Physics - First Tome.

(A friendly approach.)

ILM

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Why taking notes like this?

Well damn, i just want to have notes that i can read easily, are non-judgemental and let you learn at your own page, this is by no means better than a specialized textbook, made by an actual professional with a degree in physics or some other natural science that just so happens to involve physics.

Can i use this text in some way?

I mean, as long as you're conscious of it's limitations and are able to work around them, i see no reason why i would get angry about you using this text for your own means, just remember to do no harm.

There's a mistake in this book! What do i do?

Tell me what it is, just let me know and maybe even correct it yourself, i have no reservations on making changes in case it happens to be necessary or otherwise useful.

Can i share this?

Go ahead! These notes are open source, everybody should be able to access them i think.

(Shoutout to Yenny Hernandez, who i studied physics with @Uniandes.)

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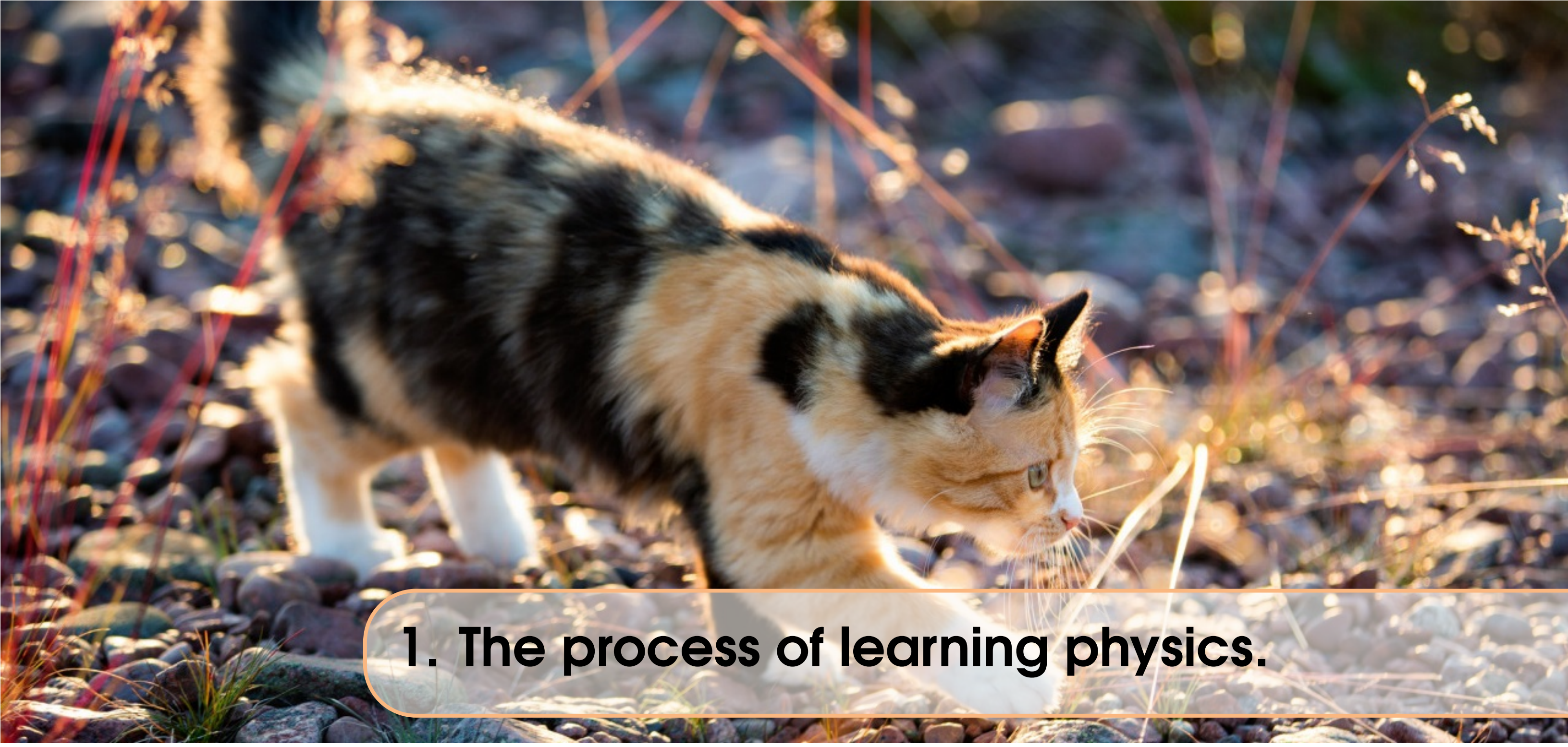


1. The process of learning physics.

1.1 So, how AND WHY do we learn physics, anyways?

(...like, really, why?)

Eh, it depends.
In reality, the process of learning physics is also the process of learning how to observe a phenomenon in a bunch of ways. It's an experimental science that is concerned with the study of energy, materials and their mutual interactions. We can explain a bunch of things regarding the way our universe works with it.

In any case, learning physics is not a marathon, cramping before an exam stresses me out, and it probably does to you too. And we should try to learn it bit by bit, going through topics slowly and surely. Rome wasn't built in a day, and neither should you try to learn a solid understanding in one.

When learning physics, remember:

- **Do exercises.** You shouldn't try to just learn by reading the notes of some other guy, you might not even know me. It's
- **Be critical of your own solution.** you won't always have problems right your first time around. If something doesn't seem to make sense, it's because it is probably not right. (you probably shouldn't be getting a negative velocity when trying to calculate a bike going in a straight line, for example)
- **Understand the topics enough to explain them to someone else.** If you can teach physics to someone else, you will probably understand it a lot better yourself, give it a try!
- **Be prepared.** Check your notes, do your homework, and try to study by yourself. Prepare the things that you need to learn before learning them becomes a point of stress.
- **You're not alone.** Look for help when you need it, it's not shameful.

1.2 Important considerations.

2. Measurements.

2.1 Uncertainty

There is always a level of uncertainty when measuring an object, given by the instrument of measurement we use, an atomic clock is not the same as your uncle's watch, and even though they both measure time, there's a certain level of uncertainty caused by the device, keep it in mind when doing experimental work. We can express this with the symbol ' $\pm$ '

for example, if we had a milimeter of uncertainty on a measurement of 9,5 cm, we could indicate it as such:

$$l = 9,5 \pm 0,1cm \tag{2.1}$$

2.2 Scientific Notation

It is usual to measure both inmense and infimal quantities of mass, time, or any other thing. For that, we shall apply the scientific notation, both on this book and elsewhere.

EXERCISES

How many years older will you be in a thousand million seconds?

We'll start by measuring how many seconds there are in a year.

$\frac{1min}{60s} * \frac{1h}{60min} * \frac{1d}{24h} * \frac{1yr.}{365d} = 1x10^9 s$

And now we'll take our given time on years

$time = 1,000,000s * 1,000s = 1000x10^6 s$

Answer: $31 yrs.$

How many nanoseconds does the speed of light need to travel 1ft in the void?

$1ft = 30,48cm$

Even though not specifically notified in the exercise, we must remember that:

$$1m = 100cm \tag{2.2}$$

2.3 Types of units

In the realm of physics, there is a bunch of ways to categorize units, be it by what do they measure or how do they measure it

2.4 Scalar units

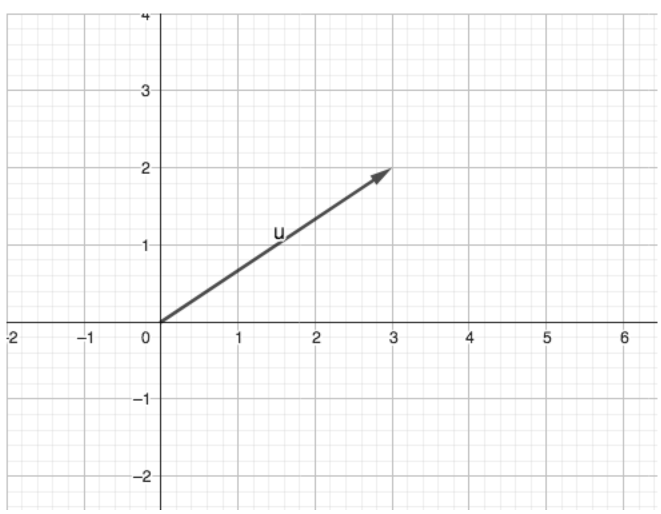
A scalar unit is defined as an unit with magnitude, but no direction. Such measurements aren't really concerned with being tied with a specific force acting over a particle.

Examples of such units include:

- Natural numbers.
- Constants.
- Acceleration

2.5 Vectors

A vector is a measurement with both a magnitude and a direction. that can exist on a specific set of dimensions. On this course, we won't be concerning ourselves with hyperplanes and might sparingly cover 3-dimensional planes, they would both be useful for you to learn eventually, so i encourage you to learn about them on your own accord, however. Our examples will be two-dimensional, for a simple, introductory example, consider the following vector; $\vec{A} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$:



Vector with x coordinates '3' and y coordinates '2'

This is what we call a cartesian vector, for it's magnitudes are defined by the coordinates in a cartesian plane, however, we can turn it into a polar vector, measured by it's magnitude and angle, through the following formulas:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \tag{2.3}$$

$$\tan\theta = \frac{A_y}{A_x} \tag{2.4}$$

Exercise: Turning our example vector 'A' into a Polar vector.
Magnitude

$$|\vec{A}| = \sqrt{3^2+2^2}|\vec{A}| = \sqrt{9+4} = \sqrt{12} = 2\sqrt{3} \tag{2.5}$$

Angle

$$\tan\theta = \frac{2}{3} \tag{2.6}$$

$$\theta = \tan^{-}1(\frac{2}{3}) \tag{2.7}$$

$$\theta = 33,69^o \tag{2.8}$$

Equally, we can turn a polar vector into a cartesian one.

So, given an angle θ and a magnitude $|\vec{A}|$, we can suppouse the measurements of a two-directional vector as:

$$A_x = |\vec{A}|\cos\theta \tag{2.9}$$

$$A_y = |\vec{A}|\sin\theta \tag{2.10}$$

And given this, we can consider the vector '$\vec{A}$' as, $\vec{A} = A_x i + A_y j$, where 'i' and 'j', are what we're going to call a **unit vector**, a vector in a specific direction that has a value of 1. (this is considered an identity value for multiplication operations, and lets us do some vector sum operations more intuitively)

Exercise: Turning our example vector 'A' back into a cartesian vector.

$$A_x = |2\sqrt{3}|\cos 33,69^o = 2,88 \approxeq 3 \tag{2.11}$$

$$A_y = |2\sqrt{3}|\sin 33,69^o = 1,92 \approxeq 2 \tag{2.12}$$

As it might be evident, the conversion isn't exact, this happens because of the angle not being an exact conversion, this will happen when you disregard part of a value for whatever reason. The more exact of a measurement you keep, the less uncertainty you will end up with.

2.5.1 Vector sum

We can take any $\vec{a}$ and $\vec{b}$ vectors on the same space and add them to each other in the form:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{pmatrix} \tag{2.13}$$

Such form remains in the case we can do subtraction, which is expressed on the equation:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1-b_1 \\ a_2-b_2 \\ a_3-b_3 \end{pmatrix} \tag{2.14}$$

This kind of operations have certain properties, shown as:

$$(\alpha+\beta)\vec{[v]} = \alpha\vec{v}+\beta\vec{v} \tag{2.15}$$

$$\vec{v}*1 = \vec{v} \tag{2.16}$$

$$\vec{v}*\vec{0} = \vec{0} \tag{2.17}$$

$$\beta\vec{v} = \begin{pmatrix}\beta a_1\\ \beta a_2\\ \beta a_3\end{pmatrix} \tag{2.18}$$

Two vectors $\vec{a}$ and $\vec{b}$ are equal if and only if:

$$\begin{cases}\vec{a}\exists\mathbb{R}^3\\ \vec{b}\exists\mathbb{R}^3\end{cases} \implies \begin{pmatrix}a_1=b_1\\ a_2=b_2\\ a_3=b_3\end{pmatrix} \textit{ note: this can be generalized to 'n' dimensions larger than 0} \tag{2.19}$$

in either case, $\vec{0}$ is the identity of the operation, therefore:

$$\vec{a}+\vec{0} = \vec{a} \tag{2.20}$$

2.5.2 Scalar/dot product

We can multiply vectors between each other with the following formula:

$$|\vec{A}|\cdot|\vec{B}| = |\vec{A}||\vec{B}|\cos\theta$$

We can also write it as such for cartesian vectors:

$$A_xB_x+A_yB_y+A_zB_z$$

This is a commutative operation, that won't be affected by neither A nor B's

2.5.3 Cross/Vectorial product

We can define the formula for the cross product of two vectors in a 3-dimensional space as:

$$\vec{A}x\vec{B} = \begin{pmatrix}(A_y-B_z-A_zB_y)i\\ (A_xB_z-B_xA_z)j\\ (A_xB_y-B_xA_y)k\end{pmatrix} \tag{2.21}$$

This will come handy when studying Torque;

2.5.4 Solved Exercises

Vector Sum

An espeologist explores a cave and follows a

2.5.5 Dot/Cross Product

2.5.6 Scientific Notation

how many nanoseconds does a ray of light require to travel 3 meters in the void?

Solution

Estimate how many gallons of gasoline are consumed by private cars in Colombia in a year. Assume that on average there is one car for every 10 inhabitants and that on average each car travels 10^4 km per year, with an average engine efficiency of 25 km/gallon.

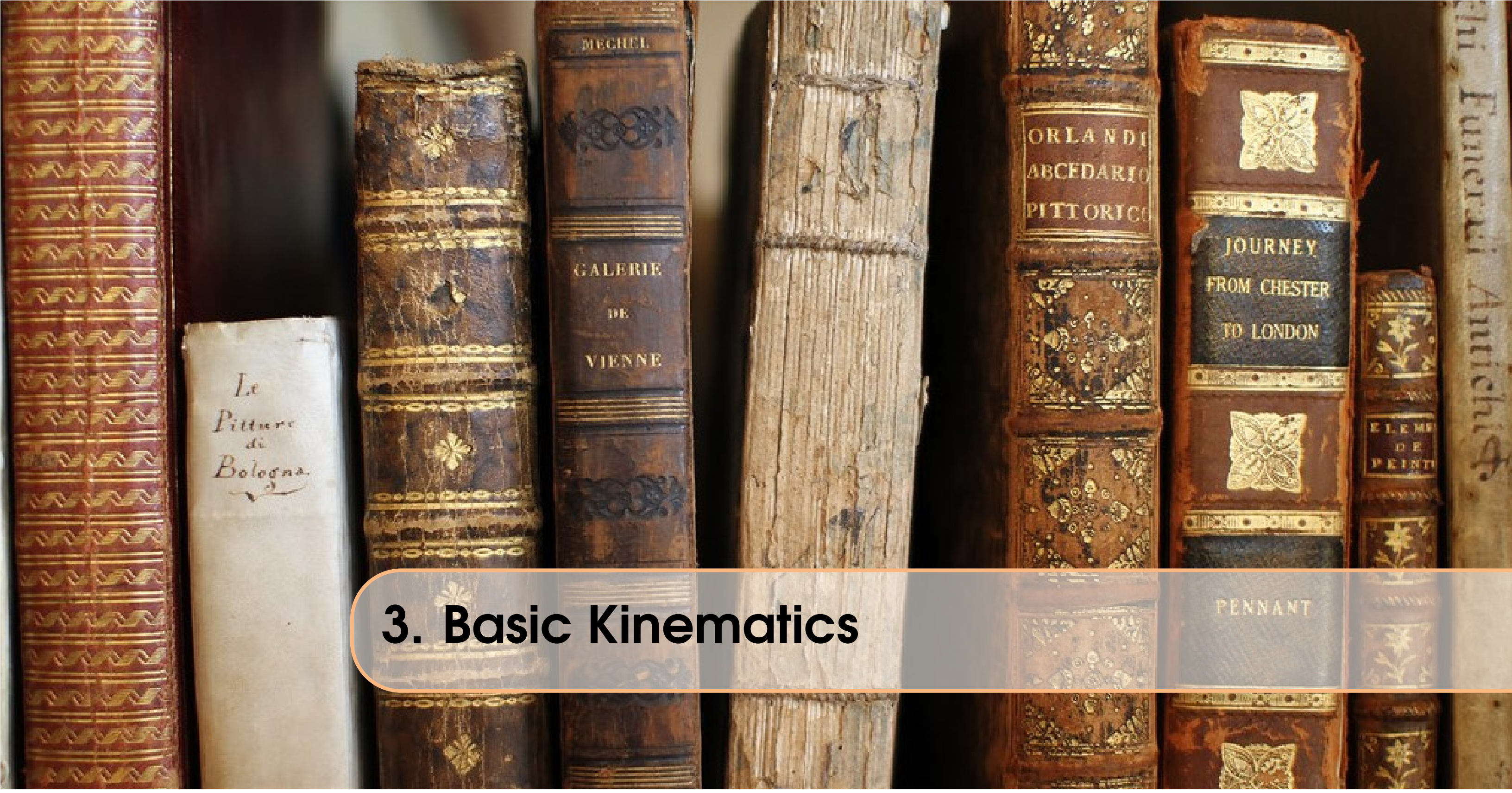
Solution.

$$\text{Population of Colombia as of 2020} = 51520000 \tag{2.22}$$

$$\text{Number of cars} = 51520000/10 = 5152000\ cars \tag{2.23}$$

$$\text{Kilometers traveled} = 5152000\ cars * 10^4 \frac{km}{yr} = 51520000000 \frac{km}{yr} = 5,152x10^{10} \frac{km}{yr} \tag{2.24}$$

$$\text{Gallons per year} = \frac{5,152x10^{10} \frac{km}{yr}}{25 \frac{km}{gallon}} = 2060800000 \frac{\text{gallon}}{yr} = 2,0608x10^9 \frac{\text{gallon}}{yr} \tag{2.25}$$



3. Basic Kinematics

3.1 One-Dimensional Movement

One-dimensional movement is probably the most basic kind of movement, as unbound by the expectations of more than two directions to move to, it allows itself to simply define the route from a point 'A' to a point 'B' in a straight line.

Such movement can be defined as:

$$\Delta x = x_2 - x_1 \tag{3.1}$$

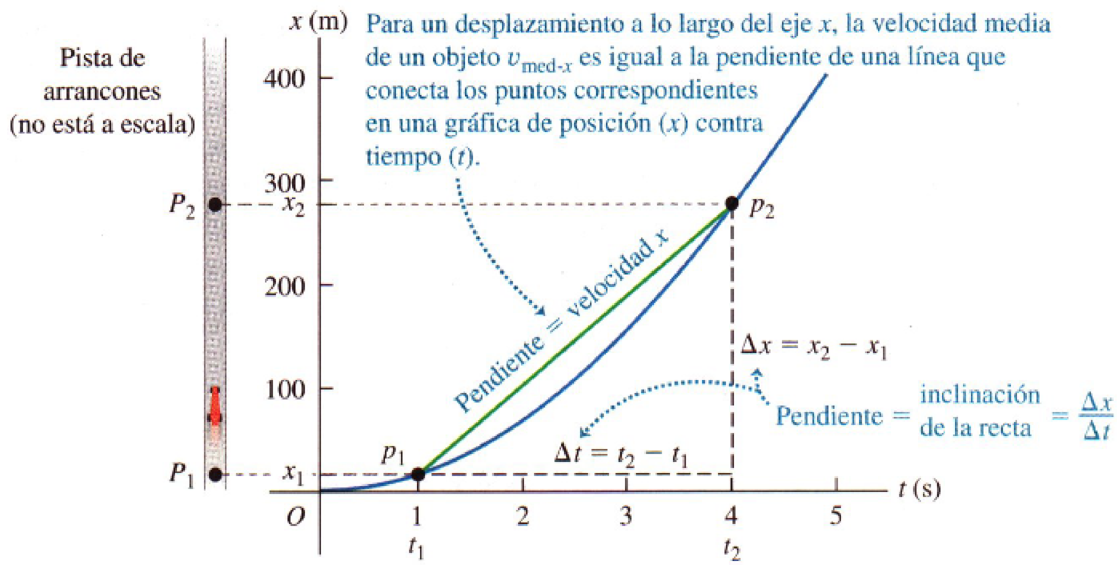
medium velocity on this kind of movement can be calculated as:

$$v_{med} = \frac{\Delta x}{\Delta y} = \frac{x_2 - x_1}{t_2 - t_1} \tag{3.2}$$

Keep in mind, we can define negative velocity in such systems to move in a negative direction (i.e moving from B=7 to A=3, for example.) and besides that **This is not the same as speed, because speed is concerned with how fast the object moves while this is concerned with how fast the object arrives somewhere, and that's not the same.**

3.2 instantaneous speed and velocity

It should now be evident that we can assume a straight line that can be interpreted as the velocity of an object, as we could see in this two-dimensional graph.



We define the instantaneous velocity of an object as a derivative of the form:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{3.3}$$

The simplest formula for such a calculation is, however:

$$x = x_0 + vt \tag{3.4}$$

3.3 Acceleration

Of course, not all objects will move at a constant pace, most times there will be a change in speed happening at some moment in their movement. This is a scalar variable that can be used to calculate the overall distance when taking into account such changes on movement. We can calculate the acceleration of an object as follows:

$$a = \frac{\Delta v}{\Delta t} \tag{3.5}$$

we can integrate distance as a function of acceleration as it follows, as well:

$$\int_t^0 v\,dt = \int_x^{x_0} dx \textit{ Note: we can integrate the result of this equation to prove correctness.} \tag{3.6}$$

Through a mathematical proof that

3.3.1 Constant Acceleration

An object can be said to have constant acceleration when acceleration is not a variable, but a constant. When working on such a system we can affirm velocity as:

$$at = v - v_0 \tag{3.7}$$

and we can indicate distance as:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{3.8}$$

From these two we can gather the following formulas:

- $v = v_0 + at$
- $t = \frac{v - v_0}{a}$
- $v^2 = v_0^2 + 2a(x - x_0) If\ we\ don't\ know\ time$
- $x - x_0 = (\frac{v + v_0}{2})t If\ we\ don't\ know\ acceleration$

3.4 Two-dimensional movement

Again, this is a pretty self explainatory title. As it just refers to the representation of movement in more than a two independent axis. We never were living in a straight line anyways so it shouldn't come as a surprise to you that physics doesn't live there either.

We can imagine two vectors to indicate the beginning and end of our specified movement, and yes, this could potentially include (0,0), in any case, we can write them as:

$$\vec{r}_i = x_i i + y_i j \tag{3.9}$$

$$\vec{r}_f = x_f i + y_f j \tag{3.10}$$

To describe a movement in two dimensions with these vectors, we can use the following equations:

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \tag{3.11}$$

$$\Delta\vec{r} = (x_f - x_i)i + (y_f - y_i)j \tag{3.12}$$

we can also affirm:

$$|\vec{r}_i| = \sqrt{x_i^2 + y_i^2} \tag{3.13}$$

$$\theta = \arctan(\frac{y_i}{x_i}) \tag{3.14}$$

$$\vec{r}_i = |\vec{r}_i| \cos\theta i + |\vec{r}_i| \sin\theta j \tag{3.15}$$

These are just vectorial operations, and they apply for either vector, nothing really changes from how we already expected vectors to work.

3.4.1 Projectile movement.

We can assume, on this version of a two-dimensional movement, that:

$$x = x_0 + v_0 \cos\theta t \tag{3.16}$$
$$v_x = v_0 \cos\theta_0 = v_0 x \tag{3.17}$$
$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2 \tag{3.18}$$
$$v_y = v_0 \sin\theta - g t \tag{3.19}$$
$$v_y^2 = v_0^2 \sin^2\theta_0 - 2g(y - y_0) \tag{3.20}$$

We can also assume an 'R' distance that will be the maximum possible distance in the x axis with 'y' as 0. We can get it as:

$$v_0 \sin\theta_0 = g t \tag{3.21}$$
$$t = \frac{v_0 \sin\theta_0}{g} \tag{3.22}$$
$$R = 0 + v_0 \cos\theta_0 \cdot \frac{2 v_0 \sin\theta_0}{g} \tag{3.23}$$
$$R = \frac{v_0^2 \sin(2\theta_0)}{g} \tag{3.24}$$

This is the maximum possible distance that you can get for a displacement given certain parameters to be replaced in such a formula. Keep in mind that when θ is 45 degrees, the value of 'R' is at its peak for that specific configuration, so given a question regarding angles, the closest to 45 it is, the furthest it will make it. A few other useful formulas are:

$$x = x_0 + v_0 t \tag{3.25}$$
$$y = h + v_0 \sin\theta_0 t - \frac{1}{2} g t^2 \tag{3.26}$$
$$t = \sqrt{\frac{2h}{g}} \tag{3.27}$$
$$x = v_0 \sqrt{\frac{2h}{g}} \tag{3.28}$$

When working on an elevated end point, we can imagine that, given a distance 'D' and an elevated endpont 'H':

$$x = x_0 + v_0 \cos \theta_0 t \tag{3.29}$$

$$y = y_0 + v_0 \sin \theta_0 t - \frac{1}{2} g t^2 \tag{3.30}$$

$$v_y^2 + v^2 \sin^2 \theta_0 - 2g(y - y_0) \tag{3.31}$$

$$D = v_0 \cos \theta_0 t \tag{3.32}$$

$$t = \frac{D}{V_0 \cos \theta_0} \tag{3.33}$$

$$H = \frac{v_0 \sin \theta_0 D}{v_0 \cos \theta_0} - \frac{1}{2} g (\frac{D^2}{v_0^2 \cos^2 \theta_0}) \tag{3.34}$$

$$H = D \tan \theta_0 - \frac{1}{2} g \frac{D^2}{v_0^2 \cos^2 \theta_0} \tag{3.35}$$

$$V_0^2 = \frac{g D^2}{2 \cos^2 \theta_0 (D \tan \theta_0 - H)} \tag{3.36}$$

What's the secret on getting these problems right?

Writing all of the possible equations and looking at:

- What do i have?
- What do we need?
- How can i rewrite the equations that i have in order to get what i need?

Circular Movement

$$\Delta s = R \Delta \phi \tag{3.37}$$

$$\frac{\Delta s}{R} = \frac{|\Delta v|}{v_1} \tag{3.38}$$

$$|\Delta v| = v_1 \Delta \phi \tag{3.39}$$

$$a = \frac{v^2}{r} \tag{3.40}$$

$$v = \frac{2 \pi R}{T} \tag{3.41}$$

$$T = \frac{2 \pi R}{v} \tag{3.42}$$

3.5 Exercises

One-dimensional movement

3.5.1 Acceleration

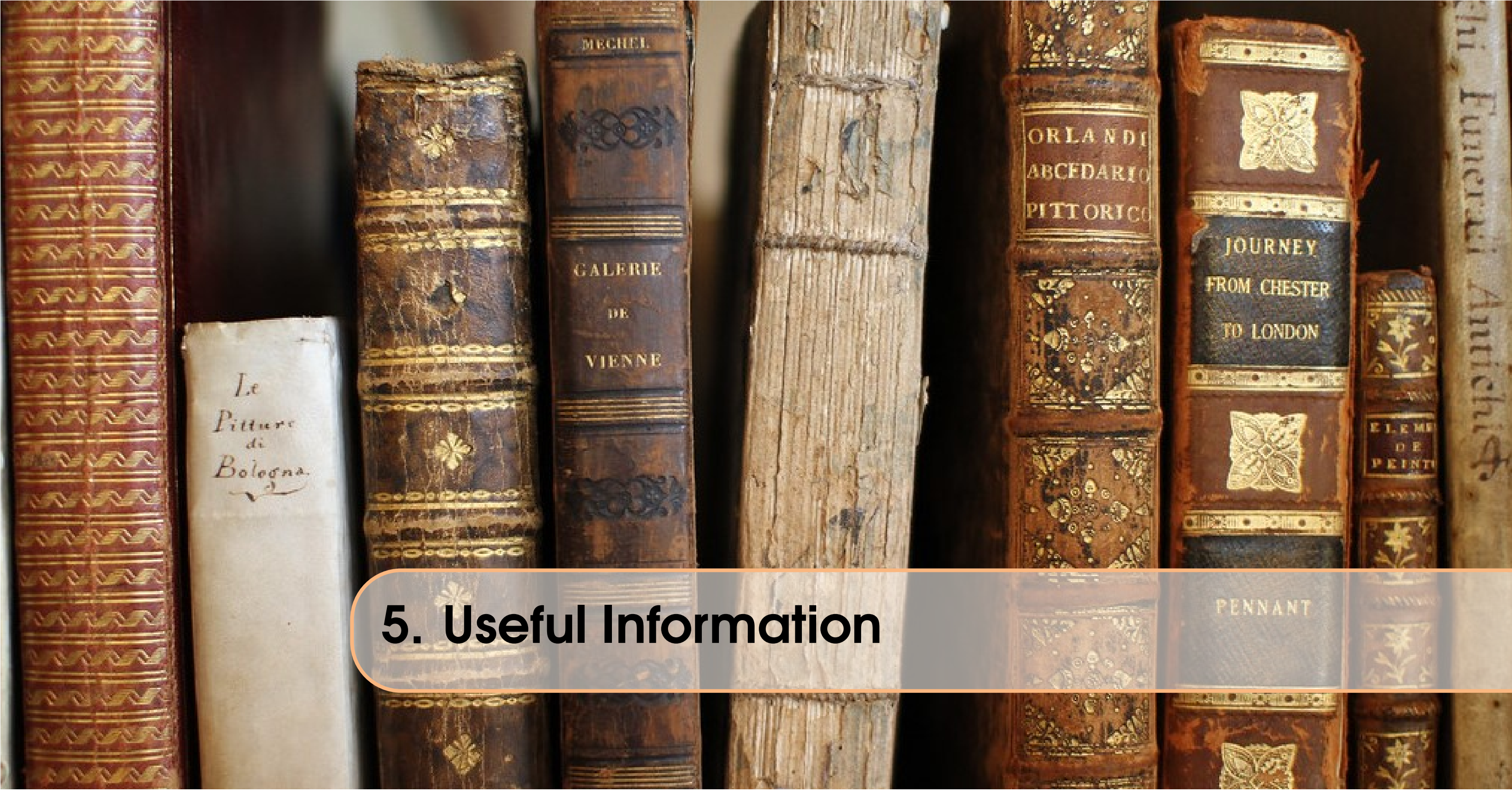
what's the maximum possible height of a marker being thrown at a velocity of $0,5 \frac{m}{s}$?

We know that:

- gravity (acceleration): $-9,8 \frac{m}{s^2}$
- initial position: 0
- initial velocity = $0,5 \frac{m}{s}$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{3.43}$$
$$0 = v_0^2 - 2g(y - 0) \tag{3.44}$$
$$v_0^2 = 2gy \tag{3.45}$$
$$y = \frac{v_0^2}{2g} \tag{3.46}$$
$$y = -\frac{0,5\frac{m}{s}^2}{2(9,8\frac{m}{s^2})} \tag{3.47}$$

4. Two-dimensional Movement



5. Useful Information

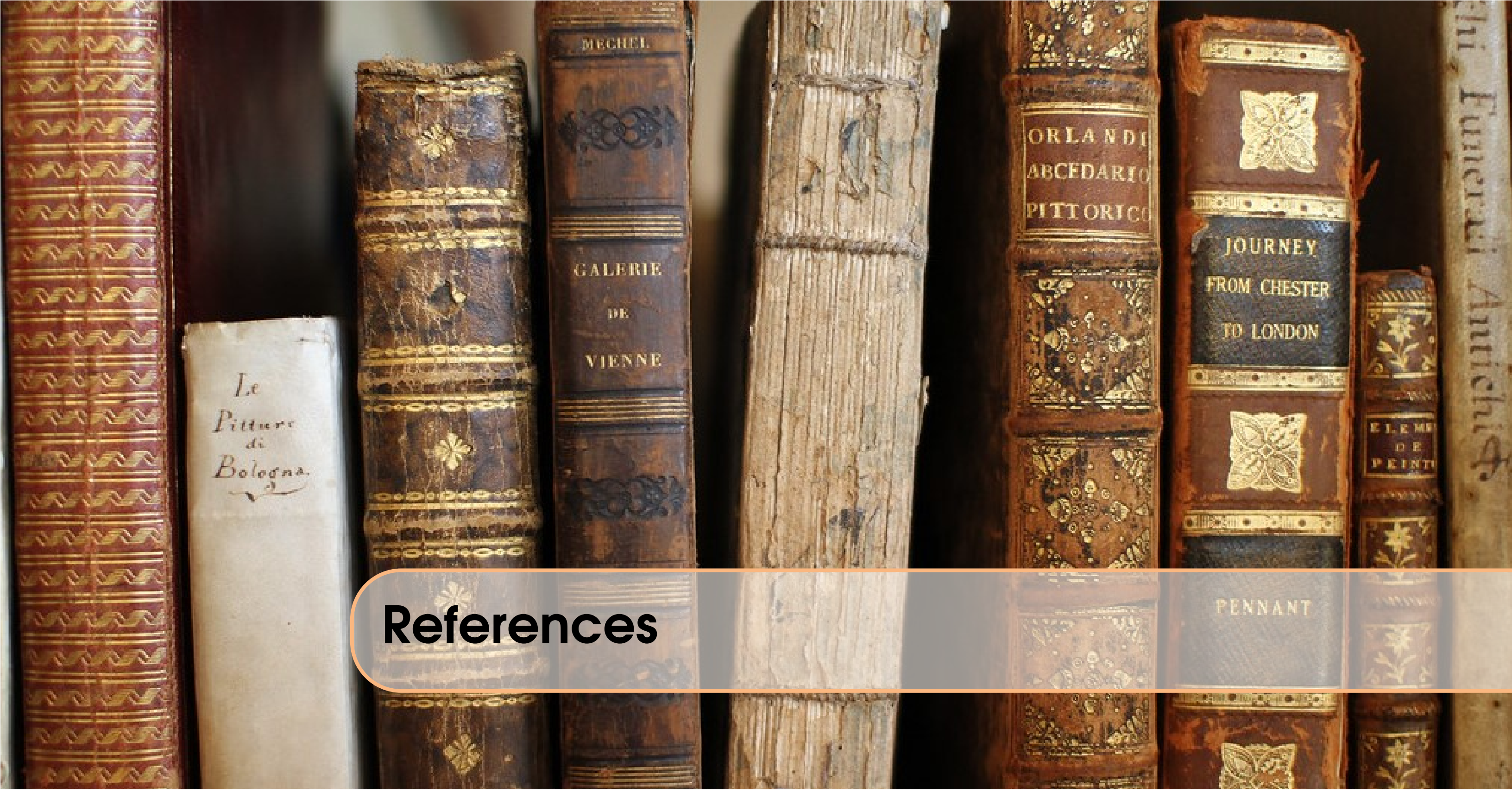
5.1 Constants

- Speed of light:
  $299'792.458\frac{km}{s}$, can be approximated to $3x10^8 m/s$ when you're not concerned with precision.
- Gravity in earth:
  $9,81\frac{km}{s^2}$ (A bit oversimplified, but for now it should suffice)

5.2 Definitions

- Delta:
  noted as Δ, it indicates change in a variable, taking x as the variable that changes, x_0 as it initial state and x_f as the final one, we can indicate how much it changes as follows:

$$\Delta x = x_f - x_0$$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References