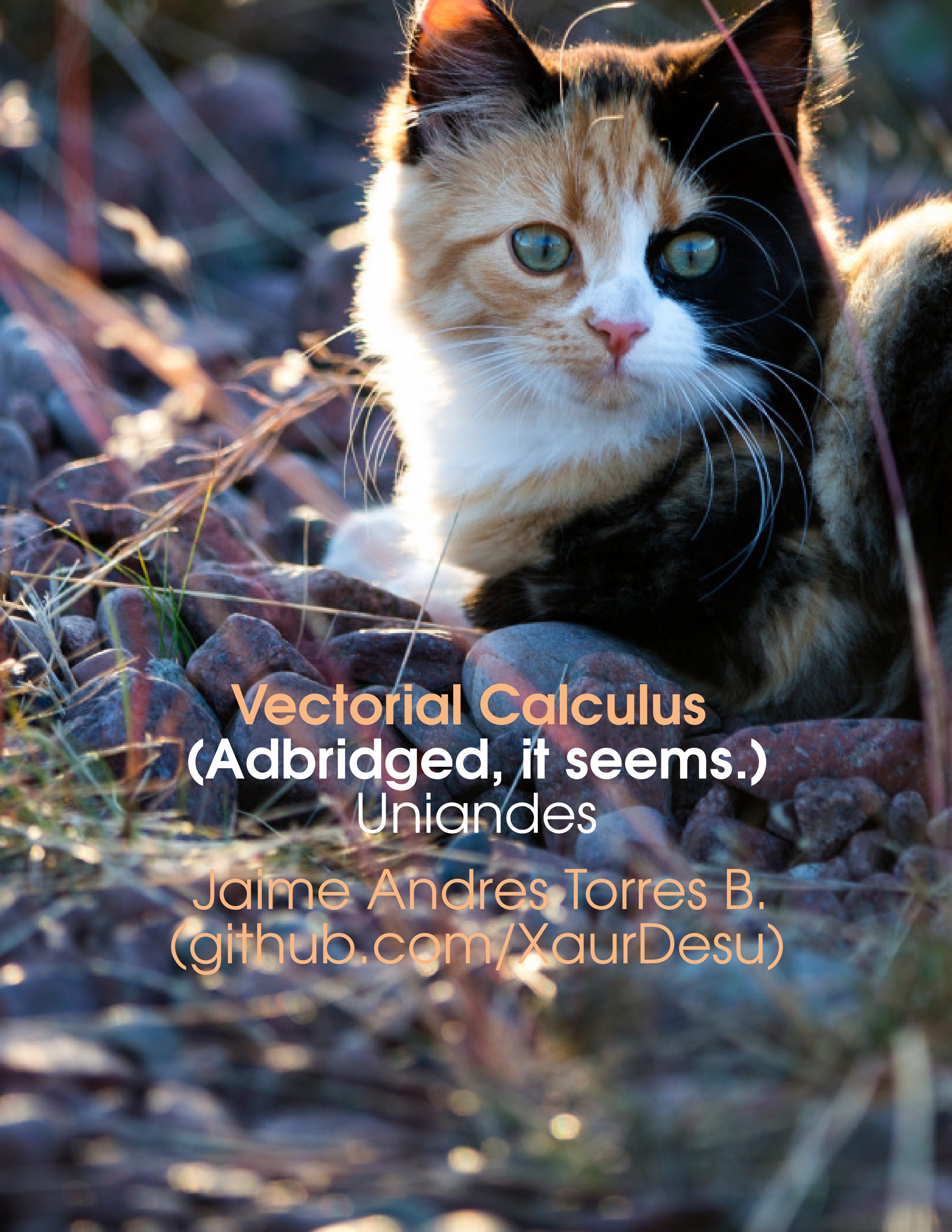


Vectorial Calculus
(Adbridged, it seems.)
Uniandes

Jaime Andres Torres B.
(github.com/XaurDesu)

Vectorial Calculus -

First release, August 9, 2023

Contents

1. Introduction

Go big or go home.

Vectorial calculus is what the title says pretty much, the act of using methods proper to calculus on vectorial spaces, for the topic of this class generally referring to merely 3-dimensional ones, at the end of this book you should be able to:

-

2. Linear Algebra Concepts

2.1 Vectors on a three-dimensional space.

given an $\mathbb{R}^3$ space and a point in that space $P = (a,b,c)$, we can describe a vector by either connecting the point P to another point Q, or by assuming the origin of this space (point $(0,0,0)$), this is a mathematical object with both a direction and a magnitude. The direction is given by an angle and the magnitude is given by $\sqrt{a_1^2 + a_2^2 + a_3^2}$

2.1.1 Addition and Subtraction

We can take any $\vec{a}$ and $\vec{b}$ vectors on the same space and add them to each other in the form:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix} \tag{2.1}$$

Such form remains in the case we can do subtraction, which is expressed on the equation:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{pmatrix} \tag{2.2}$$

This kind of operations have certain properties, shown as:

$$(\alpha + \beta)\vec{[v]} = \alpha\vec{v} + \beta\vec{v} \tag{2.3}$$

$$\vec{v} * 1 = \vec{v} \tag{2.4}$$

$$\vec{v} * \vec{0} = \vec{0} \tag{2.5}$$

$$\beta\vec{v} = \begin{pmatrix} \beta a_1 \\ \beta a_2 \\ \beta a_3 \end{pmatrix} \tag{2.6}$$

Two vectors $\vec{a}$ and $\vec{b}$ are equal if and only if:

$$\begin{cases} \vec{a}\exists\mathbb{R}^3 \\ \vec{b}\exists\mathbb{R}^3 \end{cases} \implies \begin{pmatrix} a_1 = b_1 \\ a_2 = b_2 \\ a_3 = b_3 \end{pmatrix} \textit{note: this can be generalized to 'n' dimensions larger than 0} \quad (2.7)$$

in either case, $\vec{0}$ is the identity of the operation, therefore:

$$\vec{a}+\vec{0}=\vec{a} \quad (2.8)$$

2.1.2 Bases

A base in R^n can be found though n vectors on that plane, such as it would happen in R^2 with:

$$\lambda\vec{u}+\mu\vec{v}|\lambda,\mu\exists\mathbb{R} \quad (2.9)$$

this equation will form a parallelogram that can express the distorsion of space when compared to a reference system, which generally is the canonical base formed by the identity.

2.1.3 Dot product

Assume two equal-length vectors of the sort:

$$\begin{cases} \vec{a}=(a_i*n|n\exists\mathbb{R});|\vec{a}|\exists\mathbb{R} \\ \vec{b}=(b_i*n|n\exists\mathbb{R});|\vec{b}|\exists\mathbb{R} \end{cases} \quad (2.10)$$

in case we wanted to do obtain a scalar number, that corresponded to the sum of the internal products we could obtain:

$$A*B=|\vec{A}|*|\vec{B}|*\cos\theta \quad (2.11)$$

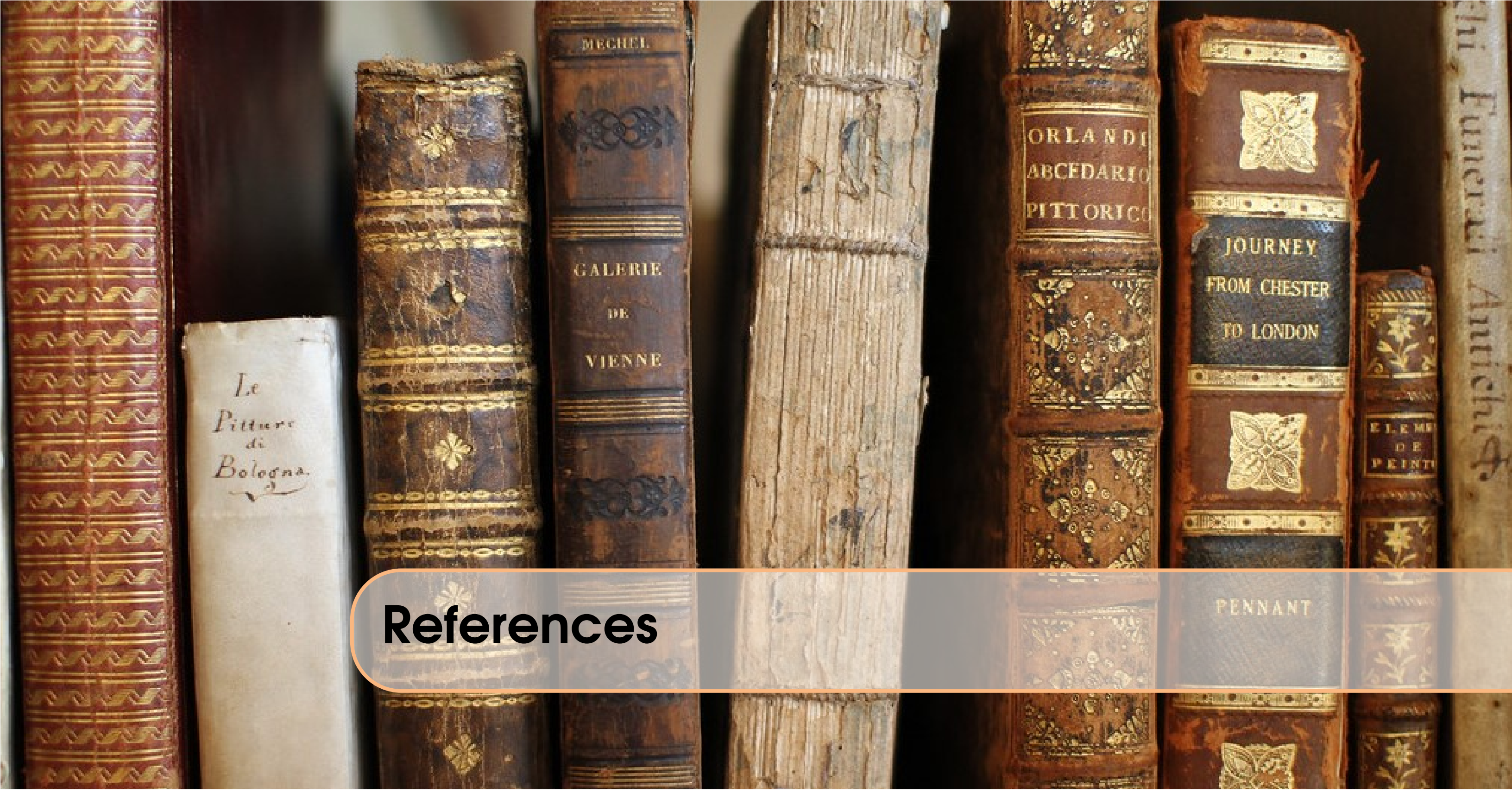
where θ is the angle between both vectors.

2.1.4 Cross product

A cross product is a

2.2 Describing objects in a space.

References


MECHEL
GALERIE
DE
VIENNE
ORLANDI
ABCEDARIO
PITTORICO
JOURNEY
FROM CHESTER
TO LONDON
PENNANT