Physics - First Tome.
(A friendly approach.)

ILM

Jaime Andres Torres Bermejo
(github.com/XaurDesu)

Why taking notes like this?

Well damn, i just want to have notes that i can read easily, are non-judgemental and let you learn at your own page, this is by no means better than a specialized textbook, made by an actual professional with a degree in physics or some other natural science that just so happens to involve physics.

Can i use this text in some way?

I mean, as long as you're conscious of it's limitations and are able to work around them, i see no reason why i would get angry about you using this text for your own means, just remember to do no harm.

There's a mistake in this book! What do i do?

Tell me what it is, just let me know and maybe even correct it yourself, i have no reservations on making changes in case it happens to be necessary or otherwise useful.

Can i share this?

Go ahead! These notes are open source, everybody should be able to access them i think.

(Shoutout to Yenny Hernandez, who i studied physics with @Uniandes.)

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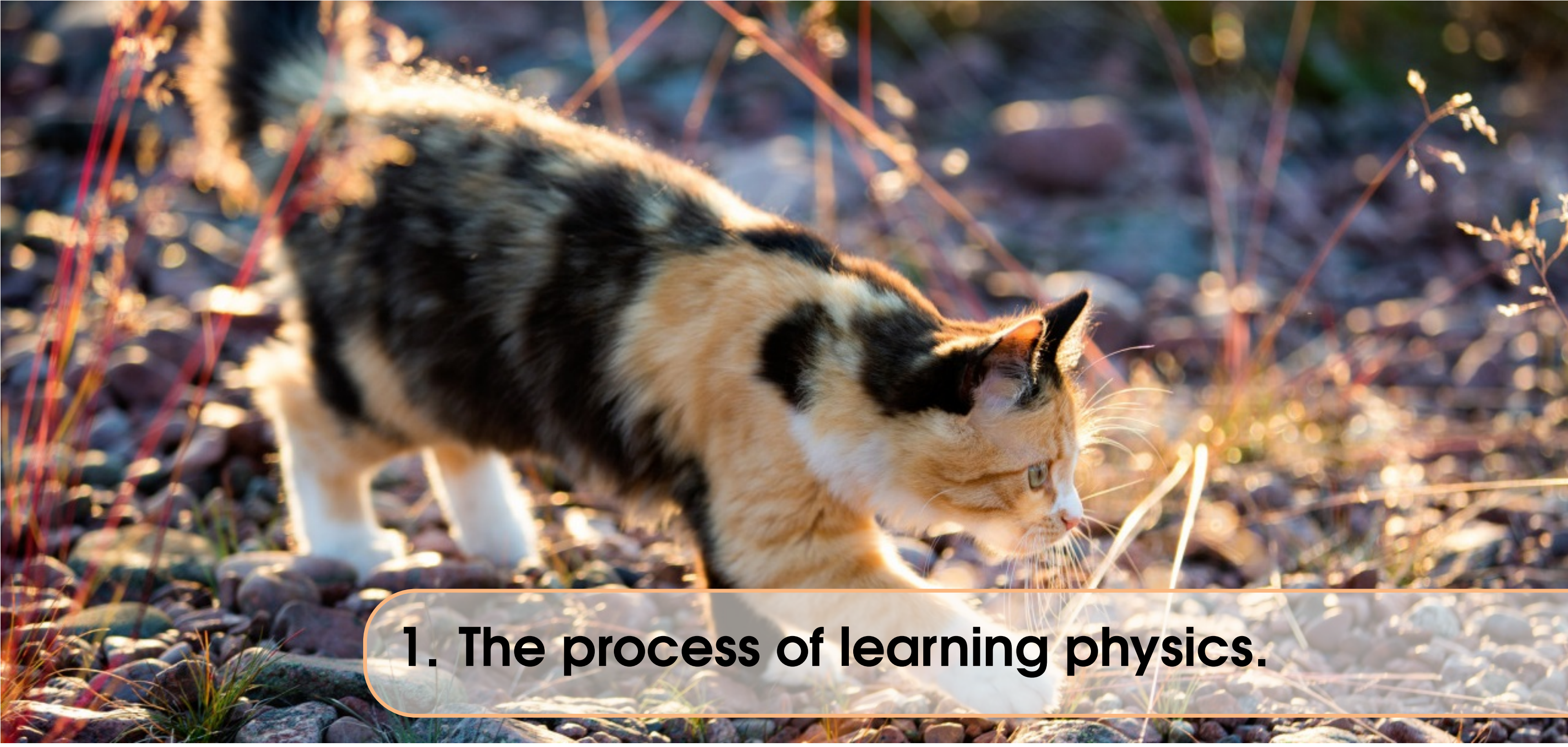
Contents



1. The process of learning physics.

1.1 So, how AND WHY do we learn physics, anyways?

(...like, really, why?)

Eh, it depends.
In reality, the process of learning physics is also the process of learning how to observe a phenomenon in a bunch of ways. It's an experimental science that is concerned with the study of energy, materials and their mutual interactions. We can explain a bunch of things regarding the way our universe works with it.

In any case, learning physics is not a marathon, cramping before an exam stresses me out, and it probably does to you too. And we should try to learn it bit by bit, going through topics slowly and surely. Rome wasn't built in a day, and neither should you try to learn a solid understanding in one.

When learning physics, remember:

- **Do exercises.** You shouldn't try to just learn by reading the notes of some other guy, you might not even know me. It's
- **Be critical of your own solution.** you won't always have problems right your first time around. If something doesn't seem to make sense, it's because it is probably not right. (you probably shouldn't be getting a negative speed of light velocity when trying to calculate a bike going in a straight line, for example)
- **Understand the topics enough to explain them to someone else.** If you can teach physics to someone else, you will probably understand it a lot better yourself, give it a try!
- **Be prepared.** Check your notes, do your homework, and try to study by yourself. Prepare the things that you need to learn before learning them becomes a point of stress.
- **You're not alone.** Look for help when you need it, it's not shameful.

1.2 Important considerations.

2. Measurements.

2.1 Uncertainty

There is always a level of uncertainty when measuring an object, given by the instrument of measurement we use, an atomic clock is not the same as your uncle's watch, and even though they both measure time, there's a certain level of uncertainty caused by the device, keep it in mind when doing experimental work. We can express this with the symbol ' $\pm$ '

for example, if we had a milimeter of uncertainty on a measurement of 9,5 cm, we could indicate it as such:

$$l = 9,5 \pm 0,1cm \tag{2.1}$$

2.2 Scientific Notation

It is usual to measure both inmense and infimal quantities of mass, time, or any other thing. For that, we shall apply the scientific notation, both on this book and elsewhere.

EXERCISES

How many years older will you be in a thousand million seconds?

We'll start by measuring how many seconds there are in a year.

$\frac{1min}{60s} * \frac{1h}{60min} * \frac{1d}{24h} * \frac{1yr.}{365d} = 1x10^9 s$

And now we'll take our given time on years

$time = 1,000,000s * 1,000s = 1000x10^6 s$

Answer:$31yrs.$

How many nanoseconds does the speed of light need to travel 1ft in the void?

$1ft = 30,48cm$

Even though not specifically notified in the exercise, we must remember that:

$$1m = 100cm \tag{2.2}$$

2.3 Types of units

In the realm of physics, there is a bunch of ways to categorize units, be it by what do they measure or how do they measure it

2.4 Scalar units

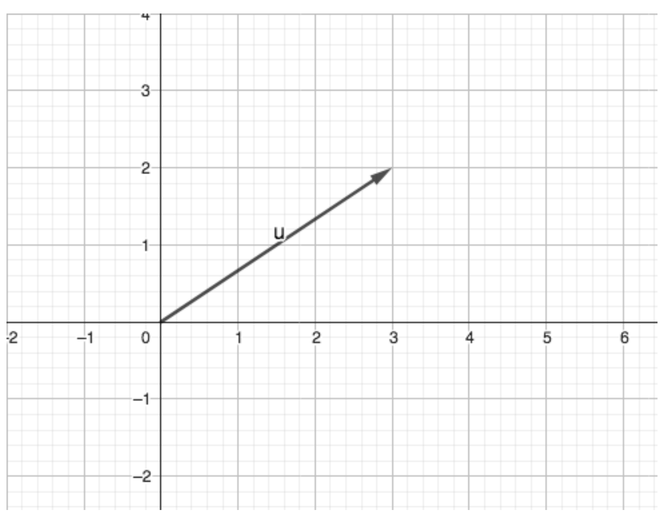
A scalar unit is defined as an unit with magnitude, but no direction. Such measurements aren't really concerned with being tied with a specific force acting over a particle.

Examples of such units include:

- Natural numbers.
- Constants.
- Acceleration

2.5 Vectors

A vector is a measurement with both a magnitude and a direction. that can exist on a specific set of dimensions. On this course, we won't be concerning ourselves with hyperplanes and might sparingly cover 3-dimensional planes, they would both be useful for you to learn eventually, so i encourage you to learn about them on your own accord, however. Our examples will be two-dimensional, for a simple, introductory example, consider the following vector; $\vec{A} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$:



Vector with x coordinates '3' and y coordinates '2'

This is what we call a cartesian vector, for it's magnitudes are defined by the coordinates in a cartesian plane, however, we can turn it into a polar vector, measured by it's magnitude and angle, through the following formulas:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \tag{2.3}$$

$$\tan\theta = \frac{A_y}{A_x} \tag{2.4}$$

Exercise: Turning our example vector 'A' into a Polar vector.
Magnitude

$$|\vec{A}| = \sqrt{3^2+2^2}|\vec{A}| = \sqrt{9+4} = \sqrt{12} = 2\sqrt{3} \tag{2.5}$$

Angle

$$\tan\theta = \frac{2}{3} \tag{2.6}$$
$$\theta = \tan^-1(\frac{2}{3}) \tag{2.7}$$
$$\theta = 33,69^o \tag{2.8}$$

Equally, we can turn a polar vector into a cartesian one.

So, given an angle θ and a magnitude $|\vec{A}|$, we can suppouse the measurements of a two-directional vector as:

$$A_x = |\vec{A}|\cos\theta \tag{2.9}$$
$$A_y = |\vec{A}|\sin\theta \tag{2.10}$$

And given this, we can consider the vector '$\vec{A}$' as, $\vec{A} = A_x i + A_y j$, where 'i' and 'j', are what we're going to call a **unit vector**, a vector in a specific direction that has a value of 1. (this is considered an identity value for multiplication operations, and lets us do some vector sum operations more intuitively)

Exercise: Turning our example vector 'A' back into a cartesian vector.

$$A_x = |2\sqrt{3}|\cos 33,69^o = 2,88 \approxeq 3 \tag{2.11}$$
$$A_y = |2\sqrt{3}|\sin 33,69^o = 1,92 \approxeq 2 \tag{2.12}$$

As it might be evident, the conversion isn't exact, this happens because of the angle not being an exact conversion, this will happen when you disregard part of a value for whatever reason. The more exact of a measurement you keep, the less uncertainty you will end up with.

2.5.1 Vector sum

We can take any $\vec{a}$ and $\vec{b}$ vectors on the same space and add them to each other in the form:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{pmatrix} \tag{2.13}$$

Such form remains in the case we can do subtraction, which is expressed on the equation:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1-b_1 \\ a_2-b_2 \\ a_3-b_3 \end{pmatrix} \tag{2.14}$$

This kind of operations have certain properties, shown as:

$$(\alpha+\beta)\vec{[v]} = \alpha\vec{v}+\beta\vec{v} \tag{2.15}$$

$$\vec{v}*1=\vec{v} \tag{2.16}$$

$$\vec{v}*\vec{0}=\vec{0} \tag{2.17}$$

$$\beta\vec{v} = \begin{pmatrix}\beta a_1\\ \beta a_2\\ \beta a_3\end{pmatrix} \tag{2.18}$$

Two vectors $\vec{a}$ and $\vec{b}$ are equal if and only if:

$$\begin{cases}\vec{a}\exists\mathbb{R}^3\\ \vec{b}\exists\mathbb{R}^3\end{cases} \implies \begin{pmatrix}a_1=b_1\\ a_2=b_2\\ a_3=b_3\end{pmatrix} \textit{ note: this can be generalized to 'n' dimensions larger than 0} \tag{2.19}$$

in either case, $\vec{0}$ is the identity of the operation, therefore:

$$\vec{a}+\vec{0}=\vec{a} \tag{2.20}$$

2.5.2 Scalar/dot product

We can multiply vectors between each other with the following formula:

$$|\vec{A}|\cdot|\vec{B}| = |\vec{A}||\vec{B}|\cos\theta$$

We can also write it as such for cartesian vectors:

$$A_xB_x+A_yB_y+A_zB_z$$

This is a commutative operation, that won't be affected by neither A nor B's

2.5.3 Cross/Vectorial product

We can define the formula for the cross product of two vectors in a 3-dimensional space as:

$$\vec{A}x\vec{B} = \begin{pmatrix}(A_y-B_z-A_zB_y)i\\ (A_xB_z-B_xA_z)j\\ (A_xB_y-B_xA_y)k\end{pmatrix} \tag{2.21}$$

This will come handy when studying Torque;

2.5.4 Solved Exercises

Vector Sum

An espeologist explores a cave and follows a

2.5.5 Dot/Cross Product

2.5.6 Scientific Notation

how many nanoseconds does a ray of light require to travel 3 meters in the void?

Solution

Estimate how many gallons of gasoline are consumed by private cars in Colombia in a year. Assume that on average there is one car for every 10 inhabitants and that on average each car travels 10^4 km per year, with an average engine efficiency of 25 km/gallon.

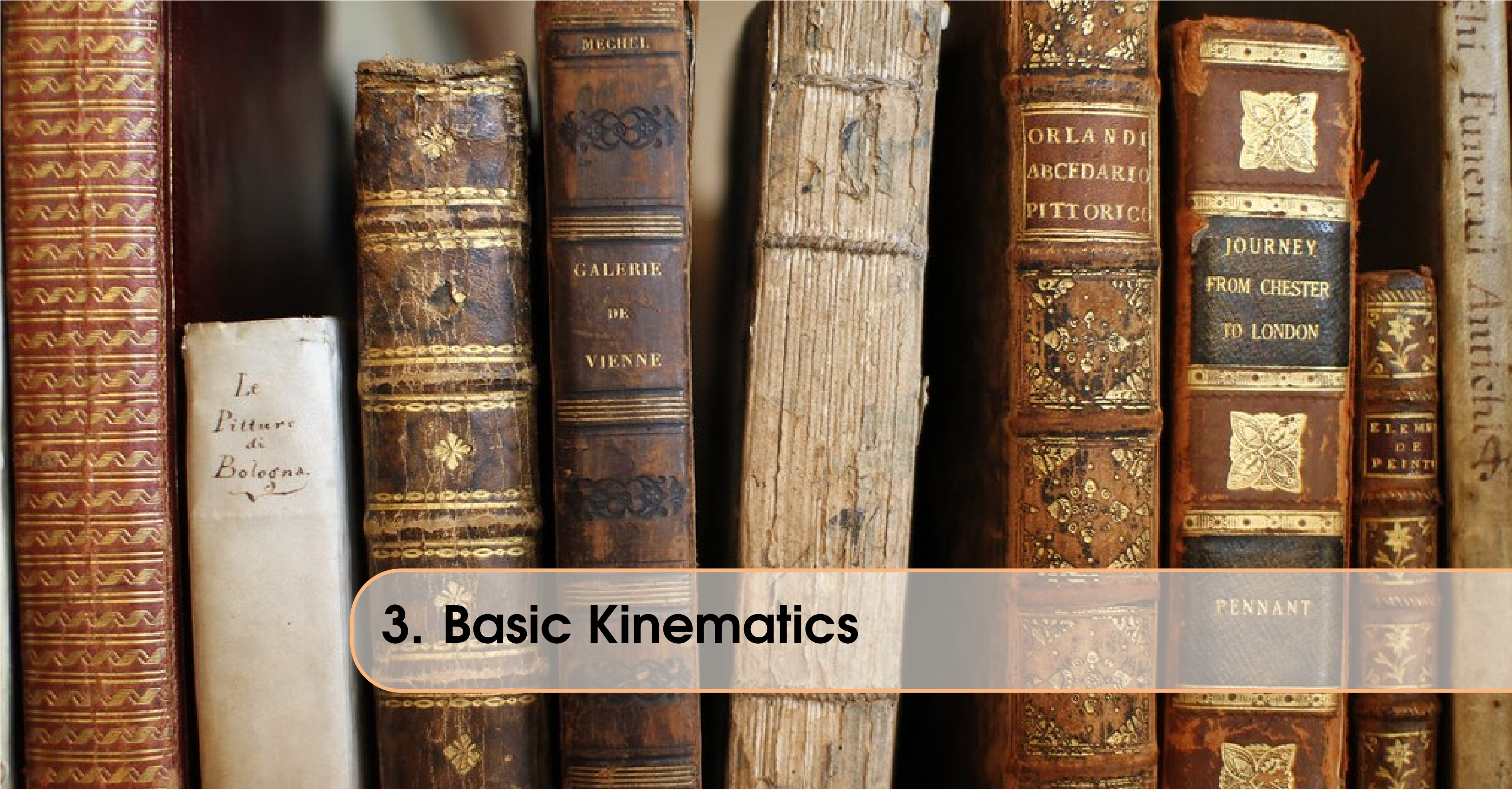
Solution.

$$\text{Population of Colombia as of 2020} = 51520000 \tag{2.22}$$

$$\text{Number of cars} = 51520000/10 = 5152000\ cars \tag{2.23}$$

$$\text{Kilometers traveled} = 5152000\ cars * 10^4 \frac{km}{yr} = 51520000000 \frac{km}{yr} = 5,152x10^{10} \frac{km}{yr} \tag{2.24}$$

$$\text{Gallons per year} = \frac{5,152x10^{10}\frac{km}{yr}}{25\frac{km}{gallon}} = 2060800000 \frac{\text{gallon}}{yr} = 2,0608x10^9 \frac{\text{gallon}}{yr} \tag{2.25}$$



3. Basic Kinematics

3.1 One-Dimensional Movement

One-dimensional movement is probably the most basic kind of movement, as unbound by the expectations of more than two directions to move to, it allows itself to simply define the route from a point 'A' to a point 'B' in a straight line.

Such movement can be defined as:

$$\Delta x = x_2 - x_1 \tag{3.1}$$

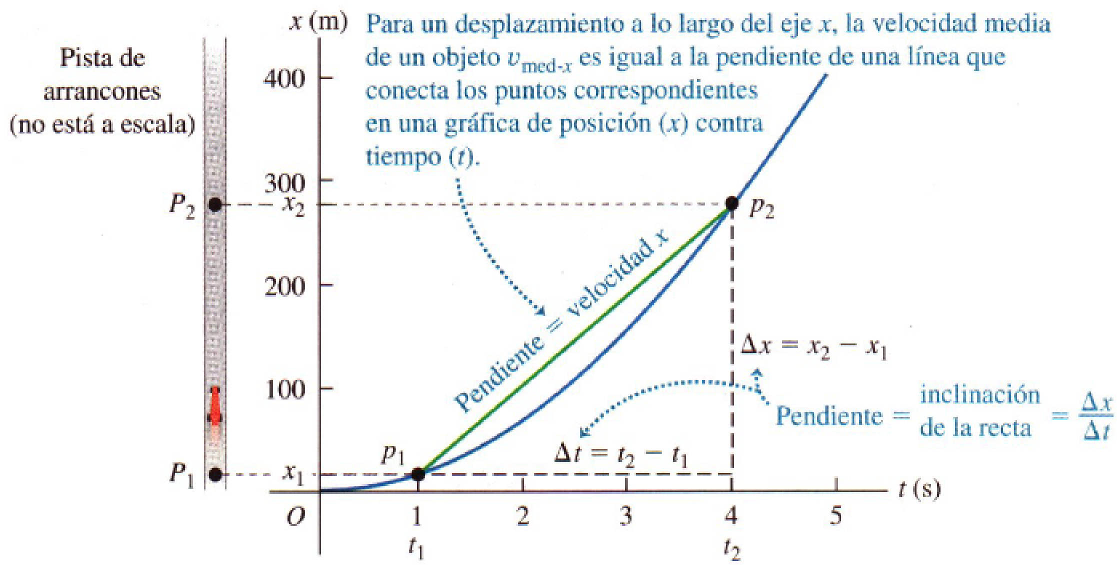
medium velocity on this kind of movement can be calculated as:

$$v_{med} = \frac{\Delta x}{\Delta y} = \frac{x_2 - x_1}{t_2 - t_1} \tag{3.2}$$

Keep in mind, we can define negative velocity in such systems to move in a negative direction (i.e moving from B=7 to A=3, for example.) and besides that **This is not the same as speed, because speed is concerned with how fast the object moves while this is concerned with how fast the object arrives somewhere, and that's not the same.**

3.2 instantaneous speed and velocity

It should now be evident that we can assume a straight line that can be interpreted as the velocity of an object, as we could see in this two-dimensional graph.



We define the instantaneous velocity of an object as a derivative of the form:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{3.3}$$

The simplest formula for such a calculation is, however:

$$x = x_0 + vt \tag{3.4}$$

3.3 Acceleration

Of course, not all objects will move at a constant pace, most times there will be a change in speed happening at some moment in their movement. This is a scalar variable that can be used to calculate the overall distance when taking into account such changes on movement. We can calculate the acceleration of an object as follows:

$$a = \frac{\Delta v}{\Delta t} \tag{3.5}$$

we can integrate distance as a function of acceleration as it follows, as well:

$$\int_t^0 v\,dt = \int_x^{x_0} dx \textit{ Note: we can integrate the result of this equation to prove correctness.} \tag{3.6}$$

Through a mathematical proof that

3.3.1 Constant Acceleration

An object can be said to have constant acceleration when acceleration is not a variable, but a constant. When working on such a system we can affirm velocity as:

$$at = v - v_0 \tag{3.7}$$

and we can indicate distance as:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{3.8}$$

From these two we can gather the following formulas:

- $v = v_0 + at$
- $t = \frac{v - v_0}{a}$
- $v^2 = v_0^2 + 2a(x - x_0) If\ we\ don't\ know\ time$
- $x - x_0 = (\frac{v + v_0}{2})t If\ we\ don't\ know\ acceleration$

3.4 Two-dimensional movement

Again, this is a pretty self explainatory title. As it just refers to the representation of movement in more than a two independent axis. We never were living in a straight line anyways so it shouldn't come as a surprise to you that physics doesn't live there either.

We can imagine two vectors to indicate the beginning and end of our specified movement, and yes, this could potentially include (0,0), in any case, we can write them as:

$$\vec{r}_i = x_i i + y_i j \tag{3.9}$$
$$\vec{r}_f = x_f i + y_f j \tag{3.10}$$

To describe a movement in two dimensions with these vectors, we can use the following equations:

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \tag{3.11}$$
$$\Delta\vec{r} = (x_f - x_i)i + (y_f - y_i)j \tag{3.12}$$

we can also affirm:

$$|\vec{r}_i| = \sqrt{x_i^2 + y_i^2} \tag{3.13}$$
$$\theta = \arctan(\frac{y_i}{x_i}) \tag{3.14}$$
$$\vec{r}_i = |\vec{r}_i|\cos\theta i + |\vec{r}_i|\sin\theta j \tag{3.15}$$

These are just vectorial operations, and they apply for either vector, nothing really changes from how we already expected vectors to work.

3.4.1 Projectile movement.

We can assume, on this version of a two-dimensional movement, that:

$$x = x_0 + v_0 \cos\theta t \tag{3.16}$$
$$v_x = v_0 \cos\theta_0 = v_0 x \tag{3.17}$$
$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2 \tag{3.18}$$
$$v_y = v_0 \sin\theta - gt \tag{3.19}$$
$$v_y^2 = v_0^2 \sin^2\theta_0 - 2g(y - y_0) \tag{3.20}$$

We can also assume an 'R' distance that will be the maximum possible distance in the x axis with 'y' as 0. We can get it as:

$$v_0 \sin\theta_0 = gt \tag{3.21}$$
$$t = \frac{v_0 \sin\theta_0}{g} \tag{3.22}$$
$$R = 0 + v_0 \cos\theta_0 \cdot \frac{2v_0 \sin\theta_0}{g} \tag{3.23}$$
$$R = \frac{v_0^2 \sin(2\theta_0)}{g} \tag{3.24}$$

This is the maximum possible distance that you can get for a displacement given certain parameters to be replaced in such a formula. Keep in mind that when θ is 45 degrees, the value of 'R' is at its peak for that specific configuration, so given a question regarding angles, the closest to 45 it is, the furthest it will make it. A few other useful formulas are:

$$x = x_0 + v_0 t \tag{3.25}$$
$$y = h + v_0 \sin\theta_0 t - \frac{1}{2}gt^2 \tag{3.26}$$
$$t = \sqrt{\frac{2h}{g}} \tag{3.27}$$
$$x = v_0\sqrt{\frac{2h}{g}} \tag{3.28}$$

When working on an elevated end point, we can imagine that, given a distance 'D' and an elevated endpont 'H':

$$x = x_0 + v_0 \cos\theta_0 t \tag{3.29}$$

$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2 \tag{3.30}$$

$$v_y^2 + v^2 \sin^2\theta_0 - 2g(y - y_0) \tag{3.31}$$

$$D = v_0 \cos\theta_0 t \tag{3.32}$$

$$t = \frac{D}{V_0 \cos\theta_0} \tag{3.33}$$

$$H = \frac{v_0 \sin\theta_0 D}{v_0 \cos\theta_0} - \frac{1}{2}g(\frac{D^2}{v_0^2 \cos^2\theta_0}) \tag{3.34}$$

$$H = D\tan\theta_0 - \frac{1}{2}g\frac{D^2}{v_0^2 \cos^2\theta_0} \tag{3.35}$$

$$V_0^2 = \frac{gD^2}{2\cos^2\theta_0 (D\tan\theta_0 - H)} \tag{3.36}$$

What's the secret on getting these problems right?

Writing all of the possible equations and looking at:

- What do i have?
- What do we need?
- How can i rewrite the equations that i have in order to get what i need?

Circular Movement

$$\Delta s = R\Delta\phi \tag{3.37}$$

$$\frac{\Delta s}{R} = \frac{|\Delta v|}{v_1} \tag{3.38}$$

$$|\Delta v| = v_1 \Delta\phi \tag{3.39}$$

$$a = \frac{v^2}{r} \tag{3.40}$$

$$v = \frac{2\pi R}{T} \tag{3.41}$$

$$T = \frac{2\pi R}{v} \tag{3.42}$$

3.5 Exercises

One-dimensional movement

3.5.1 Acceleration

what's the maximum possible height of a marker being thrown at a velocity of $0,5\frac{m}{s}$?

We know that:

- gravity (acceleration): $-9,8\frac{m}{s^2}$
- initial position: 0
- initial velocity = $0,5\frac{m}{s}$

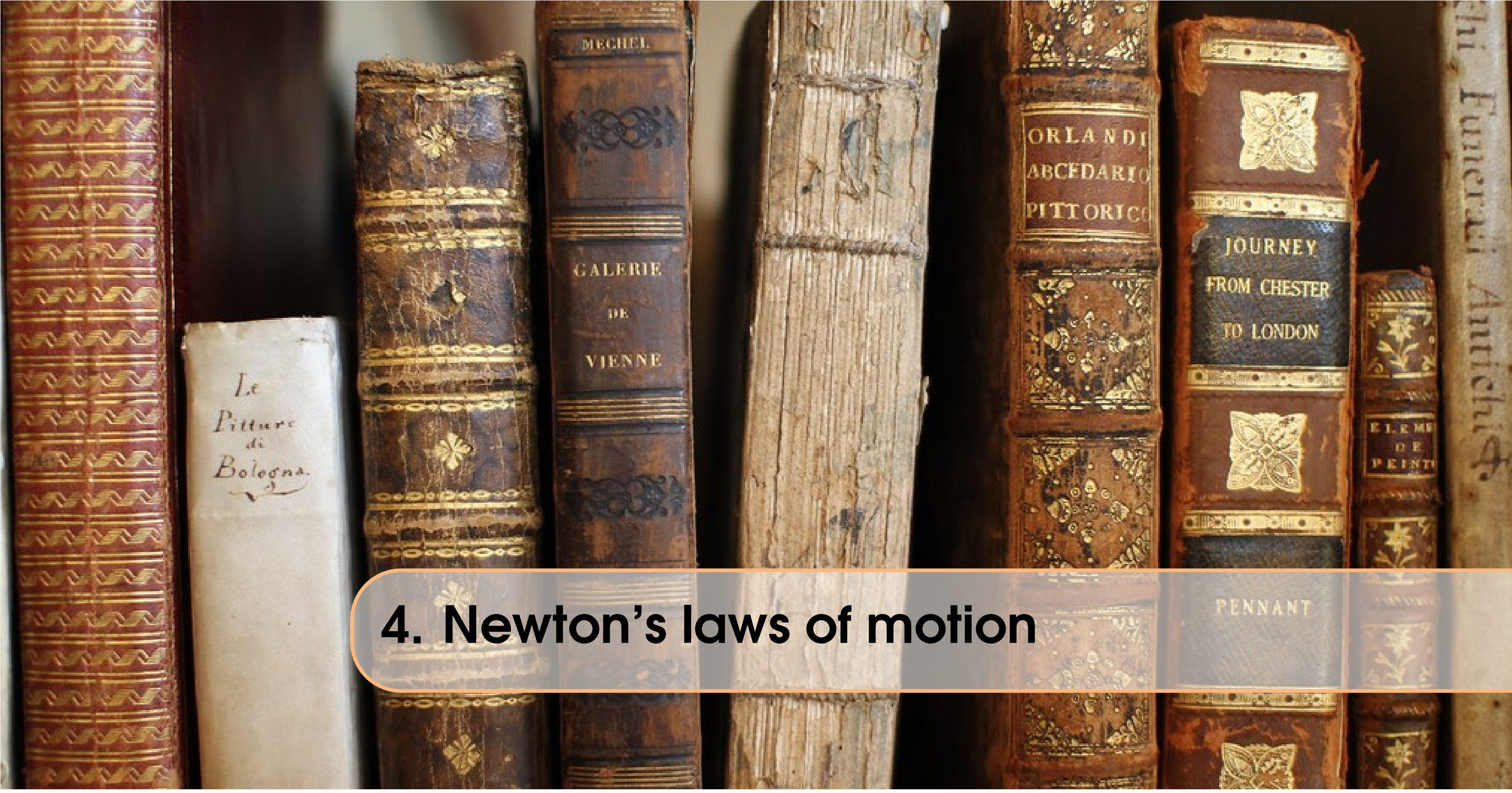
$$v^2 = v_0^2 + 2a(x - x_0) \tag{3.43}$$

$$0 = v_0^2 - 2g(y - 0) \tag{3.44}$$

$$v_0^2 = 2gy \tag{3.45}$$

$$y = \frac{v_0^2}{2g} \tag{3.46}$$

$$y = -\frac{0,5\frac{m}{s}^2}{2(9,8\frac{m}{s^2})} \tag{3.47}$$



4. Newton's laws of motion

Newton's laws of motion are three basic laws of classical mechanics that describe the relationship between the motion of an object and the forces acting on it.

4.1 Forces.

We mostly can assume forces to mantain the following

$$\vec{R} = \vec{F}_1 + \vec{F}_2 \tag{4.1}$$
$$\vec{R} = \vec{R}_x i + \vec{R}_y j \tag{4.2}$$
$$\vec{R}_x = \vec{F}_{1x} + \vec{F}_{2x} \tag{4.3}$$
$$\vec{R}_y = \vec{F}_{1y} + \vec{F}_{2y} \tag{4.4}$$

4.2 Newton's laws.

4.2.1 First Law

A body in state of rest, or in uniform motion in a straight line will have an overall summatory of forces equal to 0

$$\sum \vec{F} = 0 \tag{4.5}$$

4.2.2 Second Law

A net force that acts over a body makes it accelerate in the same direction as the net force. The magnitude of acceleration is directly proportional to the magnitude of the forces acting over it.

- if a net force acts over a body, this body accelerates
- The direction of acceleration is the same as a net force.

we can assume:

$$\vec{F}_{net} = m\vec{a} \tag{4.6}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{4.7}$$

4.2.3 Third Law

When two bodies interact their forces are always equal in magnitude and opposed in direction. This can be expressed as:

$$\vec{F}AB = -\vec{F}BA \tag{4.8}$$

4.3 Free-body diagrams

A free body diagram is a representation of a system and the forces acting over it.

4.4 Friction

Friction is the

4.5 Drag and Terminal Velocity

there is a force that linearly depends on velocity called drag. Such

4.6 Examples

4.6.1 Forces

Two horses horizontally pull strings tied to a tree trunk. The forces $\vec{F}_1$ and $\vec{F}_2$ are applied such as $\vec{R}$ has the same magnitude as $\vec{F}_1$ and is 90^o from $\vec{F}_1$; if $\vec{F}_1 = 1300N$, calculate the magnitude of $\vec{F}_2$ and it's direction.

Solution

An electron with mass $9,11x10^{-31}kg$ goes from a kinescope with an initial speed of 0 and travels in a straight line to an end point 1,80 cm away. arriving with a speed of $3x10^6\frac{m}{s}$. if net force is constant, calculate acceleration, time, and net force (in Newtons).

solution

We know:

- $V_f = 3x10^6\frac{m}{s}$

Acceleration

$$\vec{V}_f^2 = \vec{v}_0^2 + 2ad \tag{4.9}$$

$$\vec{V}_f^2 = \vec{v}_0^2 + 2a(x - x_0) \tag{4.10}$$

$$a = \frac{\vec{V}_f^2}{2d} \tag{4.11}$$

$$a = \frac{3x10^6\frac{m}{s}}{2x1,8x10^{-2}m} = 2,5x10^{14}\frac{m}{s^2} \tag{4.12}$$

A 2.00 kg object is subject to three forces that give it a total acceleration of $\vec{a} = -(8,00\frac{m}{s^2})i + (6,00\frac{m}{s^2})j$; given that two of the three forces are $(30,00N)i + (16,00N)j$ and $-(12,00N)i + (8,00N)j$, What is the third force?

solution

We know that:

$$\sum F_x = ma_x \tag{4.13}$$
$$\sum F_y = ma_y \tag{4.14}$$

and therefore:

$$\sum F_x = (2,00kg) * (-8,00\frac{m}{s^2}) = -16N \tag{4.15}$$
$$\sum F_y = (2,00kg) * (6,00\frac{m}{s^2}) = 12N \tag{4.16}$$
$$F_{x1} + F_{x2} + F_{x3} = -16N \tag{4.17}$$
$$F_{y1} + F_{y2} + F_{y3} = 12N \tag{4.18}$$
$$< replace > \tag{4.19}$$
$$30,0N - 12,00N + F_{x3} = -16N \tag{4.20}$$
$$16,00N + 8,00N + F_{y3} = 12N \tag{4.21}$$
$$F_{x3} = -16N + 12,00N - 30N \tag{4.22}$$
$$F_{y3} = 12N - 8,00N - 16,00N \tag{4.23}$$
$$F_{x3} = -34N \tag{4.24}$$
$$F_{y3} = -12N \tag{4.25}$$

An ice block that weighs 8,00kg is liberated from rest through a ramp with no friction with a length of 1,50m. It slides downwards and reaches a maximum velocity of 2,50 m/s when finishing its slide.

$$\sum_y = N - mg\cos\theta = 0 \tag{4.26}$$
$$\sum_x = -mg\sin\theta = -ma \tag{4.27}$$
$$a = g\sin\theta \tag{4.28}$$

What's the angle of the ramp?

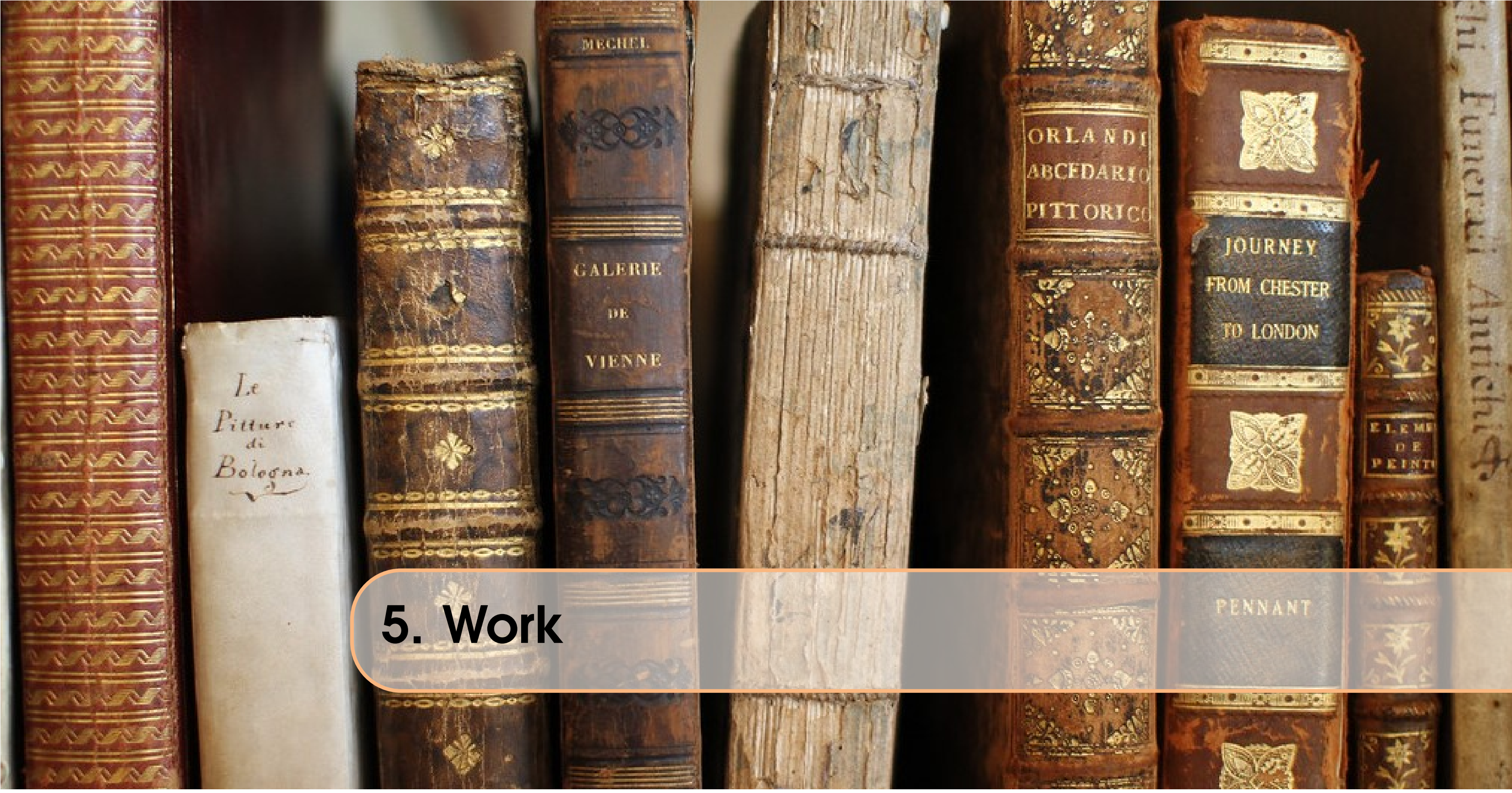
$$v_f^2 = v_i^2 + 2ad \tag{4.29}$$
$$2,5\frac{m}{s}^2 = 0\frac{m}{s} + 2ad \tag{4.30}$$
$$2,5\frac{m}{s}^2 = 2 . 1,50m(9,8\frac{m}{s^2}\sin\theta) \tag{4.31}$$
$$\frac{2,5\frac{m}{s}^2}{2 . 1,50m} = 9,8\frac{m}{s^2}\sin\theta \tag{4.32}$$
$$\theta = \arcsin\left(\frac{2,5\frac{m}{s}^2}{2 . 1,50m . 9,8\frac{m}{s^2}}\right) \tag{4.33}$$
$$\theta = 4,87^o \tag{4.34}$$

What's the speed the ice would have if there was a friction force of 10N rarallel to the surface?

$$\sum_y = N - mg\cos\theta = 0 \tag{4.35}$$

$$\sum_x = -mg\sin\theta = -ma + f_k \tag{4.36}$$

$$\tag{4.37}$$



5. Work

Work is the exertion of forces over a body, that moves from one point to another.

$$\vec{F} = \ forces \tag{5.1}$$

$$s = \ movement \tag{5.2}$$

So, to measure how effective a force is over a moving object, we must first decide:

- Magnitude
- Direction

We must therefore, understand that only components that are parallel to the movement itself are effective.

Given this, we can affirm:

$$W = F_{||}s = F(\cos\phi)s = Fs\cos\phi$$

or, our work is equal to the forces parallel to our movement. We can also derive from it, that work will be a dot product between the vectors acting, set up as $\vec{F}\cdot\vec{s}$. Let's remember a few things about this sort of operation

- The result is a scalar number in $\mathbb{R}$
- we have two ways to calculate it:

$$\begin{cases} \vec{F} = F_x i + F_y j + F_z k = (F_x, F_y, F_z) \\ \vec{s} = \Delta_x i + \Delta_y j + \Delta_z k \end{cases} \quad implies \vec{F}\cdot\vec{s} = F_x\Delta_x + F_y\Delta_y + F_z\Delta_z$$

When we have the components, and:

$$\begin{cases} F = |\vec{F}| \\ s = |\vec{s}| \end{cases} \quad \implies \vec{F}\cdot\vec{s} = Fs\cos\phi$$

When we don't, ϕ is the angle between both vectors.

- $0 \leq \phi \leq 90^o$
- Work is an energy transference, positive if towards the system and negative if coming from it.

5.1 Theorem of Work and Kinetic Energy

An object that moves with constant acceleration and over it
$\vec{W} = F\Delta x$
$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

5.2 Work by variable forces

When a force is variable, then we can affirm:

$$\Delta W_i = F_{x,i}\Delta x_i \tag{5.3}$$
$$W_T = \sum_i F_{x,i}\Delta x_i \tag{5.4}$$
$$\lim_{\Delta x \to 0} \sum_i F_{x,i}\Delta x_i = \int_{x_i}^{x_f} F_x dx \tag{5.5}$$
$$W = \int_{x_i}^{x_f} F_x dx \cos\phi \tag{5.6}$$
$$W = \int_i^f \vec{F} \cdot d\vec{s} \tag{5.7}$$

5.3 Hooke's Law

The work made by an elastic force is:

$$F = k\vec{x} \tag{5.8}$$

And:

$$W_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \tag{5.9}$$

5.4 Curved trajectory work.

$$F = w\tan\theta \tag{5.10}$$
$$s = r\theta \tag{5.11}$$
$$W = mg\tan\theta(R\theta) \tag{5.12}$$

5.5 Potency

When a force is constant, we can say

$$P = \frac{dW}{dt} = \frac{F\cos\phi dx}{dt} = Fcos\phi(\frac{dx}{dt}) \tag{5.13}$$
$$\tag{5.14}$$

5.6 Potential Energy

Potential energy is the associated energy to the configuration of a system of objects that interact between themselves. This can be, for example; expressed as either:

- Gravitational potential energy:

$$\Delta U = -\int_{y_i}^{y_f} -mgdy \tag{5.15}$$

- Elastic potential energy

$$\Delta U = -\int_{x_i}^{x_f} -kxdx \tag{5.16}$$

Such a concept allows us to write, besides our previous definition of the work-energy theorem $W = \Delta k$, A different, alternative definition:

$$W = -\Delta U \tag{5.17}$$

5.7 Examples.

A

We know that:

$$W = 12J \tag{5.18}$$

$$W = \frac{1}{2}k(3x10^{-2}m)^2 - \frac{1}{2}kx_i^2 \tag{5.19}$$

$$24J = k(3x10^{-2}m)^2 \tag{5.20}$$

$$k = 2,7x10^4\frac{N}{m} \tag{5.21}$$

Therefore:

$$F = kx \tag{5.22}$$

$$F = 810N \tag{5.23}$$

$$fr = -kx \tag{5.24}$$

$$F + fr = 0 \tag{5.25}$$

$$W = \frac{1}{2}k(-4x10^{-2})^2 \tag{5.26}$$

$$W = \frac{1}{2}2,7x10^4\frac{N}{m} \cdot 16x10^{-4}m^2 \tag{5.27}$$

$$W = 21,6J \tag{5.28}$$

B:

We can assume:

$$F_x = -[20N + 3\frac{N}{m}x] \tag{5.29}$$

$$\int_0^{6,9m} = Fdx \tag{5.30}$$

Therefore:

$$\int_0^{6,9m} -[20N + 3\frac{N}{m}x]dx = -209.4J \tag{5.31}$$

a 250g block is let fall

$$W_g = mgx \cdot \cos 0 \tag{5.32}$$
$$W_g = mgx \tag{5.33}$$
$$W_g = 0.3J \tag{5.34}$$
$$W_k = \frac{1}{2}kx^2 \tag{5.35}$$
$$W_k = 1,8J \tag{5.36}$$
$$\frac{1}{2}mv^2 + mgx = \frac{1}{2}kx^2 \tag{5.37}$$
$$\frac{1}{2}mv^2 = 1,5J \tag{5.38}$$
$$v = 3,47\frac{m}{s} \tag{5.39}$$

And, for the theoretical of the 'd' literal:

$$v_i = 2(v_0) \tag{5.40}$$
$$\frac{1}{2}m(2v_0)^2 + mgx_2 = \frac{1}{2}kx_2^2 \tag{5.41}$$
$$\frac{1}{2}kx_2^2 - mgx_2 - 2mv_0^2 = 0x_2 = 22,9cm \tag{5.42}$$

a 2kg piece of wood is let slide through figure 1: both curved sides are perfectly smooth, but the bottom is coarse and has a longitude of 30m. The coefficient of kinetic friction is 0.2 against wood. The piece has an initial height 4m above the coarse bottom.

Figure 1

$$mgh = \frac{1}{2}mv_1^2 \tag{5.43}$$
$$W_f = \mu_k mgx \cos 180 \tag{5.44}$$
$$W_f = -\mu_k mgx \tag{5.45}$$
$$W_f = \Delta k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_1^2 \tag{5.46}$$
$$-\mu_k mgx = -\frac{1}{2}m2gh \tag{5.47}$$
$$x = \frac{h}{\mu_k} \tag{5.48}$$

Coarse spring:

$$\tag{5.49}$$

6. Momentum and Collisions

Linear momentum is a vectorial quantity that can be measured as:

$\vec{p} = m\vec{v}$

The second law of Newton can be written in respect to momentum, and

6.1 Collision and Impulse

In a collision, the external force on the body is brief, considerable in magnitude and short in effect over the system, it changes the linear changes the linear momentum of the system suddenly and can be seen as:

$$d\vec{p} = \vec{F}(t)dt \tag{6.1}$$

6.2 Conservation of linear momentum

6.3 Second Law of Newton on a particle system

Let:

$$\vec{F}_{net} = M\vec{a}_{c.o.m} \tag{6.2}$$

We can then affirm:

- Net force will be the summatory of all external forces acting over the body
- M is the total mass of the system; no mass gets out of the system if the system is closed
- $\vec{a}_{c.o.m}$ is the acceleration of the center of mass in the system.

6.4 Examples

A ball weighing 1.2 kg is dropped vertically on a vertically on a floor, reaching the ground the ground with a speed of 25m/s. The ball bounces with an initial speed of 10 m/s a) What impulse acts on the ball during the contact? b) If the ball is in contact with the ground for 0.020 s what is the magnitude of the average force on the floor from the ball?

$$p_i = mv_i j \tag{6.3}$$

$$p_f = mv_f j \tag{6.4}$$

$$\Delta p = p_f - p_i = (mv_f - (-mv_i)) \tag{6.5}$$

$$\Delta p = p_f - p_i = mv_f + mv_i = m(v_f + v_i) \tag{6.6}$$

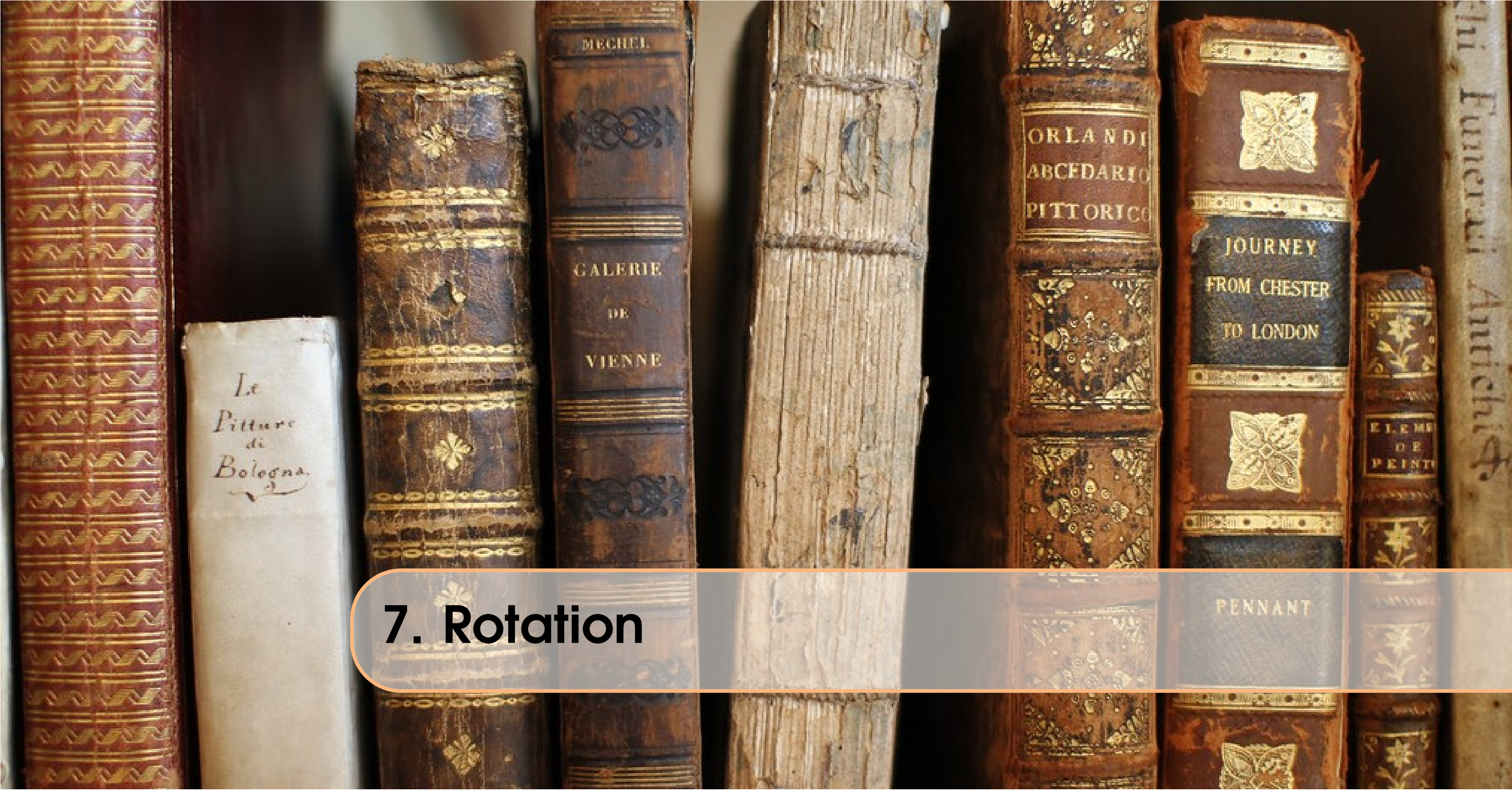
$$J = \Delta p = m(v_f + v_i) \tag{6.7}$$

$$J = \int_{t_i}^{t_f} F dt \tag{6.8}$$

$$= F(t_f - t_i) \tag{6.9}$$

$$F = \frac{J}{t_f - t_i} \Delta p = 42kg\frac{m}{s} \tag{6.10}$$

$$F = 2100N \tag{6.11}$$



7. Rotation

A rigid body can rotate and we can measure the way it does.

$v_i = \omega r_1$

where ω is the angular velocity and

7.1 Energy in rotating bodies

We can model energy in a rotating body as:

$$K = \frac{1}{2}\omega^2 \sum_i m_i (r_i)^2 \tag{7.1}$$

This can also be written as:

$$K = \frac{1}{2} I \omega^2 \tag{7.2}$$

Because the sum can be rewritten as the Inertia momentum of the object we're modeling. However, we might also model it as an integral, such as:

$$I = \int_0^r r^2 dm \tag{7.3}$$

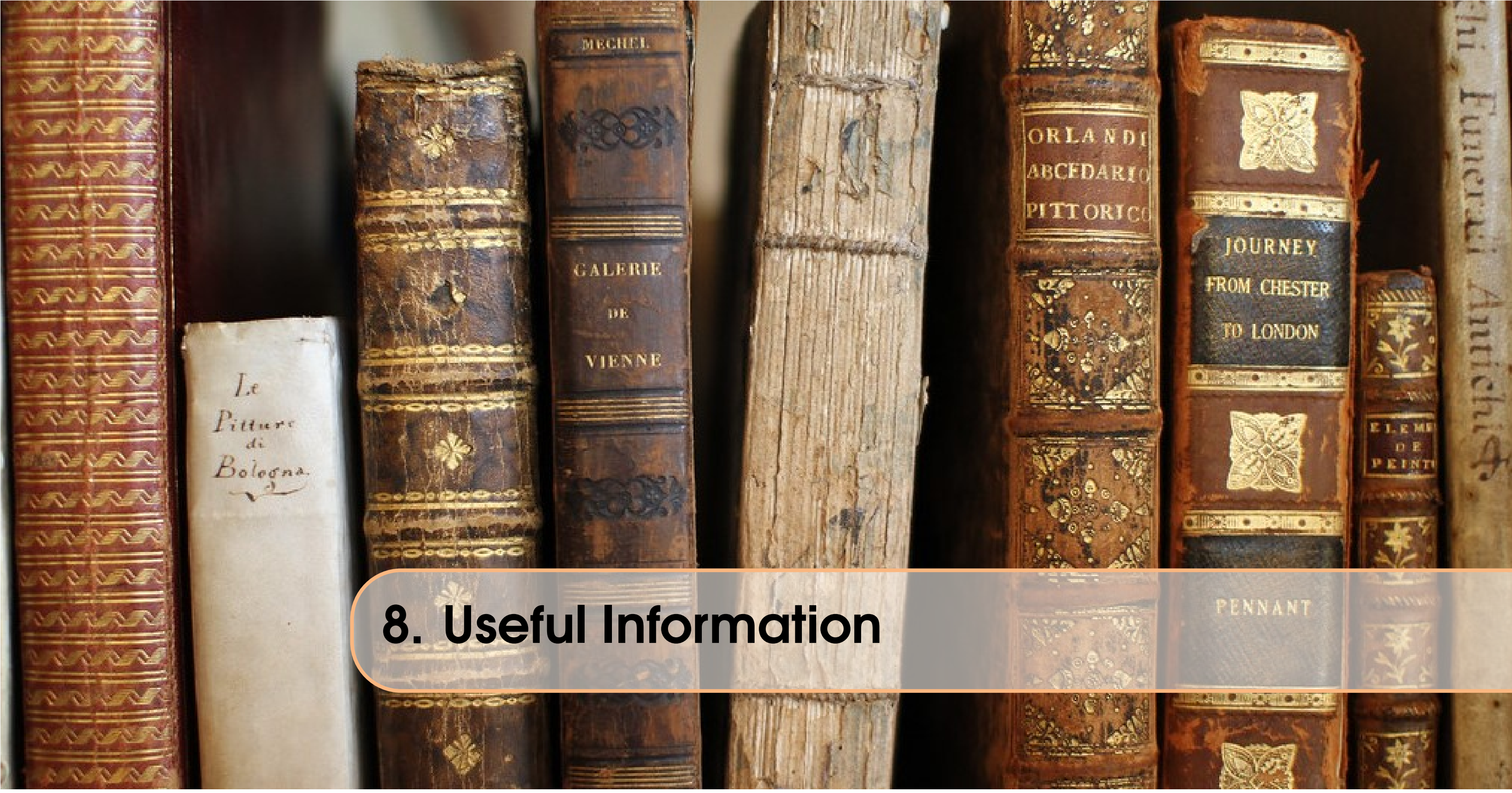
the units should always eng up being for mass and length squared, such as:

$$kg \cdot m^2$$

7.2 Parrallel axis theorem

$$I = I_{com} + Mh^2 \tag{7.4}$$

where 'h' is the distance to the center.



8. Useful Information

8.1 Constants

- Speed of light:
  $299'792.458\frac{km}{s}$, can be approximated to $3x10^8 m/s$ when you're not concerned with precision.
- Gravity in earth:
  $9,81\frac{km}{s^2}$ (A bit oversimplified, but for now it should suffice)

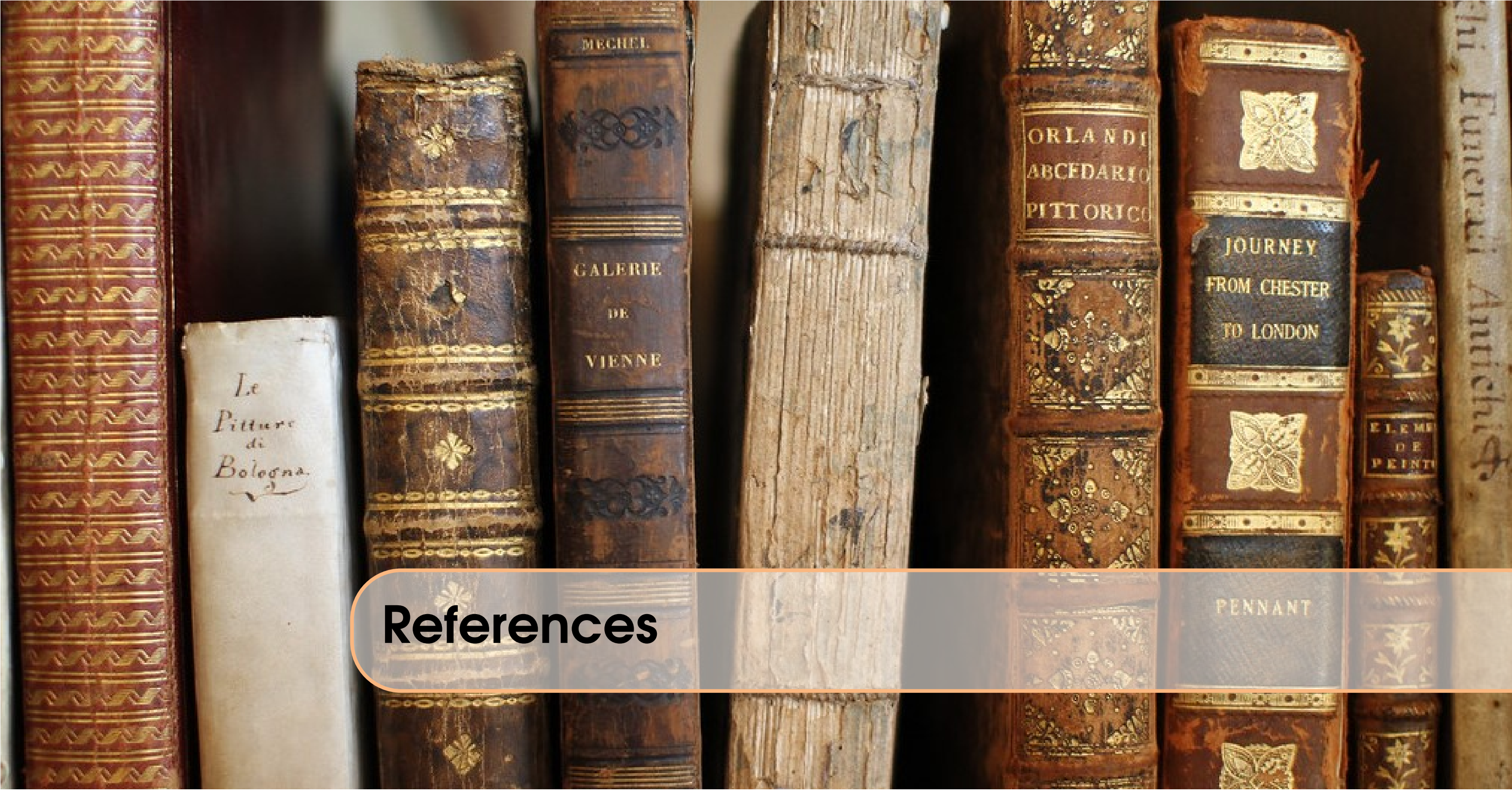
8.2 Formulas

- Newton.
  $1N = 1\frac{kg\cdot m}{s}$

8.3 Definitions

- Delta:
  noted as Δ, it indicates change in a variable, taking x as the variable that changes, x_0 as it initial state and x_f as the final one, we can indicate how much it changes as follows:

$$\Delta x = x_f - x_0$$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References