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Word count: XXXXX

November 2, 2018

Systems Modelling & Simulation Coursework 1

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List of Abbreviations

CAD Computer Aided Design

CEF Cost Escalation Factor

CER Cost Estimating Relationship

DOC Direct Operating Cost

FAL Final Assembly Line

IRR Internal Rate of Return

LCC Life Cycle Cost

MRC Marginal Recurring Cost

NPV Net Present Value

NRC Non-Recurring Costs

OWE Operational Weight Empty

RC Recurring Costs

RTDE Research Test Development and Evaluation

TFU Theoretical First Unit Cost

USD United States Dollar

Nomenclature

C_E Engineering Cost

C_F Flight Test Costs

C_M Manufacturing Costs

C_M Material Costs

C_T Tooling Cost

C_{aed_m} Airframe Engineering & Design Cost for Manufacturing Stage

$C_{aed_{rtde}}$ Airframe Engineering and Design Cost for RTDE Stage

$C_{avionics}$ Cost of Avionics per Aircraft

$C_{dst_{rtde}}$ Development Support and Test Costs for RTDE Stage

C_{ea_m} Engine and Avionics Cost for Manufacturing Stage

$C_{ea_{rtde}}$ Engines and Avionics Cost for RTDE Stage

$C_{FAL_{A320}}$ Final Assembly Line Cost for the A320

$C_{FAL_{A3U10}}$ Final Assembly Line Cost for the A3U10

C_{fta} Cost of Flight Test Aircraft

C_{fto_m} Flight Test Operations for Manufacturing Stage

C_{int_m} Aircraft Interior Cost for Manufacturing Stage

C_{man_m} Manufacturing Cost for Manufacturing Stage

$C_{man_{rtde}}$ Manufacturing Costs for RTDE Stage

C_{mat_m} Material Costs for Manufacturing Stage

$C_{mat_{rtde}}$ Material Costs for RTDE Stage

C_{prog} Program Cost

C_{qc_m} Quality Control Costs for Manufacturing Stage

$C_{qc_{rtde}}$ Quality Control Costs for RTDE Stage

C_{tool_m} Tooling Costs for Manufacturing Stage

$C_{tool_{rtde}}$ Tooling Costs for RTDE Stages

C_{tsf} Test & Simulation Facilities Cost

C_{unit} Cost per Unit (average)

F_{diff} Program Difficult Factor

F_{mat} Material Factor

FTA Number of Flight Test Aircraft Produced

N_{eng} Number of Engines per Aircraft

N_m Number Units Built To Production Standard

N_p Number of Propellers per Aircraft

N_{R_m} Monthly Production Rate for Manufactuing stage

$N_{R_{rtde}}$ Monthly Production Rate for RTDE Stage

N_{rtde} Number of Development Aircraft

$N_{Y_{A320}}$ Number of A320s Produced per Year at FAL Site

Q Number of A3U10 Aircraft Produced

V_{max} Maximum Velocity

OWE_{A320} Operating Weight Empty of the A320

OWE_{A3U10} Operating Weight Empty of the A3U10

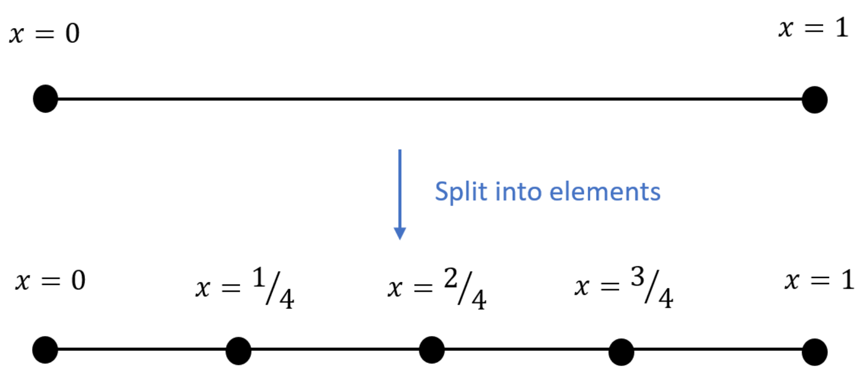
1 Part 1: Software Verification & Analytical Testing

1.1 Derivation of Element Matrix

Here we will derive the 2-by-2 element matrix for a diffusion operator for an arbitrary element e_n between the points x_0 x_1. The derivation will start from the weak form version of the diffusion integral, after performing integration by parts. This is given by equation 1 in the domain $x = 0$ to $x = 1$.

$$\int_0^1 D \frac{\partial c}{\partial x} \frac{\partial v}{\partial x} dx = \int_0^1 v f dx + \left[v D \frac{\partial c}{\partial x} \right]_0^1 \tag{1}$$

We have the domain from $x = 0$ to $x = 1$ which we can split into ne number of elements. This is shown pictorially below for the case $ne = 4$.



We can now say the integral from $x = 0$ to $x = 1$ is equivalent to the sum of the integral of the individual elements, for the $ne = 4$ case:

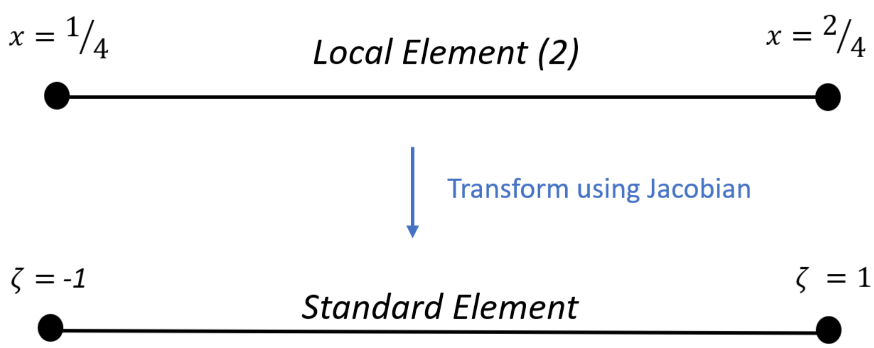
$$\int_0^1 dx = \int_0^{\frac{1}{4}} dx + \int_{\frac{1}{4}}^{\frac{2}{4}} dx + \int_{\frac{2}{4}}^{\frac{3}{4}} dx + \int_{\frac{3}{4}}^1 dx \tag{2}$$

To integrate an individual element we will use linear Lagrange nodal basis function 3 to represent c and x, the functions are shown below. The test function v is set to be equal to the basis function ψ.

$$c = c_0\psi_0(\zeta) + c_1\psi_1(\zeta) \tag{3a}$$
$$x = x_0\psi_0(\zeta) + x_1\psi_1(\zeta) \tag{3b}$$
$$v = \psi_0, \psi_1 \tag{3c}$$
$$\text{where,} \tag{3d}$$
$$\psi_0 = \frac{1-\zeta}{2} \quad , \quad \psi_1 = \frac{1+\zeta}{2} \tag{3e}$$
$$\text{and,} \tag{3f}$$
$$\zeta = 2\left(\frac{x - x_0}{x_1 - x_0}\right) - 1 \tag{3g}$$

for x in that element between x_0 and x_1.
We need to map the local element to a standard element as shown below. The Jacobian transform J is used to map from the x to the ζ coordinate system.

$$J = \left|\frac{dx}{d\zeta}\right| \tag{4}$$



Starting with the left hand side of equation 1 transforming with the Jacobian to a standard using $dx = Jd\zeta$ we get:

$$\int_{x_0}^{x_1} D\frac{\partial c}{\partial x}\frac{\partial v}{\partial x}dx = \int_{-1}^{1} D\frac{\partial c}{\partial x}\frac{\partial v}{\partial x}Jd\zeta \tag{5}$$

We need to evaluate the derivatives $\frac{\partial c}{\partial x}$ and $\frac{\partial v}{\partial x}$ which we can obtain by applying the chain rule to the definitions of c and v given by equations eq:LagrangeC and eq:LagrangeV. This gives the results

$$\frac{dc}{dx} = c_0\frac{d\psi_0}{d\zeta}\frac{d\zeta}{dx} + c_1\frac{d\psi_1}{d\zeta}\frac{d\zeta}{dx} = c_n\frac{d\psi_n}{d\zeta}\frac{d\zeta}{dx} \quad \text{for } n = 0, 1 \tag{6a}$$
$$\frac{dv}{dx} = \frac{d\psi_m}{d\zeta}\frac{d\zeta}{dx} \quad \text{for } m = 0, 1 \tag{6b}$$

We can now rewrite equation 7 as the following, recognising c is independent of x and therefore ζ.

$$c_n \int_{-1}^{1} D \frac{d\psi_n}{d\zeta} \frac{d\zeta}{dx} \frac{d\psi_m}{d\zeta} \frac{d\zeta}{dx} J d\zeta \tag{7}$$

Knowing that $\frac{d\zeta}{dx} = J^{-1}$ (for $x_1 > x_0$) from equation 4 and that for a given element J is constant, we can rewrite equation 7 as

$$c_n J^{-1} \int_{-1}^{1} D \frac{d\psi_n}{d\zeta} \frac{d\psi_m}{d\zeta} d\zeta \qquad \text{for} n = 0, 1 \& m = 0, 1 \tag{8}$$

From 8 we have two equations, one for each node, which when written in full, is clearly suitable for matrix representation.

$$J^{-1} \left[c_0 \int_{-1}^{1} D \frac{d\psi_0}{d\zeta} \frac{d\psi_0}{d\zeta} d\zeta + c_1 \int_{-1}^{1} D \frac{d\psi_1}{d\zeta} \frac{d\psi_0}{d\zeta} d\zeta) \right] \tag{9a}$$

$$J^{-1} \left[c_0 \int_{-1}^{1} D \frac{d\psi_1}{d\zeta} \frac{d\psi_0}{d\zeta} d\zeta + c_1 \int_{-1}^{1} D \frac{d\psi_1}{d\zeta} \frac{d\psi_1}{d\zeta} d\zeta) \right] \tag{9b}$$

The matrix representation is as follows where I_{nm} represents the individual integrals in the above equations 9a and 9b.

$$J^{-1} \begin{bmatrix} I_{00} & I_{01} \\ I_{10} & I_{11} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} \tag{10}$$

We now need to evaluate each Int_{nm} term individually. In order to evaluate the integrals we need to calculate the derivatives of ψ_0 and ψ_1 with respect to ζ using the definition of the basis function given by equation 3e. The results is as follows.

$$\frac{d\psi_0}{d\zeta} = \frac{d}{d\zeta}(\frac{1-\zeta}{2}) = -\frac{1}{2} \tag{11a}$$

$$\frac{d\psi_1}{d\zeta} = \frac{d}{d\zeta}(\frac{1+\zeta}{2}) = \frac{1}{2} \tag{11b}$$

<u>Int_{00}</u>

$$\begin{aligned}
Int_{00} &= \int_{-1}^{1} D\frac{d\psi_0}{d\zeta}\frac{d\psi_0}{d\zeta}d\zeta \\
&= \int_{-1}^{1} D.(-\frac{1}{2}).(-\frac{1}{2})d\zeta \\
&= \left[\frac{D}{4}\zeta\right]_{-1}^{1} \\
&= \left[(\frac{D}{4}.1) - (\frac{D}{4}.-1)\right] \\
&= \frac{D}{2}
\end{aligned} \tag{12}$$

<u>Int_{01}</u>

$$\begin{aligned}
Int_{01} &= \int_{-1}^{1} D\frac{d\psi_0}{d\zeta}\frac{d\psi_1}{d\zeta}d\zeta \\
&= \int_{-1}^{1} D.(-\frac{1}{2}).(\frac{1}{2})d\zeta \\
&= \left[-\frac{D}{4}\zeta\right]_{-1}^{1} \\
&= \left[(-\frac{D}{4}.1) - (-\frac{D}{4}.-1)\right] \\
&= -\frac{D}{2}
\end{aligned} \tag{13}$$

<u>Int_{10}</u>

$$\begin{aligned}
Int_{01} &= \int_{-1}^{1} D\frac{d\psi_1}{d\zeta}\frac{d\psi_0}{d\zeta}d\zeta \\
&= \int_{-1}^{1} D.(\frac{1}{2}).(-\frac{1}{2})d\zeta \\
&= \left[-\frac{D}{4}\zeta\right]_{-1}^{1} \\
&= \left[(-\frac{D}{4}.1) - (-\frac{D}{4}.-1)\right] \\
&= -\frac{D}{2}
\end{aligned} \tag{14}$$

$\underline{Int_{11}}$

$$
\begin{aligned}
Int_{11} &= \int_{-1}^{1} D \frac{d\psi_1}{d\zeta} \frac{d\psi_1}{d\zeta} d\zeta \\
&= \int_{-1}^{1} D.(\frac{1}{2}).(\frac{1}{2}) d\zeta \\
&= \left[\frac{D}{4} \zeta \right]_{-1}^{1} \\
&= \left[(\frac{D}{4}.1) - (\frac{D}{4}. - 1) \right] \\
&= \frac{D}{2}
\end{aligned} \tag{15}
$$

We can now assemble our local element matrix (not including the c term matrix). This is the form used in the code for LaplaceElemMatrix.m function. Where J and D are scalars (we have assumed D to be constant).

$$
J^{-1} D \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \tag{16}
$$

2 Conclusions