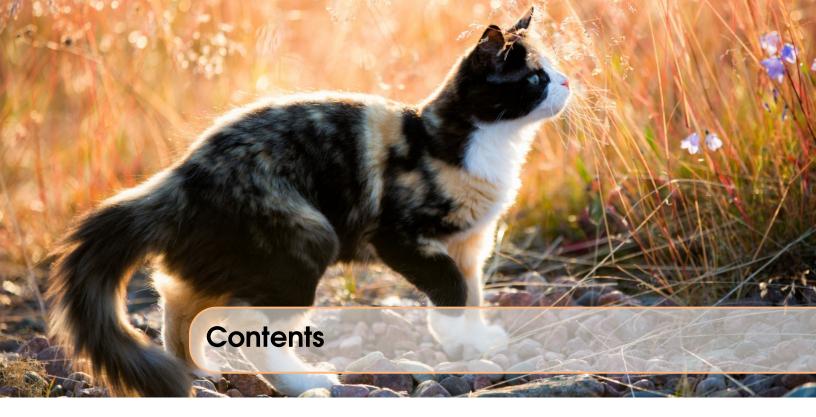


Vectorial Calculus -

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Go big or go home.

Vectorial calculus is what the title says pretty much, the act of using methods proper to calculus on vectorial spaces, for the topic of this class generally referring to merely 3-dimensional ones, at the end of this book you should be able to:

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2.1 Vectors on a three-dimensional space.

given an \mathbb{R}^3 space and a point in that space P=(a,b,c), we can describe a vector by either connecting the point P to another point Q, or by assuming the origin of this space (point (0,0,0)), this is a mathematical object with both a direction and a magnitude. The direction is given by an angle and the magnitude is given by $\sqrt{a_1^2+a_2^2+a_3^2}$

As an example, let's assume the vector given by P = (1, 2, 1):



Vector formed by P = (1,2,1)

For this vector, we can calculate the magnitude by replacing the vectorial components by the magnitudes of the individual directions, resulting in:

Note, in this course we will be mostly only concerned with \mathbb{R}^3

2.1.1 Addition and Subtraction

We can take any \vec{a} and \vec{b} vectors on the same space and add them to each other in the form:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$$
 (2.1)

Such form remains in the case we can do subtraction, which is expressed on the equation:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix}$$
 (2.2)

This kind of operations have certain properties, shown as:

$$(\alpha + \beta)|v| = \alpha v + \beta v \tag{2.3}$$

$$\vec{v} * 1 = \vec{v} \tag{2.4}$$

$$\vec{v} * \vec{0} = \vec{0} \tag{2.5}$$

$$\beta \vec{v} = \begin{pmatrix} \beta a_1 \\ \beta a_2 \\ \beta a_3 \end{pmatrix} \tag{2.6}$$

Two vectors \vec{a} and \vec{b} are equal if and only if:

$$\begin{cases} \vec{a} \exists \mathbb{R}^3 \\ \vec{b} \exists \mathbb{R}^3 \end{cases} \implies \begin{pmatrix} a_1 = b_1 \\ a_2 = b_2 \\ a_3 = b_3 \end{pmatrix} \text{ note: this can be generalized to 'n' dimensions larger than 0 (2.7)}$$

in either case, $\vec{0}$ is the identity of the operation, therefore:

$$\vec{a} + \vec{0} = \vec{a} \tag{2.8}$$

2.1.2 Bases

A base in \mathbb{R}^n can be found though n vectors on that plane, such as it would happen in \mathbb{R}^2 with:

$$\lambda \vec{u} + \mu \vec{v} | \lambda, \mu \exists \mathbb{R} \tag{2.9}$$

this equation will form a parallelogram that can express the distorsion of space when compared to a reference system, which generally is the canonical base formed by the identity.

2.1.3 Dot product

Assume two equal-length vectors of the sort:

$$\begin{cases} \vec{a} = (a_i * n | n \exists \mathbb{R}); |\vec{a}| \exists \mathbb{R} \\ \vec{b} = (b_i * n | n \exists \mathbb{R}); |\vec{b}| \exists \mathbb{R} \end{cases}$$
(2.10)

in case we wanted to do obtain a scalar number, that corresponded to the sum of the internal products we could obtain:

$$A * B = |\vec{A}| * |\vec{B}| * \cos \theta \tag{2.11}$$

where θ is the angle between both vectors. We can get it by calculating

$$\cos\theta = \frac{\vec{u} * \vec{v}}{||\vec{u}|| * ||\vec{u}||}$$

If perpendicular, we can assume:

$$\vec{u} * \vec{v} = 0$$

2.1.4 Cross product

A cross product is, much like the dot product, an operation that seeks to multiply the values between two vectors. it can be annotated as:

$$\vec{u} \times \vec{v} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} * \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \tag{2.12}$$

This is a non-commutative operation, changing the order of signs will cause the signs to invert, seen mathematically as:

$$\vec{u}x\vec{v} = -(\vec{v}x\vec{u})$$

2.2 Describing objects in a space.

So far we've been mostly concerned with planes and

2.2.1 lines

A line is a geometrical object of the form:

$$r(t) = t\vec{v} + P_t t \exists \mathbb{R} \tag{2.13}$$

Find the equation of the line that joins points P = (1,2,1) and P = (-1,3,4)



