments with incomplete data or nonsense values were removed from consideration. The data set was filtered to remove these segments as shown here prior to any modeling. The segments remaining after each filter step are noted in parenthesis.

- Consider only segments in which the land use type (i.e., urban or rural) is known (1,202,105)
- Consider only segments in which the length in miles is $0.1 \le L \le 2$ (404,620)
- Consider only segments in which AADT>0 (403,666)
- Consider only segments in which the PT is known (242,862)

- Consider only segments in which the value for the number of lanes is equal to two (221,171)
- Consider only segments in which the posted speed limit > 0 mph (220,975).

This filtering of the data set resulted in 220,975 segments being included in the modeling data set. The descriptive statistics for this data set are shown in Table 2.

Model Development

Model with Control Variables

Recall, the number of crashes per year were tabulated in the data set. A negative binomial model was fit to the data set of head-on crashes, and the log of the SegL in miles was included as an offset to allow for the frequency of opposite direction crashes to be evaluated per year per mile. The resulting parameter estimates are shown in Table 3 for the cross-over model with control variables that take this form:

$$FREQ_{COC} = \cdot e^{B1} \cdot AADT^{B2} \cdot PT^{B3} \cdot \prod_{i=4}^{N} e^{Bi \cdot Ai}$$

where

 $FREQ_{COC}$ = Frequency of cross-over crashes per year per mile,

AADT = Annual Average Daily Traffic (vpd),

 A_i = Control variable values for each segment under consideration,

Table 3. Negative Binomial Model for Cross-Over Crashes

Coefficients:	Parameter estimate	Standard error	P-value	95% confide	nce interval
(Intercept)	-9.3993	0.49	< 2e-16	-10.4910	-8.5228
log(AADT)	0.9025	0.02	< 2e-16	0.8690	0.9363
log(PT)	0.0235	0.00	1.32e-15	0.0178	0.0293
Urban	0.1343	0.03	6.69e-06	0.0758	0.1928
Rural	1.0000	NA	NA	NA	NA
PSL.20	1.0000	NA	NA	NA	NA
PSL.25	0.4377	0.47	0.353	-0.3989	1.5042
PSL.30	0.5085	0.50	0.309	-0.3970	1.6172
PSL.35	0.1129	0.47	0.810	-0.7197	1.1770
PSL.40	0.0043	0.47	0.993	-0.8350	1.0725
PSL.45	-0.0708	0.47	0.880	-0.9059	0.9948
PSL.50	-0.3035	0.47	0.521	-1.1438	0.7653
PSL.55	-0.2767	0.47	0.556	-1.1104	0.7880
PSL.60	-0.7111	0.51	0.162	-1.6382	0.4095
PSL.65	-0.6755	0.73	0.357	-2.1498	0.7372
lanewid	0.0215	0.00	1.13e-06	0.0127	0.0301
DOC	0.0454	0.01	2.73e-14	0.0326	0.0569
PG	0.0454	0.01	1.39e-08	0.0187	0.0386
SHLDR_PRE	-0.0517	0.00	< 2e-16	-0.0606	-0.0429
AIC			68,272		
BIC			68,447		
Dispersion parameter	r (α)		1.099		
Standard error			0.0730		
LL (full)			-34,119)	
Pseudo R ²			0.15		

B = Regression coefficients,

N' = Total number of control variables considered per segment.

Limiting Data set to Base Conditions to Remove Confounders

Recall that the initial data set was filtered to remove segments with missing or nonsense values, which resulted in a data set of 220,975 segments used in the model developed that included control variables. A different approach is taken in this section when the data set only includes segments that meet the base conditions of the complementary encroachment probability model for which this P(CMC) model is being explored, such that this P(CMC) model does not account for variation in highway characteristics that are already accounted for elsewhere in the encroachment probability model.

The 220,975 segment data set was further limited for this analysis as shown here, with the remaining segments after each filter step noted in parenthesis:

- Consider only segments in which the area type is rural (185,414)
- Consider only segments in which the PT \geq 10 (36,038)

- Consider only segments in which the posted speed limit ≥45mph (32,069).
- Consider only segments in which 10 ft \leq lanewid \geq 12 ft (24,690)
- Consider only segments in which the DOC = 0 (21,918)
- Consider only segments in which $-2\% \le PG \ge +2\%$ (17,443).

This base-condition limited data set resulted in 17,443 segments being included in the analysis. The descriptive statistics for this base-condition limited data set are shown in Table 4 using the same shorthand for the categorical variables previously discussed with respect to Table 2.

Again, a negative binomial model was fit to the data set of cross-over and opposite direction side-swipe crashes and the log of the SegL in miles was included as an offset to allow for the frequency of head-on crashes to be evaluated per year per mile. The resulting parameter estimates are shown in Table 5 for the cross-over model which explicitly limits confounders and takes this form:

$$FREQ_{COC} = \cdot e^{B1} \cdot AADT^{B2}$$

where

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Table 4.	Descriptive	Statistics for	Limited P	(CMC)	Data Set

Continuous variables	Min.	lst Qu.	Median	Mean	3rd Qu.	Max.
L	0.1	0.17	0.32	0.4879	0.65	2
AADT	40	1,594	2,850	3,604	5,110	20,260
PC_KABCOU	0	0	0	0.0308	0	3
HV_KABCOU	0	0	0	0.0170	0	3
MC_KABCOU	0	0	0	0.0007	0	1
KABCOU	0	0	0	0.0501	0	5
Categorical variables		Count	Categorical variables	Count	Categorical variables	Count
PSL = 45		1,143	PSL = 55	15,510	PSL = 65	107
PSL = 50		562	PSL = 60	121		

Table 5. Negative Binomial Model for Cross-Over Crashes

Coefficients:	Parameter estimate	Standard error	P-value	95% confide	ence interval
(Intercept)	-11.3901	0.49	< 2e-16	-12.3625	-10.4393
log(AADT)	1.1050	0.06	< 2e-16	0.9928	1.2193
AIC			6,14	2	
BIC			6,15	8	
Dispersion paramet	ter (α)		1.29	6	
Standard error	. ,		0.33	9	
LL (full)		-3,069.25			
Pseudo R ²			0.12	2	

FREQ_{COC} = Frequency of cross-over crashes per year per mile,

AADT = Annual Average Daily Traffic (vpd), B = Regression coefficients.

Recommended Model

Recall, a FREQ_{COC} model that controls for highway characteristics is desired and that this model is presumed to be a function of AADT. The severity of these crashes is understood through a different model and need not be considered here, rather, this model should consider all observed crashes to allow for the conversion from a frequency model to a probability model. The simpler model in which the confounders are explicitly controlled through limiting segments considered to base-condition segments, as documented in Table 5, is preferred because (1) the AIC and BIC values are lower, (2) the model makes better engineering sense, and (3) the simpler model best satisfies the principals of parsimony (i.e., Occam's Razor).

The frequency of COCs when a left encroachment over the centerline on a two-lane undivided highway has occurred is now estimated to be:

$$FREQ_{COC \cap enc} = \cdot e^{-11.3901} \cdot AADT^{1.1050}$$

It was previously derived that $P(CMC|CME) = FREQ_{coc \cap enc} / FREQ_{enc}$. The frequency of left encroaching

Table 6. Recommended P(CMC|CME)

	· ' '
Rural Divided Highways	
For 0 < AADT < 29,000: 29,000 < AADT < 43,000:	0.178 AADT $^{-0.1002}$ e $^{(0.2104+0.04128 \text{ AADT}/1000)}$ 3.042 AADT $^{1.1050} \cdot 10^{-6}$
43,000 < AADT < 43,000: 43,000 < AADT < 67,000:	1.145AADT -0.1002
67,000 < AADT < 90,000:	0.117AADT ^{0.1050}
AADT > 90,000: <u>Urban Divided Highways</u>	1.300 AADT ^{1.1050} ·10 ⁻⁶
For 0 < AADT < 26,000:	$0.052AADT^{0.0162}e^{(0.2104+0.04128AADT/1000)}$
26,000 < AADT < 45,000:	2.934AADT ^{1.1050} ·10 ⁻⁶
45,000 < AADT < 90,000:	0.335AADT ^{0.0162}
AADT > 90,000:	1.353 AADT ^{1.1050} ⋅10 ⁻⁶

vehicles by direction of travel, $FREQ_{enc}$ has been documented previously (2, 6).

This cross-over centerline model accounts for traffic volume in the opposing lanes and is only dependent on vehicles being present in the opposing lanes. This principle can be extended to medians using the conditional probability model. As in the centerline cross-over model, the probability of a CMC, given a CME, is only a function of traffic volume in the opposing lanes and vehicles being present. Therefore, this undivided model can be used to model CMC given a CME.

The $FREQ_{enc}$ is documented by direction of travel. That is to say that vehicles originating from the primary direction of travel and encroaching into the median are counted once,

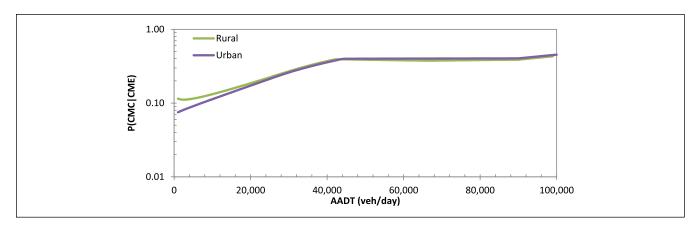


Figure 1. Recommended P(CMC|CME).

whereas those encroachments originating in the opposing direction of travel and encroaching into a median are counted separately. Until this point, the $FREQ_{COC \cap enc}$ model has considered the frequency of crashes from each direction in total for a median width of zero. This frequency model must be divided by two to allow for consideration of each direction of travel as the encroachments are considered, and as it will be used in RSAPv3. The results are shown in Table 6 in equation form, combined with the encroachment frequency model, and graphically in Figure 1.

Conclusion

The P(CMC|CME) model provides a missing piece of the conditional encroachment probability model for modeling cross-median crashes and will provide valuable insight into predicting the probability of these events and ultimately the risk of a serious or fatal (A + K) crash when these events occur. This model is proposed for use in RSAPv3 to represent the influence traffic volume in the opposing direction has on the probability of a cross-median crash, P(CMC|CME). This new P(CMC|CME) model will be added to RSAPv3 for use by any agency who wishes to consider median design on a case-by-case basis or develop agency-specific guidance.

This model is proposed to be used in conjunction with the severity model [i.e., P(A+K|CMC)] to determine the risk of an A + K for CMCs. This value is an important part of the conditional probability model for determining the frequency of severe and fatal CMCs and it is also an important aspect of comparing the severity of median barrier crashes, as median barriers should only be placed where the frequency of severe or fatal barrier crashes is less than the frequency of severe and fatal CMCs.

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