Probability and Statistics
In a friendly manner

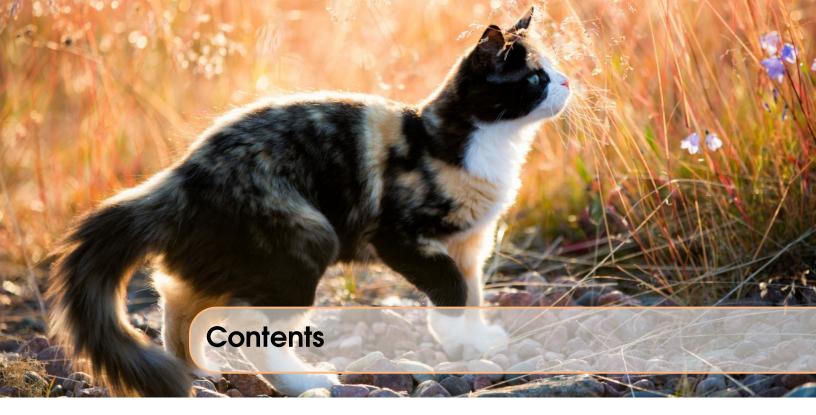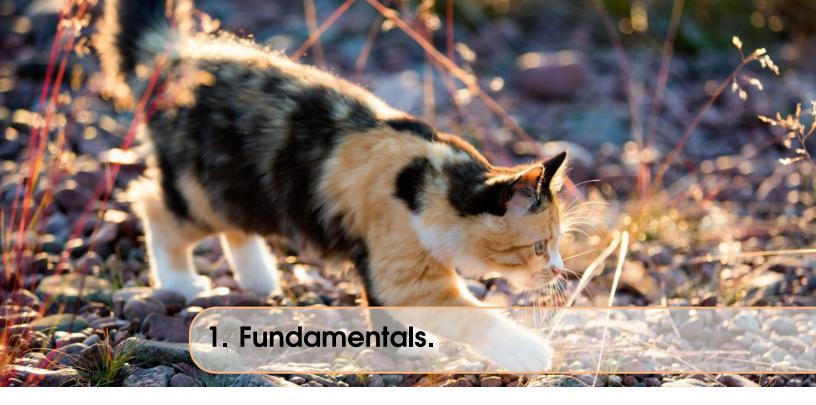Jaime Andres Torres B.
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Why take notes like this?

I dunno, it's cool to do so i guess.

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Contents

1. Fundamentals.

1.1   Experiments and Events

On the field of probability, we'll define an experiment as the process that generates a set of data. But also more broadly, it is a process that generates any data that we might find an observation or result with. We denote an experiment with the letter ζ, examples of experiments include:

- ζ 1: The number of rolls of a dice before landing on the number 6
- ζ 2: The number of calls from Bogota to New York being registered on the next hour, starting instantly.
- ζ 3: The result of a hipotecary credit (approved or rejected)
- ζ 4: The time between now and the next earthquake measured with 5.0 or more on Richter's scale somewhere in south america.

As we can see, a random experiment can cover a very broad category of measurements when we really think about it, however, they all show three particular characteristics, they must be present at all times for a random experiment to be able to have itself called as random:

- **Randomness:** We must not know the result of the experiment before it concludes
- **Unique results:** Every experiment must produce an unique result per instance.
- **Determinable results:** The result of an experiment must be able to be noticed and determined or otherwise categorized.

1.1.1   Sample space

A sample space, denoted as Ω or 'S' in most literature, is a set of possible results in a random experiment ζ. Such an experiment ζ can have multiple sample spaces Ω. Such a space can be both finite and infinite, in case of it being infinite, we must be able to distinguish wether or not it is numerable, in case it is we will call such a space 'discrete' otherwise, we call it 'continuous'.

For an example on how this could be seen, let's take into consideration the following example:

ζ: The number of times a dice is rolled with a number greater than 3 that is odd

We can find two distinct sample spaces in this example, namedly:

$$\begin{cases} \Omega_1 : \{1,2,3,4,5,6\} \\ \Omega_2 : \{\text{even, odd}\} \end{cases}$$

both of these spaces are described by ennumerating, however, we can also describe sample spaces via rules and mathematical notation, for example, we could describe our first sample space in the former example as:

$$\Omega_1 : \{x | 1 \leq x \leq 6\}$$

A sample space is not necessarily tied to a data variable, and a data variable might be implied more than once on a dataset, take this thought experiment, for example:

Ex. 1

In a probability and statistics class of 56 students, one student is selected as a representative to the Student Council and another as an alternate. Thus, the result of the experiment can be represented by the following tuple:

(president, alternate)

What's the size of the sample space here presented?

Solution:

Even though the number of students is just 56, this doesn't mean the sample size is also 56, this is because we have a combination where we use the same dataset twice on two different random experiments, noted as:

$$\begin{cases} \zeta_1 : \text{The election of a president.} \\ \zeta_2 : \text{The election of an alternate.} \end{cases}$$

Both of them will have similar, but ultimately different sizes and the final result will be a combination between the two. we could think of it as the total number of students, multiplied by the total number of students minus one (since we have to select a president and the president themselves cannot be the alternate), therefore:

$$len(S) = 56 * (56 - 1) \tag{1.1}$$
$$len(S) = 56 * 55 \tag{1.2}$$
$$len(S) = 3080 \tag{1.3}$$

the total number of possible results is 3080

1.2   Definitions.

1. **Union**
   Between a set A and a set B, we can say an Union is the collection of items that can be found in either set. it's marked as $A \bigcup B$

2. **Intersection**
   Between a set A and a set B, we can say an intersection is the collection of items that can be found in both sets. it's marked as $A \bigcap B$

3. **Compliment**
   Noted as A^c or A' on a set 'A', a compliment is in essence the inverse of said set. Every item not in A will be part of its compliment.
4. **Event**
   An event is defined as the result of a random experiment. Once the experiment ζ is finished we denote the result as a subset 'B' of the sample space defined for ζ
5. **Impossible Event**
   An event with no chance of ocurring with the parameters given. Such an event should be noted as $P() = \varnothing = 0$

2. Counting Methods.

We can count the elements in a set with different techniques, depending on what our objective is with this

References